CUET Mathematics Solved Paper-2022

Held on 30 August 2022

1. Assume P, Q, R and S are matrices of order $2 \times m$, $k \times n$, $m \times 2$ and 2×3 respectively. The restrictions on k, m and n, so that $PQ + RS$ is defined are
(a) $m = 3, n = 2$ (b) $m = n$, k is arbitrary
(c) $m = k$, n is arbitrary (d) $m = k = 2, n = 3$

2. The system of equations $3x + 4y = 5, 6x + 7y = -8$ is written in matrix from as
(a) $\begin{bmatrix} 3 & 4 \\ 6 & 7 \end{bmatrix} [x \;\; y] = [5 \;\; -8]$
(b) $\begin{bmatrix} 3 & 6 \\ 4 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -8 \end{bmatrix}$
(c) $\begin{bmatrix} x \\ y \end{bmatrix} \begin{bmatrix} 3 & 4 \\ 6 & 7 \end{bmatrix} = \begin{bmatrix} 5 \\ -8 \end{bmatrix}$
(d) $[x \;\; y] \begin{bmatrix} 3 & 6 \\ 4 & 7 \end{bmatrix} = [5 \;\; -8]$

3. If $2\begin{bmatrix} a & d \\ b & c \end{bmatrix} + 3\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} = 3\begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}$, then the value of $|a + b - c - d|$ is
(a) 3 (b) 24 (c) 6 (d) 16

4. Consider the function $f(x) = x^{\frac{1}{x}}$. Its
(a) minimum value is $e^{\frac{1}{e}}$
(b) maximum value is $e^{\frac{1}{e}}$
(c) minimum value is e^e
(d) maximum value is $\left(\frac{1}{e}\right)^e$

5. The given function $f(x) = x^5 - 5x^4 + 5x^3 - 1$; has/have
(a) Local maxima at $x = 1$
(b) Local maximum value is 0
(c) Local minimum at $x = 3$
(d) Local minimum value is -28
(e) The point of inflexion is $x = 1$
Choose the correct answer from the options given below
(a) (a), (b) only
(b) (a), (b), (c) only
(c) (a), (b), (c), (d) only
(d) (a), (c), (e) only

6. Match **List-I** with **List-II**

List-I	List-II
(A) If $x = t^2$ and $y = t^3$ then $\frac{d^2y}{dx^2}$ at $t = 1$	(i) -2
(B) If $f(x) = \sqrt{x} + 1$, the $f'(1)$	(ii) -1
(C) The minimum value of $f(x) = 9x^2 + 12x + 2$ is	(iii) $\frac{3}{4}$
(D) The point of inflexion of the function $f(x) = (x-2)^4 (x+1)^3$ is	(iv) $-\frac{1}{4}$

Choose the **correct** answer from the options given below
(a) (A) - (i), (B) - (iii), (C) - (ii), (D) - (iv)
(b) (A) - (ii), (B) - (iii), (C) - (i), (D) - (iv)
(c) (A) - (iii), (B) - (iv), (C) - (i), (D) - (ii)
(d) (A) - (iv), (B) - (i), (C) - (iii), (D) - (ii)

7. The area enclosed by the curve $y^2 = 4ax$ and its latus - rectum is
(a) $\frac{8}{3}a^2$ (b) $\frac{4}{3}a^2$ (c) $\frac{1}{3}a^2$ (d) $\frac{1}{12}a^2$

8. $\int \frac{xe^x}{(x+1)^2} dx =$
(a) $\frac{e^x}{x+1} + c$ (b) $\frac{e^x}{x-1} + c$
(c) $\frac{x}{x+1} + c$ (d) $\frac{x}{x-1} + c$

9. The solution of the differential equation $(x+1)\frac{dy}{dx} = 1 + y$ is
(a) $\frac{1 + y + y^2}{1 + x^2} = C$
(b) $\log(x+1) - \log\left(y + \frac{1}{2}\right) = C$
(c) $\frac{x+1}{y+1} = C$
(d) $\log(1+y) \frac{\sqrt{3}}{2} \log(x+1) = C$

10. Order and degree of the differential equation $y\frac{dy}{dx} + \frac{4}{\frac{dy}{dx}} = 5$ are
(a) 1, 2 respectively (b) 1, 1 respectively
(c) 1, 0 respectively (d) 2, 1 respectively

11. Derivative of $x^3 + 1$ with respect to $x^2 + 1$ is
(a) $\frac{2x}{3}$ (b) $\frac{x}{3}$ (c) $\frac{x}{2}$ (d) $\frac{3x}{2}$

12. Solution of the differential equation $(x + xy)\,dy - y(1-x^2)\,dx = 0$ is

(a) $y = \log\frac{x}{y} - \frac{x^2}{2} + C$ (b) $y = \log\frac{x}{y} + \frac{x^2}{2} + C$

(c) $y = \log xy - \frac{x^2}{2} + C$ (d) $y = \log xy + \frac{x^2}{2} + C$

13. Two numbers are selected at random (without replacement) from the first three positive integers. Let X denotes the larger of the two integers, then the probability distribution of X is

(a)

x	2	3
$P(X=x)$	1/3	2/3

(b)

x	2	3
$P(X=x)$	1/2	1/2

(c)

x	2	3
$P(X=x)$	1/3	1/3

(d)

x	2	3
$P(X=x)$	1/5	4/5

14. The probability distribution of number of doublets in three throws of a pair of dice is

(a)

x	0	1	2	3
$P(X=x)$	125/216	75/216	15/216	1/216

(b)

x	0	1	2	3
$P(X=x)$	75/216	125/216	1/216	15/216

(c)

x	0	1	2	3
$P(X=x)$	1/216	75/216	15/216	125/216

(d)

x	0	1	2	3
$P(X=x)$	1/216	15/216	75/216	125/216

15. In linear programming, the optimal value of the objective function is attained at the points given by

(a) intersection of the inequalities with the x-axis only

(b) intersection of the inequalities with the axes only

(c) corner points of the feasible region

(d) intersection of the inequalities with the y-axis only

16. If R is a relation on Z (set of all integers) defined by xRy, iff $|x-y| \le 1$, then

(1) R is reflexive (2) R is symmetric

(3) R is transitive (4) R is not symmetric

(5) R is not transitive

Choose the most appropriate answer from the options given below

(a) (1) and (4) only (b) (1), (2) and (3) only

(c) (2) and (3) only (d) (1), (2) and (5) only

17. If the vertices of a triangle ABC are $A(1, 2, 1)$, $B(4, 2, 3)$ and $C(2, 3, 1)$, then the equation of the median passing through the vertex A, is

(a) $\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-1}{2}$

(b) $x-2 = \frac{y-2}{1} = z-1$

(c) $x-1 = 2y-4 = z-1$

(d) $\frac{x-1}{2} = 2y-4 = z-1$

18. A line makes the angle θ with each of the x and z axes. If the angle β which it makes with y-axis is such that $\sin^2\beta = 3\sin^2\theta$, then the value of $\cos^2\theta$ is

(a) $\frac{2}{5}$ (b) $\frac{1}{5}$ (c) $\frac{3}{5}$ (d) $\frac{2}{5}$

19. If $x = 2\sin\theta$ and $y = 2\cos\theta$, then the value of $\frac{d^2y}{dx^2}$ at $\theta = 0$ is

(a) $-\frac{1}{2}$ (b) -1 (c) 0 (d) 1

20. If $x = e^{y+e^{y+e^{y+\dots\infty}}}$, $x > 0$, then $\frac{dy}{dx}$ is equal to

(a) $\frac{x}{1+x}$ (b) $\frac{1}{x}$ (c) $\frac{1-x}{x}$ (d) $\frac{1+x}{x}$

21. $\sin^{-1}(1-x) - 2\sin^{-1}x = \frac{\pi}{2}$, then x is equal to

(a) 0 (b) 1 (c) $\frac{1}{2}$ (d 2

Choose the **most appropriate** answer from the options given below:

(a) (a) and (b) only (b) (a) and (c) only

(c) (a) only (d) (c) only

22. The smaller of the areas enclosed by the circle $x^2 + y^2 = 4$ and the line $x + y = 2$ is

(a) $2(\pi-2)$ (b) $\pi-2$

(c) $2\pi-1$ (d) $2\pi+2$

23. If $0 < x < \pi$ and the matrix $\begin{bmatrix} 4\sin x & -1 \\ -3 & \sin x \end{bmatrix}$ is singular, then the values of x are:

(a) $\frac{\pi}{3}, \frac{2\pi}{3}$ (b) $\frac{\pi}{6}, \frac{5\pi}{6}$

(c) $\frac{\pi}{6}, \frac{\pi}{3}$ (d) $\frac{\pi}{6}, \frac{2\pi}{3}$

24. $\int_{\frac{1}{3}}^{1} \frac{(x-x^3)^{\frac{1}{3}}}{x^4}\,dx =$

(a) 3 (b) 4 (c) 6 (d) 0

26. The function $f(x) = e^{|x|}$ is

(A) continuous everywhere on R

(B) not continuous at $x = 0$

(C) Differentiable everywhere on R

(D) not differentiable at $x = 0$

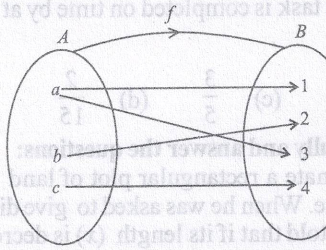
(E) continuous and differentiable on R

Choose the **most appropriate** answer from the options given below:

(a) (E) only (b) (B) and (C) only

(c) (A) and (D) only (d) (B) and (D) only

26.



Which of the following is **true** on the basis of above diagram?

(a) 'f' is a function from $A \to B$
(b) 'f' is one-one function from $A \to B$
(c) 'f' is onto function from $A \to B$
(d) 'f' is not a function from $A \to B$

27. If the points $(2, -3)$, $(\lambda, -1)$ and $(0, 4)$ are collinear, then the value of λ is

(a) $\frac{7}{10}$ (b) $\frac{3}{10}$ (c) $\frac{7}{3}$ (d) $\frac{10}{7}$

28. The value of $\sin\left[2\cot^{-1}\left(\frac{-5}{12}\right)\right]$ is:

(a) $\frac{120}{169}$ (b) $\frac{-120}{169}$ (c) $\frac{-60}{169}$ (d) $\frac{60}{169}$

29. Let $y = m\sin rx + n\cos rx$. What is the value of $\frac{d^2y}{dy^2}$?

(a) ry (b) $-ry$ (c) r^2y (d) $-r^2y$

30. The integrating factor of the differential equation $\cos x\frac{dy}{dx} + y\sin x = 1$ is

(a) $\sec x$ (b) $\cos x$
(c) $\sec x + \tan x$ (d) $\tan x$

31. The order and degree of the differential equation

$$\left[\left(\frac{d^2y}{dx^2}\right)^3\right]^{\frac{1}{3}} = 2\left(\frac{dy}{dx}\right)^{\frac{1}{4}}$$ are

(a) order = 2, degree = 2 (b) order = 2, degree = 4
(c) order = 2, degree = 8 (d) order = 1, degree = 1

32. $\int\sqrt{1-49x^2}\,dx$ is equal to

(a) $\frac{x}{2}\left(\sqrt{1-49x^2}\right)+\frac{1}{98}\sin^{-1}7x+C$

(b) $\frac{7x}{2}\sqrt{1+49x^2}+\frac{1}{49}\sin^{-1}x+C$

(c) $\frac{x}{2}\sqrt{1+\frac{1}{7x^2}}-\frac{1}{49}\sin^{-1}7x+C$

(d) $\frac{x}{2}\sqrt{1-49x^2}+\frac{1}{14}\sin^{-1}7x+C$

33. The shortest distances of the point $(1, 2, 3)$ from x, y, z axes respectively are

(a) $1, 2, 3$ (b) $\sqrt{5}, \sqrt{13}, \sqrt{10}$
(c) $\sqrt{10}, \sqrt{13}, \sqrt{5}$ (d) $\sqrt{13}, \sqrt{10}, \sqrt{5}$

34. Distance between two planes $x + 2y - z = 5$ and $2x + 4y - 2z + 2 = 0$ is

(a) $\sqrt{6}$ unit (b) 7 unit
(c) $\frac{5}{\sqrt{6}}$ unit (d) $\frac{4}{\sqrt{6}}$ unit

35. If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ are two non zero vectors inclined at an angle θ, then identify the correct option out of the given options

(A) $\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$

(B) $\vec{a}$ and $\vec{b}$ are perpendicular, if $a_1b_1 + a_2b_2 + a_3b_3 = 0$

(C) $\vec{a}$ and $\vec{b}$ are perpendicular, $\frac{a_1}{b_1} = \frac{a_2}{b_2} \neq \frac{c_1}{c_2}$

(D) for $\theta = \pi$, $\vec{a}\times\vec{b} = 0$

(E) $\cos\theta = \frac{|\vec{a}\times\vec{b}|}{|\vec{a}||\vec{b}|}$

Choose the **most appropriate** answer from the options given below

(a) (A), (B) and (D) only
(b) (A), (B) and (E) only
(c) (B), (D) and (E) only
(d) (A) and (B) only

36. If $\vec{p} = \hat{i} + \hat{j} - 2\hat{k}$ and $\vec{q} = 2\hat{i} + \hat{j} - \hat{k}$, then the area of parallelogram having diagonals $(\vec{p}+\vec{q})$ and $(\vec{p}-\vec{q})$ is

(a) $4\sqrt{11}$ sq. unit (b) $\sqrt{44}$ sq. unit
(c) $\sqrt{11}$ sq. unit (d) $3\sqrt{11}$ sq. unit

37. If $\vec{a}, \vec{b}$ and $\vec{c}$ are three unit vectors such that $\vec{a} + \vec{b} + \vec{c} = 0$ then the value of $\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}$ is

(a) 3 (b) $-\frac{3}{2}$ (c) $\frac{3}{2}$ (d) -3

38. The corner points of the feasible region for an *L.P.P.* are $(0, 10)$, $(5, 5)$, $(15, 15)$ and $(0, 20)$. If the objective function is $z = px + qy$; $p, q > 0$, then the condition on p and q so that the maximum of z occurs at $(15, 15)$ and $(0, 20)$ is

(a) $p = q$ (b) $p = 2q$
(c) $q = 3p$ (d) $q = 2p$

39. $\int x\sqrt{x+2}$ is equal to:

(a) $\frac{2}{5}(x+2)^{\frac{5}{2}} - \frac{4}{3}(x+2)^{\frac{3}{2}} + C$

(b) $\frac{2}{5}(x+2)^{\frac{5}{2}} + \frac{4}{3}(x+2)^{\frac{3}{2}} + C$

(c) $\frac{1}{5}(x+2)^{\frac{5}{2}} + \frac{2}{3}(x+2)^{\frac{3}{2}} + C$

(d) $\frac{2}{5}(x+2)^{\frac{5}{2}} + \frac{4}{3}(x+2)^{\frac{3}{2}} + C$

40. Three urns contain 6 red, 4 black; 4 red, 6 black and 5 red, 5 black marbles respectively. One of the urns is selected at random and a marble is drawn from it. If the marble drawn is red, then the probability that it is drawn from the first urn is

(a) $\frac{6}{10}$ (b) $\frac{4}{10}$ (c) $\frac{5}{10}$ (d) $\frac{2}{5}$

41. Three persons A, B and C were given a task, whose probabilities of completion their task on time are $\frac{1}{3}, \frac{1}{4}$, and $\frac{1}{5}$ respectively. They were asked to complete the task on time independently. The probability that exactly one of them complete the task on time is

(a) $\frac{2}{15}$ (b) $\frac{2}{5}$ (c) $\frac{3}{20}$ (d) $\frac{13}{20}$

42. Read the text carefully and answer the questions:
Three persons A, B and C were given a task, whose probabilities of completion their task on time are $\frac{1}{3}, \frac{1}{4}$ and $\frac{1}{5}$ respectively. They were asked to complete the task on time independently.
The probability that exactly two of them complete the task on time is

(a) $\frac{3}{20}$ (b) $\frac{13}{20}$ (c) $\frac{2}{5}$ (d) $\frac{2}{15}$

43. Read the text carefully and answer the questions:
Three persons A, B and C were given a task, whose probabilities of completion their task on time are $\frac{1}{3}, \frac{1}{4}$ and $\frac{1}{5}$ respectively. They were asked to complete the task on time independently.
The probability that B alone complete the task on time is:

(a) $\frac{13}{30}$ (b) $\frac{3}{30}$ (c) $\frac{2}{5}$ (d) $\frac{2}{15}$

44. Read the text carefully and answer the questions:
Three persons A, B and C were given a task, whose probabilities of completion their task on time are $\frac{1}{3}, \frac{1}{4}$, and $\frac{1}{5}$ respectively. They were asked to complete the task on time independently.
The probability that the task is completed on time by none of them is

(a) $\frac{3}{20}$ (b) $\frac{2}{5}$ (c) $\frac{13}{20}$ (d) $\frac{2}{15}$

45. Read the text carefully and answer the questions:
Three persons A, B and C were given a task, whose probabilities of completion their task on time are $\frac{1}{3}, \frac{1}{4}$ and $\frac{1}{5}$ respectively. They were asked to complete the task on time independently.
The probability that task is completed on time by at least one of them is:

(a) $\frac{2}{5}$ (b) $\frac{3}{20}$ (c) $\frac{3}{5}$ (d) $\frac{2}{15}$

46. Read the text carefully and answer the questions:
Mohan wants to donate a rectangular plot of land for a hospital in his village. When he was asked to give dimensions of the plot, he told that if its length (x) is decreased by 50 m and breadth (y) is increased by 50 m, then its area will remain same, but if length is decreased by 10 m and breadth is decreased by 20 m, then its area will decrease by 5300 m^2.
The equations in terms of x and y are:

(a) $x-y=50, 2x+y=550$
(b) $x+y=40, 2x-y=550$
(c) $x-y=10, 2x+y=50$
(d) $x-y=30, 2x+y=505$

47. Read the text carefully and answer the questions:
Mohan wants to donate a rectangular plot of land for a hospital in his village. When he was asked to give dimensions of the plot, he told that if its length (x) is decreased by 50 m and breadth (y) is increased by 50 m, then its area will remain same, but if length is decreased by 10 m and breadth is decreased by 20 m, then its area will decrease by 5300 m^2.
The value x is:

(a) 150 m (b) 100 m (c) 200 m (d) 300 m

48. Read the text carefully and answer the questions:
Mohan wants to donate a rectangular plot of land for a hospital in his village. When he was asked to give dimensions of the plot, he told that if its length (x) is decreased by 50 m and breadth (y) is increased by 50 m, then its area will remain same, but if length is decreased by 10 m and breadth is decreased by 20 m, then its area will decrease by 5300 m^2.
The value of y is

(a) 50 m (b) 100 m (c) 240 m (d) 150 m

49. Read the text carefully and answer the questions:
Mohan wants to donate a rectangular plot of land for a hospital in his village. When he was asked to give dimensions of the plot, he told that if its length (x) is decreased by 50 m and breadth (y) is increased by 50 m, then its area will remain same, but if length is decreased by 10 m and breadth is decreased by 20 m, then its area will decrease by 5300 m^2.

The value of the expression $\frac{x^2+y^2}{x-y}$ is:

(a) 625 (b) 1250 (c) 312.5 (d) 3125

50. Read the text carefully and answer the questions:
Mohan wants to donate a rectangular plot of land for a hospital in his village. When he was asked to give dimensions of the plot, he told that if its length (x) is decreased by 50 m and breadth (y) is increased by 50 m, then its area will remain same, but if length is decreased by 10 m and breadth is decreased by 20 m, then its area will decrease by 5300 m^2.
The area of rectangular field is:

(a) 30000 sq. m (b) 3000 sq. m
(c) 300000 sq. m (d) 60000 sq. m

Hints & Explanations

1. (d) Order of $P = 2 \times m$

Order of $Q = k \times n$

Order of $R = m \times 2$

Order of $S = 2 \times 3$

Now, for $PQ + RS$ to be defined.

PQ and RS is to be defined and PQ and RS should be of same order.

For PQ to be defined $m = k$

$\Rightarrow$ Order of $PQ = 2 \times n$

For RS to be defined $2 = 2$

$\Rightarrow$ Order of $RS = m \times 3$

If order of PQ = Order of RS

$\Rightarrow 2 \times n = m \times 3$

$\Rightarrow m = 2, n = 3, k = 2$

2. (d) Given the equations

$3x + 4y = 5$

$6x + 7y = -8$

In matrix form

$$[x\ y]\begin{bmatrix} 3 & 6 \\ 4 & 7 \end{bmatrix} = [5 -8]$$

3. (c) Given, $2\begin{bmatrix} a & d \\ b & c \end{bmatrix} + 3\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} = 3\begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}$

$$= \begin{bmatrix} 2a+3 & 2d-3 \\ 2b & 2c+6 \end{bmatrix} = \begin{bmatrix} 9 & 15 \\ 12 & 15 \end{bmatrix}$$

$\Rightarrow \quad 2a + 3 = 9 \quad \Rightarrow a = 3$

$2d - 3 = 15 \quad \Rightarrow d = 9$

$2b = 12 \quad \Rightarrow b = 6$

$2c + 6 = 18 \quad \Rightarrow c = 6$

$|3 + 6 - 9 - 6| = |-6| = 6$

4. (b) Given the function

$$f(x) = x^{\frac{1}{x}} \Rightarrow \log f(x) = \frac{1}{x}\log x$$

$$\frac{f'(x)}{f(x)} = \frac{1}{x^2} + (\log x)\left(-\frac{1}{x^2}\right)$$

$$f'(x) = \frac{x^{\frac{1}{x}}[1 - \log x]}{x^2}, \text{ Now, } f'(x) = 0$$

$\Rightarrow 1 - \log x = 0 \Rightarrow \log x = 1$

$\therefore \quad x = e$

Since, $f'(e^-) > 0$ and $f'(e^+) < 0$

So, $f(x)$ has max. at e.

$\therefore f(x)_{max} = e^{\frac{1}{e}}$

5. (c) Given, $f(x) = x^5 - 5x^4 + 5x^3 - 1$

$f'(x) = 5x^4 - 20x^3 + 15x^2$

$= 5x^2(x^2 - 4x + 3) = 0$

$= 5x^2(x-3)(x-1) = 0$

$x = 0, 3, 1$

Now, $f''(x) = 20x^3 - 60x^2 + 30x$

$= 10x(2x^2 - 6x + 3)$

$f''(x) = 0 \Rightarrow x = 0 \qquad \therefore x = 0$ is point of inflexion.

$f''(1) < 0 \Rightarrow x = 1 \qquad$ point of maxima.

$f''(3) > 0 \Rightarrow x = 3 \qquad$ point of minima.

$f(3) = (3)^5 - 5(3)^4 + 5(3)^3 - 1 = -28$

$f(1) = 1 - 5 + 5 - 1 = 0$

6. (c) (a) Given, $x = t^2, y = t^3$

$$\frac{dx}{dt} = 2t, \quad \frac{dy}{dt} = 3t^2$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} \Rightarrow \frac{dy}{dx} = \frac{3}{2}t$$

$$\frac{d^2y}{dx^2} = \frac{3}{2} \times \frac{dt}{dx} = \frac{3}{2} \times \frac{1}{2} = \frac{3}{4}$$

(b) $f(x) = \sqrt{x} + 1$

$$f'(x) = \frac{1}{2\sqrt{x}}$$

$$f''(x) = -\frac{1}{4x^{\frac{3}{2}}} \Rightarrow f''(1) = -\frac{1}{4}$$

(c) $f(x) = 9x^2 + 12x + 2$

$$f'(x) = 18x + 12 = 0 \Rightarrow x = -\frac{2}{3}$$

Now, $f''(x) = 18$ and $f''\left(-\frac{2}{3}\right) > 0$

$$\text{So, } f(x)_{min} = 9\left(-\frac{2}{3}\right)^2 + 12\left(-\frac{2}{3}\right) + 2$$

$$= 9 \times \frac{4}{9} + (-8) + 2 = 4 - 8 + 2 = -2$$

(d) Given, $f(x)=(x-2)^4(x+1)^3$

$f'(x)=3(x+1)^2(x-2)^4+4(x-2)^3(x+1)^3$

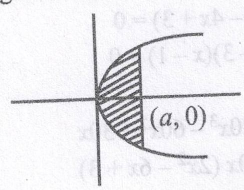
$f''(x)=6(x+1)(x-2)^4+12(x-2)^3(x+1)^2$
$\quad +12(x+1)^2(x-2)^3+12(x-2)^2(x+1)^3$

$=(x+1)[6(x-2)^4+12(x-2)^3(x+1)$
$\quad +12(x+1)(x-2)^3+12(x-2)^2(x+1)^2]=0$

$\Rightarrow x=-1$ is point of inflexion.

$\therefore$ (a) $\rightarrow$ (iii), (b) $\rightarrow$ (iv), (c) $\rightarrow$ (i), (d) $\rightarrow$ (ii)

7. (d) Graph of given curves



Area $= 2\int_0^a \sqrt{4ax}\,dx = 2.2\sqrt{a}\int_0^a \sqrt{x}\,dx$

$= 4\sqrt{a}\left[\frac{x^{\frac{3}{2}}}{3}\times 2\right]_0^a = \frac{4a^{\frac{1}{2}}}{3}\left[a^{\frac{3}{2}}-0\right]\times 2$

$= \frac{4a^{\frac{1}{2}}a^{\frac{3}{2}}}{3}\times 2 = \frac{8}{3}a^2$

8. (a) $\int \frac{xe^x}{(x+1)^2}dx = \int e^x\left[\frac{x+1-1}{(x+1)^2}\right]dx$

$\therefore \int e^x[f(x)+f'(x)]dx = e^x f(x)+C$

$=\int e^x\left[\frac{1}{x+1}-\frac{1}{(x+1)^2}\right]dx = \frac{e^x}{1+x}+C$

9. (c) Given differential equation,

$(x+1)\frac{dy}{dx}=1+y \Rightarrow \frac{dx}{1+x}=\frac{dy}{1+y}$

$\Rightarrow \ln(1+x)=\ln(1+y)+\ln C \Rightarrow \ln\left(\frac{1+x}{1+y}\right)=\ln C$

$\Rightarrow \frac{1+x}{1+y}=C$

10. (a) Given differential equation,

$y\frac{dy}{dx}+\frac{4}{\frac{dy}{dx}}=5$

$y\left(\frac{dy}{dx}\right)^2+4=5\left(\frac{dy}{dx}\right)$

$\therefore$ Order $=1$

Degree $=2$

11. (d) Let $f(x)=x^3+1$

$g(x)=x^2+1$

$\frac{f'(x)}{g'(x)}=\frac{3x^2}{2x}=\frac{3}{2}x$

12. (a) The given differential equation

$(x+xy)\,dy-y(1-x^2)\,dx=0$

$x(1+y)\,dy-y(1-x^2)\,dx=0$

$\int\frac{(1+y)}{y}dy=\int\frac{(1-x^2)}{x}dx$

$\log y+y=\log x-\frac{x^2}{2}+C$

$y=\log\frac{x}{y}-\frac{x^2}{2}+C$

13. (a) Sample space

S: {1, 2, 3}

Two numbers can be select as (1, 2) (2, 3) (1, 3)

Now,

x	2	3
$P(X=x)$	1/3	2/3

where X denotes larger of two integers.

14. (d) $X=$ getting number of doubles

$P=\frac{6}{36}=\frac{1}{6}$

$P(X=0)={}^3C_0\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^3=\frac{125}{216}$

$P(X=1)={}^3C_1\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^2=\frac{75}{216}$

$P(X=2)={}^3C_2\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^1=\frac{15}{216}$

$P(X=3)={}^3C_3\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^0=\frac{1}{216}$

15. (c) The optimal values of the objective function is attained at the corner points of the feasible region.

16. (d) Since, xRy, $|x-y|\le 1$

For reflexive $(a,a)\Rightarrow|a-a|=0\le 1$.

$\therefore$ Relation is reflexive,

For symmetric $(a,b)\Rightarrow|a-b|\le 1$.

$(b,a)\Rightarrow|b-a|\le 1$.

$\therefore$ Relation is symmetric.

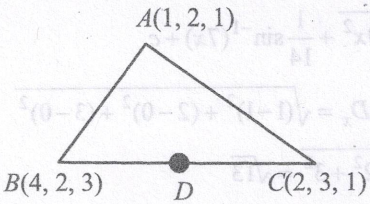
For transitive $(a,b)\Rightarrow|a-b|\le 1$

$(b, c) \Rightarrow |b - c| \le 1$

$\not\Rightarrow |a - c| \le 1$

$\therefore$ Only reflexive and symmetric and not transitive.

17. **(d)**



D is mid-point of line segment BC.

$$D \equiv \left(\frac{4+2}{2}, \frac{2+3}{2}, \frac{3+1}{2}\right) \equiv \left(3, \frac{5}{2}, 2\right)$$

Equation of median through A.

$$\left(\frac{x-1}{3-1} = \frac{y-2}{\frac{5}{2}-2} = \frac{z-1}{2-1}\right) \Rightarrow \frac{x-1}{2} = \frac{y-2}{\frac{1}{2}} = \frac{z-1}{1}$$

18. **(c)** We know that

$\cos^2\theta + \cos^2\beta + \cos^2\theta = 1$

or $\quad 2\cos^2\theta + 1 - 3\sin^2\theta = 1$

or $\quad 2\cos^2\theta = 3\sin^2\theta$

or $\quad 2\cos^2\theta = 3 - 3\cos^2\theta \Rightarrow \cos\theta = \frac{3}{5}$

19. **(a)** Now, $\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} \Rightarrow \frac{dy}{dx} = \frac{-2\sin\theta}{2\cos\theta} = -\tan\theta$

$$\frac{d^2y}{dx^2} = \frac{d}{d\theta}\left(\frac{dy}{dx}\right)\cdot\frac{d\theta}{dx} \Rightarrow \frac{d^2y}{dx^2} = -\sec^2\theta.\frac{d\theta}{dx}$$

$$= \frac{-\sec^2\theta}{2\cos\theta} = -\frac{1}{2}\sec^3\theta$$

At $\theta = 0 \Rightarrow \frac{d^2y}{dx^2} = -\frac{1}{2}$

20. **(c)** Since, $x = e^{y+x}$

Differentiating w.r.t. x, we get

$$1 = e^{x+y}\left(1 + \frac{dy}{dx}\right) \Rightarrow \frac{1}{x} = 1 + \frac{dy}{dx}$$

or $\frac{dy}{dx} = \frac{1-x}{x}$

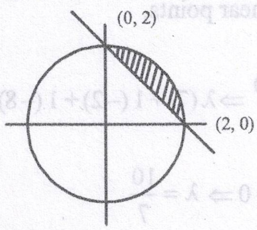
21. **(c)** Domain for $\sin^{-1}(1-x)$ is $[0, 2]$ and domain for $\sin^{-1}x$ is $[-1, 1]$.

So, Domain for x is $[0, 1]$.

$$\sin^{-1}(1-x)\big|_{max} = \frac{\pi}{2};\ \sin^{-1}(x)\big|_{max} = 0$$

So, only possible condition is when $x = 0$.

22. **(b)** Graph of given curves



Area of smaller region

$$= \frac{1}{4}\times 4\pi - \frac{1}{2}.2.2 = (\pi - 2) \text{ sq. units}$$

23. **(a)** Since, $\begin{vmatrix} 4\sin x & -1 \\ -3 & \sin x \end{vmatrix} = 0$

$$\Rightarrow 4\sin^2 x - 3 = 0 \Rightarrow \sin^2 x = \frac{3}{4}$$

$$\Rightarrow \sin x = \pm\frac{\sqrt{3}}{2} \Rightarrow x = \frac{\pi}{3} \text{ or } \frac{2\pi}{3}$$

24. **(c)** Let

$$I = \int_{1/3}^{1} \frac{x\left(\frac{1}{x^2} - 1\right)^{1/3}}{x^4} dx = \int_{1/3}^{1} \frac{\left(\frac{1}{x^2} - 1\right)^{1/3}}{x^3} dx$$

Let $\frac{1}{x^2} - 1 = t \Rightarrow \frac{-2}{x^3}dx = dt$

If $x = \frac{1}{3} \Rightarrow t = 8$

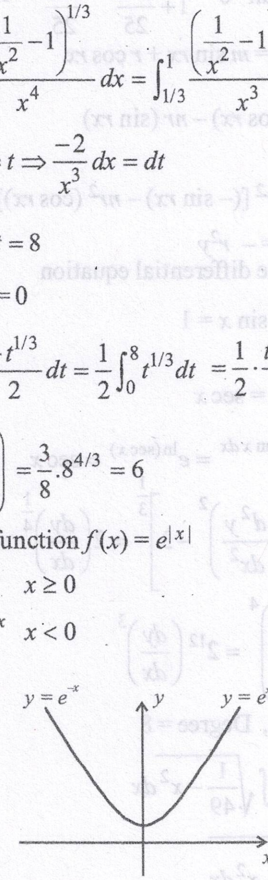
If $x = 1 \Rightarrow t = 0$

$$\text{So, } I = \int_8^0 \frac{-t^{1/3}}{2} dt = \frac{1}{2}\int_0^8 t^{1/3} dt = \frac{1}{2}\cdot\frac{t^{4/3}}{4}\Bigg|_0^8$$

$$\frac{3}{8}\left(8^{\frac{4}{3}} - 0^{\frac{4}{3}}\right) = \frac{3}{8}.8^{4/3} = 6$$

25. **(d)** Given function $f(x) = e^{|x|}$

$$f(x) = \begin{cases} e^x & x \ge 0 \\ e^{-x} & x < 0 \end{cases}$$



$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^-} f(x) = f(0) = 1$

So, $f(x)$ is continuous.

But RHD at $x=0$ is 1.

LHD at $x=0$ is -1.

So, $f(x)$ is not differentiable at $x=0$.

26. (d) $f(a)=1$ and $f(a)=3$, which is not possible for any function.

27. (d) For collinear points,

$$\begin{vmatrix} \lambda & -1 & 1 \\ 0 & 4 & 1 \\ 2 & -3 & 1 \end{vmatrix} = 0 \Rightarrow \lambda(7)+1(-2)+1(-8)=0$$

$$\Rightarrow 7\lambda - 10 = 0 \Rightarrow \lambda = \frac{10}{7}$$

28. (b) $\sin\left(2\pi - 2\cot^{-1}\frac{5}{12}\right) = -\sin\left(2\cot^{-1}\frac{5}{15}\right)$

Since, $\sin(2\pi - x) = -\sin x$

Let $\cot^{-1}\frac{15}{12} = \theta \Rightarrow \cot\theta = \frac{5}{12}$

$$\Rightarrow \tan\theta = \frac{12}{5}$$

$$\sin 2\theta = \frac{2\tan\theta}{1+\tan^3\theta} = \frac{\frac{24}{5}}{1+\frac{144}{25}} = \frac{\frac{25}{5}}{\frac{169}{25}} = \frac{120}{169}$$

29. (d) Given, $y = m\sin rx + r\cos rx$

$$\frac{dy}{dx} = m.r(\cos rx) - nr(\sin rx)$$

$$\Rightarrow \frac{d^2y}{dx^2} = mr^2[(-\sin rx) - nr^2(\cos rx)]$$

$$= -r^2y$$

30. (a) Given, the differential equation

$$\cos x\frac{dy}{dx} + y\sin x = 1$$

$$\frac{dy}{dx} + y\tan x = \sec x$$

So, I.F. $= e^{\int \tan x\,dx} = e^{\ln(\sec x)} = \sec x$

31. (c) Since, $\left[\left(\frac{d^2y}{dx^2}\right)^2 - 3\right]^{\frac{1}{3}} = 2\left(\frac{dy}{dx}\right)^{\frac{1}{4}}$

$$\left(\left(\frac{d^2y}{dx^2}\right)^2 - 3\right)^4 = 2^{12}\left(\frac{dy}{dx}\right)^3$$

So, Order $= 2$, Degree $= 8$

32. (d) Let $I = 7\int\sqrt{\frac{1}{49} - x^2}\,dx$

$$= 7\int\sqrt{\left(\frac{1}{7}\right)^2 - x^2}\,dx$$

$$= 7\left(\frac{x}{2}\sqrt{\frac{1}{49} - x^2} + \frac{1}{98}\sin^{-1}\frac{x}{\frac{1}{7}}\right) + c$$

$$= \frac{x}{2}\sqrt{1-49x^2} + \frac{1}{14}\sin^{-1}(7x) + c$$

33. (d) Since, $D_x = \sqrt{(1-1)^2 + (2-0)^2 + (3-0)^2}$

$$\Rightarrow D_x = \sqrt{2^2+3^2} = \sqrt{13}$$

$$D_y = \sqrt{1^2+3^2} = \sqrt{10}$$

$$D_z = \sqrt{1^2+2^2} = \sqrt{5}$$

34. (a) Given the planes

$P_1: 2x+4y-2z=10$

$P_2: 2x+4y-2z=-2$

$$d = \frac{12}{\sqrt{2^2+4^2+2^2}} = \frac{12}{\sqrt{24}} = \sqrt{6} \text{ units}$$

35. (a) Given the vectors

$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$; $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

If angle between $\vec{a}$ and $\vec{b}$ is θ

So, $\cos\theta = \frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}$

and if $\vec{a}$ is perpendicular to $\vec{b}$ then

$\vec{a}.\vec{b} = 0 \Rightarrow a_1b_1 + a_2b_2 + a_3b_3 = 0$

and if $\theta = \pi$, then

$\vec{a}\times\vec{b} = |\vec{a}||\vec{b}|\sin\theta\hat{n} = \vec{0}$

So, option (1) is correct.

36. (c) Given the vectors

$p = \hat{i} + \hat{j} - 2\hat{k}$ and $q = 2\hat{i} + 2\hat{j} - k$

Let $\vec{d}_1 = \vec{p} + \vec{q} = 3\hat{i} + 2\hat{j} - 3\hat{k}$

$\vec{d}_2 = \vec{p} - \vec{q} = \hat{i} - \hat{k}$

$\therefore$ Area of parallelogram $= \frac{1}{2}|\vec{d}_1 \times \vec{d}_2|$

$$\text{Now, } \vec{d}_1 \times \vec{d}_2 = \begin{vmatrix} i & \hat{j} & k \\ 3 & 2 & -3 \\ -1 & 0 & -1 \end{vmatrix} = -2\hat{i} + 6\hat{j} + 2\hat{k}$$

$$\Rightarrow |d_1 \times d_2| = \sqrt{(-2)^2 + 6^2 + 2^2}$$

$\therefore$ Area $= \frac{1}{2}\sqrt{4+36+4} = \sqrt{11}$ sq. units

37. (b) Given, $\vec{a}+\vec{b}+\vec{c}=0$

Now, $(\vec{a}+\vec{b}+\vec{c}).(\vec{a}+\vec{b}+\vec{c})=0$

$\Rightarrow |\vec{a}|+|\vec{b}|+|\vec{c}|+2(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a})=0$

Since, $|\vec{a}|=|\vec{b}|=|\vec{c}|=1$

$\Rightarrow 2(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a})=-3$

$\therefore \quad (\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a})=-\frac{3}{2}$

38. (c) Given, z has max at (15, 15) and (0, 20).

So, $z_{(15,15)}=z_{(10,20)}$

$\Rightarrow 15p+15q=20q \Rightarrow q=3p$

39. (a) $I=\int x\sqrt{x+2}\,dx$

Let $x+2=t^2 \Rightarrow dx=2t\,dt$

$\therefore \quad I=\int (t^2-2)t(2t)\,dt$

$=\int (2t^4-4t^2)dt=\frac{2}{5}t^5-\frac{4}{3}t^3+C$

$=\frac{2}{5}(x+2)^{\frac{5}{2}}-\frac{4}{3}(x+2)^{\frac{3}{2}}+C$

40. (d) Let

$\underbrace{\begin{matrix}6R\\4B\end{matrix}}_{U-1},\ \underbrace{\begin{matrix}4R\\6B\end{matrix}}_{U-2},\ \underbrace{\begin{matrix}5R\\5B\end{matrix}}_{U-3}$

E : Drawn marble is red

E_1 : Drawn marble from urn I

E_2 : Drawn marble from urn II

E_3 : Drawn marble from urn III

So, $P(E_1)=P(E_2)=P(E_3)=\frac{1}{3}$

So, by conditional probability

$$P\left(\frac{E_1}{E}\right)=\frac{P(E\cap E_1)}{P(E)}$$

$$=\frac{P(E_1)P\left(\frac{E}{E_1}\right)}{P(E_1)P\left(\frac{E}{E_1}\right)+P(E_2)P\left(\frac{E}{E_2}\right)+P(E_3)\left(\frac{E}{E_3}\right)}$$

$$=\frac{\frac{6}{10}}{\frac{6}{10}+\frac{4}{10}+\frac{5}{10}}=\frac{6}{15}=\frac{2}{5}$$

41. (d) Given, $P(A)=\frac{1}{3};P(B)=\frac{1}{4};P(C)=\frac{1}{5}$

So, required probability

$$P=\frac{1}{3}\times\frac{3}{4}\times\frac{4}{5}+\frac{2}{3}\times\frac{1}{4}\times\frac{4}{5}+\frac{2}{3}\times\frac{3}{4}\times\frac{1}{5}$$

$$=\frac{12+8+6}{60}=\frac{26}{60}=\frac{13}{30}$$

42. (a) Given, $P(A)=\frac{1}{3};P(B)=\frac{1}{4};P(C)=\frac{1}{5}$

Required probability

$$P=\frac{1}{3}\times\frac{1}{4}\times\frac{4}{5}+\frac{2}{3}\times\frac{1}{4}\times\frac{1}{5}+\frac{1}{3}\times\frac{3}{4}\times\frac{1}{5}$$

$$=\frac{4+2+3}{60}=\frac{9}{60}=\frac{3}{20}$$

43. (d) Given, $P(A)=\frac{1}{3};P(B)=\frac{1}{4};P(C)=\frac{1}{5}$

So, required probability

$$P=\frac{2}{3}\times\frac{1}{4}\times\frac{4}{5}=\frac{2}{15}$$

44. (b) Given, $P(A)=\frac{1}{3};P(B)=\frac{1}{4};P(C)=\frac{1}{5}$

So, required probability

$$P=\frac{2}{3}\times\frac{3}{4}\times\frac{4}{5}=\frac{2}{5}$$

45. (c) Given, $P(A)=\frac{1}{3};P(B)=\frac{1}{4};P(C)=\frac{1}{5}$

So, required probability

$$P=1-\frac{2}{3}\times\frac{3}{4}\times\frac{4}{5}=1-\frac{2}{5}=\frac{3}{5}$$

46. (a) Let length and breadth of plot be x and y respectively.

Then, $(x-50)(y+50)=xy$ ...(i)

and, $(x-10)(y-20)=xy-5300$ ...(ii)

From (i), $50x-50y-2500=0$

$\Rightarrow x-y=50$

From (ii), $-20x-10y+200=-5300$

$\Rightarrow 20x+10y=5500$

$\Rightarrow 2x+y=550$

47. (c) Let length and breadth of plot be x and y respectively.

Then, $(x-50)(y+50)=xy$ ...(i)

and $(x-10)(y-20)=xy-5300$ ...(ii)

From (i), $50x-50y-2500=0$

$\Rightarrow x-y=50$

From (ii), $-20x-10y+200=-5300$

$\Rightarrow 20x+10y=5500$

$\Rightarrow 2x+y=550$

$x-y=50$ ...(iii)

$2x+y=550$ ...(iv)

Equation (iii) + (iv),

$\Rightarrow 3x=600 \Rightarrow x=200$

48. (d) Let length and breadth of plot be x and y respectively.

Then, $(x-50)(y+50)=xy$ ...(i)

and $(x-10)(y-20)=xy-5300$ ...(ii)

From (i), $50x - 50y - 2500 = 0$

$\Rightarrow x - y = 50$

From (ii), $-20x - 10y + 200 = -5300$

$\Rightarrow 20x + 10y = 5500$

$\Rightarrow 2x + y = 550$

$x - y = 50$ ...(iii)

$2x + y = 550$ ...(iv)

Equation (iii) + (iv)

$\Rightarrow 3x = 600 \Rightarrow x = 200$

$x - y = 50$ and $x = 200$

then $y = 150$ m

49. **(b)** Let length and breadth of plot be x and y respectively.

Then, $(x - 50)(y + 50) = xy$ ...(i)

and $(x - 10)(y - 20) = xy - 5300$ ...(ii)

From (i), $50x - 50y - 2500 = 0$

$\Rightarrow x - y = 50$

From (ii), $-20x - 10y + 200 = -5300$

$\Rightarrow 20x + 10y = 5500$

$\Rightarrow 2x + y = 550$

$x - y = 50$ ...(iii)

$2x + y = 550$ ...(iv)

Equation (iii) + (iv),

$\Rightarrow 3x = 600 \Rightarrow x = 200$

$x = 200, y = 150$

$$\frac{x^2 + y^2}{x - y} = \frac{(200)^2 + (150)^2}{50} = \frac{62500}{50} = 1250$$

50. **(a)** Let length and breadth of plot be x and y respectively.

Then, $(x - 50)(y + 50) = xy$ ...(i)

and $(x - 10)(y - 20) = xy - 5300$ ...(ii)

From (i), $50x - 50y - 2500 = 0$

$\Rightarrow x - y = 50$

From (ii), $-20x - 10y + 200 = -5300$

$\Rightarrow 20x + 10y = 5500$

$\Rightarrow 2x + y = 550$

$x - y = 50$ ...(iii)

$2x + y = 550$ ...(iv)

Equation (iii) + (iv)

$\Rightarrow 3x = 600 \Rightarrow x = 200$

$x = 200, y = 150$

Area $= x \times y$

Area $= 200 \times 150 = 30000$ sq. m.