Ratio and Proportion

- In many situations, comparison between quantities is made by using division i.e., by observing how many times one quantity is in relation to the other quantity. This comparison is known as **ratio.** We denote it by using the symbol ‘:’.

- A ratio may be treated as a fraction. For example, 3:11 can be treated as $\frac{3}{11}$.

- We can compare two quantities in terms of ratio, if these quantities are in the same unit. If they are not, then they should be expressed in the same unit before the ratio is taken.

  For example, if we want to compare 70 paise and Rs 3 in terms of ratio then we have to convert Rs 3 into paise.

  Rs 3 = 300 paise

  Hence, required ratio $\frac{70}{300} = 7:30$

- The same ratio may occur in different situations.

  To understand this concept, let us consider the following situations.

  - Distances of Lata’s home and Ravi’s home from their school are 12 km and 21 km respectively. Therefore, the ratio of the distance of Lata’s home to the distance of Ravi’s home from their school is $\frac{12}{21} = \frac{12 \div 3}{21 \div 3} = \frac{4}{7} = 4:7$
  - Neha has Rs 20 and Saroj has Rs 35. Therefore, the ratio of the amount of money that Neha has to that of Saroj is $\frac{20}{35} = \frac{20 \div 5}{35 \div 5} = \frac{4}{7} = 4:7$

  In this way, we can come across many situations where the ratio would be 4:7.

- The order of ratio is important.

  For example, let us consider that the length and breadth of a rectangle are 80 m and 30 m respectively. The ratio of length to the breadth of rectangle is $\frac{80}{30}$. This ratio can be written as 8: 3. However, it cannot be written as 3:8.

  Therefore, the order in which quantities are taken to express their ratio is important.

- Four quantities are said to be in proportion, if the ratio of first and second quantities is equal to the ratio of third and fourth quantities.

  For example, to check whether 8, 22, 12, and 33 are in proportion or not, we have to find the ratio of 8 to 22 and the ratio of 12 to 33.

  $$8:22 = \frac{8}{22} = \frac{4}{11} = 4:11$$
  $$12:33 = \frac{12}{33} = \frac{4}{11} = 4:11$$

  Therefore, 8, 22, 12, and 33 are in proportion as 8:22 and 12:33 are equal.

- When four terms are in proportion, the first and fourth terms are known as extreme terms and the second and third terms are known as middle terms.

  In the above example, 8, 22, 12, and 33 were in proportion. Therefore, 8 and 33 are known as extreme terms while 22 and 12 are known as middle terms.

- If two ratios are equal then we say that they are in proportion and use the symbol '::' or '=' to equate the two ratios.

  For example, 8:36 and 14:63 are equal as $\frac{8}{36} = \frac{2}{9}$ 836 = 29 and 14:63 = $\frac{14}{63} = \frac{2}{9}$

  Since 8:36 and 14:63 are in proportion, we write it as 8:36 :: 14:63 or 8:36 = 14:63.

- If *a*, *b*, *c*, *d*… are some (non-zero) quantities of the same kind then *a*, *b*, *c*, *d* … are said to be in **continued proportion,** if

$$\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = \ldots$$

For example, 2, 6 and 18 are in continued proportion as $\frac{2}{6} = \frac{6}{18}$.

- The method in which we first find the value of one unit and then the value of the required number of units is known as **unitary method**.

  Example:

  If 15 men can do a piece of work in 10 days, then in how many days can 6 men do the same work?

  Solution:

  This is the case of indirect variation since more the number of men, less will be the number of days required to finish the work.

  It is given that 15 men can do the work in 10 days.

  ∴One man can do the work in (10 × 15) days.

  Hence, 6 men can do the work in $\frac{10 \times 15}{6}$ days = 25 days

- Important formulae to solve problems related to time and work:

  One day's work = $\frac{1}{\text{Number of days to complete the work}}$

  Number of days to complete the work = $\frac{1}{\text{One day's work}}$

  Example:

  Nitika and Ruchika together can type 100 pages in 20 hours. Nitika alone can type 100 pages in 25 hours. How much time would Ruchika take to type 100 pages alone?

Solution:

Nitika and Ruchika together type 100 pages in 20 hours.

∴One hour's work done by Nitika and Ruchika together $=\frac{1}{20}$ hours

Now, Nitika alone can type 100 pages in 25 hours.

∴Nitika's one hour's work $=\frac{1}{25}$ hours

∴Ruchika one hour's work $=\frac{1}{20}-\frac{1}{25}$

$=\frac{5-4}{100}=\frac{1}{100}$

Thus, Ruchika alone would type 100 pages in 100 hours.

- Speed of an object can be calculated using the formula:

$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$ Speed=DistanceTime

- If an object covers equal distances in equal intervals of time, then its speed is said to be **uniform** (or constant), otherwise its speed is said to be **variable**.
- If the speed is given in terms of km/hour, then we can convert it into m/sec as:

1 km/hour $=\frac{5}{18}$ **m/sec**

- If the speed is given in terms of m/sec, then we can convert it into km/hour as:

1 m/sec $=\frac{18}{5}$ **km/hour**

Example:

A car covers a distance of 3 km in 10 minutes. How much time will it take to travel 24 km?

Solution:

$$\text{Speed} = \frac{3\text{ km}}{10\text{ minute}} = \frac{3}{10}\text{ km/min}$$

Now, distance = 24 km

$$\therefore \text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

$$= \frac{24\text{ km}}{\frac{3}{10}\text{ km/min}}$$

$$= \frac{24 \times 10}{3}\text{ min}$$

= 80 minutes

= 1 hour 20 minutes ($\because$ 1 hour = 60 minutes)

Thus, the time taken by the car to travel 24 km is 1 hour 20 minutes.