

## Coefficient of Correlation

**Q.1.** The data for marks in Physics and History obtained by a ten students are given below :

|                         |    |    |    |    |    |    |    |    |   |    |
|-------------------------|----|----|----|----|----|----|----|----|---|----|
| <b>Marks in Physics</b> | 15 | 12 | 8  | 8  | 7  | 7  | 7  | 6  | 5 | 3  |
| <b>Marks in History</b> | 10 | 25 | 17 | 11 | 13 | 17 | 20 | 13 | 9 | 15 |

Using this data compute Karl Pearson's coefficient of correlation between the marks in Physics and History obtained by 10 students.

**Solution : 1**

| Physics (x) | History (y) | xy          | x <sup>2</sup> | y <sup>2</sup> |
|-------------|-------------|-------------|----------------|----------------|
| 15          | 10          | 150         | 225            | 100            |
| 12          | 25          | 300         | 144            | 625            |
| 8           | 17          | 136         | 64             | 289            |
| 8           | 11          | 88          | 64             | 121            |
| 7           | 13          | 91          | 49             | 169            |
| 7           | 17          | 119         | 49             | 189            |
| 7           | 20          | 140         | 49             | 400            |
| 6           | 13          | 78          | 36             | 169            |
| 5           | 9           | 45          | 25             | 81             |
| 3           | 15          | 45          | 9              | 225            |
| <b>78</b>   | <b>150</b>  | <b>1192</b> | <b>714</b>     | <b>2468</b>    |

Karl Pearson's coefficient ,

$$r = \{1/n[\Sigma xy - (\Sigma x \Sigma y)/n]\} / \sqrt{\{(\Sigma x^2)/n - (\Sigma x/n)^2\}\{(\Sigma y^2)/n - (\Sigma y/n)^2\}}.$$

$$= [ \{1192 - (78 \times 150)/10\} ] / [ \sqrt{\{714 - (78 \times 78)/10\}\{2468 - (150 \times 150)/10\}} ]$$

$$= (1192 - 1170) / [ \sqrt{\{714 - 608.4\}\{2468 - 2250\}} ]$$

$$= 22 / \sqrt{(105.6 \times 218)} = 22 / 151.7 = 0.14.$$

**Q.2.** Calculate the coefficient of correlation between X and Y from the following data using Karl Pearson's method :

|          |   |   |   |   |   |
|----------|---|---|---|---|---|
| <b>X</b> | 1 | 2 | 3 | 4 | 5 |
| <b>Y</b> | 2 | 5 | 3 | 8 | 7 |

**Solution : 2**

|  |  |  |  |  |  |
|--|--|--|--|--|--|
|  |  |  |  |  |  |
|--|--|--|--|--|--|

| X  | Y  | x <sup>2</sup> | y <sup>2</sup> | xy |
|----|----|----------------|----------------|----|
| 1  | 2  | 1              | 4              | 2  |
| 2  | 5  | 4              | 25             | 10 |
| 3  | 3  | 9              | 9              | 9  |
| 4  | 8  | 16             | 64             | 32 |
| 5  | 7  | 25             | 49             | 35 |
| 15 | 25 | 55             | 151            | 88 |

Karl Pearson's correlation coefficient

$$\begin{aligned}
 r &= \{\Sigma xy - (\Sigma x \Sigma y)/n\} / \sqrt{[\{\Sigma x^2 - (\Sigma x)^2/n\} \{\Sigma y^2 - (\Sigma y)^2/n\}]} \\
 &= [88 - (15 \times 25)/5] / [\sqrt{\{55 - 225/5\} \{151 - 625/5\}}] \\
 &= (88 - 75) / \sqrt{\{(10)(26)\}} \\
 &= 13 / \sqrt{(260)} \\
 &= 0.806 = 0.81 .
 \end{aligned}$$

**Q.3.** The following table shows the sales and advertisement expenditure of a firm :

|                           | Sales Rs. (in crores) | Advertisement Expenditure Rs. (in crores) |
|---------------------------|-----------------------|-------------------------------------------|
| <b>Mean</b>               | 40                    | 6                                         |
| <b>Standard Deviation</b> | 10                    | 1.5                                       |

Coefficient of correlation =  $\gamma = 0.9$  .

Estimate the likely sales for a proposed advertisement expenditure of Rs. 10 crores.

**Solution : 3**

|                           | Sales Rs. (in crores) | Advertisement Expenditure Rs. (in crores) |
|---------------------------|-----------------------|-------------------------------------------|
| <b>Mean</b>               | <b>40</b>             | <b>6</b>                                  |
| <b>Standard Deviation</b> | <b>10</b>             | <b>1.5</b>                                |

Coefficient of correlation =  $\gamma = 0.9$

Regression coefficient x on y =  $b_{xy} = \gamma(\sigma_x/\sigma_y) = 0.9 (10/1.5) = 90/15 = 6$

Equation required (x on y)  $(x - M_x) = b_{xy} (y - M_y)$

Or,  $x - 40 = 6 (y - 6)$

Or,  $x - 40 = 6y - 36$

Or,  $x = 6y + 4$

Therefore, when  $y = 10$ ,  $x = 6 \times 10 + 4 = 64$

Therefore, Sales for expenditure of Rs. 10 crores = Rs.64Crores.