Class XII Session 2024-25
Subject - Applied Mathematics
Sample Question Paper - 6

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This question paper contains five sections A, B, C, D and E. Each section is compulsory.
2. Section - A carries 20 marks weightage, Section - B carries 10 marks weightage, Section - C carries 18 marks weightage, Section - D carries 20 marks weightage and Section - E carries 3 case-based with total weightage of 12 marks.
3. **Section – A:** It comprises of 20 MCQs of 1 mark each.
4. **Section – B:** It comprises of 5 VSA type questions of 2 marks each.
5. **Section – C:** It comprises of 6 SA type of questions of 3 marks each.
6. **Section – D:** It comprises of 4 LA type of questions of 5 marks each.
7. **Section – E:** It has 3 case studies. Each case study comprises of 3 case-based questions, where 2 VSA type questions are of 1 mark each and 1 SA type question is of 2 marks. Internal choice is provided in 2 marks question in each case-study.
8. Internal choice is provided in 2 questions in Section - B, 2 questions in Section – C, 2 questions in Section - D. You have to attempt only one of the alternatives in all such questions.

Section A

1. If A, B are two $n \times n$ non-singular matrices, then **[1]**

a) $(AB)^{-1}$ does not exist

b) AB is non-singular

c) $(AB)^{-1} = A^{-1} B^{-1}$

d) AB is singular

2. An observed set of the population that has been selected for analysis is called **[1]**

a) a forecast

b) a sample

c) a process

d) a parameter

3. Assume that the year-end revenues of a business over a three period, are mentioned in the following table: **[1]**

Year-End	31-12-2018	31-12-2021
Year-End Revenue	9,000	13,000

Calculate the CAGR of revenues over, three-years period spanning the "end" of 2018 to the **end** of 2021. Given that

$\left(\frac{13}{9}\right)^{\frac{1}{3}} = 1.13$

a) 13% b) 16%

c) 15% d) 14%

4. Z = 7x + y, subject to 5x + y $\geq$ 5, x + y $\geq$ 3, x $\geq$ 0, y $\geq$ 0. The minimum value of Z occurs at **[1]**

a) (7, 0) b) (0, 5)

c) $\left(\frac{1}{2}, \frac{5}{2}\right)$ d) (3, 0)

5. The function f(x) = $\frac{x}{2} + \frac{2}{x}$ has a local minimum at **[1]**

a) x = -1 b) x = -2

c) x = 2 d) x = 1

6. A random variable X takes the values 0, 1, 2, 3 and its mean is 1.3. If P(X = 3) = 2 P(X = 1) and P(X = 2) = 0.3, then P(X = 0) is: **[1]**

a) 0.2 b) 0.1

c) 0.3 d) 0.4

7. A coin is tossed 10 times. The probability of getting exactly six heads is **[1]**

a) $\frac{512}{513}$ b) $^{10}C_6$

c) $\frac{105}{512}$ d) $\frac{100}{153}$

8. The order and the degree of the differential equation of the family of curves given by y = Ax + A^3, where A is arbitrary constant, are **[1]**

a) 2, 3 b) 1, 2

c) 1, 1 d) 1, 3

9. An outlet pipe can empty a cistern in 3 hours. The time taken by it to empty $\frac{3}{2}$ rd of the cistern is **[1]**

a) 4 hours b) 6 hours

c) 2 hours d) 3 hours

10. The solution of the differential equation $\frac{dy}{dx} = \frac{2y}{x}$ = 0 with y(1) = 1 is given by: **[1]**

a) x = $\frac{1}{y}$ b) y = $\frac{1}{x^2}$

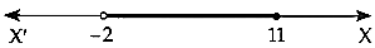
c) x = $\frac{1}{y^2}$ d) y = $\frac{1}{x}$

11. What is the least value of 'x' that satisfies x $\equiv$ 27 (mod 4), when 27 < x $\leq$ 36? **[1]**

a) 31 b) 30

c) 35 d) 27

12. The solution of the linear inequality in x represented on number line as **[1]**



i. $(-\infty$, -2) $\cup$ [11, ∞)

ii. $(-\infty$, -2] $\cup$ [11, ∞)

iii. (-2, 11)

iv. (-2, 11]

a) Option (iv) b) Option (ii)

c) Option (i)

d) Option (iii)

13. A man can row upstream at 10 km/hr and downstream at 18 km/hr. Man's rate in still water in km/hr is **[1]**

a) 10

b) 14

c) 12

d) 4

14. The graph of the inequality 2x + 3y > 6 is **[1]**

a) half plane that contains the origin

b) half plane that neither contains origin nor the points of the line 2x + 3y = 6

c) whole XOY-plane excluding the points on the line 2x + 3y = 6

d) entire XOY-plane.

15. If x $\leq$ 8, then **[1]**

a) -x $\leq$ -8

b) -x > -8

c) -x < -8

d) -x $\geq$ -8

16. What is the standard deviation of a sampling distribution called? **[1]**

a) Simple error

b) Sampling error

c) Sample error

d) Standard error

17. If the supply function for a commodity is p = $\sqrt{9+x}$ and the market price p_0 = 4, then producer's surplus is **[1]**

a) 15

b) 3

c) $\frac{10}{3}$

d) 10

18. For the given values 15, 23, 28, 36, 41, 46, the 3-yearly moving averages are: **[1]**

a) 22, 29, 35, 41

b) 24, 29, 35, 41

c) 24, 28, 35, 41

d) 22, 28, 35, 41

19. **Assertion (A):** If A = $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix}$, then |3A| = 27|A| **[1]**

Reason (R): If A is a square matrix of order n, then $|kA| = k^n |A|$.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. Let a, b $\in$ R be such that the function f given by f(x) = log |x| + bx^2 + ax, x $\neq$ 0 has extreme values: at x = -1 and x = 2. **[1]**

Assertion (A):f has local maximum at x = -1 and at x = 2

Reason (R): a = $\frac{1}{2}$ and b = $-\frac{1}{4}$

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. Calculate the 3-yearly moving averages of the following data: **[2]**

Year	1	2	3	4	5	6	7
Value	2	4	5	7	8	10	13

22. Abhay bought a mobile phone for ₹ 30,000. The mobile phone is estimated to have a scrap value of ₹ 3,000 after a span of 3 years. Using the linear depreciation method, find the book value of the mobile phone at the end of 2 years. **[2]**

OR

Find the present value of a sequence of payments of ₹8,000 made at the end of each 6 months and continuing forever if money is worth 4% compounded semi-annually.

23. By using properties of definite integral, evaluate: $\int_{-1}^{1} e^{|x|} dx$ **[2]**

24. How much money is needed to endure a series of lectures costing ₹2,500 at the beginning of each year indefinitely, if money is worth 5% compounded annually? **[2]**

OR

Mr. Bharti wishes to purchase a flat for ₹ 6000000 with a down payment of ₹ 1000000 and balance in equal monthly payments for 20 years. If bank charges 7.5 % p.a. compounded monthly, calculate the EMI. (Given $(1.00625)^{240}$ = 4.4608)

25. Prove that: $3^{500} \equiv 2 \pmod 7$ **[2]**

Section C

26. Solve the initial value problem: $x\frac{dy}{dx} + y = x \log x$, $y(1) = \frac{1}{4}$ **[3]**

OR

If $\sqrt{1-x^2} + \sqrt{1-y^2} = 4(x - y)$, then show that $\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$.

27. A machine costing ₹ 30,000 is expected to have a useful life of 4 years and a final scrap value of ₹ 4000. Find the annual depreciation charge using the straight-line method. Prepare the depreciation schedule. **[3]**

28. The marginal cost of production of x units of a commodity is 30 + 2x. It is known that fixed costs are ₹ 120. Find **[3]**

   i. the total cost of producing 100 units
   ii. the cost of increasing output from 100 to 200 units.

29. In a precision bombing attack, there is a 50% chance that any one bomb will strike the target. Two direct hits are required to destroy the target completely. How many bombs must be dropped to give a 99% chance or better, of completely destroying the target? **[3]**

OR

Two biased dice are thrown together. For the first die P(6) = $\frac{1}{2}$, other scores being equally 2 likely while for the second die, P(1) = $\frac{2}{5}$ and other scores are equally likely. Find the probability distribution of 'the number of ones seen'.

30. Fit the straight line trend to the following series data: **[3]**

Year	2017	2018	2019	2020	2021
Sales of sugar (in thousand kg)	80	90	92	83	94

Also, tabulate the trend values.

31. Consider the following hypothesis test: **[3]**

$H_0 : \mu = 15$

$H_a: \mu \neq 15$.

A sample of 50 provided a sample mean of 14.15. The population standard deviation is 3.

i. Compute the value of the test statistic.

ii. What is the p-value?

iii. At α = 0.05, what is your conclusion?

iv. What is the rejection rule using the critical value? What is your conclusion?

Section D

32. Express the matrix A = $\begin{bmatrix} 4 & 2 & -1 \\ 3 & 5 & 7 \\ 1 & -2 & 1 \end{bmatrix}$ as the sum of a symmetric and a skew-symmetric matrix. **[5]**

OR

Find A^{-1}, where A = $\begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}$. Hence solve the system of equations: x + 2y - 3z = -4, 2x + 3y + 2z = 2, 3x - 3y - 4z = 11

33. In a 1000-metre race, A, B and C get Gold, Silver and Bronze medals respectively. If A beats B by 100 metres and B beats C by 100 metres, then by how many metres does A beat C? **[5]**

34. The probability that Rohit will hit a shooting target is $\frac{2}{3}$. While preparing for an international shooting competition. Rohit aims to achieve the probability of hitting the target atleast once to be 0.99. What is the minimum number of chances must he shoot to attain this probability? **[5]**

OR

A die is thrown 5 times. Find the probability that an odd number will turn up

i. exactly 3 times

ii. atleast 4 times

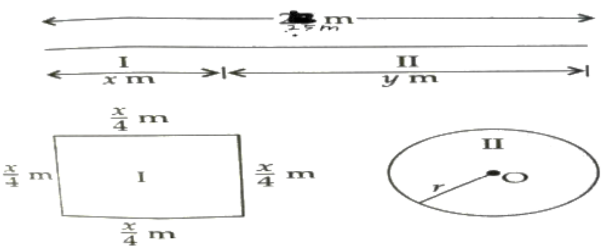
iii. maximum 3 times

35. Anil plans to send his daughter for higher studies abroad after 10 years. He expects the cost of the studies to be ₹ 2,00,000. How much must he set aside at the end of each quarter for 10 years to accumulate this amount, if money is worth 6% compounded quarterly? [Given: $(1.015)^{40} = 1.8140$] **[5]**

Section E

36. **Read the text carefully and answer the questions:** **[4]**

A piece of wire of length 25 cm is to be cut into pieces one of which is to bent into the form of a square and other into the form of a circle.



(a) What is the total area of the square and circle?

(b) What is the relation between r and y?

(c) If we consider total length of wire then what is the relation between x and y?

OR

When $\frac{dA}{dy} = 0$, then find the value of y.

37. **Read the text carefully and answer the questions:** **[4]**

EMI or equated monthly installment, as the name suggests, is one part of the equally divided monthly outgoes to clear off an outstanding loan within a stipulated time frame. The EMI is dependent on multiple factors, such as:

- Principal borrowed
- Rate of interest
- Tenure of the loan
- Monthly/annual resting period

For a fixed interest rate loan, the EMI remains fixed for the entire tenure of the loan, provided there is no default or part-payment in between. The EMI is used to pay off both the principal and interest components of an outstanding loan.

Example:

A person amortizes a loan of ₹1500000 for renovation of his house by 8 years mortgage at the rate of 12% p.a. compounded monthly.

(Given $(1.01)^{96} = 2.5993$, $(1.01)^{57} = 1.7633$)

(a) Find the equated monthly installment.

(b) Find the principal outstanding at the beginning of 40th month.

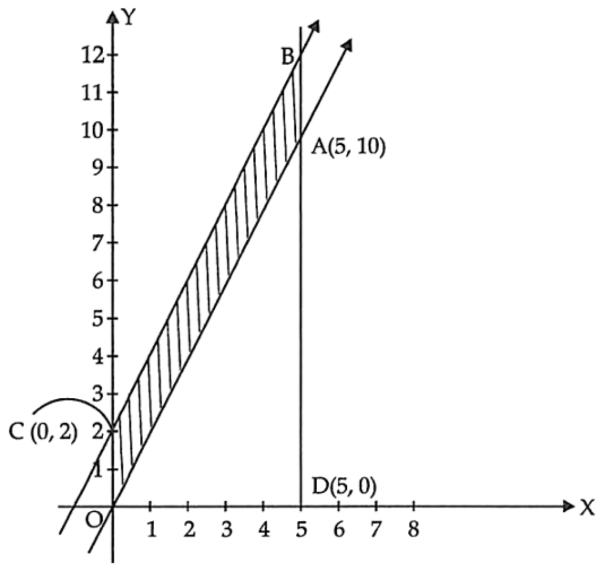
(c) Find the interest paid in 40th payment.

OR

Find the principal contained in 40th payment.

38. **Read the following text carefully and answer the questions that follow:** **[4]**

The feasible region for an LPP is shown in the following figure. The CB is parallel to OA.



i. What is the equation of line OA? (1)

ii. What is the equation of line BC? (1)

iii. What is the co-ordinates of point B? (2)

OR

What are the constraints for the L.P.P.? (2)

OR

A bakery in an establishment produces and sells flour-based food baked in an oven such as bread, cakes, pastries, etc. Ujjwal cake store makes two types of cake. First kind of cake requires 200g of flour and 25 g of fat and 2nd type of cake requires 100g of flour and 50 g of fat.

Based on above information answer the following questions.

i. If the bakery make x cakes of first type and y cakes of 2nd type and it can use maximum 5 kg flour, then write the constraint.
ii. If Bakery can use maximum 1 kg fat, then write the constraint.
iii. Represent total number of cakes made by bakery which is represented by Z.
iv. What is the maximum number of total cakes which can be made by bakery, assuming that there is no shortage of ingredients used in making the cakes?
v. What are number of first and second type of cakes?

Solution

Section A

1.

(b) AB is non-singular

Explanation: If A and B are non - singular then $|AB| \neq 0$

= AB is non - singular matrix

As |AB| = |A||B|

2.

(b) a sample

Explanation: An observed set of the population that has been selected for analysis is called a sample. A sample is a small part of the whole information.

3. **(a)** 13%

Explanation: The CAGR of the revenues over the three years period spanning the "end" of 2018 to "end" of 2021 is

$\left(\frac{\text{Final value}}{\text{Initial Value}}\right)^{\frac{1}{11}}$ - 1 = $\left(\frac{13000}{9000}\right)^{\frac{1}{3}}$ - 1

= 1.13 - 1

= 0.13

= 13%

4.

(b) (0, 5)

Explanation:

Corner Points	**Z = 7x + y**
(3, 0)	21
$\left(\frac{1}{2}, \frac{5}{2}\right)$	6
(7, 0)	49 (minimum)
(0, 5)	5

5.

(c) x = 2

Explanation: f(x) = $\frac{x}{2} + \frac{2}{x} \Rightarrow$ f'(x) = $\frac{1}{2} + \frac{2}{x^2}$ and f''(x) = $\frac{4}{x^3}$

Now, f'(x) = 0 $\Rightarrow x^2 = 4 \Rightarrow x = \pm 2$

$\because$ f''(2) = $\frac{4}{2^3} = \frac{1}{2}$ > 0

$\Rightarrow$ f(x) has a local minimum at x = 2

6.

(d) 0.4

Explanation: Let P(X = 0) = m

P(X = 1) = k

Now,

P(X = 3) = 2k

x_i	p_i	$p_i x_i$
0	m	0
1	k	k
2	0.3	0.6

3	2k	6k

Mean = $\sum p_i x_i$

0 + k + 0.6 + 6k = 1.3

$\Rightarrow$ 7k = 1.3 - 0.6

$\Rightarrow$ k = $\frac{0.7}{7}$ = 0.1

We know that the sum of probabilities in a probability distribution is always 1.

$\therefore$ P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) = 1

$\Rightarrow$ m + 0.1 + 0.3 + 0.2 = 1

$\Rightarrow$ m + 0.6 = 1

$\Rightarrow$ m = 0.4

7.

(c) $\frac{105}{512}$

Explanation: n = 10, X = 6, p = q = $\frac{1}{2}$

P(X = 6) = ${}^{10}C_6\left(\frac{1}{2}\right)^{10} = \frac{105}{512}$

8.

(d) 1, 3

Explanation: y = Ax + A^3 $\Rightarrow$ $\frac{dy}{dx}$ = A

$\therefore$ The differential equation of family of curves is

y = x$\left(\frac{dy}{dx}\right)$ + $\left(\frac{dy}{dx}\right)^3$

$\therefore$ Order = 1, degree = 3

9.

(c) 2 hours

Explanation: The outlet pipe empties the one complete cistern in 3 hours

Time taken to empty $\frac{2}{3}$ Part of the cistern

$= \frac{2}{3} \times 3$

= 2 hours

10.

(b) y = $\frac{1}{x^2}$

Explanation: We have,

$\frac{dy}{dx} + \frac{2y}{x}$ = 0

$\Rightarrow \frac{dy}{dx} = -\frac{2y}{x}$

q

$\Rightarrow \frac{dy}{2y} = -\frac{dx}{x}$

$\Rightarrow \int \frac{dy}{2y} = -\int \frac{dx}{x}$

$\Rightarrow \frac{1}{2}$log |y| = -log |x| + log c

$\Rightarrow \sqrt{y}x$ = c

$\Rightarrow$ yx^2 = c

Given that y(1) = 1 $\Rightarrow$ x = y = 1

$\Rightarrow$ c = 1

$\Rightarrow$ yx^2 = 1

$\Rightarrow$ y = $\frac{1}{x^2}$

11. **(a)** 31

Explanation: Given x $\equiv$ 27 (mod 4)

$\Rightarrow$ x - 27 = 4λ, where $\lambda \in$ I

$\Rightarrow$ x = 27 + 4λ

Putting x = 0, $\pm$1, $\pm$2, ..., we get

x = ..., 19, 23, 27, 31, 35, ...

But $27 < x \leq 36$,
so, least value of x = 31.

12. **(a)** Option (iv)
Explanation: (-2, 11]

13.
(b) 14
Explanation: u = 10 km/h
d = 18 km/h
Speed of man is still water = $\frac{1}{2}$(d + u)
= $\frac{1}{2}$(10 + 18)
= 14 km/h

14.
(b) half plane that neither contains origin nor the points of the line 2x + 3y = 6
Explanation: half plane that neither contains origin nor the points of the line 2x + 3y = 6

15.
(d) $-x \geq -8$
Explanation: $-x \geq -8$

16.
(d) Standard error
Explanation: Standard error

17.
(c) $\frac{10}{3}$
Explanation: Given P = $\frac{10}{x}$ and $p_0 = 4$
So, 4 = $\sqrt{9+x_0} \Rightarrow x_0 = 7$
$$\text{P.S. } = 7 \times 4 - \int_0^7 \sqrt{9+x}dx = 28 - \left[\frac{2}{3}(9+x)^{\frac{3}{2}}\right]_0^7$$
$$= 28 - \left(\frac{128}{3} - \frac{54}{3}\right) = \frac{10}{3}$$

18. **(a)** 22, 29, 35, 41
Explanation: 22, 29, 35, 41

19. **(a)** Both A and R are true and R is the correct explanation of A.
Explanation: We know that if A is square matrix of order n, then $|kA| = k^n |A|$ (see properties)
$\therefore$ Reason is true.
Given A = $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix}$ (Expand by C_1)
$\Rightarrow$ |A| = 1(4 - 0) - 0 + 0
$\Rightarrow$ |A| = 4
Now, 3A = $\begin{bmatrix} 3 & 0 & 3 \\ 0 & 3 & 6 \\ 0 & 0 & 12 \end{bmatrix}$ (Expand by C_1)
$\Rightarrow$ |3A| = 3(36 - 0) - 0 + 0
$\Rightarrow$ |3A| = 108
$\Rightarrow |3A| = 27 \times 4 \Rightarrow |3A| = 27|A|$
$\therefore$ Assertion is true.
Hence, both Assertion and Reason are true and Reason is the correct explanation of Assertion.

20. **(a)** Both A and R are true and R is the correct explanation of A.
Explanation: $f(x) = \log IxI + bx^2 + ax, x \neq 0$
$\Rightarrow f'(x) = \frac{1}{x} + 2bx + a, x \neq 0$
Given x = -1 and x = 2 are extreme values of f(x).
So, f'(-1) = 0 and f'(2) = 0

$\Rightarrow$ -1 - 2b + a = 0 and $\frac{1}{2}$ + 4b + a = 0

Solving these equations, we get a = $\frac{1}{2}$, b = $-\frac{1}{4}$

$\therefore$ Reason is true.

Now, f'(x) $\frac{1}{x} - \frac{1}{2}x + \frac{1}{2} = \frac{2-x^2+x}{2x}$

$\Rightarrow$ f'(x) = 0 $\Rightarrow$ x = -1 and x = 2

f''(x) = $-\frac{1}{x^2} - \frac{1}{2} \Rightarrow$ f''(-1) = $\frac{1}{1} - \frac{1}{2} = -\frac{3}{2}$ < 0

$\Rightarrow$ x = -1 is a point of local maxima.

Also f''(2) = $-\frac{1}{4} - \frac{1}{2} = -\frac{3}{4}$ < 0

$\Rightarrow$ x = 2 is a point of local maxima.

Hence, Assertion and Reason both are true and Reason is the correct explanation of Assertion.

Section B

21. **Calculation of 3-year moving averages:**

Year	Value	3-year moving total	3-year moving average ($\frac{1}{3}$)
1	2		
2	4	11	3.667
3	5	16	5.333
4	7	20	6.667
5	8	25	8.333
6	10	31	10.333
7	13		

22. We have,

$r = \frac{10}{100} = 0.1$

m = 12

$r_e = \left(1 + \frac{r}{m}\right)^m - 1$

$= \left(1 + \frac{0.1}{12}\right)^{12} - 1$

$= (1.00833)^{12} - 1$

= 1.1047 - 1

= 0.1047

Thus, the effective rate of interest is 10.47%, which means that the rate of 10.47% compounded annually yield the same interest as the nominal rate 10% compounded monthly.

OR

Let P be the present value of the given perpetuity. It is given that R = 8,000 and i = $\frac{4}{200}$ = 0.02

$\therefore$ P = $\frac{R}{i} \Rightarrow$ P = ₹$\frac{8,000}{0.02}$ = ₹400,000

Hence the present value of the given perpetuity is ₹400,000. It means that a sum of ₹400,000 invested now at 4% compounded semi-annually will fetch ₹8,000 semi-annually forever.

23. Let f(x) = $e^{|x|} \Rightarrow$ f(-x) = $e^{|-x|} = e^{|x|}$ = f(x)

$\Rightarrow$ f(x) is an even function; therefore,

$\therefore \int_{-1}^{1} e^{|x|}dx = 2\int_{0}^{1} e^{|x|}dx = 2\int_{0}^{1} e^{x}dx$ ($\because 0 \leq x \leq 1 \Rightarrow |x| = x$)

$= 2[e^x]_0^1$ = 2(e^1 - e^0) = 2(e - 1)

24. Here we have to find how much money should be invested now that would provide for an unlimited number of payments of ₹2,500 each year, the first due now. So, it is a perpetuity of ₹2,500 payable at the beginning each year, if money is worth 5% compounded annually. Thus, we have

R = 2,500 and i = $\frac{5}{100}$ = 0.05

Let P be the present value of this annuity. Then,

P = R + $\frac{R}{i} \Rightarrow$ P = ₹(2,500 + $\frac{2,500}{0.05}$) = ₹52,500

Hence, required sum of money is ₹52,500.

OR

Cost of flat = ₹ 6000000, cash payment ₹ 1000000

So, balance = ₹ 6000000 - ₹ 1000000 = ₹ 5000000

Given P = ₹ 5000000, n = 12 $\times$ 20 = 240 months, i = $\frac{7.5}{1200}$ = 0.00625

$\therefore$ EMI = $\frac{5000000 \times 0.00625 \times (1.00625)^{240}}{(1.00625)^{240}-1}$

= $\frac{5000000 \times 0.00625 \times 4.4608}{3.4608}$ = ₹40279.70

25. In order to prove $3^{500} \equiv 2$ (mod 7), let us first find an integer k such that $3^k \equiv \pm 1$ (mod 7).

We know that $3^1 \equiv 3$ (mod 7)

$\Rightarrow 3^2 \equiv 3 \times 3 = 9 \equiv 2$ (mod 7)

$\Rightarrow 3^3 \equiv 3 \times 2$(mod7)

$\Rightarrow 3^3 \equiv 6 \equiv -1$ (mod 7)

Thus, we find that $3^3 \equiv -1$ (mod 7). Let us now express 3^{500} in terms of 3^3.

$3^{500} = (3^3)^{166} \times 3^2$

Now,

$3^3 \equiv -1$ (mod 7)

$\Rightarrow (3^3)^{166} \equiv (-1)^{166}$ (mod 7) [$\because$ a $\equiv$ b (mod m) $\Rightarrow a^n \equiv b^n$ (mod m)]

$\Rightarrow (3^3)^{166} \times 3^2 \equiv (-1)^{166} \times 3^2$ (mod 7) [$\because$ a $\equiv$ b (mod m) $\Rightarrow$ ax $\equiv$ bx (mod m)]

$\Rightarrow 3^{500} \equiv 9$ (mod 7)

But, $9 \equiv 2$ (mod 7). Thus, we obtain

$3^{500} \equiv 9$ (mod 7) and $9 \equiv 2$ (mod 7)

$\Rightarrow 3^{500} \equiv 2$ (mod 7) [$\because$ a $\equiv$ b (mod m), b $\equiv$ c (mod m) $\Rightarrow$ a $\equiv$ c (mod m)]

Section C

26. We have, $x\frac{dy}{dx}$ + y = x log x

$\Rightarrow \frac{dy}{dx} + \frac{1}{x}y$ = log x ...(i)

This is linear differential equation of the form $\frac{dy}{dx}$ + Py = Q with P = $\frac{1}{x}$ and Q = log x

$\therefore$ I.F. = $e^{\int \frac{1}{x} dx}$ = $e^{\log x}$ = x [$\because$ x > 0]

Multiplying both sides of (i) by I.F. = x, we get

$x\frac{dy}{dx}$ + y = x log x

Integrating with respect to x, we ge

yx = $\int \underset{II}{x} \underset{I}{\log x}\, dx$ [Using: y (I.F.) = $\int$ Q (I.F.) dx + C]

$\Rightarrow$ yx = $\frac{x^2}{2}$(log x)$\frac{1}{2}\int$ x dx

$\Rightarrow$ yx = $\frac{x^2}{2}$(log x) - $\frac{x^2}{4}$ + C ...(ii)

It is given that y (1) = $\frac{1}{4}$ i.e. y = $\frac{1}{4}$ where x = 1. Putting x = 1 and y = $\frac{1}{4}$ in (ii), we get

$\frac{1}{4}$ = 0 - $\frac{1}{4}$ + C $\Rightarrow$ C = $\frac{1}{2}$

Putting C = $\frac{1}{2}$ in (ii), we get

xy = $\frac{x^2}{2}$(log x) - $\frac{x^2}{2}$ + $\frac{1}{2} \Rightarrow$ y = $\frac{1}{2}$x log x - $\frac{x}{4} + \frac{1}{2x}$

Hence, y = $\frac{1}{2}$ x log x - $\frac{x}{4} + \frac{1}{2x}$ is the solution of the given differential equation.

OR

$\sqrt{1-x^2} + \sqrt{1-y^2}$ = 4(x - y)

put x = sin θ, y = sin θ

θ = $\sin^{-1}$x ϕ = $\sin^{-1}$y

$\sqrt{1-\sin^2\theta} + \sqrt{1-\sin^2\phi}$ = 4 (sin θ - sin ϕ)

$\cos\theta + \cos\phi = 4\sin\theta - \sin\phi$

$2\cos\left(\frac{\theta+\phi}{2}\right)\cos\frac{\theta-\phi}{2} = 2 \cdot 4\cos\left(\frac{\theta+\phi}{2}\right)\sin\left(\frac{\theta-\phi}{2}\right)$

$\frac{\cos\theta-\phi}{2} = 4 \cdot \sin\frac{\theta-\phi}{2}$

$\frac{\cos\left(\frac{\theta-\phi}{2}\right)}{\sin\left(\frac{\theta-\phi}{2}\right)} = 4$

$\frac{\theta-\phi}{2}$ = $\cot^{-1}4$

$\theta - \phi = 2\cot^{-1}4$

$\sin^{-1}x - \sin^{-1}y = 2\cot^{-1}4$

diff. w.r.t. x we get

$\frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-y^2}} \cdot \frac{dy}{dx} = 0$

$\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$

Hence proved

27. We are given that

C = 30,000; n = 4; S = 4000

Annual depreciation = $\frac{C-S}{n}$

$= \frac{30000-4000}{4}$

= 6500

Depreciation schedule

Year	**Annual depreciation**	**Accumulated depreciation**	**Book Value**
0	0	0	30,000
1	6500	6500	23,500
2	6500	13000	17,000
3	6500	19,500	10,500
4	6500	26,000	4000

28. i. MC = 30 + 2x.

As MC = $\frac{dC}{dx}$,

C(x) = $\int$ (MC) dx = $\int$ (30 + 2x) dx

= $30x + x^2 + k$, where k is constant of integration.

Given fixed cost (in ₹) = 120 i.e. when x = 0, C(x) = 120

$\Rightarrow 30 \times 0 + 0^2 + k = 120 \Rightarrow k = 120$.

$\therefore C(x) = 120 + 30x + x^2$

$\therefore$ Total cost of producing 100 units = $120 + 30 \times 100 + 100^2 = 13120$ (in ₹).

ii. Cost of increasing output from 100 to 200 = C(200) - C(100)

= $(120 + 30 \times 200 + 200^2) - 13120 = 33000$ (in ₹).

Alternatively, we can obtain it as

$\int_{100}^{200} (MC)dx = \int_{100}^{200} (30 + 2x)dx = \left[30x + x^2\right]_{100}^{200}$

= $(30 \times 200 + 200^2) - (30 \times 100 + 100^2) = 33000$ (in ₹).

29. p = $50\% = \frac{1}{2} \therefore q = \frac{1}{2}$

Let n be the number of bombs to be draped.

n $\rightarrow$ atleast 2 bombs should hit target

Probability $\geq$ 0.99 [i.e. 99%]

$P(x \geq 2) \geq 0.99$

$1 - P(x < 2) \geq 0.99$

$1 - [P(n = 0) + P(n = 1)] \geq 0.99$

$1 - \left[{}^nc_0 p^0 q^n + n_{c_1} p^1 q^{n-1}\right] \geq 0.99$

$1 - \left[\frac{1}{2^n} + n \times \frac{1}{2} \times \left(\frac{1}{2}\right)^{n-1}\right] \geq 0.99$

$1 - \frac{1}{2^n}(1+n) \geq 0.91$

$0.01 \geq \frac{1+n}{2^n}$

OR

For the first die, it is given that P(6) = $\frac{1}{2}$ and other scores are equally likely.

i.e., P(1) = P (2) = P(3) = P(4) = P(5) = p_1 (say)

$\therefore$ P(1) + P(2) + P(3) + P(4) + P(5) + P(6) = 1

$\Rightarrow 5p_1 + \frac{1}{2} = 1 \Rightarrow p_1 = \frac{1}{10}$

So, for the first die, we have

P(1) = P(2) = P(3) = P(4) = P(5) = $\frac{1}{10}$ and P(6) = $\frac{1}{2}$

For the second die, it is given that P(1) = $\frac{2}{5}$ and other scores are equally likely.

i.e., P(2) = P(3) = P(4) = P(5) = P(6) = p_2 (say)

$\therefore$ P(1) + P(2) + P(3) + P(4) + P(5) + P (6) = 1

$\Rightarrow \frac{2}{5} + 5p_2 = 1 \Rightarrow p_2 = \frac{3}{25}$

So, for the second die, we have

P(1) = $\frac{2}{5}$ and P(2) = P(3) = P(4) = P(5) = P(6) = $\frac{3}{25}$

P(1) = $\frac{2}{5}$ and P(2) = P(3) = P(4) = P(5) = P(6) = $\frac{3}{25}$

When two dice are thrown, there may not be one on both the dice or one of the dice may show one or both of them show one.

This, if X denotes 'the number of ones seen'. Then, X can take values 0, 1 and 2 such that

P(X = 0) = Probability of not getting one on both dice

= (Probability of not getting one on first die) $\times$ (Probability of not getting one on second die)

$= \left(1 - \frac{1}{10}\right) \times \left(1 - \frac{2}{5}\right) = \frac{9}{10} \times \frac{3}{5} = \frac{27}{50}$

P(X = 1) = Probability of getting one on one die and another number on the other die

$= \frac{1}{10} \times \left(1 - \frac{2}{5}\right) + \left(1 - \frac{1}{10}\right) \times \frac{2}{5} = \frac{21}{50}$

P(X = 2) = Probability of getting one on both dice = $\frac{1}{10} \times \frac{2}{5} = \frac{2}{50}$

Thus, the probability distribution of X is as given below:

X	0	1	2
P(X)	$\frac{27}{50}$	$\frac{21}{50}$	$\frac{2}{50}$

30.

Year t	**Sale** y	**x = t - 2019**	**xy**	x^2	y_t
2017	80	-2	-160	4	83.6
2018	90	-1	-90	1	85.7
2019	92	0	0	0	87.8
2020	83	1	83	1	89.9
2021	94	2	188	4	92.0
n = 5	$\sum y$ = 439	$\sum x$ = 0	$\sum xy$ = 21	$\sum x^2$ = 10	

Now, $a = \frac{\Sigma y}{n} = \frac{439}{5} = 87.8$

$b = \frac{\Sigma xy}{\Sigma x^2} = \frac{21}{10} = 2.1$

Hence, trend equation is y_t = 87.8 + 2.1x

y_{2017} = 87.8 + 2.1(-2) = 83.6

y_{2018} = 87.8 + 2.1(-1) = 85.7

y_{2019} = 87.8 + 2.1(0) = 87.8

y_{2020} = 87.8 + 2.1(1) = 89.9

y_{2021} = 87.8 + 2.1(2) = 92.0

31. Given μ_0 = 15, n = 50, $\bar{x}$ = 14.15, σ = 3

i. Z = $\frac{\bar{x}-\mu_0}{\frac{\sigma}{\sqrt{n}}} = \frac{14.15-15}{\frac{3}{\sqrt{50}}} = \frac{-0.85\times\sqrt{50}}{3}$

= -2.003

$\therefore$ Z = -2

ii. $\because$ Z = -2 < 0

So, p-value = 2(Area under the standard normal curve to the left of Z)

= 2 $\times$ (0.0228) = 0.0456

$\therefore$ p-value = 0.0456

iii. $\because$ p-value < 0.05 (Given α = 0.05)

So, reject H_0

iv. Reject H_0 if Z $\leq -Z_{\frac{\alpha}{2}}$

$\because -Z_{\frac{\alpha}{2}}$ = $-Z_{0.025}$ = -1.96

$\because$ -2 < -1.96

So, reject H_0

Section D

32. Given, A = $\begin{bmatrix} 4 & 2 & -1 \\ 3 & 5 & 7 \\ 1 & -2 & 1 \end{bmatrix}$ Then A^T = $\begin{bmatrix} 4 & 3 & 1 \\ 2 & 5 & -2 \\ -1 & 7 & 1 \end{bmatrix}$

X = $\frac{1}{2}(A + A^T)$

$= \frac{1}{2}\left(\begin{bmatrix} 4 & 2 & -1 \\ 3 & 5 & 7 \\ 1 & -2 & 1 \end{bmatrix} + \begin{bmatrix} 4 & 3 & 1 \\ 2 & 5 & -2 \\ -1 & 7 & 1 \end{bmatrix}\right)$

$= \frac{1}{2}\begin{bmatrix} 4+4 & 2+3 & -1+1 \\ 3+2 & 5+5 & 7-2 \\ 1-1 & -2+7 & 1+1 \end{bmatrix}$

$= \frac{1}{2}\begin{bmatrix} 8 & 5 & 0 \\ 5 & 10 & 5 \\ 0 & 5 & 2 \end{bmatrix}$

X = $\begin{bmatrix} 4 & \frac{5}{2} & 0 \\ \frac{5}{2} & 5 & \frac{5}{2} \\ 0 & \frac{5}{2} & 1 \end{bmatrix}$

X^T = $\begin{bmatrix} 4 & \frac{5}{2} & 0 \\ \frac{5}{2} & 5 & \frac{5}{2} \\ 0 & \frac{5}{2} & 1 \end{bmatrix}^T = \begin{bmatrix} 4 & \frac{5}{2} & 0 \\ \frac{5}{2} & 5 & \frac{5}{2} \\ 0 & \frac{5}{2} & 1 \end{bmatrix}$ = X

$\therefore$ X is a symmetric matrix.

Y = $\frac{1}{2}$ $(A - A^T)$

$= \frac{1}{2}\left(\begin{bmatrix} 4 & 2 & -1 \\ 3 & 5 & 7 \\ 1 & -2 & 1 \end{bmatrix} - \begin{bmatrix} 4 & 3 & 1 \\ 2 & 5 & -2 \\ -1 & 7 & 1 \end{bmatrix}\right)$

$= \frac{1}{2}\begin{bmatrix} 4-4 & 2-3 & -1-1 \\ 3-2 & 5-5 & 7+2 \\ 1+1 & -2-7 & 1-1 \end{bmatrix}$

$= \frac{1}{2}\begin{bmatrix} 0 & -1 & -2 \\ 1 & 0 & 9 \\ 2 & -9 & 0 \end{bmatrix}$

Y = $\begin{bmatrix} 0 & -\frac{1}{2} & -1 \\ \frac{1}{2} & 0 & \frac{9}{2} \\ 1 & -\frac{9}{2} & 0 \end{bmatrix}$

Y^T = $\begin{bmatrix} 0 & -\frac{1}{2} & -1 \\ \frac{1}{2} & 0 & \frac{9}{2} \\ 1 & -\frac{9}{2} & 0 \end{bmatrix}^T = \begin{bmatrix} 0 & -\frac{1}{2} & -1 \\ \frac{1}{2} & 0 & \frac{9}{2} \\ 1 & -\frac{9}{2} & 0 \end{bmatrix}$ = Y

$\Rightarrow$ Y is a skew-symmetric matrix.

Now,

X + Y = $\begin{bmatrix} 4 & \frac{5}{2} & 0 \\ \frac{5}{2} & 5 & \frac{5}{2} \\ 0 & \frac{5}{2} & 1 \end{bmatrix} + \begin{bmatrix} 0 & -\frac{1}{2} & -1 \\ \frac{1}{2} & 0 & \frac{9}{2} \\ 1 & -\frac{9}{2} & 0 \end{bmatrix}$

$$= \begin{bmatrix} 4+0 & \frac{5}{2}-\frac{1}{2} & 0-1 \\ \frac{5}{2}+\frac{1}{2} & 5+0 & \frac{5}{2}+\frac{9}{2} \\ 0+1 & \frac{5}{2}-\frac{9}{2} & 1+0 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 2 & -1 \\ 3 & 5 & 7 \\ 1 & -2 & 1 \end{bmatrix} = A$$

Hence, X + Y = A.

Thus matrix A is expressed as the sum of symmetric and skew-symmetric matrices.

OR

We have,

$$A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}$$

$$\therefore |A| = \begin{vmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{vmatrix} = -6 + 28 + 45 = 67 \neq 0$$

So, A is invertible

Let C_{ij} be the co-factors of a_{ij} in $A = [a_{ij}]$. Then,

$C_{11} = (-1)^{1+1} \begin{vmatrix} 3 & 2 \\ -3 & -4 \end{vmatrix} = -6$, $C_{12} = (-1)^{1+2} \begin{vmatrix} 2 & 2 \\ 3 & -4 \end{vmatrix} = 14$,

$C_{13} = (-1)^{1+3} \begin{vmatrix} 2 & 3 \\ 3 & -3 \end{vmatrix} = -15$, $C_{21} = (-1)^{2+1} \begin{vmatrix} 2 & -3 \\ -3 & -4 \end{vmatrix} = 17$,

$C_{22} = (-1)^{2+2} \begin{vmatrix} 1 & -3 \\ 3 & -4 \end{vmatrix} = 5$, $C_{23} = (-1)^{2+3} \begin{vmatrix} 1 & 2 \\ 3 & -3 \end{vmatrix} = 9$,

$C_{31} = (-1)^{3+1} \begin{vmatrix} 2 & -3 \\ 3 & 2 \end{vmatrix} = 13$, $C_{32} = (-1)^{3+2} \begin{vmatrix} 1 & -3 \\ 2 & 2 \end{vmatrix} = -8$,

and $C_{33} = (-1)^{3+3} \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = -1$

$$\therefore \text{adj } A = \begin{bmatrix} -6 & 14 & -15 \\ 17 & 5 & 9 \\ 13 & -8 & -1 \end{bmatrix} = \begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}$$

So, $A^{-1} = \frac{1}{|A|} \text{ adj } A = \frac{1}{67} \begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}$ ...(i)

The given system of equations is

x + 2y - 3z = -4

2x + 3y + 2z = -2

3x - 3y - 4z = 11

or, AX = B, where $A = \begin{bmatrix} 1 & 1 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} -4 \\ 2 \\ 11 \end{bmatrix}$

As discussed above A is non-singular and so invertible. The inverse of A is given by (i)

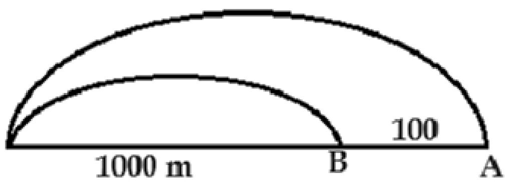
The solution of the given system of equations is given by

$X = A^{-1} B$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{67} \begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix} \begin{bmatrix} -4 \\ 2 \\ 11 \end{bmatrix} = \frac{1}{67} \begin{bmatrix} 24 & +34 & +143 \\ -56 & +10 & -88 \\ 60 & +18 & -11 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$$

$\Rightarrow$ x = 3, y = -2 and z = 1 is the required solution.

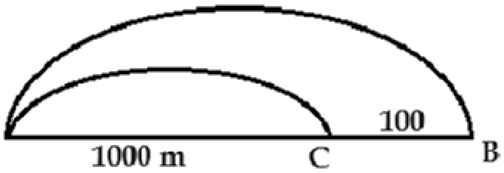
33. Distance covered by A = 1000 m



Distance covered by B = 900 m

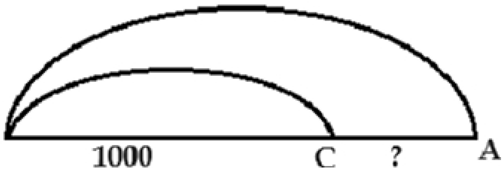
Speed of A: speed of B = 10 : 9

Distance covered by B = 1000



Distance covered by C = 900

Speed of B: Speed of C = 10 : 9



$\therefore$ A : B : C = 100 : 90 : 81

= 1000 : 900 : 81

A : B = 10 : 9

10 : 9.

When A covers 1000 meter C covers 810 metes

$\therefore$ Required distance cover = 1000 - 810

= 190 metre.

34. Given probability of hitting a shooting target = $p = \frac{2}{3}$.

So, q = 1 - p = $1 - \frac{2}{3} = \frac{1}{3}$.

Let the number of trials be n.

The probability of hitting target atleast once = P(X $\geq$ 1) = 1 - P(0)

$= 1 - {}^nC_0 q^n = 1 - \left(\frac{1}{3}\right)^n$

According to given,

$1 - \left(\frac{1}{3}\right)^n > 0.99 \Rightarrow 1 - \frac{1}{3^n} > \frac{99}{100}$

$\Rightarrow 1 - \frac{99}{100} > \frac{1}{3^n} \Rightarrow \frac{1}{100} > \frac{1}{3^n}$

$\Rightarrow 100 < 3^n$

$\Rightarrow 3^n > 100$, which is satisfied if n is atleast 5.

Hence, Rohit must shoot the target at 5 times.

OR

When a die is thrown, sample space = {1, 2, 3, 4, 5, 6}. It has six equally likely outcomes.

p = probability of an odd number $= \frac{3}{6} = \frac{1}{2}$, so q $= 1 - \frac{1}{2} = \frac{1}{2}$.

As the die is thrown 5 times, so there are 5 trials i.e. n = 5.

P(r) $= {}^5C_r p^r q^{5-r} = {}^5C_r \left(\frac{1}{2}\right)^r \left(\frac{1}{2}\right)^{5-r} = {}^5C_r \left(\frac{1}{2}\right)^5$

i. Probability of an odd number exactly 3 times = P(3)

   $= {}^5C_3 \left(\frac{1}{2}\right)^5 = \frac{10}{32} = \frac{5}{16}$

ii. Probability of an odd number atleast 4 times = P(X $\geq$ 4)

   = P(4) + P(5) $= {}^5C_4 \left(\frac{1}{2}\right)^5 + {}^5C_5 \left(\frac{1}{2}\right)^5$

   $= \left({}^5C_4 + {}^5C_5\right) \left(\frac{1}{2}\right)^5 = (5 + 1) \times \frac{1}{32} = \frac{3}{16}$.

iii. Probability of an odd number maximum 3 times = P (X $\leq$ 3)

   = 1 - (P(4) + P(5)) = $1 - \frac{3}{16}$ (see part (ii))

   $= \frac{13}{16}$

35. FC = P $\times \left(\frac{(1+r)^{nt} - 1}{r}\right)$

2,00,000 = P $\times \left(\frac{(1+0.015)^{4\times10} - 1}{0.015}\right)$

Now, calculate the value inside the parentheses:

$(1.015)^{40}$- 1 = 1.8140 - 1= 0.8140

2,00,000 = P $\times \left(\frac{0.8140}{0.015}\right)$

Now, calculate the value inside the second set of parentheses:

$\frac{0.8140}{0.015} \approx 54.267$

Now, solve for P:

P = $\frac{2,00,000}{54.267}$

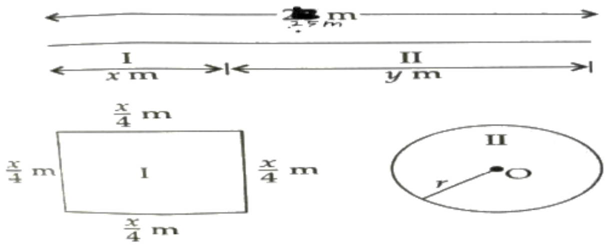
P $\approx$ ₹ 3,684.81

So, Anil must set aside approximately ₹ 3,684.81 at the end of each quarter for 10 years to accumulate ₹ 2,00,000 with a 6% quarterly compounded interest rate.

Section E

36. **Read the text carefully and answer the questions:**

A piece of wire of length 25 cm is to be cut into pieces one of which is to bent into the form of a square and other into the form of a circle.



(i) $(\frac{x}{2})^2 + \pi r^2$

(ii) r = $\frac{y}{2\pi}$

(iii) x + y = 25

OR

$\frac{25\pi}{\pi+4}$

37. **Read the text carefully and answer the questions:**

EMI or equated monthly installment, as the name suggests, is one part of the equally divided monthly outgoes to clear off an outstanding loan within a stipulated time frame. The EMI is dependent on multiple factors, such as:

- Principal borrowed
- Rate of interest
- Tenure of the loan
- Monthly/annual resting period

For a fixed interest rate loan, the EMI remains fixed for the entire tenure of the loan, provided there is no default or part-payment in between. The EMI is used to pay off both the principal and interest components of an outstanding loan.

Example:

A person amortizes a loan of ₹1500000 for renovation of his house by 8 years mortgage at the rate of 12% p.a. compounded monthly.

(Given $(1.01)^{96}$ = 2.5993, $(1.01)^{57}$ = 1.7633)

(i) ₹ 24379.10

(ii) ₹ 1055326.20

(iii) ₹ 10553.26

OR

₹ 13825.84

38. i. The point A(5, 10) lies on the equation y - 2x = 0, therefore the equation of line OA is y - 2x = 0.

ii. Point on line BC i.e., C(0, 2) lies on the equation y - 2x = 2, therefore equation of line BC is y - 2x = 2.

iii. Point B is the intersection point of line BC and BD.

So, substituting x = 5 in y - 2x = 2,

we get y = 12

Thus, required coordinates are (5, 12).

OR

The required constraints for L.P.P. are

y $\geq$ 2x

y - 2x $\leq$ 2

x $\leq$ 5

x $\geq$ 0, y $\geq$ 0

OR

i. Maximum quantity of flour that can be used by bakery = 5 kg

$\Rightarrow 200x + 100y \leq 5000$

$\Rightarrow 2x + y \leq 50$

ii. Maximum quantity of fat that can be used by bakery = 1 kg

$\Rightarrow 25x + 50y \leq 1000$

$\Rightarrow x + 2y \leq 40$

iii. Total No. of cake of first type = x

Total No. of cake of second type = y

$\therefore$ Total no. of cakes = x + y

$\therefore Z = x + y$

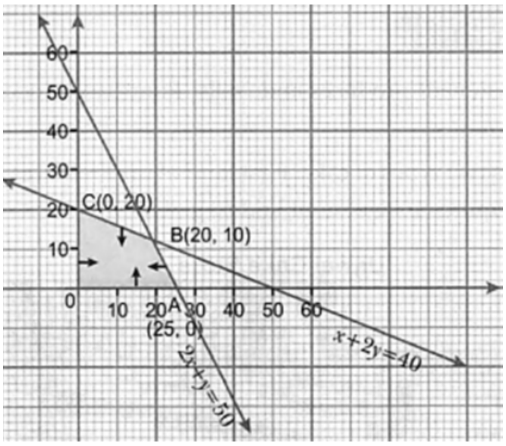
iv. We have

Z = x + y, which is to be maximise under constraints

$2x + y \leq 50$

$x + 2y \leq 40$

$x, y \geq 0$



Here, OABC is the feasible region which is bounded.

The co-ordinates of comer points are O(0,0), A (25, 0), B (20,10), C(0, 20)

Now we evaluate Z at each corner points.

Comer Point	**Z = x + y**
O(0, 0)	0
A(25, 0)	25
B (20, 10)	30 $\leftarrow$ Maximum
C(0, 20)	20

Hence, maximum no. of cakes = 30

v. From above table we get

Maximum number of cakes are 30

x = 20 and y = 10

i.e. No. of first kind of cakes = 20

No. of second kind of cakes = 10