2. Fractions

Questions Pg-24

1. Question

Explain each of the patterns below and write the general principle in algebra.

$$\frac{1^2+1}{1+1}=1 \quad \frac{2^2+2}{2+2}=1\frac{1}{2} \quad \frac{3^2+3}{3+3}=2 \quad \frac{4^2+4}{4+4}=2\frac{1}{2}$$

Answer

$$\frac{1^2 + 1}{1 + 1} = \frac{1(1 + 1)}{1(1 + 1)} = \frac{2}{2} = 1$$

$$\frac{2^2 + 2}{2 + 2} = \frac{2(2 + 1)}{2(1 + 1)} = \frac{3}{2} = 1\frac{1}{2}$$

$$\frac{3^2 + 3}{3 + 3} = \frac{3(3 + 1)}{3(1 + 1)} = \frac{4}{2} = 2$$

$$\frac{4^2 + 4}{4 + 4} = \frac{4(4 + 1)}{4(1 + 1)} = \frac{5}{2} = 2\frac{1}{2}$$

Generally, we can write,

$\frac{n^2+n}{n+n} = \frac{n(n+1)}{n(1+1)} = \frac{n+1}{2}$ where, n can be any natural number.

2. Question

Explain each of the patterns below and write the general principle in algebra.

$$\frac{2^2-2}{2-1}=2 \quad \frac{3^2-3}{3-1}=3 \quad \frac{4^2-4}{4-1}=4 \quad \frac{5^2-5}{5-1}=5$$

Answer

$$\frac{2^2-2}{2-1} = \frac{2(2-1)}{(2-1)} = 2$$

$$\frac{3^2-3}{3-1} = \frac{3(3-1)}{(3-1)} = 3$$

$$\frac{4^2-4}{4-1} = \frac{4(4-1)}{(4-1)} = 4$$

$$\frac{5^2-5}{5-1} = \frac{5(5-1)}{(5-1)} = 5$$

Generally, we can write,

$\frac{n^2-n}{n-1} = \frac{n(n-1)}{(n-1)} = n$ where, n can be any natural number.

3. Question

Explain each of the patterns below and write the general principle in algebra.

$$\frac{2^2-1}{2-1}=3 \quad \frac{3^2-1}{3-1}=4 \quad \frac{4^2-1}{4-1}=5 \quad \frac{5^2-1}{5-1}=6$$

Answer

using identity, $a^2 - b^2 = (a + b)(a-b)$, in the numerator.

$$\frac{2^2 - 1}{2 - 1} = \frac{(2 + 1)(2 - 1)}{(2 - 1)} = 3$$

$$\frac{3^2 - 1}{3 - 1} = \frac{(3 + 1)(3 - 1)}{(3 - 1)} = 4$$

$$\frac{4^2 - 1}{4 - 1} = \frac{(4 + 1)(4 - 1)}{(4 - 1)} = 5$$

$$\frac{5^2 - 1}{5 - 1} = \frac{(5 + 1)(5 - 1)}{(5 - 1)} = 6$$

Generally, we can write,

$\frac{n^2 - 1}{n - 1} = \frac{(n + 1)(n - 1)}{(n - 1)} = (n + 1)$ where, n can be any natural number.

Questions Pg-28

1. Question

Look at this method of making a pair of equal fraction from another such pair.

$$\frac{1}{3} = \frac{2}{6} \rightarrow \frac{1}{2} = \frac{3}{6}$$

$$\frac{3}{4} = \frac{9}{12} \rightarrow \frac{3}{9} = \frac{4}{12}$$

i) Check some more pairs of equal fractions. By interchanging the numerator of one with the denominator of the other, do you get equal fractions?

ii) Write this as a general principle using algebra and explain it.

Answer

i) $\frac{1}{4} = \frac{2}{8} \rightarrow \frac{1}{2} = \frac{4}{8}$

$$\frac{2}{3} = \frac{10}{15} \rightarrow \frac{2}{10} = \frac{3}{15}$$

$$\frac{2}{5} = \frac{4}{10} \rightarrow \frac{2}{4} = \frac{5}{10}$$

So, in all these examples we get equal fractions.

ii) General principle using algebra,

We know that,

When $\frac{a}{b} = \frac{p}{q}$ then, aq = pb .... (1)

Now, on dividing both sides by pq, we get

$$\frac{aq}{pq} = \frac{pb}{pq}$$

$$\Rightarrow \frac{a}{p} = \frac{b}{q}$$

2. Question

Look at these calculations:

$$\frac{1}{2} = \frac{2}{4} \quad \frac{(3\times1)+(4\times2)}{(3\times2)+(4\times4)} = \frac{11}{22} = \frac{1}{2}$$

$$\frac{1}{2} = \frac{3}{6} \quad \frac{(3\times1)+(4\times3)}{(3\times2)+(4\times6)} = \frac{15}{30} = \frac{1}{2}$$

i) Take some more fractions equal to $\frac{1}{2}$ and form fractions by multiplying the numerators and denominators by 3 and 4 and adding.

Do you get fractions equal to $\frac{1}{2}$?

ii) Take some other pairs of equal fractions and check this

iii) In all these, instead of multiplying numerators and denominators by 3 and 4, multiply by some other numbers and add. Do you still get equal fractions?

iv) Explain why, if the fraction $\frac{p}{q}$ is equal to the fraction $\frac{a}{b}$, then for any pair of natural numbers m and n, the fractions $\frac{ma+np}{mb+nq}$ is equal to $\frac{a}{b}$.

Answer

i) $\frac{1}{2} = \frac{4}{8}\ \frac{(3\times1)+(4\times4)}{(3\times2)+(4\times8)} = \frac{19}{38} = \frac{1}{2}$

$$\frac{1}{2} = \frac{5}{10}\ \frac{(3\times1)+(4\times5)}{(3\times2)+(4\times10)} = \frac{23}{46} = \frac{1}{2}$$

We observe that, in these cases also, we obtain $\frac{1}{2}$

ii) $\frac{2}{3} = \frac{4}{6}\ \frac{(3\times2)+(4\times4)}{(3\times3)+(4\times6)} = \frac{22}{33} = \frac{2}{3}$

$$\frac{2}{3} = \frac{10}{15}\ \frac{(3\times2)+(4\times10)}{(3\times3)+(4\times15)} = \frac{46}{69} = \frac{2}{3}$$

$$\frac{2}{3} = \frac{6}{9}\ \frac{(3\times2)+(4\times6)}{(3\times3)+(4\times9)} = \frac{30}{45} = \frac{2}{3}$$

We observe in these cases also, we obtain equal fraction, $\frac{2}{3}$

iii) let us now take 2 and 5 instead of 3 and 4 in part(ii).

$$\frac{2}{3} = \frac{4}{6}\ \frac{(2\times2)+(5\times4)}{(2\times3)+(5\times6)} = \frac{24}{36} = \frac{2}{3}$$

$$\frac{2}{3} = \frac{10}{15}\ \frac{(2\times2)+(5\times10)}{(2\times3)+(5\times15)} = \frac{54}{81} = \frac{2}{3}$$

$$\frac{2}{3} = \frac{6}{9}\ \frac{(2\times2)+(5\times6)}{(2\times3)+(5\times9)} = \frac{34}{51} = \frac{2}{3}$$

So, again we get equal fractions.

iv) We know that,

When $\frac{a}{b} = \frac{p}{q}$ then, aq = pb .... (1)

∴ for $\frac{ma + np}{mb + nq} = \frac{a}{b}$

(ma + np) × b = a × (mb + nq) →this must be satisfied

∴ (ma + np) × b = mab + npb ……….. (2)

And a × (mb + nq) = mab + nqa

= mab + npb ……….. using (1)

∴ a × (mb + nq) = mab + npb …………. (3)

Since, (2) is equal to (3)

∴ $\frac{ma + np}{mb + nq} = \frac{a}{b}$

3. Question

The sum of the square of a number and one, divided by the difference of 1 from the square gives $\frac{221}{220}$. What is the number?

Answer

Let the number be x.

⇒ $\frac{x^2 + 1}{x^2 - 1} = \frac{221}{220}$

⇒ $220(x^2 + 1) = 221(x^2 – 1)$

⇒ $220x^2 + 220 = 221x^2 – 221$

⇒ $x^2 = 441$

⇒ x = ±21

∴ The required number can be 21 or -21.

4. Question

The sum of a number and its square is one and a half times their difference. What is the number?

Answer

Let the number be x.

As per the condition,

⇒ $x + x^2 = \frac{1}{2}(x - x^2)$

⇒ $2x + 2x^2 = x – x^2$

⇒ $3x^2 + x = 0$

⇒ $3x\left(x + \frac{1}{3}\right) = 0$

⇒ x = 0 or $x = -\frac{1}{3}$

Questions Pg-34

1. Question

Find the larger of each pair of fractions below, without multiplication:

i) $\frac{13}{17}, \frac{14}{15}$ ii) $\frac{13}{17}, \frac{11}{18}$ iii) $\frac{14}{15}, \frac{11}{18}$

Answer

We know that,

When the denominators of the fractions are equal ,

then the fraction having greater numerator is larger than

the other. Example: $\frac{3}{4} > \frac{2}{4}$

Also, When the numerators of the fractions are equal ,

then the fraction having smaller denominator is larger

than the other. Example: $\frac{6}{4} > \frac{6}{5}$

on combining these two statements we can conclude that,

The fraction having larger numerator and also smaller denominator is greater.

Now,

i) Among $\frac{13}{17}$ and $\frac{14}{15}$

since, 14 is greater than 13 and 15 is smaller than 17.

$\therefore \frac{13}{17} < \frac{14}{15}$

ii) Among $\frac{13}{17}$ and $\frac{11}{18}$

since, 13 is greater than 11 and 17 is smaller than 18.

$\therefore \frac{13}{17} > \frac{11}{18}$

iii) Among $\frac{14}{15}$ and $\frac{11}{18}$

since, 14 is greater than 11 and 15 is smaller than 18.

$\therefore \frac{11}{18} < \frac{14}{15}$

2. Question

Find the larger of each pair of fractions below, without pen and paper.

i) $\frac{3}{5}, \frac{8}{13}$ ii) $\frac{3}{5}, \frac{6}{11}$ iii) $\frac{101}{102}, \frac{98}{99}$

Answer

i) $\frac{3}{5}, \frac{8}{13}$

We know that,

$\frac{a}{b} < \frac{p}{q}$ when aq<pb

3 × 13 = 39

5 × 8 = 40

Since, 39<40

$\therefore \frac{3}{5} < \frac{8}{13}$

ii) $\frac{3}{5}, \frac{6}{11}$

We know that,

$\frac{a}{b} > \frac{p}{q}$ when aq>pb

3 × 11 = 33

5 × 6 = 30

Since, 33>30

$\therefore \frac{3}{5} > \frac{6}{11}$

iii) $\frac{98}{99}, \frac{101}{102}$

In $\frac{98}{99}$ adding 3 to both numerator and denominator gives $\frac{101}{102}$ and since, 98<99

$\therefore \frac{98}{99} < \frac{101}{102}$ ……… using, $\frac{a}{b} < \frac{a+n}{b+n}$, when b>a

3. Question

i) Find three fractions larger than $\frac{1}{3}$ and smaller than $\frac{1}{2}$.

ii) Find three such fractions, all with the denominator 24.

iii) Find three such fractions, all with the numerator 4.

Answer

i) we know that, when $\frac{a}{b} < \frac{p}{q}$ then $\frac{a}{b} < \frac{a+p}{b+q} < \frac{p}{q}$ …….. (1)

$$\frac{1}{3} < \frac{1}{2}$$

$\Rightarrow \frac{1}{3} < \frac{2}{5} < \frac{1}{2}$ ……….using (1)

$\Rightarrow \frac{1}{3} < \frac{3}{8} < \frac{2}{5} < \frac{3}{7} < \frac{1}{2}$ ………..using (1)

$\therefore \frac{3}{8}, \frac{2}{5}, \frac{3}{7}$ are the required fractions.

ii) Lets make the denominator of $\frac{1}{3}$ and $\frac{1}{2}$ equal to 24.

$$\frac{1 \times 8}{3 \times 8} = \frac{8}{24}$$

$$\frac{(1 \times 12)}{2 \times 12} = \frac{12}{24}$$

Now, we can say that,

$$\frac{8}{24} < \frac{9}{24} < \frac{10}{24} < \frac{11}{24} < \frac{12}{24}$$

$\therefore \frac{9}{24}, \frac{10}{24}$ and $\frac{11}{24}$ are the required fractions.

iii) Lets make the numerator of $\frac{1}{3}$ and $\frac{1}{2}$ equal to 4.

$$\frac{1 \times 4}{3 \times 4} = \frac{4}{12}$$

$$\frac{1 \times 4}{2 \times 4} = \frac{4}{8}$$

Now, we can say that,

$$\frac{4}{12} < \frac{4}{11} < \frac{4}{10} < \frac{4}{9} < \frac{4}{8}$$

$\therefore \frac{4}{11}, \frac{4}{10}, \frac{4}{9}$ are the required fractions.

4. Question

From a fraction, a new fraction is formed by adding the same natural number to both the numerator and the denominator.

i) In what kind of fractions does this give a larger fraction?

ii) In what kind of fractions does this give a smaller fraction?

Answer

Let the fraction be $\frac{a}{b}$

Now, fraction obtained by adding a natural number n to both numerator and denominator = $\frac{a+n}{b+n}$

i) $\frac{a}{b} < \frac{a+n}{b+n}$ when b>a

Proof: $\frac{a}{b} < \frac{a+n}{b+n}$

⇒ a(b + n) < (a + n)b

⇒ ab + an < ab + bn

⇒ an < bn

⇒ a < b

ii) $\frac{a}{b} > \frac{a+n}{b+n}$ when b<a

Proof: $\frac{a}{b} > \frac{a+n}{b+n}$

⇒ a(b + n) > (a + n)b

⇒ ab + an > ab + bn

⇒ an > bn

⇒ a > b

Questions Pg-38

1. Question

Find the general principle of each of the patterns below and explain it using algebra.

$$1-\frac{1}{3}=\frac{2}{3}=\frac{2}{2^2-1}; \frac{1}{2}-\frac{1}{4}=\frac{2}{8}=\frac{2}{3^2-1};$$

$$\frac{1}{3}-\frac{1}{5}=\frac{2}{15}=\frac{2}{4^2-1}.$$

Answer

On observing the pattern we get general formula as,

$$\frac{1}{n-1}-\frac{1}{n+1}=\frac{2}{n^2-1}$$

To prove this algebraically,

We know that $\frac{a}{b} - \frac{p}{q} = \frac{aq-bp}{bq}$ so,

$$\frac{1}{n-1} - \frac{1}{n+1} = \frac{(n+1)-(n-1)}{(n-1)(n+1)}$$

$$= \frac{n+1-n+1}{n^2-1}$$

Using, (a-b)(a + b) = $a^2 - b^2$

$$= \frac{2}{n^2-1}$$

Hence, proved!

2. Question

Find the general principle of each of the patterns below and explain it using algebra.

$$\frac{1}{2}+\frac{2}{1}=\frac{5}{2}=2+\frac{1}{1\times 2};\frac{2}{3}+\frac{3}{2}=\frac{13}{6}$$
$$=2+\frac{1}{2\times 3};\frac{3}{4}+\frac{4}{3}=\frac{25}{12}=2+\frac{1}{3\times 4}$$

Answer

On observing the pattern we get general formula as

$$\frac{n}{n+1} + \frac{n+1}{n} = 2 + \frac{1}{n(n+1)}$$

To prove this algebraically,

We know that $\frac{a}{b} - \frac{p}{q} = \frac{aq-bp}{bq}$ so,

$$\frac{n}{n+1} + \frac{n+1}{n} = \frac{n^2 + (n+1)^2}{n(n+1)}$$

$$= \frac{n^2 + n^2 + 2n + 1}{n(n+1)}$$

$$= \frac{2n^2 + 2n + 1}{n(n+1)}$$

$$= \frac{2n(n+1) + 1}{n(n+1)}$$

$$= \frac{2n(n+1)}{n(n+1)} + \frac{1}{n(n+1)}$$

$$= 2 + \frac{1}{n(n+1)}$$

Hence, proved!

3. Question

Find the general principle of each of the patterns below and explain it using algebra.

$$\frac{1}{2}+\frac{1}{3}=\frac{5}{6} \qquad \frac{3}{2}-\frac{2}{3}=\frac{5}{6}$$

$$\frac{1}{3}+\frac{1}{4}=\frac{7}{12} \qquad \frac{4}{3}-\frac{3}{4}=\frac{7}{12}$$

$$\frac{1}{4}+\frac{1}{5}=\frac{9}{20} \qquad \frac{5}{4}-\frac{4}{5}=\frac{9}{20}$$

Answer

On observing the pattern we get general formula as

$$\frac{1}{n}+\frac{1}{n+1}=\frac{(n+1)+n}{n(n+1)}=\frac{n+1}{n}-\frac{n}{n+1}$$

To prove this algebraically,

We know that $\frac{a}{b}-\frac{p}{q}=\frac{aq-bp}{bq}$ so,

$\frac{1}{n}+\frac{1}{n+1}=\frac{(n+1)+n}{n(n+1)}$ ....(1)

$$\frac{n+1}{n}-\frac{n}{n+1}=\frac{(n+1)^2-n^2}{n(n+1)}$$

$$=\frac{(n+1-n)(n+1+n)}{n(n+1)}$$

∵ using, $a^2-b^2=(a-b)(a+b)$

$$=\frac{n+1+n}{n(n+1)}$$

$=\frac{(n+1)+n}{n(n+1)}$ ....(2)

From (1) and (2) , we get

$$\frac{1}{n}+\frac{1}{n+1}=\frac{(n+1)+n}{n(n+1)}=\frac{n+1}{n}-\frac{n}{n+1}$$

Hence, proved!

4. Question

Find the general principle of each of the patterns below and explain it using algebra.

$$4\frac{1}{4}-1\frac{1}{2}=3 \qquad 4\frac{1}{2}-1\frac{1}{2}=3$$

$$5\frac{1}{3}-1\frac{1}{3}=4 \qquad 5\frac{1}{3}-1\frac{1}{3}=4$$

$$6\frac{1}{4}-1\frac{1}{4}=5 \qquad 6\frac{1}{4}-1\frac{1}{4}=5$$

Answer

On observing the pattern we get general formula as

$$(n+2)\frac{1}{n}-1\frac{1}{n}=(n+1)=(n+2)\frac{1}{n}\div 1\frac{1}{n}$$

To prove this algebraically,

We know that $\frac{a}{b} - \frac{p}{q} = \frac{aq - bp}{bq}$ so,

$$(n + 2)\frac{1}{n} - 1\frac{1}{n} = \frac{n(n + 2) + 1}{n} - \frac{n + 1}{n}$$

$$= \frac{n^2 + 2n + 1}{n} - \frac{n + 1}{n}$$

$$= \frac{n^2 + 2n + 1 - n - 1}{n}$$

$$= \frac{n^2 + n}{n}$$

$$= \frac{n(n + 1)}{n} = (n + 1) \text{ ....(1)}$$

$$(n + 2)\frac{1}{n} \div 1\frac{1}{n} = \frac{n(n + 2) + 1}{n} \div \frac{n + 1}{n}$$

$$= \frac{n(n + 2) + 1}{n} \times \frac{n}{n + 1}$$

$$= \frac{n^2 + 2n + 1}{n + 1}$$

$$= \frac{(n + 1)^2}{n + 1} = (n + 1) \text{ ....(2)}$$

From (1) and (2) , we get

$$(n + 2)\frac{1}{n} - 1\frac{1}{n} = (n + 1) = (n + 2)\frac{1}{n} \div 1\frac{1}{n}$$

Hence, proved!

Questions Pg-45

1. Question

Find the fraction of denominator is a power of 10 equal to each of the fractions below, and then write their decimal forms:

i) $\frac{1}{50}$ ii) $\frac{3}{40}$ iii) $\frac{5}{16}$ iv) $\frac{12}{625}$

Answer

i) $\frac{1}{50} = \frac{1 \times 2}{5 \times 10 \times 2} = \frac{2}{100} = 0.02$

ii) $\frac{3}{40} = \frac{3 \times 5^2}{2^2 \times 10 \times 5^2} = \frac{75}{1000} = 0.075$

iii) $\frac{5}{16} = \frac{5 \times 5^4}{2^4 \times 5^4} = \frac{3125}{10000} = 0.3125$

iv) $\frac{12}{625} = \frac{12 \times 2^4}{5^4 \times 2^4} = \frac{192}{10000} = 0.0192$

2 A. Question

Find fractions of denominators which are power of 10, getting closer and closer to each of the fractions below and then write their decimal form.

$\frac{5}{3}$

Answer

$$\frac{5}{6} = \frac{1}{10} \times \frac{50}{6}$$

$$= \frac{1}{10}\left(8 + \frac{2}{6}\right) = \frac{8}{10} + \frac{1}{30}$$

$$\Rightarrow \frac{5}{6} - \frac{8}{10} = \frac{1}{30}$$

Thus, we get a fraction of denominator 10, close to $\frac{5}{6}$

Now, to get a fraction of denominator 100, close to $\frac{5}{6}$,

start with

$$\frac{5}{6} = \frac{1}{100} \times \frac{500}{6}$$

$$= \frac{1}{100}\left(83 + \frac{2}{6}\right) = \frac{83}{100} + \frac{1}{300}$$

$$\Rightarrow \frac{5}{6} - \frac{83}{100} = \frac{1}{300}$$

Similarly, we can have

$$\frac{5}{6} = \frac{1}{1000} \times \frac{5000}{6}$$

$$= \frac{1}{1000}\left(833 + \frac{2}{6}\right) = \frac{833}{1000} + \frac{1}{3000}$$

$$\Rightarrow \frac{5}{6} - \frac{833}{1000} = \frac{1}{3000}$$

We observe that, the fractions $\frac{8}{10}, \frac{83}{100}, \frac{833}{1000}$ get closer and closer to $\frac{5}{6}$

∴ we can write,

$$\frac{5}{6} = 0.833$$

2 B. Question

Find fractions of denominators which are power of 10, getting closer and closer to each of the fractions below and then write their decimal form.

$$\frac{3}{11}$$

Answer

$$\frac{3}{11} = \frac{1}{10} \times \frac{30}{11}$$

$$= \frac{1}{10}\left(2 + \frac{8}{11}\right) = \frac{2}{10} + \frac{8}{110}$$

$$\Rightarrow \frac{3}{11} - \frac{2}{10} = \frac{8}{110}$$

Thus, we get a fraction of denominator 10, close to $\frac{3}{11}$

Now, to get a fraction of denominator 100, close to $\frac{3}{11}$, start with

$$\frac{3}{11} = \frac{1}{100} \times \frac{300}{11}$$

$$= \frac{1}{100}\left(27 + \frac{3}{11}\right) = \frac{27}{100} + \frac{3}{1100}$$

$$\Rightarrow \frac{3}{11} - \frac{27}{100} = \frac{3}{1100}$$

Similarly, we can have

$$\frac{3}{11} = \frac{1}{1000} \times \frac{3000}{11}$$

$$= \frac{1}{1000}\left(272 + \frac{8}{11}\right) = \frac{272}{1000} + \frac{8}{11000}$$

$$\Rightarrow \frac{3}{11} - \frac{272}{1000} = \frac{8}{11000}$$

We observe that, the fractions $\frac{2}{10}, \frac{27}{100}, \frac{272}{1000}$ get closer and closer to $\frac{3}{11}$

∴ we can write,

$$\frac{3}{11} = 0.272$$

2 C. Question

Find fractions of denominators which are power of 10, getting closer and closer to each of the fractions below and then write their decimal form.

$$\frac{23}{11}$$

Answer

$$\frac{23}{11} = \frac{1}{10} \times \frac{230}{11}$$

$$= \frac{1}{10}\left(20 + \frac{10}{11}\right) = \frac{20}{10} + \frac{1}{11}$$

$$\Rightarrow \frac{23}{11} - \frac{20}{10} = \frac{1}{11}$$

Thus, we get a fraction of denominator 10, close to $\frac{23}{11}$

Now, to get a fraction of denominator 100, close to $\frac{23}{11}$,

start with

$$\frac{23}{11} = \frac{1}{100} \times \frac{2300}{11}$$

$$= \frac{1}{100}\left(209 + \frac{1}{11}\right) = \frac{209}{100} + \frac{1}{1100}$$

$$\Rightarrow \frac{23}{11} - \frac{209}{100} = \frac{1}{1100}$$

Similarly, we can have

$$\frac{23}{11} = \frac{1}{1000} \times \frac{23000}{11}$$

$$= \frac{1}{1000}\left(2090 + \frac{10}{11}\right) = \frac{2090}{1000} + \frac{1}{1100}$$

$$\Rightarrow \frac{23}{11} - \frac{2090}{1000} = \frac{1}{1100}$$

$$\frac{23}{11} = \frac{1}{10000} \times \frac{230000}{11}$$

$$= \frac{1}{10000}\left(20909 + \frac{1}{11}\right) = \frac{20909}{10000} + \frac{1}{110000}$$

$$\Rightarrow \frac{23}{11} - \frac{20909}{10000} = \frac{1}{110000}$$

We observe that, the fractions $\frac{20}{10}, \frac{209}{100}, \frac{2090}{1000}, \frac{20909}{10000}$ get closer and closer to $\frac{23}{11}$

∴ we can write,

$$\frac{23}{11} = 2.0909$$

2 D. Question

Find fractions of denominators which are power of 10, getting closer and closer to each of the fractions below and then write their decimal form.

$$\frac{1}{13}$$

Answer

$$\frac{1}{13} = \frac{1}{10} \times \frac{10}{13}$$

$$= \frac{1}{10}\left(0 + \frac{10}{13}\right) = \frac{0}{10} + \frac{1}{13}$$

$$\Rightarrow \frac{1}{13} - \frac{0}{10} = \frac{1}{13}$$

Thus, we get a fraction of denominator 10, close to $\frac{1}{13}$

Now, to get a fraction of denominator 100, close to $\frac{1}{13}$,

start with

$$\frac{1}{13} = \frac{1}{100} \times \frac{100}{13}$$

$$= \frac{1}{100}\left(7 + \frac{9}{13}\right) = \frac{7}{100} + \frac{9}{1300}$$

$$\Rightarrow \frac{1}{13} - \frac{7}{100} = \frac{9}{1300}$$

Similarly, we can have

$$\frac{1}{13} = \frac{1}{1000} \times \frac{1000}{13}$$

$$= \frac{1}{1000}\left(76 + \frac{12}{13}\right) = \frac{76}{1000} + \frac{12}{13000}$$

$$\Rightarrow \frac{5}{6} - \frac{76}{1000} = \frac{12}{13000}$$

We observe that, the fractions $\frac{0}{10}, \frac{7}{100}, \frac{76}{1000}$ get closer and closer to $\frac{1}{13}$

∴ we can write,

$$\frac{1}{13} = 0.076$$

3. Question

i) Explain using algebra, that the fractions $\frac{1}{10}, \frac{11}{100}, \frac{111}{1000}, \ldots$ gets closer and closer to $\frac{1}{9}$.

ii) Using the general principle above on single digit numbers, find the decimal forms of $\frac{2}{9}, \frac{4}{9}, \frac{5}{9}, \frac{7}{9}, \frac{8}{9}$ (why $\frac{2}{9}$ and $\frac{6}{9}$ are left out in this?)

iii) What can we say in general about those decimal forms in which a single digit repeats?

Answer

i) We can easily see that

$$\frac{1}{9} - \frac{1}{10} = \frac{1}{90}$$

$$\frac{1}{9} - \frac{11}{100} = \frac{1}{900}$$

$$\frac{1}{9} - \frac{111}{1000} = \frac{1}{9000}$$

Since, $\frac{1}{9000} < \frac{1}{900} < \frac{1}{90}$

Thus, $\frac{1}{10}, \frac{11}{100}, \frac{111}{1000}$ get closer and closer to $\frac{1}{9}$

ii) From part (i),

We can write $\frac{1}{9} = 0.111$

∴ $\frac{2}{9} = 2 \times \frac{1}{9} = 0.222$

$$\frac{4}{9} = 4 \times \frac{1}{9} = 0.444$$

$$\frac{5}{9} = 5 \times \frac{1}{9} = 0.555$$

$$\frac{7}{9} = 7 \times \frac{1}{9} = 0.777$$

$$\frac{8}{9} = 8 \times \frac{1}{9} = 0.888$$

$\frac{3}{9}$ and $\frac{6}{9}$ are excluded because they in these fractions numerator and denominator have common factors.

iii) The decimal forms in which a single digit repeat are generally those in which we get the same remainder after each step. These forms are called repeating or recurring decimal forms.