Short Answer Questions-I (PYQ)

[2 Mark]

Q.1. If $\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix} \begin{bmatrix} 1 & -3 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & x \end{bmatrix}$, then write the value of x.

Ans.

Given $\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix} . \begin{bmatrix} 1 & -3 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & x \end{bmatrix}$

$\Rightarrow \quad \begin{bmatrix} 2\times1+3\times(-2) & 2\times(-3)+3\times4 \\ 5\times1+7\times(-2) & 5\times(-3)+7\times4 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & x \end{bmatrix} \quad \Rightarrow \quad \begin{bmatrix} -4 & 6 \\ -9 & 13 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & x \end{bmatrix}$

Equating the corresponding elements, we get

$x = 13$

Q.2. Find the value of *x* + *y* from the following equation:

$$2\begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$$

Ans.

Given, $2\begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$

$\Rightarrow \quad \begin{bmatrix} 2x & 10 \\ 14 & 2y-6 \end{bmatrix} + \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix} \quad \Rightarrow \quad \begin{bmatrix} 2x+3 & 6 \\ 15 & 2y-4 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$

Equating the corresponding elements, we get

$2x + 3 = 7$ and $2y - 4 = 14 \quad \Rightarrow \quad x = \frac{7-3}{2}$ and $y = \frac{14+4}{2}$

$\Rightarrow \quad x = 2 \quad$ and $\quad y = 9$

$\therefore \quad x + y = 2 + 9 = 11$

Q.3. If matrix $A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ **and $A^2 = kA$, then write the value of *k*.**

Ans.

Given: $A^2 = kA$

$$\Rightarrow \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = k \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \qquad \Rightarrow \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} = k \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$\Rightarrow 2\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = k \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \qquad \Rightarrow k = 2$$

Q.4. If $\begin{bmatrix} 3 & 4 \\ 2 & x \end{bmatrix} \begin{bmatrix} x \\ 1 \end{bmatrix} = \begin{bmatrix} 19 \\ 15 \end{bmatrix}$**, then find the value of *x*.**

Ans.

Given,

$$\Rightarrow \begin{bmatrix} 3 & 4 \\ 2 & x \end{bmatrix} \begin{bmatrix} x \\ 1 \end{bmatrix} = \begin{bmatrix} 19 \\ 15 \end{bmatrix} \qquad \Rightarrow \begin{bmatrix} 3(x)+4(1) \\ (2)(x)+(x)(1) \end{bmatrix} = \begin{bmatrix} 19 \\ 15 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3x+4 \\ 3x \end{bmatrix} = \begin{bmatrix} 19 \\ 15 \end{bmatrix}$$

Equating the corresponding elements, we get

$3x + 4 = 19$ and $3x = 15$

$\Rightarrow \quad 3x = 19 - 4, \qquad 3x = 15$

$\Rightarrow \quad 3x = 15, \qquad x = 5$

$\therefore \quad x = 5$

Q.5. If matrix $A = \begin{bmatrix} 3 & -3 \\ -3 & 3 \end{bmatrix}$ **and $A^2 = \lambda A$, then write the value of λ.**

Ans.

Here, $A = \begin{bmatrix} 3 & -3 \\ -3 & 3 \end{bmatrix}$

Given, $A^2 = \lambda A$

$\Rightarrow \quad \begin{bmatrix} 3 & -3 \\ -3 & 3 \end{bmatrix} \begin{bmatrix} 3 & -3 \\ -3 & 3 \end{bmatrix} = \lambda \begin{bmatrix} 3 & -3 \\ -3 & 3 \end{bmatrix} \quad \Rightarrow \quad \begin{bmatrix} 18 & -18 \\ -18 & 18 \end{bmatrix} = \lambda \begin{bmatrix} 3 & -3 \\ -3 & 3 \end{bmatrix}$

$\Rightarrow \quad 6\begin{bmatrix} 3 & -3 \\ -3 & 3 \end{bmatrix} = \lambda \begin{bmatrix} 3 & -3 \\ -3 & 3 \end{bmatrix} \quad \Rightarrow \quad \lambda = 6$

Q.6. If A is a square matrix such that $A^2 = I$, then find the simplified value of $(A - I)^3 + (A + I)^3 - 7A$.

Ans.

We have $A^2 = I$

Now, $(A - I)^3 + (A + I)^3 - 7A = A^3 - 3A^2I + 3AI^2 - I^3 + A^3 + 3A^2I + 3AI^2 + I^3 - 7A$

$= 2A^3 + 6AI^2 - 7A$

$= 2A^3 + 6AI - 7A \qquad [\because I^2 = I]$

$= 2A^2 . A + 6A - 7A \qquad [\because AI = A]$

$= 2I.A + 6A - 7A \qquad [\because A^2 = I]$

$= 2A + 6A - 7A = A \qquad [\because IA = A]$

Q.7. Matrix $A = \begin{bmatrix} 0 & 2b & -2 \\ 3 & 1 & 3 \\ 3a & 3 & -1 \end{bmatrix}$ **is given to be symmetric, find values of *a* and *b*.**

Ans.

We have $A = \begin{bmatrix} 0 & 2b & -2 \\ 3 & 1 & 3 \\ 3a & 3 & -1 \end{bmatrix}$

$\because$ A is symmetric matrix

$\Rightarrow$ $A^T = A$

$$\Rightarrow \begin{bmatrix} 0 & 3 & 3a \\ 2b & 1 & 3 \\ -2 & 3 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 2b & -2 \\ 3 & 1 & 3 \\ 3a & 3 & -1 \end{bmatrix}$$

Equating both sides, we get

$$2b = 3 \quad \text{and} \quad 3a = -2 \quad \Rightarrow \quad b = \frac{3}{2} \quad \text{and } a = -\frac{2}{3}.$$

Q.8. If $A = \begin{bmatrix} 1 & -2 & 3 \\ -4 & 2 & 5 \end{bmatrix}$ **and** $B = \begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{bmatrix}$ **and BA = (bij), find b21 + b32.**

Ans.

We have, $A = \begin{bmatrix} 1 & -2 & 3 \\ -4 & 2 & 5 \end{bmatrix}$ and $\begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{bmatrix}$

$$\therefore \quad BA = \begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{bmatrix}_{3 \times 2} \begin{bmatrix} 1 & -2 & 3 \\ -4 & 2 & 5 \end{bmatrix}_{2 \times 3}$$

$$[b_{ij}] = \begin{bmatrix} 2-12 & -4+6 & 6+15 \\ 4-20 & -8+10 & 12+25 \\ 2-4 & -4+2 & 6+5 \end{bmatrix}_{3\times3} \quad \Rightarrow \quad [b_{ij}] = \begin{bmatrix} -10 & 2 & 21 \\ -16 & 2 & 37 \\ -2 & -2 & 11 \end{bmatrix}_{3\times3}$$

Now, $b_{21} = -16$; $b_{32} = -2$

$\therefore$ $b_{21} + b_{32} = -16 - 2 = -18$

Short Answer Questions-I (OIQ)

[2 Mark]

Q.1. Solve for x, $[1 \quad x] \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \end{bmatrix} = [0]$.

Ans.

Given: $[1 \quad x] \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \end{bmatrix} = [0]$

$\Rightarrow \quad [1 \quad x] \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \end{bmatrix} = [0] \quad \Rightarrow \quad [2+x \quad -1+2x] \begin{bmatrix} 1 \\ 3 \end{bmatrix} = [0]$

$\Rightarrow \quad [2 + x - 3 + 6x] = [0]$

$\Rightarrow \quad -1 + 7x = 0$

$\Rightarrow \quad x = \frac{1}{7}$

Q.2. If $[2 \quad 1 \quad 3] \begin{bmatrix} -1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = A$, then find the value of A.

Ans.

Given, $A = [2 \quad 1 \quad 3] \begin{bmatrix} -1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$

$= [-2-1+0 \quad 0+1+3 \quad -2+0+3] \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = [-3 \quad 4 \quad 1] \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$

$A = [-3 + 0 - 1] = [-4]$

Q.3. Find the value of *x* and *y* if

$$\begin{bmatrix} x+10 & y^2+2y \\ 0 & -4 \end{bmatrix} = \begin{bmatrix} 3x+4 & 3 \\ 0 & y^2-5y \end{bmatrix}$$

Ans.

$$x + 10 = 3x + 4 \quad \text{and} \quad y^2 + 2y = 3$$

$\Rightarrow \quad 3x - x = 10 - 4 \qquad \Rightarrow \quad y^2 + 2y - 3 = 0$

$\Rightarrow \quad 2x = 6 \qquad \Rightarrow \quad y^2 + 3y - y - 3 = 0$

$\Rightarrow \quad x = 3 \qquad \Rightarrow \quad y(y + 3) - 1(y + 3) = 0$

$\Rightarrow \quad (y + 3)\ (y - 1) = 0$

$\Rightarrow \quad y = 1, -3 \qquad ...(i)$

Also, $y^2 - 5y = -4$

$\Rightarrow \quad y^2 - 5y + 4 = 0 \qquad \Rightarrow \quad y^2 - 4y - y + 4 = 0$

$\Rightarrow \quad y(y - 4) - 1(y - 4) = 0 \qquad \Rightarrow \quad (y - 4)\ (y - 1) = 0$

$\Rightarrow \quad y = 4, 1 \qquad ...(ii)$

From (i) and (ii) $y = 1$

i.e., $x = 3$ and $y = 1$

Q.4. If $\begin{bmatrix} a+b & 2 \\ 5 & ab \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$

Ans.

$$\begin{bmatrix} a+b & 2 \\ 5 & ab \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$$

According to definition of equality of matrix

$\Rightarrow \quad a + b = 6 \quad$ and $\quad ab = 8 \quad \Rightarrow \quad b = \frac{8}{a}$

$\Rightarrow \quad a + \frac{8}{a} = 6 \quad \Rightarrow \quad \frac{a^2+8}{a} = 6$

$\Rightarrow \quad a^2 + 8 = 6a \quad \Rightarrow \quad a^2 - 6a + 8 = 0$

$\Rightarrow \quad a^2 - 4a - 2a + 8 = 0$

$\Rightarrow \quad a(a - 4) - 2(a - 4) = 0 \quad \Rightarrow \quad (a - 2)(a - 4) = 0$

$\Rightarrow \quad a = 2, 4 \quad \therefore \quad b = 4, 2$

i.e., if $a = 2$ then $b = 4$ and if $a = 4$ then $b = 2$