# CBSE Test Paper 05
## Chapter 5 Continuity and Differentiability

1. $\frac{d}{dx}\left(\log\left|\tan\frac{x}{2}\right|\right)$ is equal to

   a. $\frac{2}{\sin x}$
   b. None of these
   c. $\frac{1}{\tan\frac{x}{2}}$
   d. cosec x

2. $\frac{d}{dx}\left(\tan^{-1}(\cot x)\right)$ is equal to

   a. None of these
   b. -1
   c. sin2x
   d. -cosec2x

3. Let f(x) $= \begin{cases} 1+x\ if\ x>0 \\ x\quad if\ x\leqslant 0\end{cases}$ then $\underset{x\to 0}{Lt}$ f(x) is equal to

   a. 1
   b. 0
   c. $\frac{1}{2}$
   d. None of these

4. If f(x) is a polynomial of degree m $(\geqslant 1)$ , then which of the following is not true?

   a. None of these
   b. f is derivable at all $x \in \mathbf{R}$
   c. $\frac{d^n y}{dx^n} = 0$ for all n > m
   d. f is continuous at all $x \in \mathbf{R}$

5. In case of strict increasing functions, slope of the tangent and hence derivative is

   a. either positive or zero
   b. zero

c. positive
d. negative

6. The number of points at which the function $f(x) = \frac{1}{\log|x|}$ is discontinuous is ________.
7. An example of a function which is continuous everywhere but fails to be differentiable exactly at two points is ________.
8. If f(x) = $x^2 \sin\frac{1}{x}$, where x $\neq$ 0, then the value of the function f at x = 0, so that the function is continuous at x = 0, is ________.
9. Find $\frac{dy}{dx}$ if $y = \frac{\sin(ax+b)}{\cos(cx+d)}$.
10. Differentiate the following function with respect to x: $e^{\sin^{-1}x}$.
11. Differentiate $\sin^2 x$ w.r.t $e^{\cos x}$.
12. Differentiate $y = 2\sqrt{\cot x^2}$.
13. Let f(x) = x|x| for all $x \in R$ Discuss the derivability of f(x) at x = 0.
14. If $x^y = e^{x-y}$, prove that $\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$.
15. If x = sint and y = sin pt prove that $\left(1-x^2\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + p^2y = 0$.
16. Find the value of k, for which f(x) = $\begin{cases} \frac{\sqrt{1+kx}-\sqrt{1-kx}}{x}, & \text{if } -1 \leq x < 0 \\ \frac{2x+1}{x-1}, & \text{if } 0 \leq x < 1 \end{cases}$ is continuous at x = 0.
17. If y = $\operatorname{cosec}^{-1}x$, x> 1, then show that $x\left(x^2-1\right)\frac{d^2y}{dx^2} + \left(2x^2-1\right)\frac{dy}{dx} = 0$.
18. Find $\frac{dy}{dx}$ of the following function expressed in parametric form in $x = e^{\theta}\left(\theta + \frac{1}{\theta}\right), y = e^{-\theta}\left(\theta - \frac{1}{\theta}\right)$.

## Solution

1. d. cosec x, **Explanation:** $\frac{d}{dx}\left(\log\left|\tan\frac{x}{2}\right|\right)=\frac{1}{\left(\left|\tan\frac{x}{2}\right|\right)}\frac{d}{dx}\left(\tan\frac{x}{2}\right)=\frac{1}{\left(\left|\tan\frac{x}{2}\right|\right)}\frac{1}{2}\sec^2\frac{x}{2}$
$=\frac{1}{2\sin\frac{x}{2}\cos\frac{x}{2}}=\frac{1}{\sin x}=cosecx$
2. b. -1, **Explanation:** $\frac{d}{dx}(\tan^{-1}(\cot x))$ =
$\frac{d}{dx}(\tan^{-1}(\tan(\frac{\pi}{2}-x)))=\frac{d}{dx}(\frac{\pi}{2}-x)=-1$
3. d. None of these, **Explanation:** $\lim\limits_{x\to 0^+}f(x)=\lim\limits_{x\to 0^+}(1+x)=1$
$\lim\limits_{x\to 0^-}f(x)=\lim\limits_{x\to 0^-}x=0,$
$\therefore\lim\limits_{x\to 0}f(x)$ does not exist
4. b. f is derivable at all $x\in\mathbf{R}$, **Explanation:** Because a polynomial is everywhere differentiable
5. a. either positive or zero, **Explanation:** If f is strictly increasing function , then f ‘ (x) can be 0 also. For example, f(x) = $x^3$ is strictly increasing, but its derivative is 0 at x = 0. As another example, take f(x) = x + cosx; here f ‘(x) = 1 – sinx, which is either +ve or 0 and the function x + cos x is strictly increasing.
6. 3
7. |x| + |x + 1|
8. 0
9. $y=\frac{\sin(ax+b)}{\cos(cx+d)}$
$\frac{dy}{dx}=\frac{\cos(cx+d)\frac{d}{dx}\sin(ax+b)-\sin(ax+b)\frac{d}{dx}\cos(cx+d)}{\cos^2(cx+d)}$
$\frac{dy}{dx}=\frac{\cos(cx+d)\cos(ax+b).a+\sin(ax+b)\sin(cx+d).c}{\cos^2(cx+d)}$
10. Let $y=e^{\sin^{-1}x}$
$\therefore\frac{dy}{dx}=e^{\sin^{-1}x}.\frac{d}{dx}\sin^{-1}x$
$=e^{\sin^{-1}x}.\frac{1}{\sqrt{1-x^2}}\left[\because\frac{d}{dx}e^{f(x)}=e^{f(x)}\frac{d}{dx}f(x)\right]$
11. $u=\sin^2 x$

$\frac{du}{dx} = 2\sin x . \cos x$
$v = e^{\cos x}$
$\frac{dv}{dx} = e^{\cos x}(-\sin x)$
$\frac{du}{dv} = \frac{2\sin x . \cos x}{-e^{\cos x} . \sin x}$
$= -\frac{2\cos x}{e^{\cos x}}$

12. $y = 2\sqrt{\cot x^2}$
$\frac{dy}{dx} = 2 . \frac{1}{2}\left(\cot x^2\right)^{-\frac{1}{2}} . \frac{d}{dx}\left(\cot x^2\right)$
$= \frac{1}{\sqrt{\cot x^2}} - \cos ec^2 x^2 . 2x$
$= \frac{-2x . \cos ec^2 x^2}{\sqrt{\cot x^2}}$

13. We may rewrite f as $f(x) = \begin{cases} x^2, if\ x \geqslant 0 \\ -x^2,\ if\ x < 0 \end{cases}$
Now, $Lf'(0) = \lim\limits_{h \to 0^-} \frac{f(0+h)-f(0)}{h} = \lim\limits_{h \to 0^-} \frac{-h^2-0}{h} = \lim\limits_{h \to 0^-} -h = 0$
Now, $Rf'(0) = \lim\limits_{h \to 0^+} \frac{f(0+h)-f(0)}{h} = \lim\limits_{h \to 0^+} \frac{h^2-0}{h} = \lim\limits_{h \to 0^+} h = 0$
Since the left-hand derivative and right hand derivative both are equal, hence f is differentiable at x = 0.

14. We have $x^y = e^{x-y}$
Taking logarithm on both sides, we get
$y \log x = x - y \Rightarrow y(1 + \log x) = x$
i.e. $y = \frac{x}{1+\log x}$
Differentiating both sides w.r.t. x, we get
$\frac{dy}{dx} = \frac{(1+\log x).1 - x\left(\frac{1}{x}\right)}{(1+\log x)^2} = \frac{\log x}{(1+\log x)^2}$

15. We have, x = sint and y = sin pt,
$\therefore \frac{dx}{dt} = \cos t$ and $\frac{dy}{dt} = \cos pt . p$
$\Rightarrow \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{p . \cos pt}{\cos t}$ ...(i)
Again, differentiating both sides w.r.t. x, we get
$\frac{d^2 y}{dx^2} = \frac{\cos t . \frac{d}{dt}(p . \cos pt)\frac{dt}{dx} - p\cos pt . \frac{d}{dt}\cos t . \frac{dt}{dx}}{\cos^2 t}$
$= \frac{[\cos t . p . (-\sin pt) . p - p\cos pt . (-\sin t)]\frac{dt}{dx}}{\cos^2 t}$
$= \frac{\left[-p^2 \sin pt . \cos t + p\ sint . cospt\right] . \frac{1}{\cos t}}{\cos^2 t}$

$\Rightarrow \frac{d^2y}{dx^2} = \frac{-p^2 \sin pt.\cos t + p\cos pt.\sin t}{\cos^3 t}$ ...(ii)

Since, we have to prove

$\left(1-x^2\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + p^2 y = 0$

$\therefore LHS = \left(1-\sin^2 t\right)\frac{\left[-p^2\ sinpt.\ cost + p\ cospt.\sin t\right]}{\cos^3 t} - sint.\frac{p\cos pt}{\cos t} + p^2 \sin pt$

$= \frac{1}{\cos^3 t}\begin{bmatrix}\left(1-\sin^2 t\right)\left(-p^2\sin pt.\cos t + p\cos pt.\sin t\right)\\ -p\cos pt.\sin t.\cos^2 t + p^2\sin pt.\cos^3 t\end{bmatrix}$

$= \frac{1}{\cos^3 t}\begin{bmatrix}-p^2\sin pt.\cos^3 t + p\cos pt.\sin t.\cos^2 t\\ -p\cos pt.\sin t.\cos^2 t + p^2\sin pt.\cos^3 t\end{bmatrix}\left[\because 1-\sin^2 t = \cos^2 t\right]$

$= \frac{1}{\cos^3 t}.0$

= 0 Hence proved.

16. According to the question,we are given that, f(x) = $\begin{cases}\frac{\sqrt{1+kx}-\sqrt{1-kx}}{x}, \text{ if } -1 \leq x < 0\\ \frac{2x+1}{x-1}, \quad \text{if } 0 \leq x < 1\end{cases}$ is continuous at x = 0 and we have to find the value of k.

Now, f(0) = $\frac{2(0)+1}{0-1} = \frac{1}{-1}$ =-1

and LHL = $\lim\limits_{h\to 0} f(0-h)$

$= \lim\limits_{h\to 0}\frac{\sqrt{1-kh}-\sqrt{1+kh}}{-h}$

$= \lim\limits_{h\to 0}\frac{\sqrt{1-kh}-\sqrt{1+kh}}{-h} \times \frac{(\sqrt{1-kh}+\sqrt{1+kh})}{(\sqrt{1-kh}+\sqrt{1+kh})}$

$= \lim\limits_{h\to 0}\frac{(1-kh)-(1+kh)}{-h(\sqrt{1-kh}+\sqrt{1+kh})}$ [(a + b)(a - b) = $a^2$ - $b^2$]

$= \lim\limits_{h\to 0}\frac{-2kh}{-h(\sqrt{1-kh}+\sqrt{1+kh})}$

$= \lim\limits_{h\to 0}\frac{2k}{\sqrt{1-kh}+\sqrt{1+kh}} = \frac{2k}{1+1} = \frac{2k}{2}$ = 2

Since, f(x) is continuous at x = 0.

$\therefore$ f(0) = LHL $\Rightarrow$ - 1 = k $\Rightarrow$ k = -1

17. According to the question,if y = $\text{cosec}^{-1}$x, x> 1, then we have to show that

$x\left(x^2-1\right)\frac{d^2y}{dx^2} + \left(2x^2-1\right)\frac{dy}{dx} = 0.$

Now, we have , y= $\text{cosec}^{-1}$x

Therefore,on differentiating both sides w.r.t x, we get,

$\frac{dy}{dx} = \frac{-1}{x\sqrt{x^2-1}}$

$\Rightarrow \quad x\sqrt{x^2-1}\cdot\frac{dy}{dx} = -1$

Again,differentiating both sides w.r.t x, we get,

$\left(x\sqrt{x^2-1}\right)\cdot\frac{d}{dx}\left(\frac{dy}{dx}\right)+\frac{dy}{dx}\cdot\frac{d}{dx}\left(x\sqrt{x^2-1}\right) = \frac{d}{dx}(-1)$[ using product rule of derivative]

$\Rightarrow x\sqrt{x^2-1}\cdot\frac{d^2y}{dx^2}+\frac{dy}{dx}\left\{x\times\frac{d}{dx}\sqrt{x^2-1}+\sqrt{x^2-1}\times\frac{d}{dx}(x)\right\}=0$

$\Rightarrow x\sqrt{x^2-1}\frac{d^2y}{dx^2}+\frac{dy}{dx}\left\{\frac{x}{2\sqrt{x^2-1}}\frac{d}{dx}\left(x^2-1\right)+\sqrt{x^2-1}\times 1\right\}=0$[ using chain rule of derivative]

$\Rightarrow x\sqrt{x^2-1}\frac{d^2y}{dx^2}+\frac{dy}{dx}\left\{\frac{x\cdot 2x}{2\sqrt{x^2-1}}+\sqrt{x^2-1}\right\}=0$

$\Rightarrow x\sqrt{x^2-1}\frac{d^2y}{dx^2}+\frac{dy}{dx}\left\{\frac{x^2}{\sqrt{x^2-1}}+\sqrt{x^2-1}\right\}=0$

$\Rightarrow \quad x\sqrt{x^2-1}\frac{d^2y}{dx^2}+\frac{dy}{dx}\left\{\frac{x^2+x^2-1}{\sqrt{x^2-1}}\right\}=0$

$\Rightarrow \quad x\left(x^2-1\right)\frac{d^2y}{dx^2}+\left(x^2+x^2-1\right)\frac{dy}{dx}=0$

$\therefore \quad x\left(x^2-1\right)\frac{d^2y}{dx^2}+\left(2x^2-1\right)\frac{dy}{dx}=0$

18. $\therefore x=e^{\theta}\left(\theta+\frac{1}{\theta}\right)$ and $y=e^{-\theta}\left(\theta-\frac{1}{\theta}\right)$

$\therefore \frac{dx}{d\theta}=\frac{d}{d\theta}\left[e^{\theta}.\left(\theta+\frac{1}{\theta}\right)\right]$

$=e^{\theta}.\frac{d}{d\theta}\left(\theta+\frac{1}{\theta}\right)+\left(\theta+\frac{1}{\theta}\right)\cdot\frac{d}{d\theta}e^{\theta}$

$=e^{\theta}\left(1-\frac{1}{\theta^2}\right)+\left(\theta+\frac{1}{\theta}\right)e^{\theta}$

$=e^{\theta}\left(1-\frac{1}{\theta^2}+\theta+\frac{1}{\theta}\right)$

$=e^{\theta}\left(\frac{\theta^2-1+\theta^3+\theta}{\theta^2}\right)$ ...(i)

and $\frac{dy}{d\theta}=\frac{d}{d\theta}\left[e^{-\theta}.\left(\theta-\frac{1}{\theta}\right)\right]$

$=e^{-\theta}.\frac{d}{d\theta}\left(\theta-\frac{1}{\theta}\right)+(\theta-\frac{1}{\theta}).\frac{d}{d\theta}e^{-\theta}$

$=e^{-\theta}\left(1+\frac{1}{\theta^2}\right)+\left(\theta-\frac{1}{\theta}\right)e^{-\theta}.\frac{d}{d\theta}(-\theta)$

$=e^{-\theta}\left[\frac{\theta^2+1}{\theta^2}-\frac{\theta^2-1}{\theta}\right]=e^{-\theta}\left[\frac{\theta^2+1-\theta^3+\theta}{\theta^2}\right]$ ...(ii)

$\therefore \frac{dy}{dx}=\frac{dy/d\theta}{dx/d\theta}=\frac{e^{-\theta}\left(\frac{\theta^2+1-\theta^3+\theta}{\theta^2}\right)}{e^{\theta}\left(\frac{\theta^2-1+\theta^3+\theta}{\theta^2}\right)}$

$=e^{-2\theta}\left(\frac{-\theta^3+\theta^2+\theta+1}{\theta^3+\theta^2+\theta-1}\right)$