4 Influence Line Diagram.

4.1. Need of ILD:

ILD is used for analysis of a structure if load changes its position over the structure.

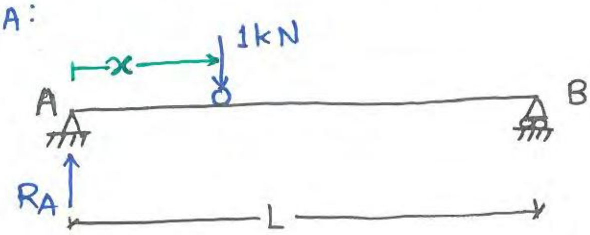
4.2 What is ILD?

It is a graphical representation of variation of reaction, shear force, bending moment, axial force, deflection etc. due to different positions of moving unit point load over span.

4.3 Derivation of ILD:

Considering a simply supported beam of span L.

For R_A:



$$\Sigma M_B = 0$$

$$\Rightarrow R_A \times L - 1(L-x) = 0$$

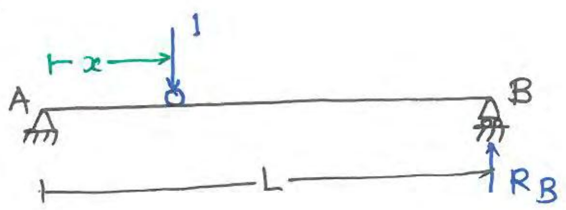
$$\Rightarrow R_A = 1 - \frac{x}{L}$$

at $x=0$, $R_A = 1$

$x = L$, $R_A = 0$

ILD of R_A

For R_B:



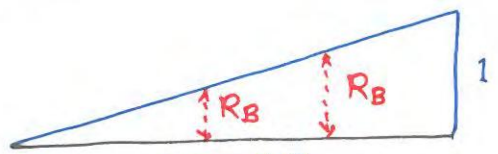
$$\Sigma M_A = 0$$

$$\Rightarrow -R_B \times L + 1 \times x = 0$$

$$R_B = \frac{x}{L}$$

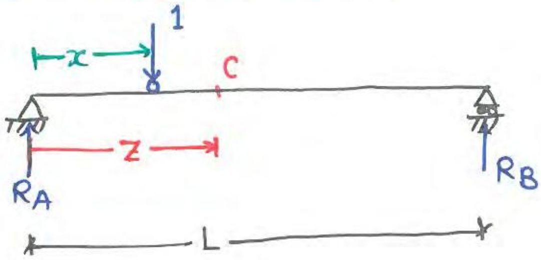
at $x=0$, $R_B = 0$

$x = L$, $R_B = 1$



ILD of R_B

- For SF_C :- case I: $0 \leq x < z$

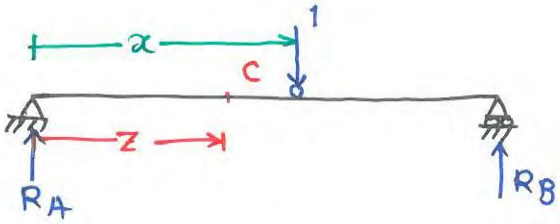


$$SF_C = R_A - 1$$
$$= \frac{L-x}{L} - 1$$
$$= -\frac{x}{L}$$

at $x = 0$, $SF_C = 0$

$x = z$, $SF_C = -\frac{z}{L}$

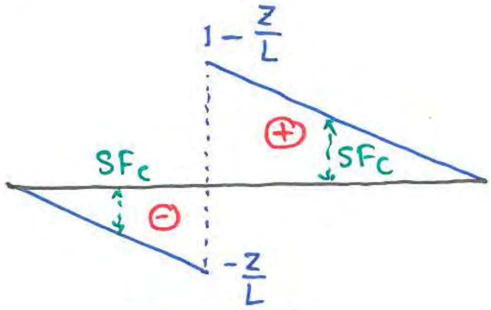
Case II: $z < x \leq L$



$$SF_C = R_A$$
$$= \frac{L-x}{L}$$

at $x = z$, $SF_C = 1 - \frac{z}{L}$

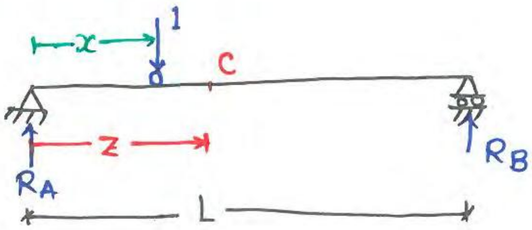
$x = L$, $SF_C = 0$



ILD of SF_C

- For BM_C :-

case I: $0 \leq x \leq z$

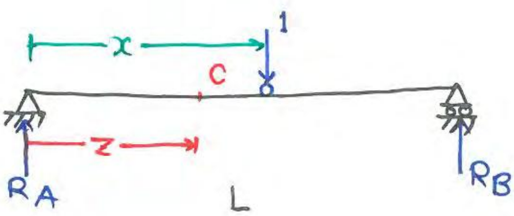


$$BM_C = R_A \times z - 1(z-x)$$
$$= \left(\frac{L-x}{L}\right) \times z - (z-x)$$
$$= \frac{x(L-z)}{L}$$

at $x = 0$, $BM_C = 0$

$x = z$, $BM_C = \frac{z(L-z)}{L} = \frac{ab}{L}$

Case II: $z \leq x \leq L$

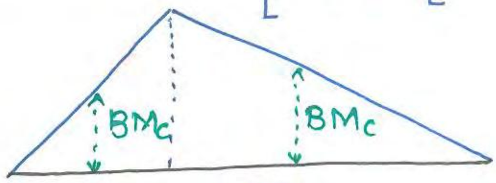


$$BM_C = R_A \times z$$
$$= \left(\frac{L-x}{L}\right) z$$

at $x = z$, $BM_C = \frac{(L-z)z}{L}$

$x = L$, $BM_C = 0$

$$\frac{z(L-z)}{L} = \frac{ab}{L}$$



ILD of BM_c

• Note:

Difference between SFD and ILD of SF at C.

- SFD:-
  It is the variation of shear force along a spane due to particular position of load.
- ILD of SF:-
  It is the variation of shear force at particular section due to different position of load along span.

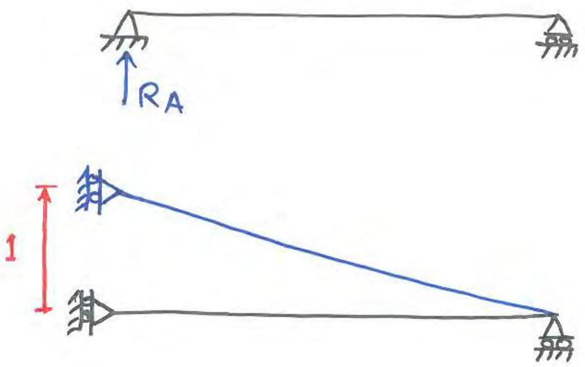
4.4 Muller Breslau's Principle:

It states that ILD of any stress function (Reaction, SF, BM) is the deflected shape of a structure after removing a stress function from the structure and applying unit displacement (deflection or rotation) in the positive direction of stress function.

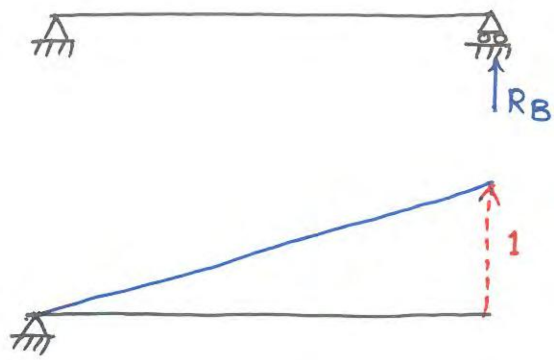
• Note:

- It is valid for all types of statically determinate and linearly elastic indeterminate structure (truss, arch, frame, cable structure etc)
- It is not valid for moving unit point moment.
- Proof of this principle is by virtual work method.
- ILD of deflection is not plotted by Muller Breslau's principle.

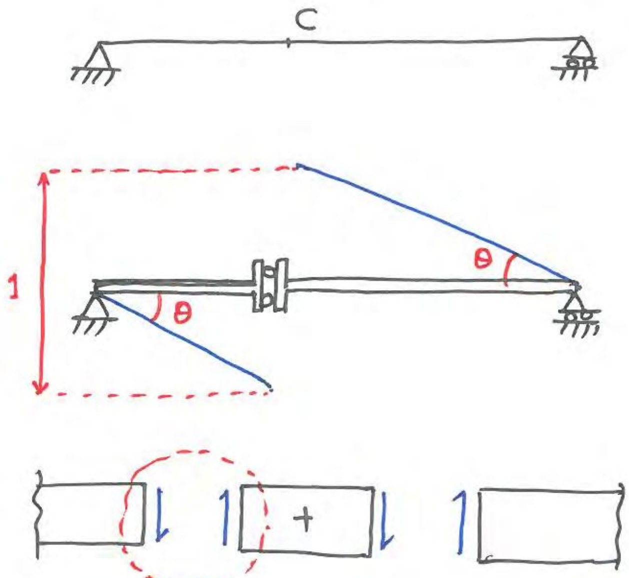
For R_A:



For R_B

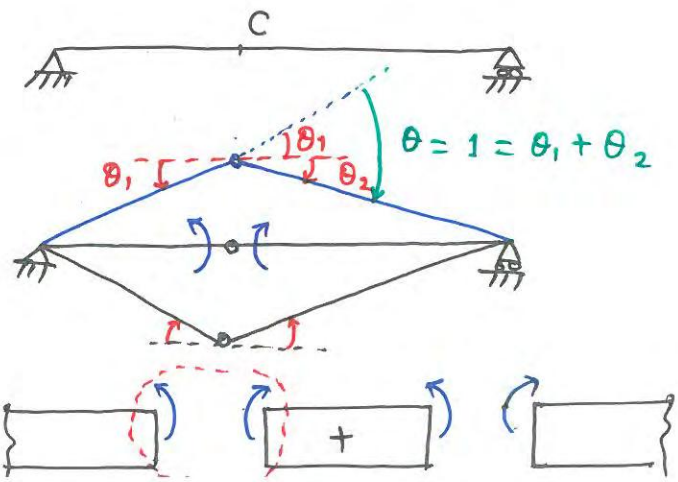


For SF_C :



Both segments must be parllel becoz there is no bending release.

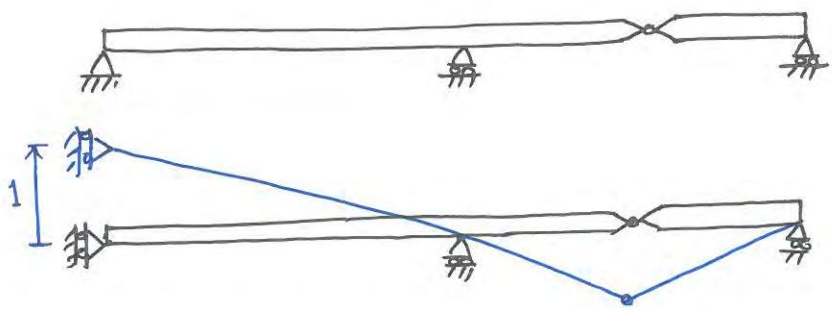
For BM_C:



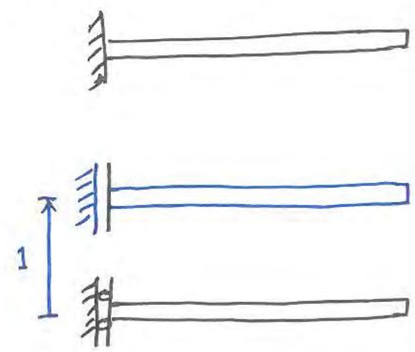
• Note:-

- ILD of statically determinate structure is always linear.
- ILD of deflection of statically determinate structure is non-linear because Muller Breslau's principle is not used for deflection.
- ILD of statically indeterminate structure is non-linear or combination of linear and non-linear.

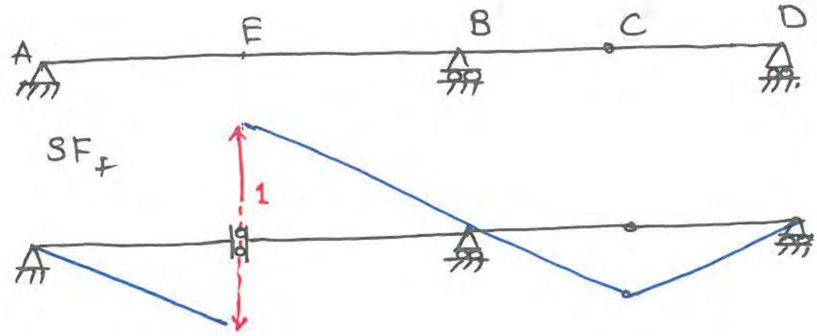
Ex. 1.



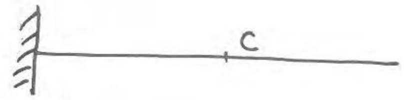
Ex. 2.



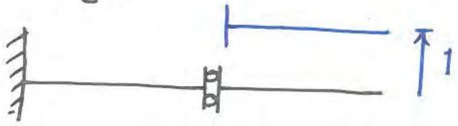
Ex.



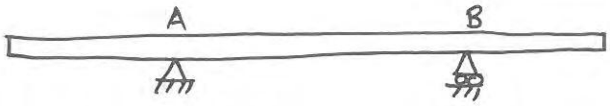
Ex. 4.



SF_C :-

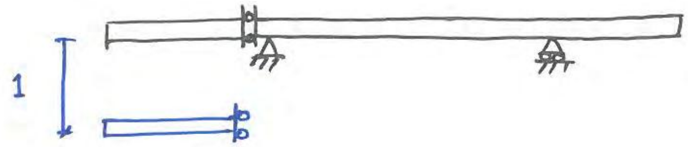


Ex. 5.

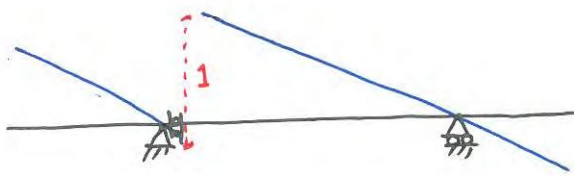


SF just Left and right to A

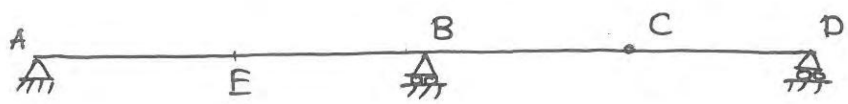
Just Left to A:



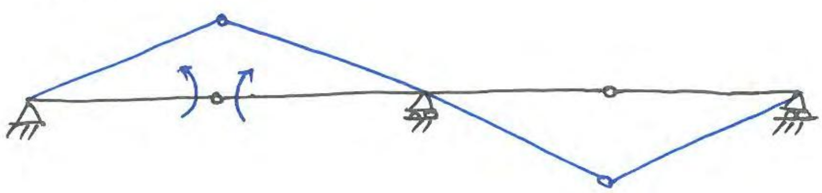
Just Right to A:



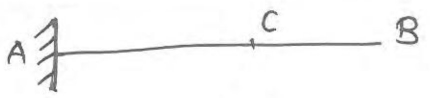
Ex. 6.



BM_E:

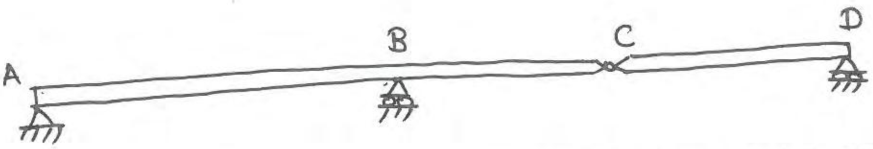


Ex. 7



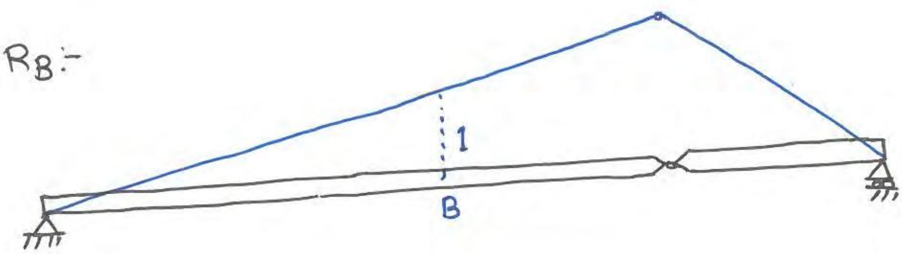
BM_C

Ex. 8.

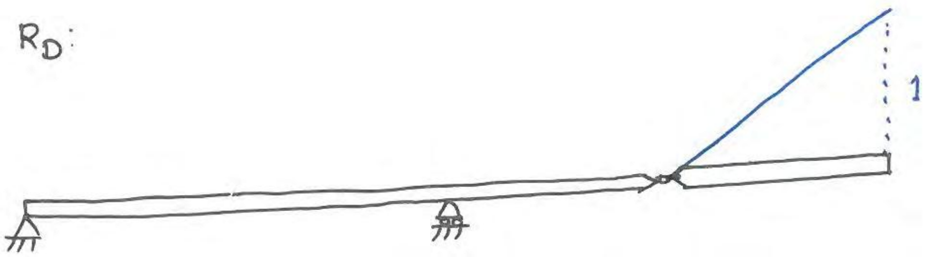


R_B, R_D, SF_C, SF just Left and right to B, BM_B.

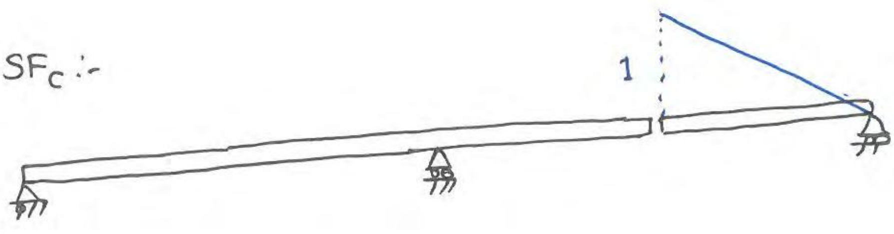
R_B:-



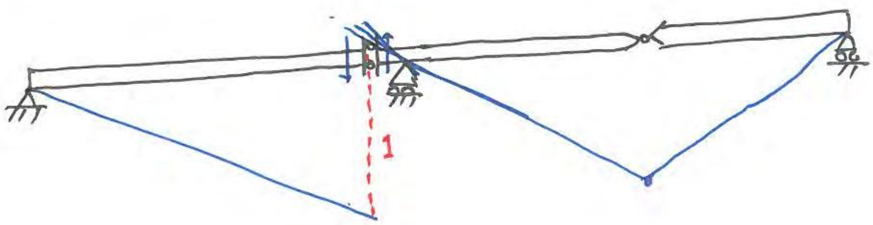
R_D:



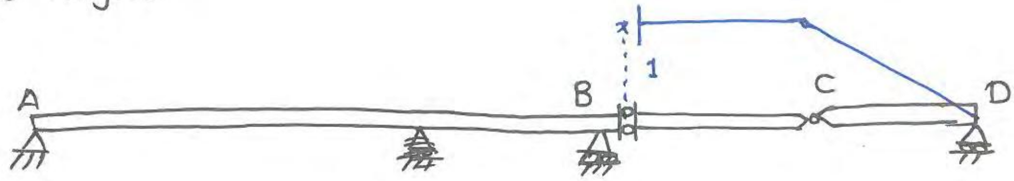
SF_C:-



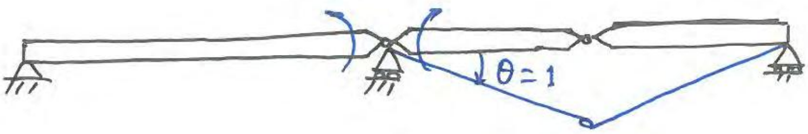
SF just Left to B:-



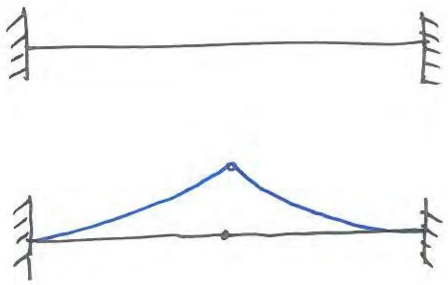
Just Right to B



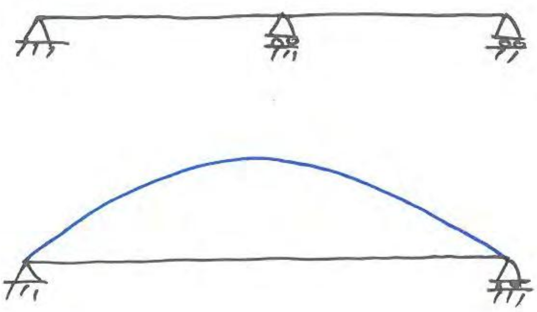
BM_B :-



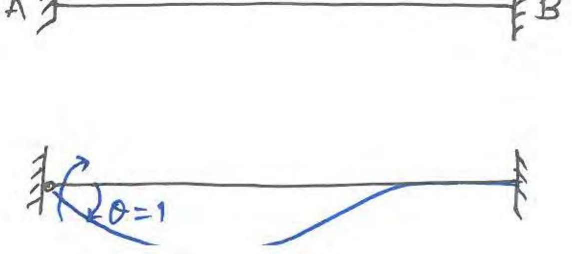
Ex. 1



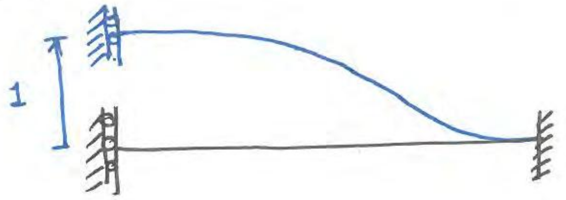
Ex 2.



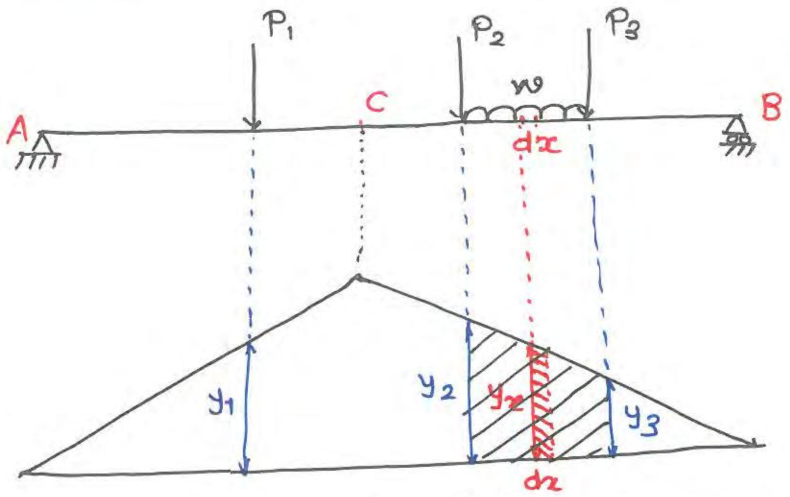
Ex 3



Ex. 4.



4.5. Application of ILD:



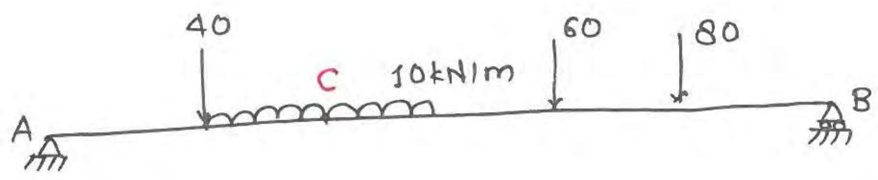
ILD of BM_C

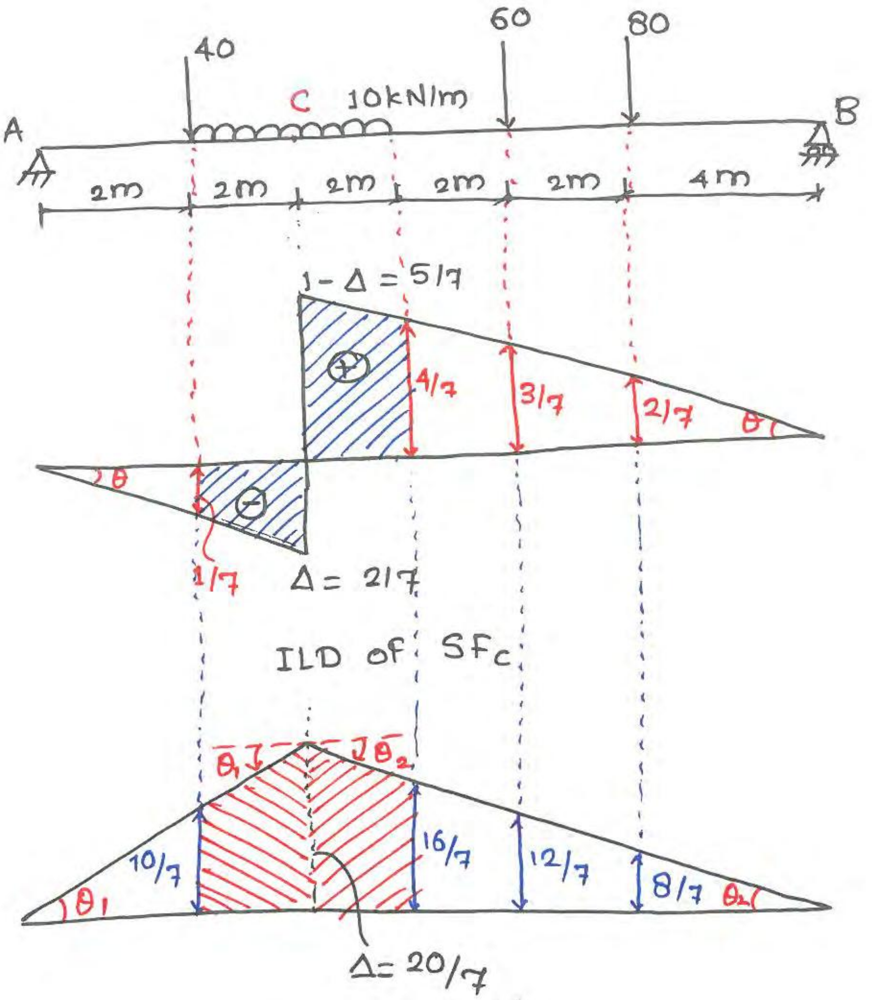
$$BM_C = P_1 y_1 + P_2 y_2 + P_3 y_3 + \Sigma (\omega dx) \cdot y_x$$

$$= P_1 y_1 + P_2 y_2 + P_3 y_3 + \omega \Sigma y_x \cdot dx$$

$$= P_1 y_1 + P_2 y_2 + P_3 y_3 + \omega \times \text{Shaded Area.}$$

Ex. Calculate SF and BM at C.





ILD of BMc.

From ILD of SF_C :-

$$\theta = \theta$$

$$\frac{\Delta}{4} = \frac{1-\Delta}{10}$$

$$\Delta = 2/7$$

$$SF_c = 40\left(-\frac{1}{7}\right) + 60\left(\frac{3}{7}\right) + 80\left(\frac{2}{7}\right) + 10 \times \frac{1}{2} \times 2\left(-\frac{1}{7} - \frac{2}{7}\right)$$

$$+ 10 \times \frac{1}{2} \times 2\left(\frac{5}{7} + \frac{4}{7}\right)$$

$$SF_c = 51.42 \text{ kN}$$

From ILD of BM_C :-

$$\Rightarrow \quad \theta_1 + \theta_2 = 1$$

$$\Rightarrow \quad \frac{\Delta}{4} + \frac{\Delta}{10} = 1$$

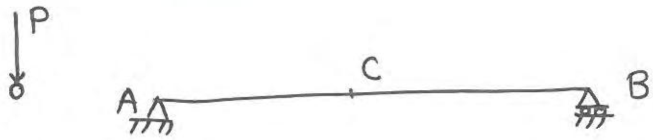
$$\Rightarrow \quad \Delta = \frac{20}{7}$$

$$BM_C = 40\left(\frac{10}{7}\right) + 60\left(\frac{12}{7}\right) + 80\left(\frac{8}{7}\right) + 10\times\frac{1}{2}\times 2\left(\frac{10}{7}+\frac{20}{7}\right) + 10\times\frac{1}{2}\times 2\left(\frac{20}{7}+\frac{16}{7}\right)$$

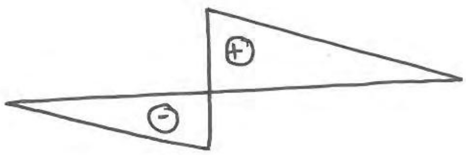
$$BM_C = 345.7\ \text{kNm}$$

4.6 Effect of Moving Load:

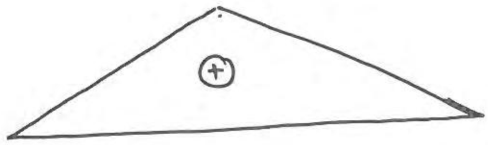
4.6.1 Single point load:



ILD of SF_c



ILD of BM_c

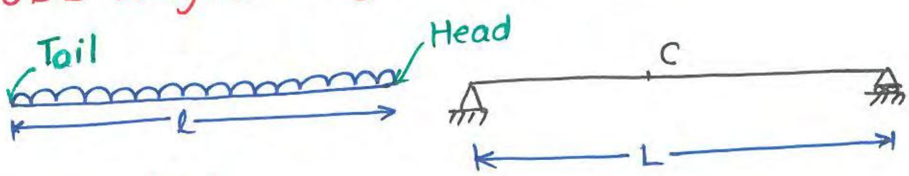


Max^m +ve SF_c = Load is placed just right to C

Max^m −ve SF_c = Load is placed just left to C.

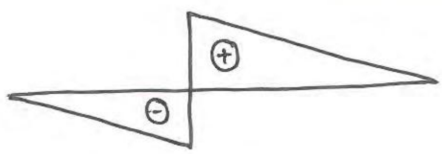
Max^m sagging BM_c = Load is placed at C.

4.6.2 UDL length longer than span:

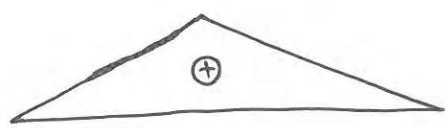


$l > L$

ILD of SF_c



ILD of BM_c

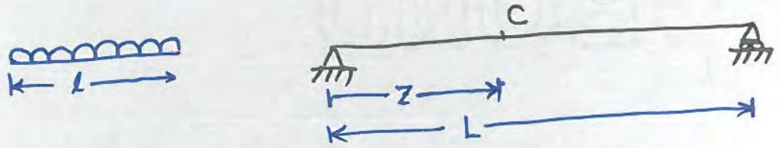


Max^m +ve SF_c = Tail is placed just right to C.

Max^m -ve SF_c = Head is placed just Left to C.

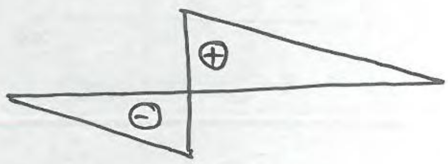
Max^m sagging BM_c = Load is placed over entire span.

4.6.3 UDL length shorter than span:

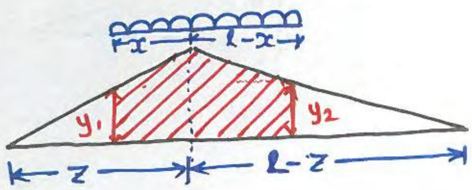


$l < L$

ILD of SF_c



ILD of BM_c



$\boxed{y_1 = y_2}$

Max^m +ve SF_c = Tail is placed just right to C

Max^m -ve SF_c = Head is placed just left to C.

Max^m sagging BM_c = Load is placed in such a way that section should divide load length and span in equal ratio.

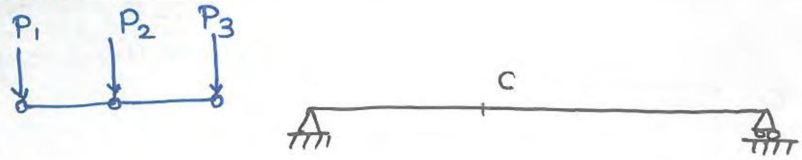
$$\frac{x}{l-x} = \frac{z}{L-z}$$

$$\Rightarrow \frac{x}{z} = \frac{l-x}{L-z}$$

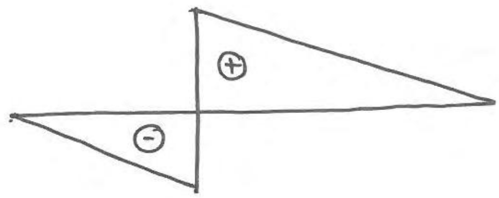
$$\Rightarrow \frac{\omega x}{z} = \frac{\omega(l-x)}{L-z}$$

$\Rightarrow$ Avg. Load on Left side of the section = Avg. Load on Right side of the section.

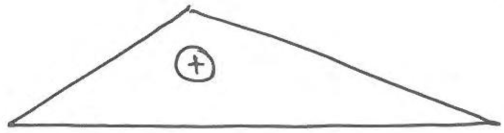
4.6.4 Series of Point Load:

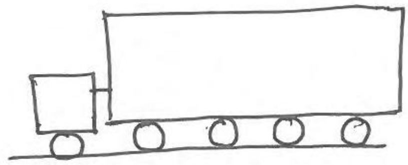


ILD of SF_c



ILD of BM_c



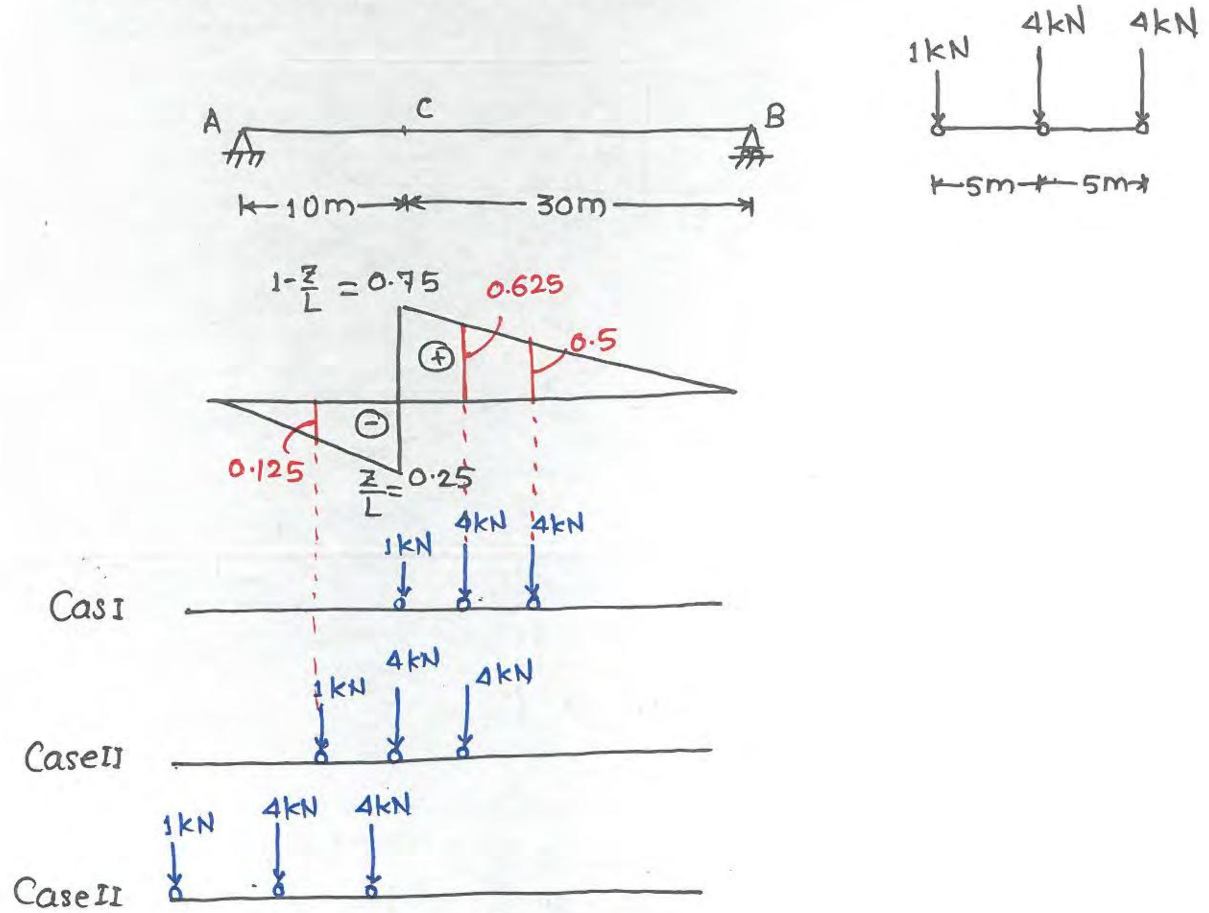


Max^m +ve SF_c = By Trials

Max^m −ve SF_c = By Trials

Max^m sagging BM_c = It can be achieved by placing equal avg. load on either side of section but this cannot be done in the case of series of point load so trials are performed corresponding to approx equal avg. load on either side of section. In all trials, atleast one point load must be placed exactly at section

Ex. A simply supported beam is subjected to series of point load moving from right to left as shown in fig. Calculate maximum +ve SF and maximum sagging BM at 10m from left support.



• Case I: 1 kN is just right to C.

$$SF_c = 1\times(0.75) + 4\times(0.625) + 4\times(0.5) = 5.25\,kN$$

• Case II: Middle 4kN is just right to C.

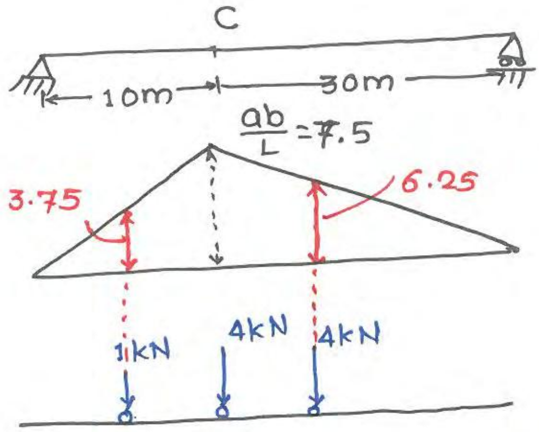
$$SF_c = 1\times(-0.125) + 4\times(0.75) + 4\times(0.625) = 5.375\,kN$$

• Case III: Rear 4 kN is just right to C.

$$SF_c = 1\times(0) + 4\times(-0.125) + 4\times(0.75) = 2.5\,kN$$

so maximum +ve SF_c is 5.375 kN corresponding to case II.

By visual inspection, we can say that case II will give maximum sagging BM_c.



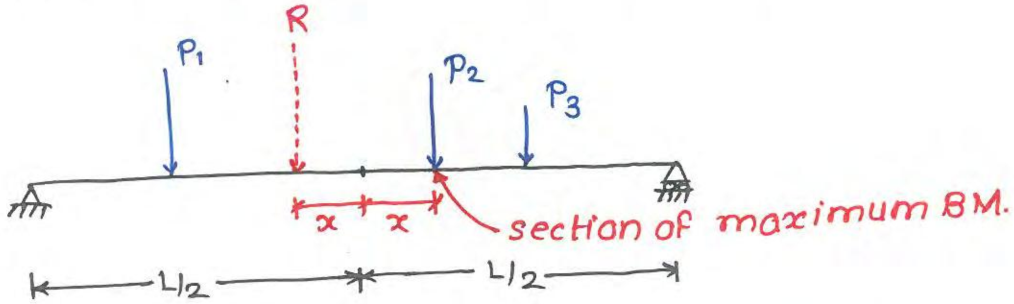
Case II·

Case II: Middle 4 kN is just right to C.

$$BM_c = 1(3.75) + 4(7.5) + 4(6.25) = 58.75\text{ kN}$$

4.7 Absolute Maximum Bending Moment:

If series of point load is moving over simply supported beam then absolute maximum BM is achieved by placing load in such a way that resultant of all point loads and one adjacent load should be at equidistance from centre of span and maximum BM is obtained just below to adjacent load considered.

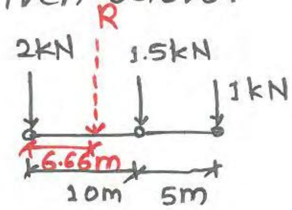


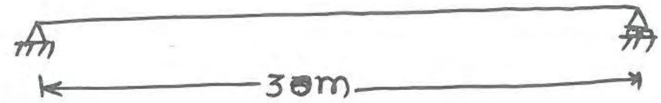
Case I: If resultant coincides with any point load then resultant is placed at centre of span and absolute maximum BM is at centre of span.

Case II: If resultant is closer to heavier adjustent load then, only heavier load is considered as adjucent load.

Case III: If resultant is closer to lighter load then calculation is done corresponding to both adjucent loads.

Ex. Determine absolute maximum BM for beam given below.





⇒

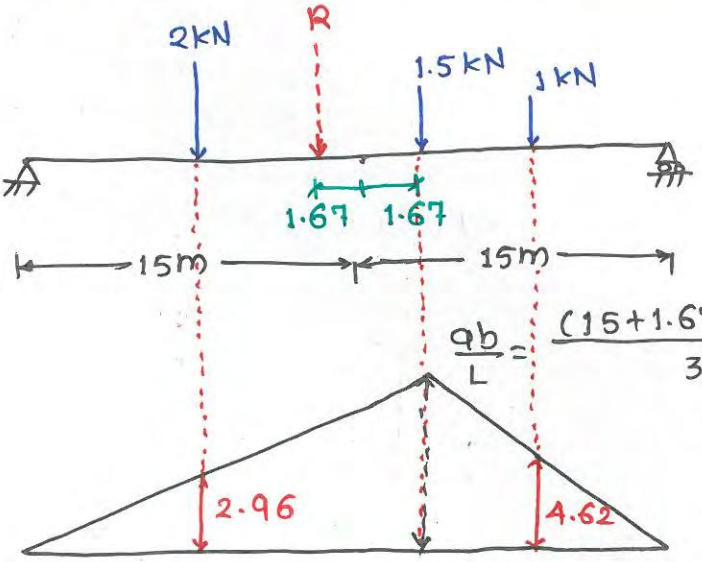
$R = 2 + 1.5 + 1 = 4.5\,kN$

Position of R from 2kN $= \dfrac{P_1x_1 + P_2x_2 + P_3x_3}{P_1 + P_2 + P_3}$

$= \dfrac{2(0) + 1.5\times(10) + 1\times 15}{4.5}$

$= 6.66\,m$

Case I: Considering 1.5kN as adjucent load.

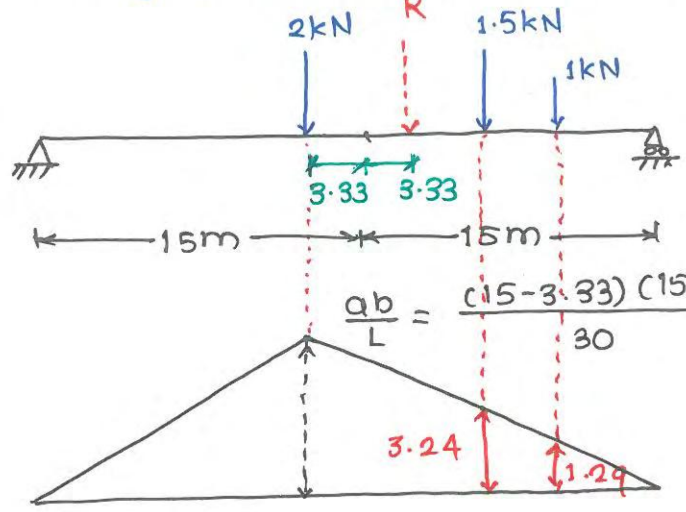


$\dfrac{ab}{L} = \dfrac{(15+1.67)(15-1.67)}{30} = 7.4$

$BM_{max} = 2(2.96) + 1.5(7.4) + 1(4.62)$

$= 21.64\ kNm.$

Case II: Considering 2 kN as adjucent load.



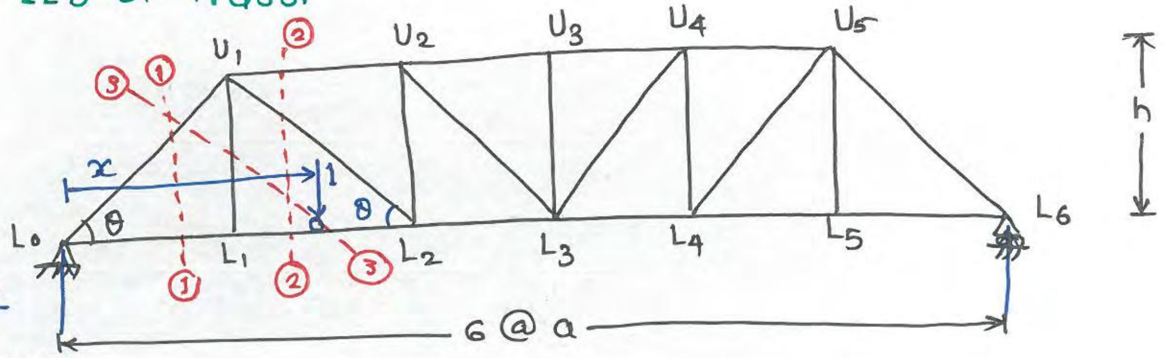
$$\frac{ab}{L} = \frac{(15-3.33)(15+3.33)}{30} = 7.13$$

$$BM_{max} = 2(7.1) + 1.5(3.24) + 1(1.29)$$

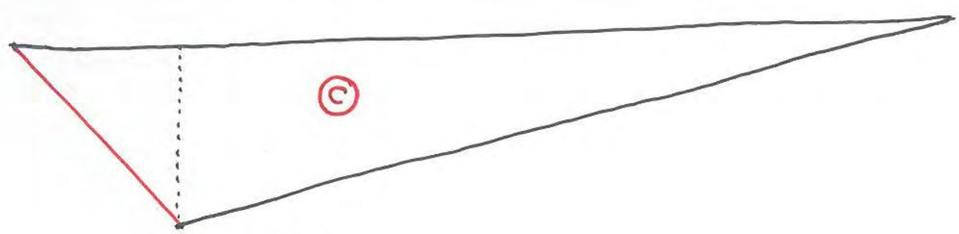
$$= 20.41 \text{ kNm}$$

So, absolute maximum BM is 21.64 kNm.

4.8 ILD of Truss:



$\frac{Pb}{L} = \frac{(6a-x)}{6a}$



$-\frac{5}{6}\operatorname{cosec}\theta$

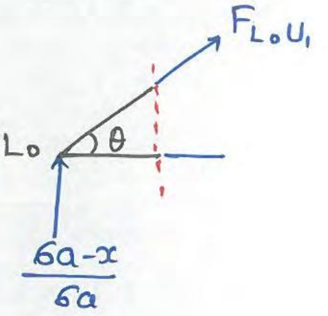
For L_0U_1:-

considering section 1 1 :-

Case I:- $x = 0$

$F_{L_0U_1} = 0$

Case II: $a \le x \le 6a$



$\Sigma F_y = 0$

$$\Rightarrow \frac{6a-x}{6a} + F_{L_0U_1} \cdot \sin\theta = 0$$

$$F_{L_0U_1} = -\left(\frac{6a-x}{6a}\right)\operatorname{cosec}\theta$$

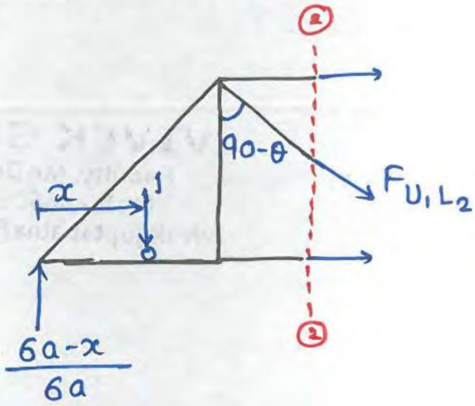
At $x = a$, $F_{L_0U_1} = -\frac{5}{6}\operatorname{cosec}\theta$

$x = 6a$, $F_{L_0U_1} = 0.$

For U_1L_2:

considering section 2-2

Case I: $0 \le x \le a$



$\Sigma F_y = 0$

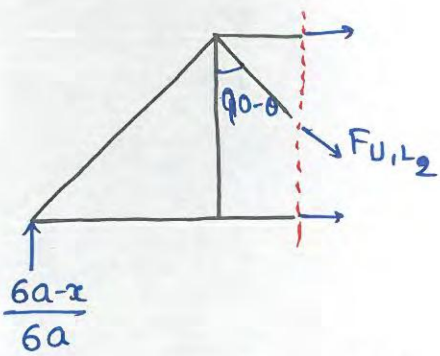
$$\Rightarrow \frac{6a-x}{6a} - 1 - F_{U_1L_2}\sin\theta = 0$$

$$\Rightarrow F_{U_1L_2} = -\frac{x}{6a}\operatorname{cosec}\theta$$

At $x = 0$, $F_{U_1L_2} = 0$

$x = a$, $F_{U_1L_2} = -\frac{1}{6}\operatorname{cosec}\theta$

Case II: $2a \le x \le 6a$



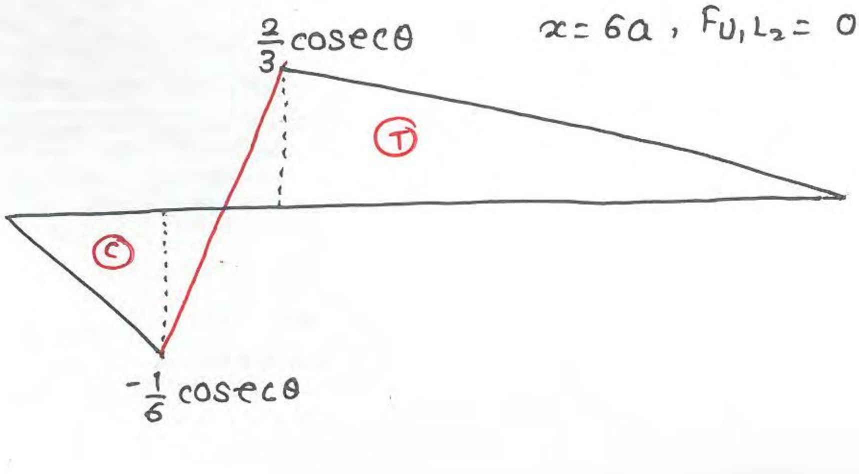
$\Sigma F_y = 0$

$$\Rightarrow \frac{6a-x}{6a} - F_{U_1L_2}\sin\theta = 0$$

$$F_{U_1L_2} = +\frac{(6a-x)}{6a}\operatorname{cosec}\theta.$$

At $x = 2a$, $F_{U_1L_2} = \frac{2}{3}\operatorname{cosec}\theta$

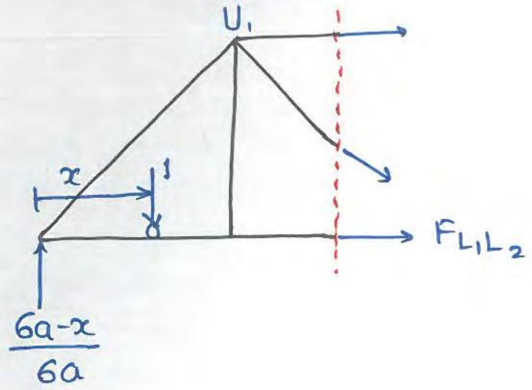
$x = 6a$, $F_{U_1L_2} = 0$



For L_1L_2:

considering section ② - ②

Case I: $0 \le x \le a$



$$\Sigma M_{U_1} = 0$$

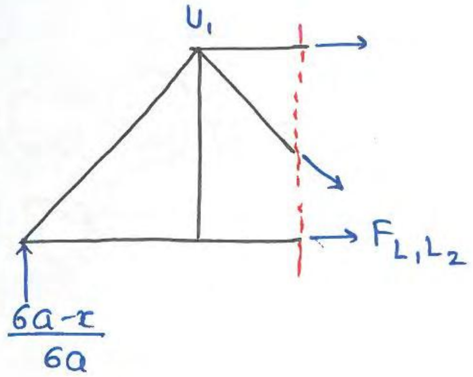
$$\Rightarrow \left(\frac{6a-x}{6a}\right).a - 1(x-a) - F_{L_1L_2} \times h = 0$$

$$\Rightarrow \quad F_{L_1L_2} = \frac{5x}{6h}$$

At $x = 0$, $F_{L_1L_2} = 0$

$x = a$, $F_{L_1L_2} = \frac{5a}{6h}$

Case II: $2a \le x \le 6a$



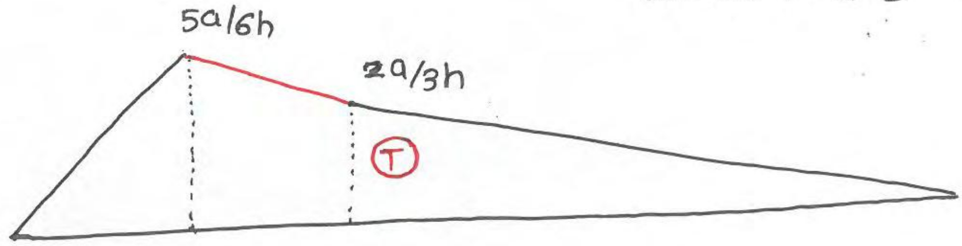
$$\Sigma M_{U_1} = 0$$

$$\Rightarrow \left(\frac{6a-x}{6a}\right) \cdot a - F_{L_1L_2} \times h = 0$$

$$\Rightarrow \quad F_{L_1L_2} = \frac{6a-x}{6h}$$

At $x = 2a$, $F_{L_1L_2} = \frac{2a}{3h}$

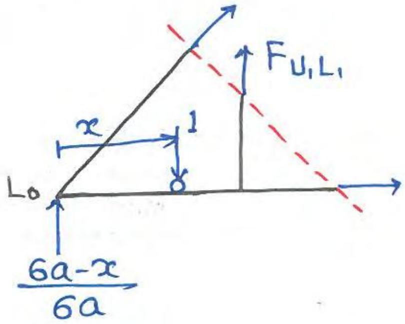
$x = 6a$, $F_{L_1L_2} = 0$



For U_1L_1:

Considering section ③ - ③

Case I: $0 \le x \le a$



$$\Sigma M_{L_0} = 0$$

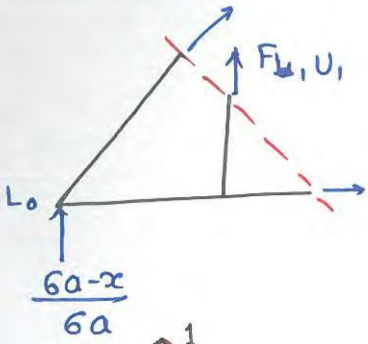
$$\Rightarrow \quad 1 \times x - F_{U_1L_1} \times a = 0$$

$$\Rightarrow \quad F_{U_1L_1} = \frac{x}{a}$$

At $x = 0$, $F_{U_1L_1} = 0$

$x = a$, $F_{U_1L_1} = 1$

Case II: $2a \le x \le 6a$



$$\Sigma M_{L_0} = 0$$

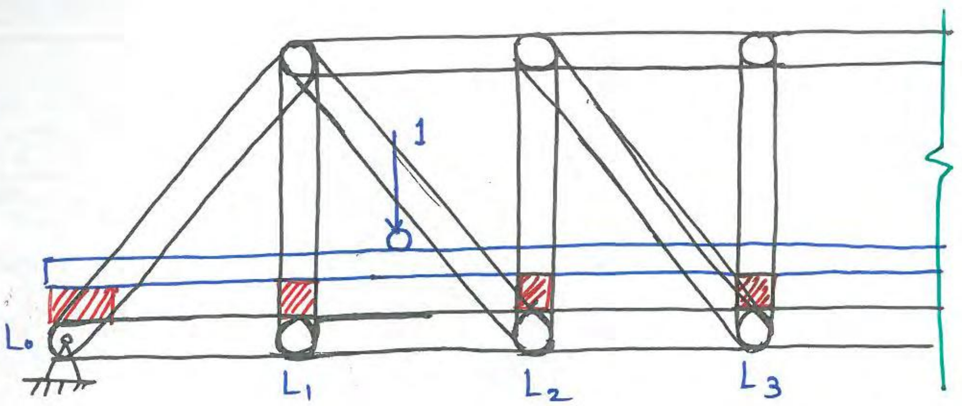
$$\Rightarrow -F_{L_1,U_1} \times a = 0$$

$$F_{L_1,U_1} = 0$$

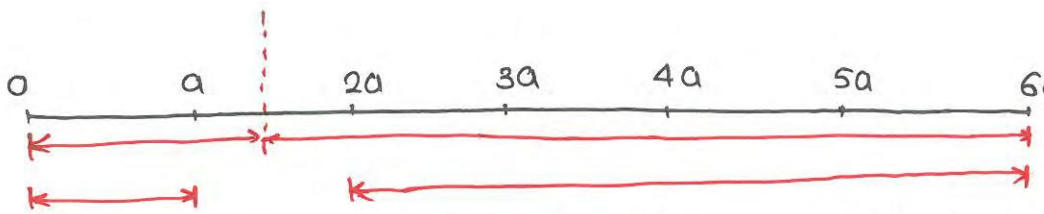
at $x = 2a$, $F_{L_1,U_1} = 0$

$x = 6a$, $F_{L_1,U_1} = 0$

* Note:



If Load is between L_0 to L_2 then some part of load is transfered at L_1 which will be balanced by member L_1U_1 so there is ILD of L_1U_1



Since position of section itself varies within any panel so it is better to not move load in that panel