Chapter 3 Systems of Linear Equations and Inequalities

Ex 3.7

Answer 1e.

In a 2×2 matrix, the difference of the products of the elements on the diagonals is its determinant. Therefore, the given statement can be completed as "The determinant of a 2×2 matrix is the difference of the products of the elements on the diagonals."

Answer 1gp.

In a 2×2 matrix, the difference of the products of the elements on the diagonals is its determinant.

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - cb$$

Thus,

$$\begin{vmatrix} 3 & -2 \\ 6 & 1 \end{vmatrix} = 3(1) - (-2)(6).$$

Evaluate.

$$\begin{aligned} 3(1) - (-2)(6) &= 3 + 12 \\ &= 15 \end{aligned}$$

Therefore, the determinant of the given matrix is 15.

Answer 2e.

To solve a system of equations 'Cramer's rule' is one of the methods. This rule uses the coefficient matrix.

For a system of two linear equations in two variables, $\begin{matrix} ax + by = e \\ cx + dy = f \end{matrix}$,

let A be the coefficient matrix, $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

If $\det A \neq 0$, then the system has exactly one solution. The solution is:

$$x = \frac{\begin{vmatrix} e & b \\ f & d \end{vmatrix}}{\det A} \text{ and } y = \frac{\begin{vmatrix} a & e \\ c & f \end{vmatrix}}{\det A}$$

The numerators for x and y are the determinants of the matrices formed by replacing the coefficients of x and y, respectively, with the column of constants.

Similarly for solving a system of three linear equations in three variables, we use a 3×3 matrix to solve the system.

Answer 2gp.

The determinant of a 3×3 matrix is the difference of the sum of the product of the elements on the diagonal for each column. That is,

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = (aei + bfg + cdh) - (gec + hfa + idb)$$

$$\begin{vmatrix} 4 & -1 & 2 \\ -3 & -2 & -1 \\ 0 & 5 & 1 \end{vmatrix}\begin{matrix} 4 & -1 \\ -3 & -2 \\ 0 & 5 \end{matrix} = (-8+0-30)-(0-20+3)$$

$$= -38+17$$

$$= -21$$

Therefore, the determinant of the given matrix is $\boxed{-21}$.

Answer 3e.

In a 2×2 matrix, the difference of the products of the elements on the diagonals is its determinant.

$$\begin{vmatrix} 2 & -1 \\ 4 & -5 \end{vmatrix} = 2(-5) - 4(-1)$$

Evaluate.

$$2(-5) - 4(-1) = -10 + 4$$

$$= -6$$

Therefore, the determinant of the given matrix is -6.

Answer 3gp.

The determinant of a 3×3 matrix is defined as

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = (aei + bfg + cdh) - (gec + hfa + idb).$$

Thus,

$$\begin{vmatrix} 10 & -2 & 3 \\ 2 & -12 & 4 \\ 0 & -7 & -2 \end{vmatrix} = [10(-12)(-2) + (-2)(4)(0) + 3(2)(-7)] - [0(-12)(3) + (-7)(4)(10) + (-2)(2)(-2)].$$

Evaluate.

$$\left[10(-12)(-2)+(-2)(4)(0)+3(2)(-7)\right]-\left[0(-12)(3)+(-7)(4)(10)+(-2)(2)(-2)\right]$$
$$=(240+0-42)-(0-280+8)$$
$$=198+272$$
$$=470$$

Therefore, the determinant of the given matrix is 470.

Answer 4e.

The determinant of a 2×2 matrix is the difference of the products of the elements on the diagonals.

$$\begin{vmatrix} 7 & 1 \\ 0 & 3 \end{vmatrix} = 21-0$$
$$=21$$

Therefore, the determinant of the given matrix is $\boxed{21}$.

Answer 4gp.

The area of a triangle with vertices (x_1, y_1), (x_2, y_2), and (x_3, y_3) is given by

$$\text{Area} = \pm\frac{1}{2}\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$$

where the symbol $\pm$ indicates that the appropriate sign should be chosen to yield a positive value.

The given coordinates of the vertices of the triangle are $A(5,11)$, $B(9,2)$, and $C(1,3)$. So the area of the triangle is

$$\text{Area} = \pm\frac{1}{2}\begin{vmatrix} 5 & 11 & 1 \\ 9 & 2 & 1 \\ 1 & 3 & 1 \end{vmatrix}$$
$$=\pm\frac{1}{2}\begin{vmatrix} 5 & 11 & 1 \\ 9 & 2 & 1 \\ 1 & 3 & 1 \end{vmatrix}\begin{matrix} 5 & 11 \\ 9 & 2 \\ 1 & 3 \end{matrix}$$
$$=\pm\frac{1}{2}\left[(10+11+27)-(2+15+99)\right]$$
$$=\pm\frac{1}{2}(48-116)$$
$$=\pm\frac{1}{2}(-68)$$
$$=34$$

Therefore, the area of the triangle is $\boxed{34}$.

Answer 5e.

In a 2×2 matrix, the difference of the products of the elements on the diagonals is its determinant.

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - cb$$

Thus,

$$\begin{vmatrix} -4 & 3 \\ 1 & -7 \end{vmatrix} = -4(-7) - 1(3).$$

Evaluate.

$$\begin{aligned} -4(-7) - 1(3) &= 28 - 3 \\ &= 25 \end{aligned}$$

Therefore, the determinant of the given matrix is 25.

Answer 5gp.

We know that the coefficient matrix of the linear system $\begin{aligned} ax + by &= e \\ cx + dy &= f \end{aligned}$ is $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$. The coefficient matrix of the given linear system is $\begin{bmatrix} 3 & -4 \\ 2 & 5 \end{bmatrix}$.

STEP 1 Evaluate the determinant of the coefficient matrix.

In a 2×2 matrix, the difference of the products of the elements on the diagonals is its determinant.

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - cb$$

Thus,

$$\begin{vmatrix} 3 & -4 \\ 2 & 5 \end{vmatrix} = 3(5) - (-4)(2).$$

Evaluate.

$$\begin{aligned} 3(5) - (-4)(2) &= 15 + 8 \\ &= 23 \end{aligned}$$

STEP 2 Since the determinant is not 0, we can apply Cramer's rule.

If $\det A \neq 0$, then the system has exactly one solution. The solution is

$$x = \frac{\begin{vmatrix} e & b \\ f & d \end{vmatrix}}{\det A} \text{ and } y = \frac{\begin{vmatrix} a & e \\ c & f \end{vmatrix}}{\det A}.$$

Substitute the known values in $x = \frac{\begin{vmatrix} e & b \\ f & d \end{vmatrix}}{\det A}$ and evaluate.

$$\begin{aligned} x &= \frac{\begin{vmatrix} -15 & -4 \\ 13 & 5 \end{vmatrix}}{23} \\ &= \frac{(-15)(5) - 13(-4)}{23} \\ &= -1 \end{aligned}$$

Substitute the known values in $y = \frac{\begin{vmatrix} a & e \\ c & f \end{vmatrix}}{\det A}$ and evaluate.

$$\begin{aligned} y &= \frac{\begin{vmatrix} 3 & -15 \\ 2 & 13 \end{vmatrix}}{23} \\ &= \frac{3(13) - (-15)(2)}{23} \\ &= 3 \end{aligned}$$

CHECK

Replace x with -1, and y with 3 in the original equation and evaluate.

$$\begin{aligned} 3x - 4y &= -15 \\ 3(-1) - 4(3) &\stackrel{?}{=} -15 \\ -3 - 12 &\stackrel{?}{=} -15 \\ -15 &= -15 \ \checkmark \end{aligned} \qquad \begin{aligned} 2x + 5y &= 13 \\ 2(-1) + 5(3) &\stackrel{?}{=} 13 \\ -2 + 15 &\stackrel{?}{=} 13 \\ 13 &= 13 \ \checkmark \end{aligned}$$

Therefore, the solution is (–1, 3).

Answer 6e.

The determinant of a 2×2 matrix is the difference of the products of the elements on the diagonals.

$$\begin{aligned}\begin{vmatrix}1 & -3\\2 & 6\end{vmatrix} &= 6-(-6)\\ &= 12\end{aligned}$$

Therefore, the determinant of the given matrix is $\boxed{12}$.

Answer 6gp.

To solve the given linear system using Cramer's rule, we have to find the determinant of the coefficient matrix.

$$\begin{aligned}4x+7y&=2\\-3x-2y&=-8\end{aligned}$$

The determinant of the coefficient matrix is

$$\begin{aligned}\begin{vmatrix}4 & 7\\-3 & -2\end{vmatrix} &= -8-(-21)\\ &= 13\end{aligned}$$

Since the determinant is not 0, apply Cramer's rule to find x and y

$$\begin{aligned}x &= \frac{\begin{vmatrix}2 & 7\\-8 & -2\end{vmatrix}}{13}\\ &= \frac{-4-(-56)}{13}\\ &= \frac{52}{13}\\ &= 4\end{aligned}$$

$$\begin{aligned}y &= \frac{\begin{vmatrix}4 & 2\\-3 & -8\end{vmatrix}}{13}\\ &= \frac{-32-(-6)}{13}\\ &= \frac{-26}{13}\\ &= -2\end{aligned}$$

Therefore, the solution is $\boxed{(4,-2)}$.

Answer 7e.

In a 2×2 matrix, the difference of the products of the elements on the diagonals is its determinant.

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - cb$$

Thus,

$$\begin{vmatrix} 10 & -6 \\ -7 & 5 \end{vmatrix} = 10(5) - (-7)(-6).$$

Evaluate.

$$\begin{aligned} 10(5) - (-7)(-6) &= 50 - 42 \\ &= 8 \end{aligned}$$

Therefore, the determinant of the given matrix is 8.

Answer 7gp.

We know that the coefficient matrix of the linear system $\begin{aligned} ax + by + cz &= j \\ dx + ey + fz &= k \\ gx + hy + iz &= l \end{aligned}$ is

$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$. The coefficient matrix of the given linear system is $\begin{bmatrix} 3 & -4 & 2 \\ 4 & 1 & -5 \\ 2 & -3 & 1 \end{bmatrix}$.

STEP 1 Evaluate the determinant of the coefficient matrix.

The determinant of a 3×3 matrix is defined as

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = (aei + bfg + cdh) - (gec + hfa + idb).$$

Thus,

$$\begin{vmatrix} 3 & -4 & 2 \\ 4 & 1 & -5 \\ 2 & -3 & 1 \end{vmatrix} = [3 + 40 - 24] - [4 + 45 - 16].$$

Evaluate.

$$\begin{aligned} [3 + 40 - 24] - [4 + 45 - 16] &= 19 - 33 \\ &= -14 \end{aligned}$$

STEP 2 Since the determinant is not 0, we can apply Cramer's rule.
If $\det A \neq 0$, then the system has exactly one solution. The solution is

$$x = \frac{\begin{vmatrix} j & b & c \\ k & e & f \\ l & h & i \end{vmatrix}}{\det A},\ y = \frac{\begin{vmatrix} a & j & c \\ d & k & f \\ g & l & i \end{vmatrix}}{\det A},\ \text{and } z = \frac{\begin{vmatrix} a & b & j \\ d & e & k \\ g & h & l \end{vmatrix}}{\det A}.$$

Substitute the known values in $x = \frac{\begin{vmatrix} j & b & c \\ k & e & f \\ l & h & i \end{vmatrix}}{\det A}$ and evaluate.

$$\begin{aligned} x &= \frac{\begin{vmatrix} 18 & -4 & 2 \\ -13 & 1 & -5 \\ 11 & -3 & 1 \end{vmatrix}}{-14} \\ &= \frac{[18 + 220 + 78] - [22 + 270 + 52]}{-14} \\ &= \frac{316 - 344}{-14} \\ &= 2 \end{aligned}$$

Substitute the known values in $y = \frac{\begin{vmatrix} a & j & c \\ d & k & f \\ g & l & i \end{vmatrix}}{\det A}$ and evaluate.

$$\begin{aligned} y &= \frac{\begin{vmatrix} 3 & 18 & 2 \\ 4 & -13 & -5 \\ 2 & 11 & 1 \end{vmatrix}}{-14} \\ &= \frac{[(-39) + (-180) + 88] - [(-52) + (-165) + 72]}{-14} \\ &= \frac{-131 + 145}{-14} \\ &= -1 \end{aligned}$$

Substitute the known values in $z = \dfrac{\begin{vmatrix} a & b & j \\ d & e & k \\ g & h & l \end{vmatrix}}{\det A}$ and evaluate.

$$z = \frac{\begin{vmatrix} 3 & -4 & 18 \\ 4 & 1 & -13 \\ 2 & -3 & 11 \end{vmatrix}}{-14}$$

$$= \frac{[33 + 104 + (-216)] - [36 + 117 + (-176)]}{-14}$$

$$= \frac{-79 + 23}{-14}$$

$$= 4$$

CHECK

Replace x with 2, y with -1, and z with 4 in the original equation and evaluate.

$$\begin{aligned} 3x - 4y + 2z &= 18 \\ 3(2) - 4(-1) + 2(4) &\stackrel{?}{=} 18 \\ 6 + 4 + 8 &\stackrel{?}{=} 18 \\ 18 &= 18 \quad \checkmark \end{aligned}$$

$$\begin{aligned} 4x + y - 5z &= -13 \\ 4(2) + (-1) - 5(4) &\stackrel{?}{=} -13 \\ 8 - 1 - 20 &\stackrel{?}{=} -13 \\ -13 &= -13 \quad \checkmark \end{aligned}$$

$$\begin{aligned} 2x - 3y + z &= 11 \\ 2(2) - 3(-1) + 4 &\stackrel{?}{=} 11 \\ 4 + 3 + 4 &\stackrel{?}{=} 11 \\ 11 &= 11 \quad \checkmark \end{aligned}$$

Therefore, the solution is $(2, -1, 4)$.

Answer 8e.

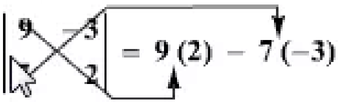
The determinant of a 2×2 matrix is the difference of the products of the elements on the diagonals.

$$\begin{vmatrix} 0 & 3 \\ 5 & -3 \end{vmatrix} = 0 - 15$$
$$= -15$$

Therefore, the determinant of the given matrix is $\boxed{-15}$.

Answer 9e.

In a 2×2 matrix, the difference of the products of the elements on the diagonals is its determinant.



Evaluate.

$$9(2) - 7(-3) = 18 + 21$$
$$= 39$$

Therefore, the determinant of the given matrix is 39.

Answer 10e.

The determinant of a 2×2 matrix is the difference of the products of the elements on the diagonals.

$$\begin{vmatrix} -5 & 12 \\ 4 & 6 \end{vmatrix} = -30 - 48$$
$$= -78$$

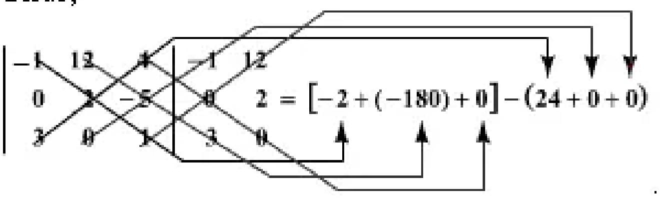
Therefore, the determinant of the given matrix is $\boxed{-78}$.

Answer 11e.

The determinant of a 3×3 matrix is defined as

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = (aei + bfg + cdh) - (gec + hfa + idb).$$

Thus,



Evaluate.

$$[-2 + (-180) + 0] - (24 + 0 + 0) = -182 - 24$$
$$= -206$$

Therefore, the determinant of the given matrix is −206.

Answer 12e.

The determinant of a 3×3 matrix is the difference of the sum of the product of the elements on the diagonal for each column. That is,

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = (aei + bfg + cdh) - (gec + hfa + idb)$$

$$\begin{vmatrix} 1 & 2 & 3 \\ 5 & -8 & 1 \\ 2 & 4 & 3 \end{vmatrix} \begin{matrix} 1 & 2 \\ 5 & -8 \\ 2 & 4 \end{matrix} = (-24+4+60)-(-48+4+30)$$

$$= -40+16$$

$$= -24$$

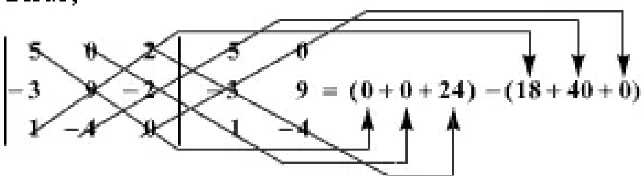
Therefore, the determinant of the given matrix is $\boxed{-24}$.

Answer 13e.

The determinant of a 3×3 matrix is defined as

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = (aei + bfg + cdh) - (gec + hfa + idb).$$

Thus,



Evaluate.

$$[0 + 0 + 24] - (18 + 40 + 0) = 24 - 58$$

$$= -34$$

Therefore, the determinant of the given matrix is -34.

Answer 14e.

The determinant of a 3×3 matrix is the difference of the sum of the product of the elements on the diagonal for each column. That is,

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = (aei + bfg + cdh) - (gec + hfa + idb)$$

$$\begin{vmatrix} -7 & 4 & 5 \\ 1 & 2 & -4 \\ -10 & 1 & 6 \end{vmatrix} \begin{matrix} -7 & 4 \\ 1 & 2 \\ -10 & 1 \end{matrix} = (-84+160+5)-(-100+28+24)$$

$$= 81+48$$

$$= 129$$

Therefore, the determinant of the given matrix is $\boxed{129}$.

Answer 15e.

The determinant of a 3×3 matrix is defined as

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = (aei + bfg + cdh) - (gec + hfa + idb).$$

Thus,

$$\begin{vmatrix} 12 & 5 & 8 \\ 0 & 6 & -8 \\ 1 & 10 & 4 \end{vmatrix} = [12(6)(4) + 5(-8)(1) + 8(0)(10)] - [1(6)(8) + 10(-8)(12) + 4(0)(5)].$$

Evaluate.

$$\begin{aligned} &[12(6)(4) + 5(-8)(1) + 8(0)(10)] - [1(6)(8) + 10(-8)(12) + 4(0)(5)] \\ &\quad = [288 + (-40) + 0] - [48 + (-960) + 0] \\ &\quad = 248 + 912 \\ &\quad = 1160 \end{aligned}$$

Therefore, the determinant of the given matrix is 1160.

Answer 16e.

The determinant of a 3×3 matrix is the difference of the sum of the product of the elements on the diagonal for each column. That is,

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = (aei + bfg + cdh) - (gec + hfa + idb)$$

$$\begin{vmatrix} -4 & 3 & -9 \\ 12 & 6 & 0 \\ 8 & -12 & 0 \end{vmatrix} \begin{matrix} -4 & 3 \\ 12 & 6 \\ 8 & -12 \end{matrix} = (0+0+1296)-(-432+0+0)$$

$$= 1296 + 432$$

$$= 1728$$

Therefore, the determinant of the given matrix is $\boxed{1728}$.

Answer 17e.

The determinant of a 3×3 matrix is defined as

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = (aei + bfg + cdh) - (gec + hfa + idb).$$

Thus,

$$\begin{vmatrix} -2 & 6 & 0 \\ 8 & 15 & 3 \\ 4 & -1 & 7 \end{vmatrix} = [-2(15)(7) + 6(4)(3) + 0(8)(-1)] - [4(15)(0) + (-1)(3)(-2) + 7(8)(6)].$$

Evaluate.

$$\left[-2(15)(7)+6(4)(3)+0(8)(-1)\right]-\left[4(15)(0)+(-1)(3)(-2)+7(8)(6)\right]$$
$$=(-210+72+0)-(0+6+336)$$
$$=-138-342$$
$$=-480$$

Therefore, the determinant of the given matrix is –480.

Answer 18e.

The determinant of a 3×3 matrix is the difference of the sum of the product of the elements on the diagonal for each column. That is,

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}=(aei+bfg+cdh)-(gec+hfa+idb)$$

$$\begin{vmatrix} 5 & 7 & 6 \\ -4 & 0 & 8 \\ 1 & 8 & 7 \end{vmatrix}\begin{matrix} 5 & 7 \\ -4 & 0 \\ 1 & 8 \end{matrix}=(0+56-192)-(0+320-196)$$
$$=-136-124$$
$$=-260$$

Therefore, the determinant of the given matrix is $\boxed{-260}$.

Answer 19e.

The sum of the products for the diagonals that go up should be subtracted from the sum of the products for the diagonals that goes down. But the values in the diagonal that go up are added to the values along the diagonal that go down. The error is that addition is used instead of subtraction.

The determinant of a 3×3 matrix is defined as

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}=(aei+bfg+cdh)-(gec+hfa+idb).$$

Thus,

$$\begin{vmatrix} 2 & 0 & -1 \\ 4 & 1 & 6 \\ -3 & 2 & 5 \end{vmatrix}=\left[10+0+(-8)\right]-(3+24+0).$$

Evaluate.

$$\left[10+0+(-8)\right]-(3+24+0) = 2-27$$
$$= -25$$

Therefore, the determinant of the given matrix is −25.

Answer 20e.

The determinant of a 3×3 matrix is the difference of the sum of the product of the elements on the diagonal for each column.

That is, we should repeat the first two columns to the right of the determinant. But in the given problem, the first two columns are repeated to the left of the determinant. This will give a wrong determinant of the matrix.

Answer 21e.

First, we have to find the determinant of the matrix in choice **A**.
In a 2×2 matrix, the difference of the products of the elements on the diagonals is its determinant.

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - cb$$

Thus,

$$\begin{vmatrix} -4 & 1 \\ 6 & 3 \end{vmatrix} = -4(3)-6(1).$$

Evaluate.

$$-4(3)-6(1) = -12-6$$
$$= -18$$

The determinant of the matrix in choice **A** is −18.

Now, find the determinant of the matrix in choice **B**.

$$\begin{vmatrix} 1 & 6 \\ 3 & 8 \end{vmatrix} = 1(8)-3(6)$$
$$= 8-18$$
$$= -10$$

The determinant is −10.

Find the determinant of the matrix in choice **C**.

$$\begin{vmatrix} 5 & -3 \\ 7 & -1 \end{vmatrix} = 5(-1) - 7(-3)$$

$$= -5 + 21$$

$$= 16$$

Finally, find the determinant of the matrix in choice **D**.

$$\begin{vmatrix} 5 & -2 \\ 1 & 5 \end{vmatrix} = 5(5) - 1(-2)$$

$$= 25 + 2$$

$$= 27$$

On comparing, we can see that the matrix $\begin{bmatrix} 5 & -2 \\ 1 & 5 \end{bmatrix}$ has the greatest determinant.

Therefore, the correct answer is choice **C**.

Answer 22e.

The area of a triangle with vertices (x_1, y_1), (x_2, y_2), and (x_3, y_3) is given by

$$\text{Area} = \pm\frac{1}{2}\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$$

where the symbol $\pm$ indicates that the appropriate sign should be chosen to yield a positive value.

The given coordinates of the vertices of the triangle are $A(1,5)$, $B(4,6)$, and $C(7,3)$. So the area of the triangle is

$$\text{Area} = \pm\frac{1}{2}\begin{vmatrix} 1 & 5 & 1 \\ 4 & 6 & 1 \\ 7 & 3 & 1 \end{vmatrix}$$

$$= \pm\frac{1}{2}\begin{vmatrix} 1 & 5 & 1 \\ 4 & 6 & 1 \\ 7 & 3 & 1 \end{vmatrix}\begin{matrix} 1 & 5 \\ 4 & 6 \\ 7 & 3 \end{matrix}$$

$$= \pm\frac{1}{2}[(6+35+12)-(42+3+20)]$$

$$= \pm\frac{1}{2}(53-65)$$

$$= \pm\frac{1}{2}(-12)$$

$$= 6$$

Therefore, the area of the triangle is $\boxed{6}$.

Answer 23e.

The area of a triangle with vertices (x_1, y_1), (x_2, y_2), and (x_3, y_3) is

$$\text{Area} = \pm\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

where the symbol $\pm$ indicates that the appropriate sign should be chosen to yield a positive value.

The area of the triangle with vertices (4, 2), (4, 8), and (8, 5) is

$$\text{Area} = \pm\frac{1}{2}\begin{vmatrix} 4 & 2 & 1 \\ 4 & 8 & 1 \\ 8 & 5 & 1 \end{vmatrix}.$$

The determinant of a 3×3 matrix is defined as

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = (aei + bfg + cdh) - (gec + hfa + idb).$$

Thus,

$$\pm\frac{1}{2}\begin{vmatrix} 4 & 2 & 1 \\ 4 & 8 & 1 \\ 8 & 5 & 1 \end{vmatrix} = \pm\frac{1}{2}\left[(32 + 16 + 20) - (64 + 20 + 8)\right].$$

Evaluate.

$$\begin{aligned} \pm\frac{1}{2}\left[(32 + 16 + 20) - (64 + 20 + 8)\right] &= \pm\frac{1}{2}(68 - 92) \\ &= \pm\frac{1}{2}(-24) \\ &= \pm(-12) \end{aligned}$$

Since area is always positive, choose the negative sign to yield a positive value.

Area $= -(-12) = 12$.

Therefore, the area of the triangle with the given vertices is 12 square units.

Answer 24e.

The area of a triangle with vertices (x_1, y_1), (x_2, y_2), and (x_3, y_3) is given by

$$\text{Area} = \pm\frac{1}{2}\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$$

where the symbol $\pm$ indicates that the appropriate sign should be chosen to yield a positive value.

The given coordinates of the vertices of the triangle are $A(-4,6)$, $B(0,3)$, and $C(6,6)$. So the area of the triangle is

$$\begin{aligned}
\text{Area} &= \pm\frac{1}{2}\begin{vmatrix} -4 & 6 & 1 \\ 0 & 3 & 1 \\ 6 & 6 & 1 \end{vmatrix} \\
&= \pm\frac{1}{2}\begin{vmatrix} -4 & 6 & 1 \\ 0 & 3 & 1 \\ 6 & 6 & 1 \end{vmatrix}\begin{matrix} -4 & 6 \\ 0 & 3 \\ 6 & 6 \end{matrix} \\
&= \pm\frac{1}{2}\left[(-12+36+0)-(18-24+0)\right] \\
&= \pm\frac{1}{2}(24+6) \\
&= \pm\frac{1}{2}(30) \\
&= 15
\end{aligned}$$

Therefore, the area of the triangle is $\boxed{15}$.

Answer 25e.

The area of a triangle with vertices (x_1, y_1), (x_2, y_2), and (x_3, y_3) is

$$\text{Area} = \pm\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}.$$

The symbol $\pm$ indicates that the appropriate sign should be chosen to yield a positive value.

The area of the triangle with vertices $(-4, -4)$, $(-1, 2)$, and $(2, -6)$ is

$$\text{Area} = \pm\frac{1}{2}\begin{vmatrix} -4 & -4 & 1 \\ -1 & 2 & 1 \\ 2 & -6 & 1 \end{vmatrix}.$$

The determinant of a 3×3 matrix is defined as

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = (aei + bfg + cdh) - (gec + hfa + idb).$$

Thus,

$$\pm\frac{1}{2}\begin{vmatrix} -4 & -4 & 1 \\ -1 & 2 & 1 \\ 2 & -6 & 1 \end{vmatrix} = \pm\frac{1}{2}\left[(-8 + (-8) + 6) - (4 + 24 + 4)\right].$$

Evaluate.

$$\begin{aligned} \pm\frac{1}{2}\left[(-8 + (-8) + 6) - (4 + 24 + 4)\right] &= \pm\frac{1}{2}(-10 - 32) \\ &= \pm\frac{1}{2}(-42) \\ &= \pm(-21) \end{aligned}$$

Since area is always positive, choose the negative sign to yield a positive value.
Area $= -(-21) = 21$.

Therefore, the area of the triangle with the given vertices is 21 square units.

Answer 26e.

The area of a triangle with vertices (x_1, y_1), (x_2, y_2), and (x_3, y_3) is given by

$$\text{Area} = \pm\frac{1}{2}\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$$

where the symbol $\pm$ indicates that the appropriate sign should be chosen to yield a positive value.

The given coordinates of the vertices of the triangle are $A(5,-4)$, $B(6,3)$, and $C(8,-1)$. So the area of the triangle is

$$\begin{aligned} \text{Area} &= \pm\frac{1}{2}\begin{vmatrix} 5 & -4 & 1 \\ 6 & 3 & 1 \\ 8 & -1 & 1 \end{vmatrix} \\ &= \pm\frac{1}{2}\begin{vmatrix} 5 & -4 & 1 \\ 6 & 3 & 1 \\ 8 & -1 & 1 \end{vmatrix}\begin{matrix} 5 & -4 \\ 6 & 3 \\ 8 & -1 \end{matrix} \\ &= \pm\frac{1}{2}\left[(15 - 32 - 6) - (24 - 5 - 24)\right] \\ &= \pm\frac{1}{2}(-23 + 5) \\ &= \pm\frac{1}{2}(-18) \\ &= 9 \end{aligned}$$

Therefore, the area of the triangle is $\boxed{9}$.

Answer 27e.

The area of a triangle with vertices (x_1, y_1), (x_2, y_2), and (x_3, y_3) is

$$\text{Area} = \pm\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}.$$

The symbol $\pm$ indicates that the appropriate sign should be chosen to yield a positive value.

The area of the triangle with vertices $(-6, 1)$, $(-2, -6)$, and $(0, 3)$ is

$$\text{Area} = \pm\frac{1}{2}\begin{vmatrix} -6 & 1 & 1 \\ -2 & -6 & 1 \\ 0 & 3 & 1 \end{vmatrix}.$$

The determinant of a 3×3 matrix is defined as

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = (aei + bfg + cdh) - (gec + hfa + idb).$$

Thus,

$$\pm\frac{1}{2}\begin{vmatrix} -6 & 1 & 1 \\ -2 & -6 & 1 \\ 0 & 3 & 1 \end{vmatrix} = \pm\frac{1}{2}\{[36 + 0 + (-6)] - [0 + (-18) + (-2)]\}.$$

Evaluate.

$$\begin{aligned} \pm\frac{1}{2}\{[36 + 0 + (-6)] - [0 + (-18) + (-2)]\} &= \pm\frac{1}{2}(30 + 20) \\ &= \pm\frac{1}{2}(50) \\ &= \pm(25) \end{aligned}$$

Since area is always positive, choose the positive sign to yield a positive value.
Therefore, the area of the triangle with the given vertices is 25 square units.

Answer 28e.

The area of a triangle with vertices (x_1, y_1), (x_2, y_2), and (x_3, y_3) is given by

$$\text{Area} = \pm\frac{1}{2}\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$$

where the symbol $\pm$ indicates that the appropriate sign should be chosen to yield a positive value.

The given coordinates of the vertices of the triangle are $A(-3,4)$, $B(6,3)$, and $C(2,-1)$. So the area of the triangle is

$$\begin{aligned} \text{Area} &= \pm\frac{1}{2}\begin{vmatrix} -3 & 4 & 1 \\ 6 & 3 & 1 \\ 2 & -1 & 1 \end{vmatrix} \\ &= \pm\frac{1}{2}\begin{vmatrix} -3 & 4 & 1 \\ 6 & 3 & 1 \\ 2 & -1 & 1 \end{vmatrix}\begin{matrix} -3 & 4 \\ 6 & 3 \\ 2 & -1 \end{matrix} \\ &= \pm\frac{1}{2}\left[(-9+8-6)-(6+3+24)\right] \\ &= \pm\frac{1}{2}(-7-33) \\ &= \pm\frac{1}{2}(-40) \\ &= 20 \end{aligned}$$

Therefore, the area of the triangle is $\boxed{20}$.

Answer 29e.

We know that the coefficient matrix of the linear system $\begin{aligned} ax + by &= e \\ cx + dy &= f \end{aligned}$ is $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$. The coefficient matrix of the given linear system is $\begin{bmatrix} 3 & 5 \\ -1 & 2 \end{bmatrix}$.

STEP 1 Evaluate the determinant of the coefficient matrix.

In a 2×2 matrix, the difference of the products of the elements on the diagonals is its determinant.

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - cb$$

Thus,

$$\begin{vmatrix} 3 & 5 \\ -1 & 2 \end{vmatrix} = 3(2) - (-1)(5).$$

Evaluate.

$$\begin{aligned} 3(2) - (-1)(5) &= 6 + 5 \\ &= 11 \end{aligned}$$

STEP 2 Since the determinant is not 0, we can apply Cramer's rule.

If $\det A \neq 0$, then the system has exactly one solution. The solution is

$$x = \frac{\begin{vmatrix} e & b \\ f & d \end{vmatrix}}{\det A} \text{ and } y = \frac{\begin{vmatrix} a & e \\ c & f \end{vmatrix}}{\det A}.$$

Substitute the known values in $x = \dfrac{\begin{vmatrix} e & b \\ f & d \end{vmatrix}}{\det A}$ and evaluate.

$$\begin{aligned} x &= \frac{\begin{vmatrix} 3 & 5 \\ 10 & 2 \end{vmatrix}}{11} \\ &= \frac{3(2) - 10(5)}{11} \\ &= -4 \end{aligned}$$

Substitute the known values in $y = \dfrac{\begin{vmatrix} a & e \\ c & f \end{vmatrix}}{\det A}$ and evaluate.

$$\begin{aligned} y &= \frac{\begin{vmatrix} 3 & 3 \\ -1 & 10 \end{vmatrix}}{11} \\ &= \frac{3(10) - (-1)(3)}{11} \\ &= 3 \end{aligned}$$

CHECK

Replace x with −4, and y with 3 in the original equation and evaluate.

$$\begin{aligned} 3x + 5y &= 3 \\ 3(-4) + 5(3) &\stackrel{?}{=} 3 \\ -12 + 15 &\stackrel{?}{=} 3 \\ 3 &= 3 \quad \checkmark \end{aligned} \qquad \begin{aligned} -x + 2y &= 10 \\ -(-4) + 2(3) &\stackrel{?}{=} 10 \\ 4 + 6 &\stackrel{?}{=} 10 \\ 10 &= 10 \quad \checkmark \end{aligned}$$

Therefore, the solution is (−4, 3).

Answer 30e.

To solve the given linear system using Cramer's rule, we have to find the determinant of the coefficient matrix.

$$2x - y = -2$$
$$x + 2y = 14$$

The determinant of the coefficient matrix is

$$\begin{vmatrix} 2 & -1 \\ 1 & 2 \end{vmatrix} = 4 - (-1)$$
$$= 5$$

Since the determinant is not 0, apply Cramer's rule to find x and y

$$x = \frac{\begin{vmatrix} -2 & -1 \\ 14 & 2 \end{vmatrix}}{5}$$
$$= \frac{-4 - (-14)}{5}$$
$$= \frac{10}{5}$$
$$= 2$$

$$y = \frac{\begin{vmatrix} 2 & -2 \\ 1 & 14 \end{vmatrix}}{5}$$
$$= \frac{28 - (-2)}{5}$$
$$= \frac{30}{5}$$
$$= 6$$

Therefore, the solution is $\boxed{(2, 6)}$.

Answer 31e.

We know that the coefficient matrix of the linear system $\begin{aligned} ax + by &= e \\ cx + dy &= f \end{aligned}$ is $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$. The coefficient matrix of the given linear system is $\begin{bmatrix} 5 & 1 \\ 2 & -5 \end{bmatrix}$.

STEP 1 Evaluate the determinant of the coefficient matrix.

In a 2×2 matrix, the difference of the products of the elements on the diagonals is its determinant.

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - cb$$

Thus,

$$\begin{vmatrix} 5 & 1 \\ 2 & -5 \end{vmatrix} = 5(-5) - 1(2).$$

Evaluate.

$$\begin{aligned} 5(-5) - 1(2) &= -25 - 2 \\ &= -27 \end{aligned}$$

STEP 2 Since the determinant is not 0, we can apply Cramer's rule.

If $\det A \neq 0$, then the system has exactly one solution. The solution is

$$x = \frac{\begin{vmatrix} e & b \\ f & d \end{vmatrix}}{\det A} \text{ and } y = \frac{\begin{vmatrix} a & e \\ c & f \end{vmatrix}}{\det A}.$$

Substitute the known values in $x = \frac{\begin{vmatrix} e & b \\ f & d \end{vmatrix}}{\det A}$ and evaluate.

$$\begin{aligned} x &= \frac{\begin{vmatrix} -40 & 1 \\ 11 & -5 \end{vmatrix}}{-27} \\ &= \frac{(-40)(-5) - 1(11)}{-27} \\ &= \frac{200 - 11}{-27} \\ &= -7 \end{aligned}$$

Substitute the known values in $y = \dfrac{\begin{vmatrix} a & e \\ c & f \end{vmatrix}}{\det A}$ and evaluate.

$$\begin{aligned} y &= \frac{\begin{vmatrix} 5 & -40 \\ 2 & 11 \end{vmatrix}}{-27} \\ &= \frac{5(11)-(-40)(2)}{-27} \\ &= \frac{55+80}{-27} \\ &= -5 \end{aligned}$$

CHECK

Replace x with -7, and y with -5 in the original equation and evaluate.

$$\begin{aligned} 5x + y &= -40 \\ 5(-7)+(-5) &\stackrel{?}{=} -40 \\ -35-5 &\stackrel{?}{=} -40 \\ -40 &= -40 \quad \checkmark \end{aligned} \qquad \begin{aligned} 2x - 5y &= 11 \\ 2(-7)-5(-5) &\stackrel{?}{=} 11 \\ -14+25 &\stackrel{?}{=} 11 \\ 11 &= 11 \quad \checkmark \end{aligned}$$

Therefore, the solution is $(-7, -5)$.

Answer 32e.

To solve the given linear system using Cramer's rule, we have to find the determinant of the coefficient matrix.

$$\begin{aligned} -x+y+z &= -3 \\ 4x-y+4z &= -14 \\ x+2y-z &= 9 \end{aligned}$$

The determinant of the coefficient matrix $A = \begin{pmatrix} -1 & 1 & 1 \\ 4 & -1 & 4 \\ 1 & 2 & -1 \end{pmatrix}$ is

$$\begin{aligned} \det A &= \begin{vmatrix} -1 & 1 & 1 \\ 4 & -1 & 4 \\ 1 & 2 & -1 \end{vmatrix}\begin{matrix} -1 & 1 \\ 4 & -1 \\ 1 & 2 \end{matrix} \\ &= (-1+4+8)-(-1-8-4) \\ &= 11+13 \\ &= 24 \end{aligned}$$

Since the determinant is not 0, let us apply Cramer's rule

$$\begin{aligned} x &= \frac{\begin{vmatrix} -3 & 1 & 1 \\ -14 & -1 & 4 \\ 9 & 2 & -1 \end{vmatrix}}{24} \\ &= \frac{(-3+36-28)-(-9-24+14)}{24} \\ &= \frac{5+19}{24} \\ &= 1 \end{aligned}$$

$$y=\frac{\begin{vmatrix}-1 & -3 & 1\\ 4 & -14 & 4\\ 1 & 9 & -1\end{vmatrix}}{24}$$
$$=\frac{(-14-12+36)-(-14-36+12)}{24}$$
$$=\frac{48}{24}$$
$$=2$$

$$z=\frac{\begin{vmatrix}-1 & 1 & -3\\ 4 & -1 & -14\\ 1 & 2 & 9\end{vmatrix}}{24}$$
$$=\frac{(9-14-24)-(3+28+36)}{24}$$
$$=\frac{-96}{24}$$
$$=-4$$

Therefore, the solution is $\boxed{(1,2,-4)}$.

Answer 33e.

We know that the coefficient matrix of the linear system $\begin{aligned} ax+by+cz &= j\\ dx+ey+fz &= k\\ gx+hy+iz &= l\end{aligned}$ is $\begin{bmatrix}a & b & c\\ d & e & f\\ g & h & i\end{bmatrix}$. The coefficient matrix of the given linear system is $\begin{bmatrix}-1 & -2 & 4\\ 1 & 1 & 2\\ 2 & 1 & -3\end{bmatrix}$.

STEP 1 Evaluate the determinant of the coefficient matrix.
The determinant of a 3×3 matrix is defined as

$$\begin{vmatrix}a & b & c\\ d & e & f\\ g & h & i\end{vmatrix}=(aei+bfg+cdh)-(gec+hfa+idb).$$

Thus,

$$\begin{vmatrix}-1 & -2 & 4\\ 1 & 1 & 2\\ 2 & 1 & -3\end{vmatrix}=[3+(-8)+4]-[8+(-2)+6].$$

Evaluate.

$$\begin{aligned}[3+(-8)+4]-[8+(-2)+6] &= -1-12\\ &= -13\end{aligned}$$

STEP 2 Since the determinant is not 0, we can apply Cramer's rule.

If $\det A \neq 0$, then the system has exactly one solution. The solution is

$$x = \frac{\begin{vmatrix} j & b & c \\ k & e & f \\ l & h & i \end{vmatrix}}{\det A},\ y = \frac{\begin{vmatrix} a & j & c \\ d & k & f \\ g & l & i \end{vmatrix}}{\det A},\ \text{and } z = \frac{\begin{vmatrix} a & b & j \\ d & e & k \\ g & h & l \end{vmatrix}}{\det A}.$$

Substitute the known values in $x = \frac{\begin{vmatrix} j & b & c \\ k & e & f \\ l & h & i \end{vmatrix}}{\det A}$ and evaluate.

$$\begin{aligned} x &= \frac{\begin{vmatrix} -28 & -2 & 4 \\ -11 & 1 & 2 \\ 30 & 1 & -3 \end{vmatrix}}{-13} \\ &= \frac{\left[84 + (-120) + (-44)\right] - \left[120 + (-56) + (-66)\right]}{-13} \\ &= \frac{-80 + 2}{-13} \\ &= 6 \end{aligned}$$

Substitute the known values in $y = \frac{\begin{vmatrix} a & j & c \\ d & k & f \\ g & l & i \end{vmatrix}}{\det A}$ and evaluate.

$$\begin{aligned} y &= \frac{\begin{vmatrix} -1 & -28 & 4 \\ 1 & -11 & 2 \\ 2 & 30 & -3 \end{vmatrix}}{-13} \\ &= \frac{\left[(-33) + (-112) + 120\right] - \left[(-88) + (-60) + 84\right]}{-13} \\ &= \frac{-25 + 64}{-13} \\ &= -3 \end{aligned}$$

Substitute the known values in $z = \dfrac{\begin{vmatrix} a & b & j \\ d & e & k \\ g & h & l \end{vmatrix}}{\det A}$ and evaluate.

$$
\begin{aligned}
z &= \frac{\begin{vmatrix} -1 & -2 & -28 \\ 1 & 1 & -11 \\ 2 & 1 & 30 \end{vmatrix}}{-13} \\
&= \frac{\left[(-30) + 44 + (-28)\right] - (-56) + 11 + (-60)}{-13} \\
&= \frac{-14 + 105}{-13} \\
&= -7
\end{aligned}
$$

CHECK

Replace x with 6, y with −3, and z with −7 in the original equation and evaluate.

$$
\begin{aligned}
-x - 2y + 4z &= -28 \\
-6 - 2(-3) + 4(-7) &\stackrel{?}{=} -28 \\
-6 + 6 - 28 &\stackrel{?}{=} -28 \\
-28 &= -28 \quad \checkmark
\end{aligned}
$$

$$
\begin{aligned}
x + y + 2z &= -11 \\
-6 + (-3) + 2(-7) &\stackrel{?}{=} -11 \\
-6 - 3 - 25 &\stackrel{?}{=} -11 \\
11 &= -11 \quad \checkmark
\end{aligned}
$$

$$
\begin{aligned}
2x + y - 3z &= 30 \\
2(6) + (-3) - 3(-7) &\stackrel{?}{=} 30 \\
12 - 3 + 21 &\stackrel{?}{=} 30 \\
30 &= 30 \quad \checkmark
\end{aligned}
$$

Therefore, the solution is (6, −3, −7).

Answer 34e.

To solve the given linear system using Cramer's rule, we have to find the determinant of the coefficient matrix.

$$4x+y+3z=7$$
$$2x-5y+4z=-19$$
$$x-y+2z=-2$$

The determinant of the coefficient matrix $A=\begin{pmatrix}4 & 1 & 3\\ 2 & -5 & 4\\ 1 & -1 & 2\end{pmatrix}$ is

$$\det A=\begin{vmatrix}4 & 1 & 3\\ 2 & -5 & 4\\ 1 & -1 & 2\end{vmatrix}\begin{matrix}4 & 1\\ 2 & -5\\ 1 & -1\end{matrix}$$
$$=(-40+4-6)-(-15-16+4)$$
$$=-15$$

Since the determinant is not 0, let us apply Cramer's rule

$$x=\frac{\begin{vmatrix}7 & 1 & 3\\ -19 & -5 & 4\\ -2 & -1 & 2\end{vmatrix}}{-15}$$
$$=\frac{(-70-8+57)-(30-28-38)}{-15}$$
$$=\frac{15}{-15}$$
$$=-1$$

$$y = \frac{\begin{vmatrix} 4 & 7 & 3 \\ 2 & -19 & 4 \\ 1 & -2 & 2 \end{vmatrix}}{-15}$$

$$= \frac{(-152+28-12)-(-57-32+28)}{-15}$$

$$= \frac{-75}{-15}$$

$$= 5$$

$$z = \frac{\begin{vmatrix} 4 & 1 & 7 \\ 2 & -5 & -19 \\ 1 & -1 & -2 \end{vmatrix}}{-15}$$

$$= \frac{(40-19-14)-(-35+76-4)}{-15}$$

$$= \frac{-30}{-15}$$

$$= 2$$

Therefore, the solution is $\boxed{(-1,5,2)}$.

Answer 35e.

We know that the coefficient matrix of the linear system $\begin{aligned} ax + by + cz &= j \\ dx + ey + fz &= k \\ gx + hy + iz &= l \end{aligned}$ is $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$. The coefficient matrix of the given linear system is $\begin{bmatrix} 5 & -1 & -2 \\ 1 & 3 & 4 \\ 2 & -4 & 1 \end{bmatrix}$.

STEP 1 Evaluate the determinant of the coefficient matrix.

The determinant of a 3×3 matrix is defined as

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = (aei + bfg + cdh) - (gec + hfa + idb).$$

Thus,

$$\begin{vmatrix} 5 & -1 & -2 \\ 1 & 3 & 4 \\ 2 & -4 & 1 \end{vmatrix} = [15 + (-8) + 8] - [(-12) + (-80) + (-1)].$$

Evaluate.

$$\begin{aligned} [15 + (-8) + 8] - [(-12) + (-80) + (-1)] &= 15 + 93 \\ &= 108 \end{aligned}$$

STEP 2 Since the determinant is not 0, we can apply Cramer's rule.

If $\det A \neq 0$, then the system has exactly one solution. The solution is

$$x = \frac{\begin{vmatrix} j & b & c \\ k & e & f \\ l & h & i \end{vmatrix}}{\det A},\ y = \frac{\begin{vmatrix} a & j & c \\ d & k & f \\ g & l & i \end{vmatrix}}{\det A},\ \text{and}\ z = \frac{\begin{vmatrix} a & b & j \\ d & e & k \\ g & h & l \end{vmatrix}}{\det A}.$$

Substitute the known values in $x = \frac{\begin{vmatrix} j & b & c \\ k & e & f \\ l & h & i \end{vmatrix}}{\det A}$ and evaluate.

$$\begin{aligned} x &= \frac{\begin{vmatrix} -6 & -1 & -2 \\ 16 & 3 & 4 \\ -15 & -4 & 1 \end{vmatrix}}{108} \\ &= \frac{[(-18) + 60 + 128] - [90 + 96 + (-16)]}{108} \\ &= \frac{170 - 170}{108} \\ &= 0 \end{aligned}$$

Substitute the known values in $y = \dfrac{\begin{vmatrix} a & j & c \\ d & k & f \\ g & l & i \end{vmatrix}}{\det A}$ and evaluate.

$$\begin{aligned}
y &= \frac{\begin{vmatrix} 5 & -6 & -2 \\ 1 & 16 & 4 \\ 2 & -15 & 1 \end{vmatrix}}{108} \\
&= \frac{[80 + (-48) + 30] - [(-64) + (-300) + (-6)]}{108} \\
&= \frac{62 + 370}{108} \\
&= 4
\end{aligned}$$

Substitute the known values in $z = \dfrac{\begin{vmatrix} a & b & j \\ d & e & k \\ g & h & l \end{vmatrix}}{\det A}$ and evaluate.

$$\begin{aligned}
z &= \frac{\begin{vmatrix} 5 & -1 & -6 \\ 1 & 3 & 16 \\ 2 & -4 & -15 \end{vmatrix}}{108} \\
&= \frac{[(-225) + (-32) + 24] - [(-36) + (-320) + 15]}{108} \\
&= \frac{-233 + 341}{108} \\
&= 1
\end{aligned}$$

CHECK

Replace x with 0, y with 4, and z with 1 in the original equation and evaluate.

$$\begin{aligned}
5x - y - 2z &= -6 \\
5(0) - 4 - 2(1) &\stackrel{?}{=} -6 \\
0 - 4 - 2 &\stackrel{?}{=} -6 \\
-6 &= -6 \quad \checkmark
\end{aligned}$$

$$\begin{aligned}
x + 3y + 4z &= 16 \\
0 + 3(4) + 4(1) &\stackrel{?}{=} 16 \\
12 + 4 &\stackrel{?}{=} 16 \\
16 &= 16 \quad \checkmark
\end{aligned}$$

$$\begin{aligned}
2x - 4y + z &= -15 \\
2(0) - 4(4) + 1 &\stackrel{?}{=} -15 \\
0 - 16 + 1 &\stackrel{?}{=} -15 \\
-15 &= -15 \quad \checkmark
\end{aligned}$$

Therefore, the solution is (0, 4, 1).

Answer 36e.

To solve the given linear system using Cramer's rule, we have to find the determinant of the coefficient matrix.

$$x + y + z = -8$$
$$3x - 3y + 2z = -21$$
$$-x + 2y - 2z = 11$$

The determinant of the coefficient matrix $A = \begin{pmatrix} 1 & 1 & 1 \\ 3 & -3 & 2 \\ -1 & 2 & -2 \end{pmatrix}$ is

$$\det A = \left|\begin{array}{ccc|cc} 1 & 1 & 1 & 1 & 1 \\ 3 & -3 & 2 & 3 & -3 \\ -1 & 2 & -2 & -1 & 2 \end{array}\right|$$
$$= (6 - 2 + 6) - (3 + 4 - 6)$$
$$= 9$$

Since the determinant is not 0, let us apply Cramer's rule

$$x = \frac{\begin{vmatrix} -8 & 1 & 1 \\ -21 & -3 & 2 \\ 11 & 2 & -2 \end{vmatrix}}{9}$$
$$= \frac{(-48 + 22 - 42) - (-33 - 32 + 42)}{9}$$
$$= \frac{-45}{9}$$
$$= -5$$

$$y=\frac{\begin{vmatrix}1 & -8 & 1\\ 3 & -21 & 2\\ -1 & 11 & -2\end{vmatrix}}{9}$$

$$=\frac{(42+16+33)-(21+22+48)}{9}$$

$$=\frac{0}{9}$$

$$=0$$

$$z=\frac{\begin{vmatrix}1 & 1 & -8\\ 3 & -3 & -21\\ -1 & 2 & 11\end{vmatrix}}{9}$$

$$=\frac{(-33+21-48)-(-24-42+33)}{9}$$

$$=\frac{-27}{9}$$

$$=-3$$

Therefore, the solution is $\boxed{(-5,0,-3)}$.

Answer 37e.

We know that the coefficient matrix of the linear system $\begin{aligned} ax+by+cz &= j\\ dx+ey+fz &= k\\ gx+hy+iz &= l\end{aligned}$ is $\begin{bmatrix}a & b & c\\ d & e & f\\ g & h & i\end{bmatrix}$. The coefficient matrix of the given linear system is $\begin{bmatrix}3 & -1 & 1\\ -1 & 2 & -3\\ 1 & 1 & 1\end{bmatrix}$.

STEP 1 Evaluate the determinant of the coefficient matrix.
The determinant of a 3×3 matrix is defined as

$$\begin{vmatrix}a & b & c\\ d & e & f\\ g & h & i\end{vmatrix}=(aei+bfg+cdh)-(gec+hfa+idb).$$

Thus,

$$\begin{vmatrix}3 & -1 & 1\\ -1 & 2 & -3\\ 1 & 1 & 1\end{vmatrix}=[6+3+(-1)]-[2+(-9)+1].$$

Evaluate.

$$\begin{aligned}[6+3+(-1)]-[2+(-9)+1] &= 8+6\\ &= 14\end{aligned}$$

STEP 2 Since the determinant is not 0, we can apply Cramer's rule.
If $\det A \neq 0$, then the system has exactly one solution. The solution is

$$x = \frac{\begin{vmatrix} j & b & c \\ k & e & f \\ l & h & i \end{vmatrix}}{\det A},\ y = \frac{\begin{vmatrix} a & j & c \\ d & k & f \\ g & l & i \end{vmatrix}}{\det A},\ \text{and } z = \frac{\begin{vmatrix} a & b & j \\ d & e & k \\ g & h & l \end{vmatrix}}{\det A}.$$

Substitute the known values in $x = \frac{\begin{vmatrix} j & b & c \\ k & e & f \\ l & h & i \end{vmatrix}}{\det A}$ and evaluate.

$$\begin{aligned} x &= \frac{\begin{vmatrix} 25 & 1 & 1 \\ -17 & 2 & -3 \\ 21 & 1 & 1 \end{vmatrix}}{14} \\ &= \frac{[50 + 63 + (-17)] - [42 + (-75) + 17]}{14} \\ &= \frac{96 + 16}{14} \\ &= 8 \end{aligned}$$

Substitute the known values in $y = \frac{\begin{vmatrix} a & j & c \\ d & k & f \\ g & l & i \end{vmatrix}}{\det A}$ and evaluate.

$$\begin{aligned} y &= \frac{\begin{vmatrix} 3 & 25 & 1 \\ -1 & -17 & -3 \\ 1 & 21 & 1 \end{vmatrix}}{14} \\ &= \frac{[(-51) + (-75) + (-21)] - [(-17) + (-189) + (-25)]}{14} \\ &= \frac{-147 + 231}{14} \\ &= 6 \end{aligned}$$

Substitute the known values in $z = \dfrac{\begin{vmatrix} a & b & j \\ d & e & k \\ g & h & l \end{vmatrix}}{\det A}$ and evaluate.

$$z = \frac{\begin{vmatrix} 3 & -1 & 25 \\ -1 & 2 & -17 \\ 1 & 1 & 21 \end{vmatrix}}{14}$$

$$= \frac{[126 + 17 + (-25)] - [50 + (-51) + 21]}{14}$$

$$= \frac{118 - 20}{14}$$

$$= 7$$

CHECK

Replace x with 8, y with 6, and z with 7 in the original equation and evaluate.

$$\begin{aligned} 3x - y + z &= 25 \\ 3(8) - 6 + 7 &\stackrel{?}{=} 25 \\ 24 - 6 + 7 &\stackrel{?}{=} 25 \\ 25 &= 25 \quad \checkmark \end{aligned}$$

$$\begin{aligned} -x + 2y - 3z &= -17 \\ -8 + 2(6) - 3(7) &\stackrel{?}{=} -17 \\ -8 + 12 - 21 &\stackrel{?}{=} -17 \\ -17 &= -17 \quad \checkmark \end{aligned}$$

$$\begin{aligned} x + y + z &= 21 \\ 8 + 6 + 7 &\stackrel{?}{=} 21 \\ 21 &= 21 \quad \checkmark \end{aligned}$$

Therefore, the solution is (8, 6, 7).

Answer 38e.

Given that the determinant of a 2×2 matrix is 5.

So, any matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ with the condition $ad - bc = 5$ is the required matrix

For instance, if $A = \begin{pmatrix} 2 & 1 \\ 5 & 5 \end{pmatrix}$ then $\det A = 5$.

Answer 39e.

a. First, we need to find the AB.
Since the dimension of the first matrix is 2×2 and that of the second matrix is 2×2, the product of the matrices is defined and it is a 2×2 matrix.

$$AB = \begin{bmatrix} 2(3) + (-1)(-2) & 2(5) + (-1)(-4) \\ 1(3) + 2(-2) & 1(5) + 2(-4) \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 14 \\ -1 & -3 \end{bmatrix}$$

Now, find det AB.
In a 2×2 matrix, the difference of the products of the elements on the diagonals is its determinant.

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - cb$$

Thus,

$$\begin{vmatrix} 8 & 14 \\ -1 & -3 \end{vmatrix} = 8(-3)-(-1)(14).$$

Evaluate.

$$\begin{aligned} 8(-3)-(-1)(14) &= -24+14 \\ &= -10 \end{aligned}$$

The determinant of AB is -10.

Similarly, we can find the determinant of A and B.
$\det A = 5$
$\det B = -2$

We can see that the value of det AB is the product of det A and det B. Therefore, $\det AB = (\det A)(\det B)$.

b. We have $\det A = 5$

Now, multiply each element of matrix A by k to find det kA.

$$\begin{aligned} kA &= k\begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix} \\ &= \begin{bmatrix} 2k & -k \\ k & 2k \end{bmatrix} \end{aligned}$$

Evaluate the determinant of the matrix kA.

$$\begin{aligned} \begin{vmatrix} 2k & -k \\ k & 2k \end{vmatrix} &= 2k(2k)-k(-k) \\ &= 4k^2+k^2 \\ &= 5k^2 \end{aligned}$$

We note that the value of det kA is the product of det A and k^2. Therefore, $\det kA = k^2 \det A$.

Answer 40e.

The area of a triangle with vertices (x_1, y_1), (x_2, y_2), and (x_3, y_3) is given by

$$\text{Area} = \pm\frac{1}{2}\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$$

where the symbol $\pm$ indicates that the appropriate sign should be chosen to yield a positive value.

The given coordinates of the vertices of the Bermuda triangle are $A(0,0)$, $B(900,-518)$, and $C(938,454)$. So the area of the Bermuda triangle is

$$\begin{aligned}\text{Area} &= \pm\frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ 900 & -518 & 1 \\ 938 & 454 & 1 \end{vmatrix} \\ &= \pm\frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ 900 & -518 & 1 \\ 938 & 454 & 1 \end{vmatrix}\begin{matrix} 0 & 0 \\ 900 & -518 \\ 938 & 454 \end{matrix} \\ &= \pm\frac{1}{2}\left[(0+0+408600)-(-485884+0+0)\right] \\ &= \pm\frac{1}{2}(894484) \\ &= 447242\end{aligned}$$

Therefore, the area of the Bermuda triangle is $\boxed{447242 \text{ sq miles}}$.

Answer 41e.

The area of a triangle with vertices (x_1, y_1), (x_2, y_2), and (x_3, y_3) is

$$\text{Area} = \pm\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

where the symbol $\pm$ indicates that the appropriate sign should be chosen to yield a positive value.

The area of the triangle with vertices (0, 0), (5, 2), and (3, 6) is

$$\text{Area} = \pm\frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ 5 & 2 & 1 \\ 3 & 6 & 1 \end{vmatrix}.$$

The determinant of a 3×3 matrix is defined as

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = (aei + bfg + cdh) - (gec + hfa + idb).$$

Thus,

$$\pm\frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ 5 & 2 & 1 \\ 3 & 6 & 1 \end{vmatrix} = \pm\frac{1}{2}\left[(0+0+30)-(6+0+0)\right].$$

Evaluate.

$$\begin{aligned} \pm\frac{1}{2}\left[(0+0+30)-(6+0+0)\right] &= \pm\frac{1}{2}(30-6) \\ &= \pm\frac{1}{2}(24) \\ &= \pm(12) \end{aligned}$$

Since area is always positive, choose the negative sign to yield a positive value.
Area = 12.

Therefore, the area of the triangular region is 12 square units.

Answer 42e.

Let x = number of people for floor seats
y = number of people for other seats

According to the given data, the total number of people is 6700, that is

$x + y = 6700$ ......(1)

Since the cost of each seat is given and the total income is \$185,500, we have

$40x + 25y = 185500$ ......(2)

Equation 1 and 2 form a system of linear equations

Using Cramer's rule let us solve the equations.

The determinant of the coefficient matrix is

$$\begin{vmatrix} 1 & 1 \\ 40 & 25 \end{vmatrix} = 25 - 40 = -15$$

Since the determinant is not 0, apply Cramer's rule

$$\begin{aligned} x &= \frac{\begin{vmatrix} 6700 & 1 \\ 185500 & 25 \end{vmatrix}}{-15} \\ &= \frac{167500 - 185500}{-15} \\ &= \frac{-18000}{-15} \\ &= 1200 \end{aligned}$$

$$\begin{aligned} y &= \frac{\begin{vmatrix} 1 & 6700 \\ 40 & 185500 \end{vmatrix}}{-15} \\ &= \frac{185500 - 268000}{-15} \\ &= \frac{-82500}{-15} \\ &= 5500 \end{aligned}$$

The solution is $\boxed{(1200, 5500)}$.

Using the substitution method, let us solve

Equation (1) is called the substitution equation, $x+y=6700$
Thus it becomes, $y=6700-x$
Let us substitute this in equation (2)

$$\begin{aligned} 40x+25y&=185500 \\ 40x+25(6700-x)&=185500 \\ 40x+167500-25x&=185500 \\ 15x&=1800 \\ x&=1200 \end{aligned}$$

Now substituting 1200 for x in the substitution equation, we can solve for y

$$\begin{aligned} y&=6700-x \\ &=6700-1200 \\ &=5500 \end{aligned}$$

The solution is $\boxed{(1200,5500)}$.

Using the elimination method, let us solve

Multiply equation (1) by –40 so that the variable x gets eliminated when added.

$$\begin{aligned} -40x-40y&=-268000 \\ 40x+25y&=185500 \\ \hline -15y&=-82500 \\ y&=5500 \end{aligned}$$

To find x, we substitute 5500 for y in equation (1) and solve for x:

$$\begin{aligned} x+y&=6700 \\ x+5500&=6700 \\ x&=1200 \end{aligned}$$

The solution is $\boxed{(1200,5500)}$.

On comparing all the methods we find the Cramer's rule is preferable for this situation.

Answer 43e.

a. Let x represent the number of single scoop cones, y represent the number of double scoop cones, and z represent the number of triple scoop cones.
Write a verbal model for the given situation.
We know that the sum of the numbers of single scoop cones, double scoop cones, and that of the triple scoop cones gives the total number of cones.

$$\begin{array}{ccccccc}
\text{Number} & & \text{Number} & & \text{Number} & & \text{Total} \\
\text{of single} & + & \text{of double} & + & \text{of triple} & = & \text{number} \\
\text{scoop cones} & & \text{scoop cones} & & \text{scoop cones} & & \text{of cones} \\
\downarrow & + & \downarrow & + & \downarrow & = & \downarrow \\
x & + & y & & z & & 120
\end{array}$$

It is given that the selling price is \$134.

$$\begin{array}{ccccccc}
0.90 \cdot \text{Number} & & 1.20 \cdot \text{Number} & & 1.60 \cdot \text{Number} & & \text{Selling} \\
\text{Of single} & + & \text{of double} & + & \text{of triple} & = & \text{price} \\
\text{scoops} & & \text{scoops} & & \text{scoops} & & \\
\downarrow & & \downarrow & & \downarrow & & \downarrow \\
0.90x & + & 1.20y & + & 1.60z & = & 134
\end{array}$$

The number of single scoops is the sum of the numbers of the double and triple scoops.

$$\begin{array}{ccccc}
\text{Number} & & \text{Number} & & \text{Number} \\
\text{of single} & = & \text{of double} & + & \text{of triple} \\
\text{scoop cones} & & \text{scoop cones} & & \text{scoop cones} \\
\downarrow & = & \downarrow & + & \downarrow \\
x & = & y & + & z
\end{array}$$

Thus, the system of equations is

$$\begin{aligned}
x + y + z &= 120 \quad (1) \\
0.90x + 1.20y + 1.60z &= 134 \quad (2) \\
x &= y + z \quad (3)
\end{aligned}$$

Rewrite equation (3). For this, subtract x from each side of equation (3).
$-x+y+z=0$

The new system of equations can be written as
$x+y+z=120$
$-x+y+z=0$
$0.90x+1.20y+1.60z=134.$

We know that the coefficient matrix of the linear system $\begin{aligned} ax+by+cz &= j \\ dx+ey+fz &= k \\ gx+hy+iz &= l \end{aligned}$ is $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$. The coefficient matrix of the new linear system is $\begin{bmatrix} 1 & 1 & 1 \\ -1 & 1 & 1 \\ 0.9 & 1.2 & 1.6 \end{bmatrix}$.

Evaluate the determinant of the coefficient matrix.
The determinant of a 3×3 matrix is defined as

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = (aei+bfg+cdh)-(gec+hfa+idb).$$

Thus,

$$\begin{vmatrix} 1 & 1 & 1 \\ -1 & 1 & 1 \\ 0.9 & 1.2 & 1.6 \end{vmatrix} = \left[1.6+0.9+(-1.2)\right]-\left[0.9+1.2+(-1.6)\right].$$

Evaluate.

$$\begin{aligned} \left[1.6+0.9+(-1.2)\right]-\left[0.9+1.2+(-1.6)\right] &= 1.3-0.5 \\ &= 0.8 \end{aligned}$$

Since the determinant is not 0, we can apply Cramer's rule.
If $\det A \neq 0$, then the system has exactly one solution. The solution is

$$x=\frac{\begin{vmatrix} j & b & c \\ k & e & f \\ l & h & i \end{vmatrix}}{\det A},\ y=\frac{\begin{vmatrix} a & j & c \\ d & k & f \\ g & l & i \end{vmatrix}}{\det A},\text{ and } z=\frac{\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}}{\det A}.$$

Substitute the known values in $x = \dfrac{\begin{vmatrix} j & b & c \\ k & e & f \\ l & h & i \end{vmatrix}}{\det A}$ and evaluate.

$$
\begin{aligned}
x &= \frac{\begin{vmatrix} 120 & 1 & 1 \\ 0 & 1 & 1 \\ 134 & 1.2 & 1.6 \end{vmatrix}}{0.8} \\
&= \frac{(192 + 134 + 0) - (134 + 144 + 0)}{0.8} \\
&= \frac{326 - 278}{0.8} \\
&= 60
\end{aligned}
$$

Substitute the known values in $y = \dfrac{\begin{vmatrix} a & j & c \\ d & k & f \\ g & l & i \end{vmatrix}}{\det A}$ and evaluate.

$$
\begin{aligned}
y &= \frac{\begin{vmatrix} 1 & 120 & 1 \\ -1 & 0 & 1 \\ 0.9 & 134 & 1.6 \end{vmatrix}}{0.8} \\
&= \frac{\left[0 + 108 + (-134)\right] - \left[0 + 134 + (-192)\right]}{0.8} \\
&= \frac{-26 + 58}{0.8} \\
&= 40
\end{aligned}
$$

Substitute the known values in $z = \dfrac{\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}}{\det A}$ and evaluate.

$$
\begin{aligned}
z &= \frac{\begin{vmatrix} 1 & 1 & 120 \\ -1 & 1 & 0 \\ 0.9 & 1.2 & 134 \end{vmatrix}}{0.8} \\
&= \frac{\left[134 + 0 + (-144)\right] - \left[108 + 0 + (-134)\right]}{0.8} \\
&= \frac{-10 + 26}{0.8} \\
&= 20
\end{aligned}
$$

The solution is (60, 40, 20).
Therefore, 60 single scoop cones, 40 double scoop cones, and 20 triple scoop cones were sold.

b. We know that 60 single scoop cones, 40 double scoop cones, and 20 triple scoop cones were sold.
First, multiply 0.9 by 0.1 to find the prices of the single scoop cones after raising the price by 10%.
$(0.9)(0.1) = 0.09$

Now, add 0.09 to 0.9.
$0.09 + 0.9 = 0.99$
After raising the price of the single scoop cones by 10%, its price is \$0.99.

Similarly, we can find the find the prices for the remaining ice cream cones.
The prices of the double scoop cones after raising their prices by 10% is \$1.32 and that of triple scoop cones is \$1.76.

It is given that the number of each size of cone sold falls by 5%.
Find the number of the single scoop cones sold after the rise in price. For this, first multiply 60 by 0.05.
$60(0.05) = 3$

Now, subtract 3 from 60.
$60 - 3 = 57$

Thus, 57 single scoop cones were sold after the rise in price.

Similarly, we can find the number of the double and triple scoop cones sold after the rise in price.
Thus, 38 double scoop cones and 19 triple scoop cones were sold after the rise in price.

Now, multiply the number of each size of cone by the corresponding price and then add each product to find the revenue from the cone sales.
$57(0.99) + 38(1.32) + 19(1.76) = 140.03$

Therefore, the revenue from the cone sales is \$140.03.

Answer 44e.

Let x, y, and z represent the atomic weights of fluorine, sodium, and chlorine.

From the given data, we can construct the following equations

$$x+y=42$$
$$y+z=5805$$
$$5x+z=130.5$$

The determinant of the coefficient matrix $A=\begin{pmatrix}1&1&0\\0&1&1\\5&0&1\end{pmatrix}$ is

$$\det A=\left|\begin{matrix}1&1&0\\0&1&1\\5&0&1\end{matrix}\right|\begin{matrix}1&1\\0&1\\5&0\end{matrix}$$
$$=(1+5+0)-(0+0+0)$$
$$=6$$

Since the determinant is not 0, let us apply Cramer's rule

$$x=\frac{\left|\begin{matrix}42&1&0\\58.5&1&1\\130.5&0&1\end{matrix}\right|}{6}$$
$$=\frac{(42+130.5+0)-(0+0+58.5)}{6}$$
$$=\frac{172.5-58.5}{6}$$
$$=\frac{114}{6}$$
$$=19$$

$$y = \frac{\begin{vmatrix} 1 & 42 & 0 \\ 0 & 58.5 & 1 \\ 5 & 130.5 & 1 \end{vmatrix}}{6}$$
$$= \frac{(58.5+210+0)-(0+130.5+0)}{6}$$
$$= \frac{268.5-130.5}{6}$$
$$= \frac{138}{6}$$
$$= 23$$

$$z = \frac{\begin{vmatrix} 1 & 1 & 42 \\ 0 & 1 & 58.5 \\ 5 & 0 & 130.5 \end{vmatrix}}{6}$$
$$= \frac{(130.5+292.5+0)-(210+0+0)}{6}$$
$$= \frac{213}{6}$$
$$= 35.5$$

Therefore, the atomic weight of fluorine, sodium, and chlorine is $\boxed{(19, 23, \text{and}, 35.5)}$ respectively.

Answer 45e.

a. The area of a triangle with vertices (x_1, y_1), (x_2, y_2), and (x_3, y_3) is

$$\text{Area} = \pm\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

where the symbol $\pm$ indicates that the appropriate sign should be chosen to yield a positive value.

The top triangular region has the vertices (0, 70), (70, 128), and (124, 36). Thus, the area is

$$\text{Area} = \pm\frac{1}{2}\begin{vmatrix} 0 & 70 & 1 \\ 70 & 128 & 1 \\ 124 & 36 & 1 \end{vmatrix}.$$

The determinant of a 3×3 matrix is defined as

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = (aei + bfg + cdh) - (gec + hfa + idb).$$

Thus,

$$\pm\frac{1}{2}\begin{vmatrix} 0 & 70 & 1 \\ 70 & 128 & 1 \\ 124 & 36 & 1 \end{vmatrix} = \pm\frac{1}{2}\left[(0 + 8680 + 2520) - (15872 + 0 + 4900)\right].$$

Evaluate.

$$\begin{aligned} \pm\frac{1}{2}\left[(0 + 8680 + 2520) - (15{,}872 + 0 + 4900)\right] &= \pm\frac{1}{2}(11{,}200 - 20{,}772) \\ &= \pm\frac{1}{2}(-9572) \\ &= \pm(-4786) \end{aligned}$$

Since area is always positive, choose the negative sign to yield a positive value.
$\text{Area} = -(-4786) = 4786$

The area of the top triangular region is 4786 mi^2.

b. The bottom triangular region has the vertices (0, 70), (67, 0), and (124, 36). Thus, the area of the top triangular region is

$$\text{Area} = \pm\frac{1}{2}\begin{vmatrix} 0 & 70 & 1 \\ 67 & 0 & 1 \\ 124 & 36 & 1 \end{vmatrix}.$$

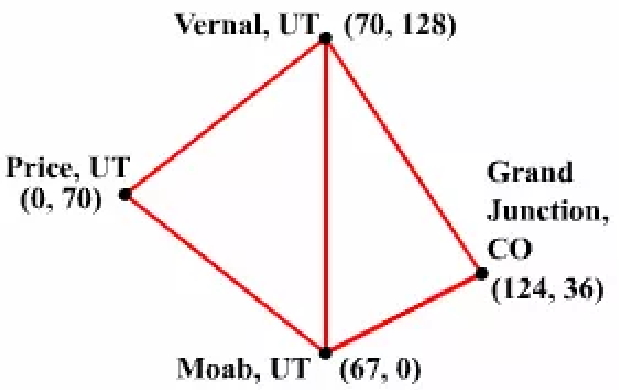
Evaluate.

$$\begin{aligned} \pm\frac{1}{2}\begin{vmatrix} 0 & 70 & 1 \\ 67 & 0 & 1 \\ 124 & 36 & 1 \end{vmatrix} &= \pm\frac{1}{2}\left[(0 + 8680 + 2412) - (0 + 0 + 4690)\right] \\ &= \pm\frac{1}{2}(11{,}092 - 4690) \\ &= \pm(3201) \end{aligned}$$

Since area is always positive, the area of the bottom triangular region is 3201 mi^2.

c. We know that the area of the top triangular region is 4786 mi^2 and that of the bottom triangular region is 3201 mi^2.
Add the areas of the top and the bottom triangular regions to find the total area.
$4786 + 3201 = 7987$
Therefore, the total area is 7987 mi^2.

d. We have to connect the vertices (70, 128) and (67, 0) with a line segment. Using this figure, we can find the area.



Answer 46e.

The area of a triangle with vertices (x_1, y_1), (x_2, y_2), and (x_3, y_3) is given by

$$\text{Area} = \pm\frac{1}{2}\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$$

where the symbol $\pm$ indicates that the appropriate sign should be chosen to yield a positive value.

The given coordinates of the vertices of the triangle are $A(0,0)$, $B(100,50)$, and $C(x,120)$. But the area of the triangular region is 5000 square feet, so

$$\text{Area} = \pm\frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ 100 & 50 & 1 \\ x & 120 & 1 \end{vmatrix}$$

$$5500 = \pm\frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ 100 & 50 & 1 \\ x & 120 & 1 \end{vmatrix}\begin{matrix} 0 & 0 \\ 100 & 50 \\ x & 120 \end{matrix}$$

$$5500 = \pm\frac{1}{2}\left[(0+0+12000)-(50x+0+0)\right]$$

$$5500 = \pm\frac{1}{2}(12000-50x)$$

$$11000 = \pm(12000-50x)$$

If $11000 = 12000 - 50x$ If $11000 = -12000 + 50x$

$$-1000 = -50x \qquad\qquad 23000 = 50x$$

$$20 = x \qquad\qquad 460 = x$$

According to the given graph, the value of x cannot be 460.

Therefore, the farmer plants the final post when $\boxed{x = 20}$.

Answer 47e.

Substitute 8 for x in the given function to evaluate $f(8)$.

$f(8) = 8 - 12$

Simplify.

$8 - 12 = -4$

The given function evaluates to -4.

Answer 48e.

The given function is $f(x) = 4x + 8$

To find $f(7)$, we have to substitute $x = 7$ in the above equation.

$$\begin{aligned} f(7) &= 4(7) + 8 \\ &= 28 + 8 \\ &= 36 \end{aligned}$$

Therefore, $f(7) = \boxed{36}$.

Answer 49e.

Substitute -5 for x in the given function to evaluate $f(-5)$.

$f(-5) = (-5)^2 - 10$

Simplify.

$$\begin{aligned} (-5)^2 - 10 &= 25 - 10 \\ &= 15 \end{aligned}$$

The given function evaluates to 15.

Answer 50e.

The given function is $f(x) = -x^2 + 2x$

To find $f(3)$, we have to substitute $x = 3$ in the above equation.

$$\begin{aligned} f(3) &= -(3)^2 + 2(3) \\ &= -9 + 6 \\ &= -3 \end{aligned}$$

Therefore, $f(3) = \boxed{-3}$.

Answer 51e.

Substitute 4 for x in the given function to evaluate $f(4)$.
$f(4) = -(4)^2 - 4 + 5$

Simplify.

$$\begin{aligned} -(4)^2 - 4 + 5 &= -16 - 4 + 5 \\ &= -15 \end{aligned}$$

The given function evaluates to -15.

Answer 52e.

The given function is $f(x) = x^2 - 2x + 4$

To find $f(-2)$, we have to substitute $x = -2$ in the above equation.

$$\begin{aligned} f(-2) &= (-2)^2 - 2(-2) + 4 \\ &= 4 + 4 + 4 \\ &= 12 \end{aligned}$$

Therefore, $f(-2) = \boxed{12}$.

Answer 53e.

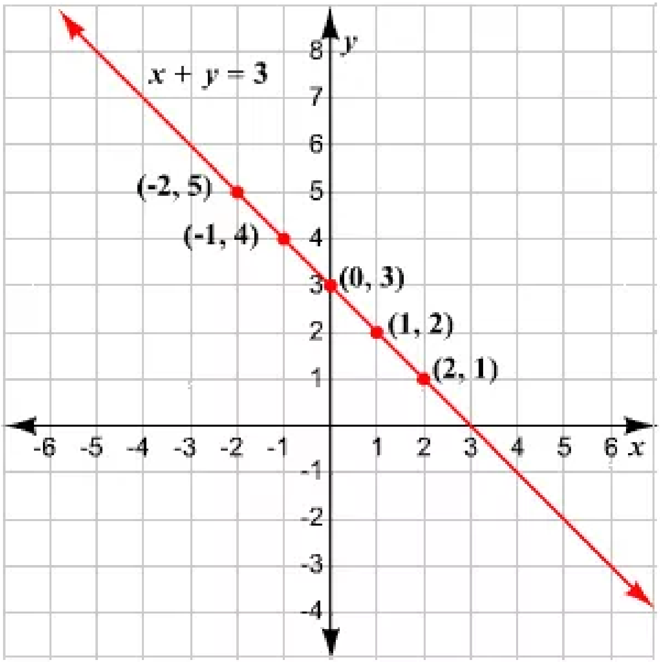
Step 1 We graph each inequality in a system separately to get the graph of that system.

Let us first graph $x + y \geq 3$. Graph the boundary line for this inequality, $x + y = 3$. For this, rewrite the equation in slope-intercept form.
$y = 3 - x$

Find some points that are the solutions of the equation. Choose some values for x and find the corresponding values of y.

x	−2	−1	0	1	2
y	5	4	3	2	1

Plot these points and draw a solid line passing through them since "≥" is the inequality symbol used.

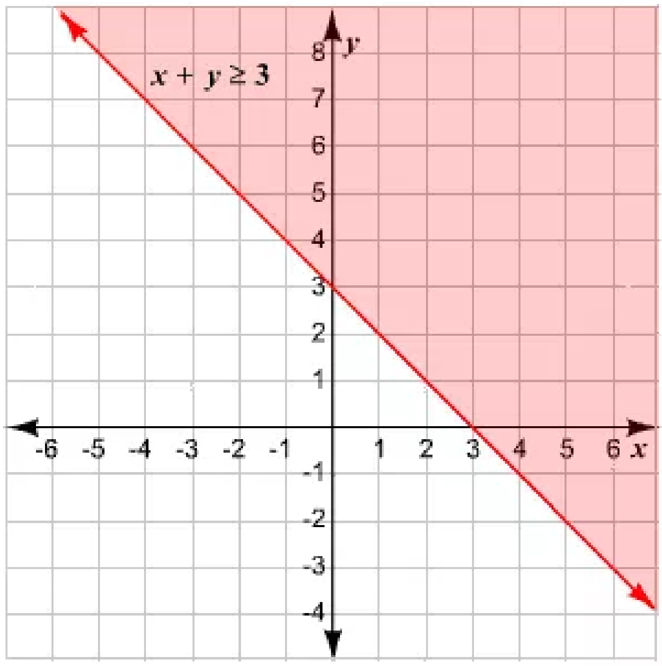


Now, test a point in either of the regions formed by the line to check which region is to be shaded. Let the point be (0, 0).

Substitute 0 for x and y in the first inequality.

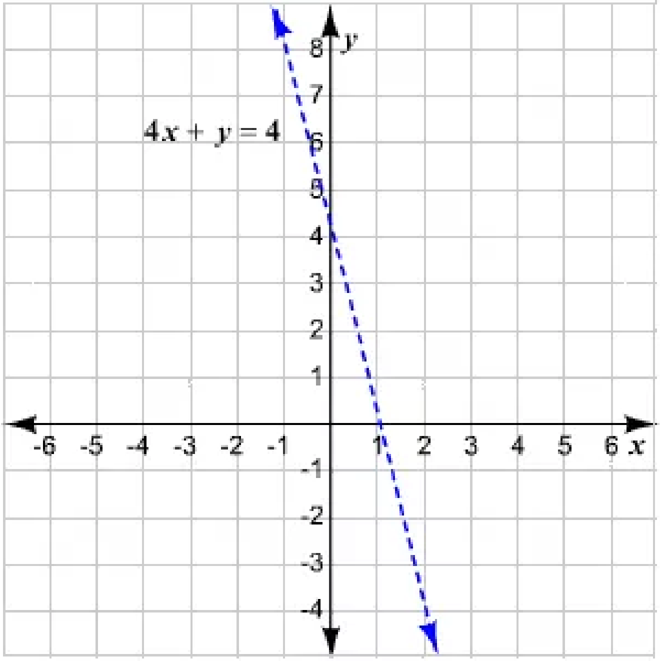
$$x + y \geq 3$$
$$0 + 0 \overset{?}{\geq} 3$$
$$0 \geq 3 \quad \times$$

Therefore, shade the region that does not contain (0, 0).



Let us first graph the second inequality. Graph the boundary line for this inequality, $4x + y < 4$.

The slope-intercept form of the equation $4x + y = 4$ is $y = 4 - 4x$. First, graph the equation $y = 4 - 4x$ and draw a dashed line passing through them since "<" is the inequality symbol used.

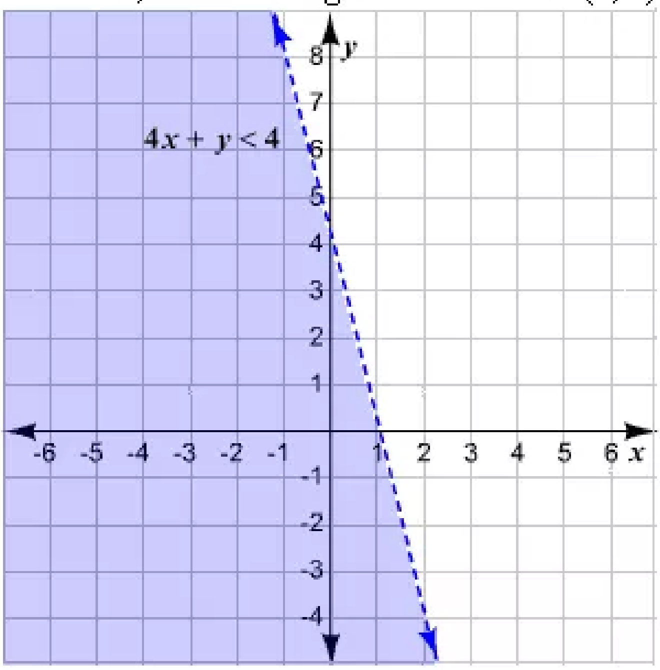


Now, test a point in either of the regions formed by the line to check which region is to be shaded. Let the point be (0, 0).

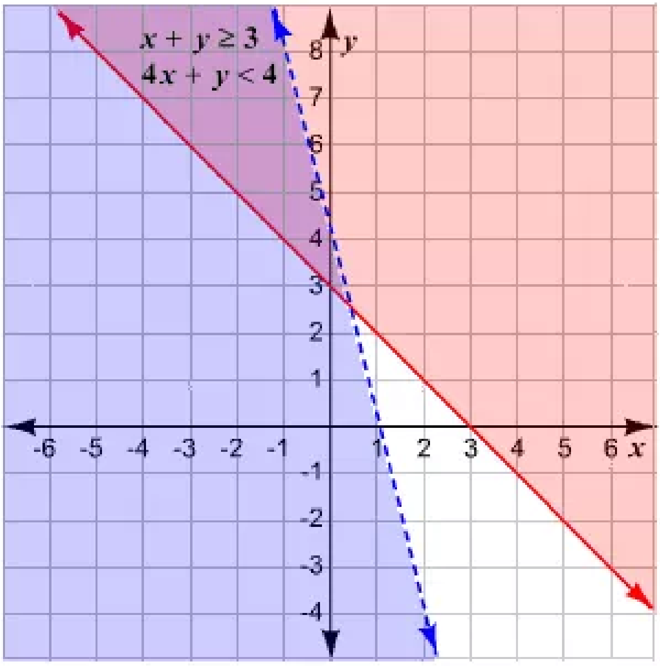
Substitute 0 for x and y in the first inequality.

$$4x + y < 4$$
$$4(0) + 0 \overset{?}{<} 4$$
$$0 \overset{?}{<} 4$$
$$0 < 4 \quad \checkmark$$

Therefore, shade the region that contains (0, 0).



Step 2 We need to identify the region that is common to both the graphs. This can be done by drawing the two graphs on the same set of coordinate axes.



The solution is the region shaded in purple.

Answer 54e.

To graph $2x-y\geq 2$, we begin by graphing the boundary line $2x-y=2$. Since the inequality contains an $\geq$ symbol, the boundary is a solid line. Because the coordinates of the test point $(0,0)$ does not satisfy $2x-y\geq 2$, we shade the side of the boundary that does not contain $(0,0)$.

Graph the boundary: $2x-y=2$

x	y	(x,y)
0	-2	$(0,-2)$
1	0	$(1,0)$

Shading: Check the test point $(0,0)$

$$2x-y\geq 2$$
$$2(0)-0\geq 2$$
$$0\geq 2$$

$(0,0)$ is not a solution of $2x-y\geq 2$

We superimpose the graph of $5x-y\geq 2$ on the graph of $2x-y\geq 2$ so that we can determine the points that the graphs have in common.

To graph $5x-y\geq 2$, we graph the boundary line $5x-y=2$. Since the test point $(0,0)$ does not satisfy $5x-y\geq 2$, we shade the half plane that does not contain $(0,0)$.

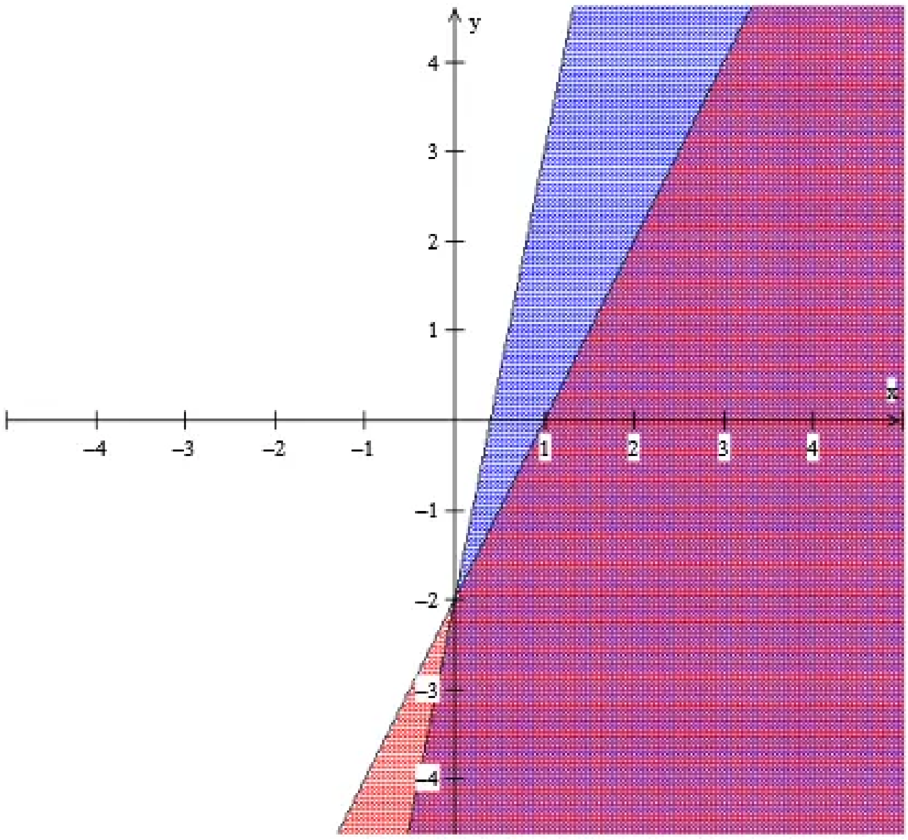
Graph the boundary: $5x - y = 2$

x	y	(x, y)
0	-2	$(0, -2)$
1	3	$(1, 3)$

Shading: Check the test point $(0,0)$

$$5x - y \geq 2$$
$$5(0) - 0 \geq 2$$
$$0 \geq 2$$

$(0,0)$ is not a solution of $5x - y \geq 2$



In the above figure, the area that is shaded twice represents the solutions of the given system. Any point in the doubly shaded region in purple has coordinates that satisfy both inequalities.

Answer 55e.

Step 1 We graph each inequality in a system separately to get the graph of that system.

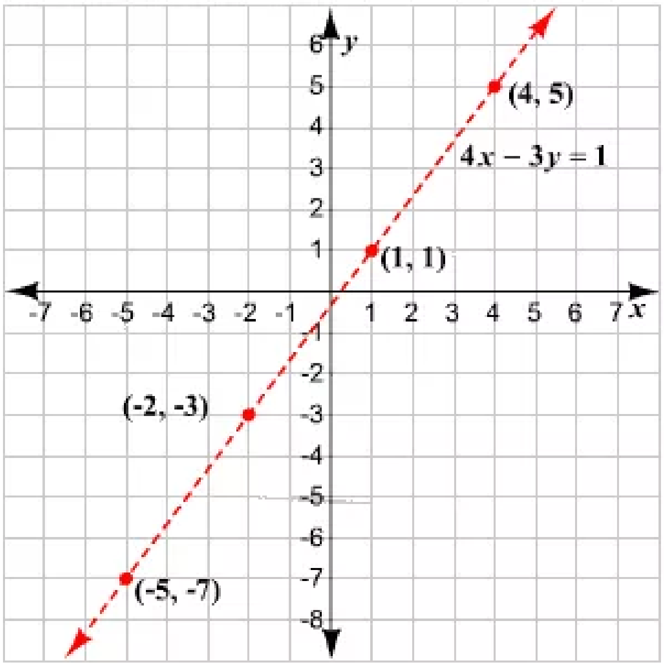
Let us first graph $4x + 3y > 1$. Graph the boundary line for this inequality, $4x + 3y = 1$. For this, rewrite the equation in slope-intercept form.

$$y = \frac{-1 + 4x}{3}$$

Find some points that are the solutions of the equation. Choose some values for x and find the corresponding values of y.

x	-5	-2	1	4
y	-7	-3	1	5

Plot these points and draw a dashed line passing through them since ">" is the inequality symbol used.

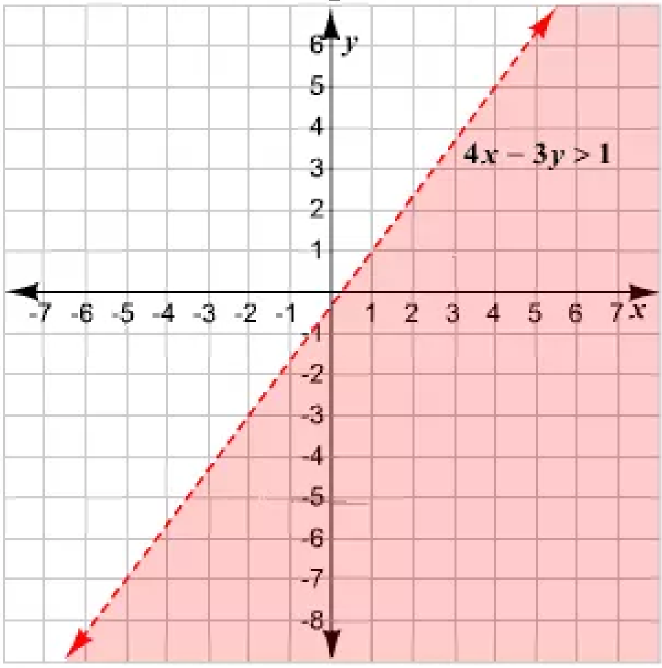


Now, test a point in either of the regions formed by the line to check which region is to be shaded. Let the point be (0, 0).

Substitute 0 for x and y in the first inequality.

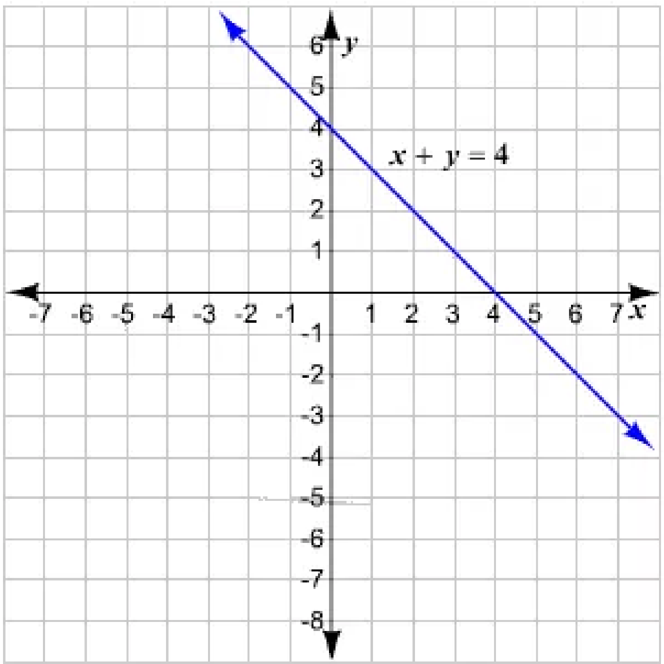
$$4x + 3y > 1$$
$$4(0) + 3(0) \overset{?}{>} 1$$
$$0 > 1 \quad \times$$

Therefore, shade the region that does not contain (0, 0).



Let us first graph the second inequality. Graph the boundary line for this inequality, $-x + y \geq 4$.

The slope-intercept form of the equation $-x + y = 4$ is $y = 4 + x$. First, graph the equation $y = 4 + x$ and draw a solid line passing through them since " ≥" is the inequality symbol used.

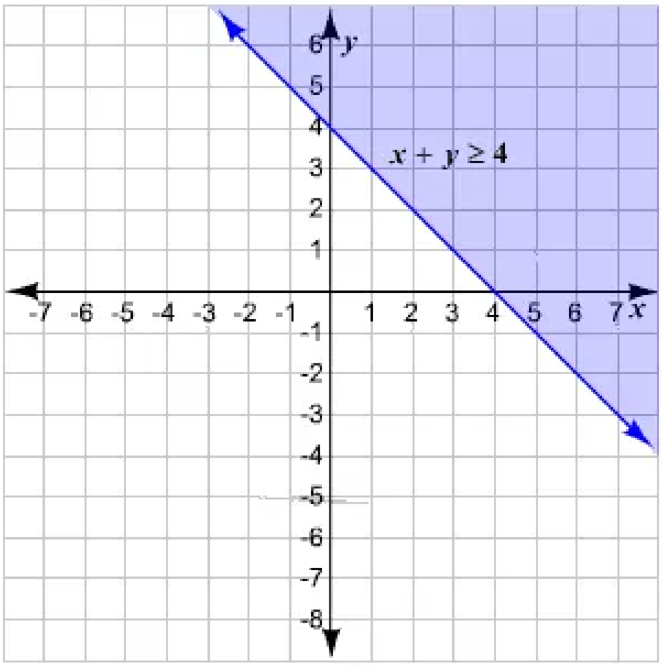


Now, test a point in either of the regions formed by the line to check which region is to be shaded. Let the point be (0, 0).

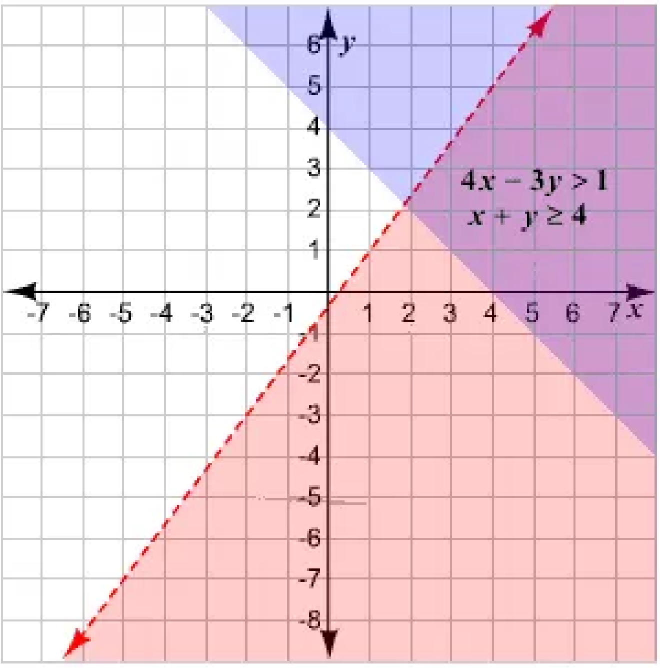
Substitute 0 for x and y in the first inequality.

$$-x + y \geq 4$$
$$-0 + 0 \overset{?}{\geq} 4$$
$$0 \overset{?}{\geq} 4$$
$$0 \geq 4 \quad \times$$

Therefore, shade the region that does not contain (0, 0).



Step 2 We need to identify the region that is common to both the graphs. This can be done by drawing the two graphs on the same set of coordinate axes.



The solution is the region shaded in purple.

Answer 56e.

To graph $y < -x - 5$, we begin by graphing the boundary line $y = -x - 5$. Since the inequality contains an < symbol, the boundary is a dashed line. Because the coordinates of the test point $(0,0)$ does not satisfy $y < -x - 5$, we shade the side of the boundary that does not contain $(0,0)$.

Graph the boundary: $y = -x - 5$

x	y	(x, y)
0	–5	$(0,-5)$
–5	0	$(-5,0)$

Shading: Check the test point $(0,0)$

$$y < -x - 5$$
$$0 < -0 - 5$$
$$0 < -5$$

$(0,0)$ is not a solution of $y < -x - 5$

We superimpose the graph of $y < 3x + 1$ on the graph of $y < -x - 5$ so that we can determine the points that the graphs have in common.

To graph $y < 3x + 1$, we graph the boundary line $y = 3x + 1$. Since the test point $(0,0)$ satisfy $y < 3x + 1$, we shade the half plane that contains $(0,0)$.

Graph the boundary: $y = 3x + 1$

x	y	(x, y)
0	1	$(0,1)$
1	4	$(1,4)$

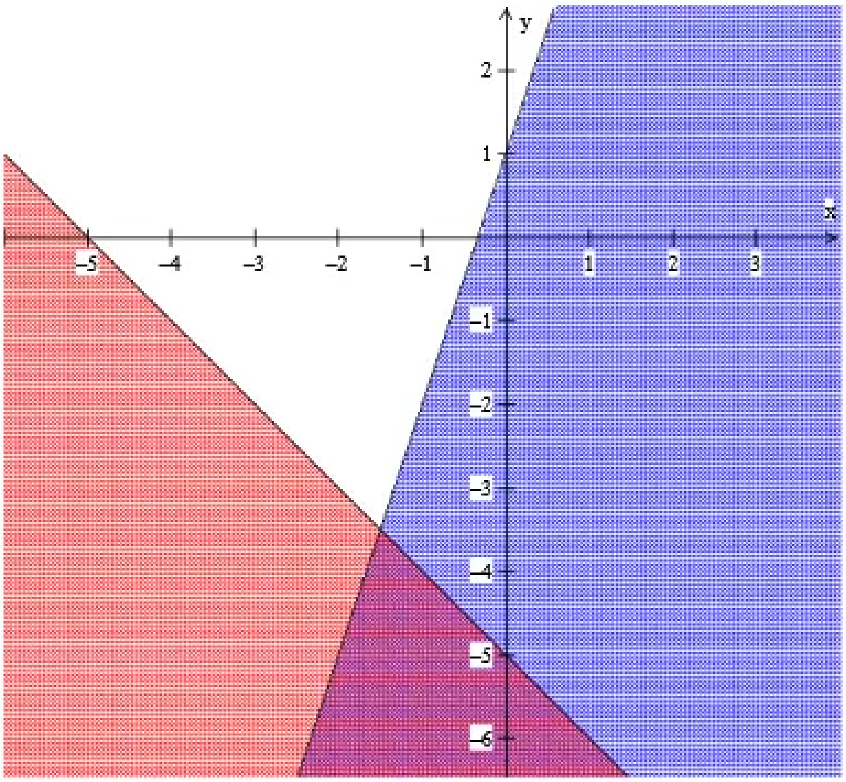
Shading: Check the test point $(0,0)$

$y < 3x + 1$

$0 < 3(0) + 1$

$0 < 1$

$(0,0)$ is a solution of $y < 3x + 1$



In the above figure, the area that is shaded twice represents the solutions of the given system. Any point in the doubly shaded region in purple has coordinates that satisfy both inequalities.

Answer 57e.

Since the dimension of the first matrix is 2×2 and that of the second matrix is 2×2, the product of the matrices is defined and it is a 2×2 matrix.

STEP 1 Multiply the numbers in the first row of the first matrix by the numbers in the first column of the second matrix. Add the products and put the result in the first row, first column of the product.

$$\begin{bmatrix} 2 & -4 \\ 6 & 1 \end{bmatrix}\begin{bmatrix} -3 & 0 \\ 1 & 7 \end{bmatrix} = \begin{bmatrix} 2(-3)+(-4)(1) & \\ & \end{bmatrix}$$

STEP 2 Multiply the numbers in the first row of the first matrix by the numbers in the second column of the second matrix. Add the products and put the result in the first row, second column of the product.

$$\begin{bmatrix} 2 & -4 \\ 6 & 1 \end{bmatrix}\begin{bmatrix} -3 & 0 \\ 1 & 7 \end{bmatrix} = \begin{bmatrix} 2(-3)+(-4)(1) & 2(0)+(-4)(7) \\ & \end{bmatrix}$$

STEP 3 Multiply the numbers in the second row of the first matrix by the numbers in the first column of the second matrix. Add the products and put the result in the second row, first column of the product.

$$\begin{bmatrix} 2 & -4 \\ 6 & 1 \end{bmatrix}\begin{bmatrix} -3 & 0 \\ 1 & 7 \end{bmatrix} = \begin{bmatrix} 2(-3)+(-4)(1) & 2(0)+(-4)(7) \\ 6(-3)+1(1) & \end{bmatrix}$$

STEP 4 Multiply the numbers in the second row of the first matrix by the numbers in the second column of the second matrix. Add the products and put the result in the second row, second column of the product.

$$\begin{bmatrix} 2 & -4 \\ 6 & 1 \end{bmatrix}\begin{bmatrix} -3 & 0 \\ 1 & 7 \end{bmatrix} = \begin{bmatrix} 2(-3)+(-4)(1) & 2(0)+(-4)(7) \\ 6(-3)+1(1) & 6(0)+1(7) \end{bmatrix}$$

STEP 5 Simplify the product matrix.

$$\begin{bmatrix} 2(-3)+(-4)(1) & 2(0)+(-4)(7) \\ 6(-3)+1(1) & 6(0)+1(7) \end{bmatrix} = \begin{bmatrix} -10 & -28 \\ -17 & 7 \end{bmatrix}$$

Therefore, $\begin{bmatrix} 2 & -4 \\ 6 & 1 \end{bmatrix}\begin{bmatrix} -3 & 0 \\ 1 & 7 \end{bmatrix} = \begin{bmatrix} -10 & -28 \\ -17 & 7 \end{bmatrix}$.

Answer 58e.

The product of an $m \times n$ matrix A and an $n \times p$ matrix B is an $m \times p$ matrix C where $c_{ij} = \sum a_{ik} b_{kj}$

Since A is a 2×2 matrix and B is a 2×2 matrix, the product of AB is defined.

Therefore,

$$\begin{pmatrix} -6 & -8 \\ 2 & -4 \end{pmatrix}\begin{pmatrix} 0 & 5 \\ 7 & 1 \end{pmatrix} = \begin{pmatrix} -6(0)+(-8)(7) & -6(5)+(-8)(1) \\ 2(0)+(-4)(7) & 2(5)+(-4)(1) \end{pmatrix}$$
$$= \begin{pmatrix} 0-56 & -30-8 \\ 0-28 & 10-4 \end{pmatrix}$$
$$= \begin{pmatrix} -56 & -38 \\ -28 & 6 \end{pmatrix}$$

The product is $\boxed{\begin{pmatrix} -56 & -38 \\ -28 & 6 \end{pmatrix}}$.

Answer 59e.

Since the dimension of the first matrix is 2×2 and that of the second matrix is 2×2, the product of the matrices is defined and it is a 2×2 matrix.

STEP 1 Multiply the numbers in the first row of the first matrix by the numbers in the first column of the second matrix. Add the products and put the result in the first row, first column of the product.

$$\begin{bmatrix} 1 & 0 \\ 3 & -2 \end{bmatrix}\begin{bmatrix} -5 & 10 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 1(-5)+0(2) & \\ & \end{bmatrix}$$

STEP 2 Multiply the numbers in the first row of the first matrix by the numbers in the second column of the second matrix. Add the products and put the result in the first row, second column of the product.

$$\begin{bmatrix} 1 & 0 \\ 3 & -2 \end{bmatrix}\begin{bmatrix} -5 & 10 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 1(-5)+0(2) & 1(10)+0(0) \\ & \end{bmatrix}$$

STEP 2 Multiply the numbers in the first row of the first matrix by the numbers in the second column of the second matrix. Add the products and put the result in the first row, second column of the product.

$$\begin{bmatrix} 1 & 0 \\ 3 & -2 \end{bmatrix}\begin{bmatrix} -5 & 10 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 1(-5)+0(2) & 1(10)+0(0) \\ & \end{bmatrix}$$

STEP 3 Multiply the numbers in the second row of the first matrix by the numbers in the first column of the second matrix. Add the products and put the result in the second row, first column of the product.

$$\begin{bmatrix} 1 & 0 \\ 3 & -2 \end{bmatrix}\begin{bmatrix} -5 & 10 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 1(-5)+0(2) & 1(10)+0(0) \\ 3(-5)+(-2)(2) & \end{bmatrix}$$

STEP 5 Simplify the product matrix.

$$\begin{bmatrix} 1(-5)+0(2) & 1(10)+0(0) \\ 3(-5)+(-2)(2) & 3(10)+(-2)(0) \end{bmatrix} = \begin{bmatrix} -5 & 10 \\ -19 & 30 \end{bmatrix}$$

Therefore, $\begin{bmatrix} 1 & 0 \\ 3 & -2 \end{bmatrix}\begin{bmatrix} -5 & 10 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} -5 & 10 \\ -19 & 30 \end{bmatrix}$.