9

Logarithms

9.01 Introduction :

We have to solve many problems from mathematical method in which expression contains larger multiple product, division or rational powers. It takes huge time to solve it, and possibility of errors. Many mathematician have paid attention for this and John Napier gave hypothesis of logrithms in 1614 B.C. By using logrithms we have convert the complex problems like large multiplication, divison and power into addition, substraction and multiplication respectively. So, complex problems have been solved easily in limited time. In this chapter, we will study the different rules of logrithms and by using logrithms we will know the importance of it.

9.02 Logarithms :

We write the continuous multiplication in form of power, like $2 \times 2 \times 2 = 8$, is also written as 2^3 . In this, 2 is called the base and 3 is called the index.

In general from, if three numbers a, x and n have relation like $a^x = n$, then a is called the base and x is called the index.

Definition : If a and n are positive real number (where $a \neq 1$) and $a^x = n$ (Index from) then x is called logrithms of n to the base a. It is written as $x = \log_a n$

Therefore, $\log_a n = x \quad [a > 0,\ a \neq 1,\ n > 0]$

Note : (i) log is brief version of logarithm.

(ii) Logarithm of 0 and negative number never be real. If $k < 0$ then $\log_a k$ is imaginary.

(iii) $\log_a 1 = 0$, Where $0 < a < 1$ or $a > 1$

(iv) $\log_a a = 1$, Where $0 < a < 1$ or $a > 1$

(v) In next studies, the base a of logarithm is always $a > 0$ and $a \neq 1$

Illustrative Examples

Example 1 : Convert the following Exponential into Logarithmic form :

(i) $3^4 = 81$ (ii) $2^{-5} = 1/32$ (iii) $(81)^{\frac{1}{4}} = 3$

Solution : (i) $3^4 = 81$

Logarithmic, form $\log_3 81 = 4$.

(ii) $2^{-5} = \frac{1}{32}$

Logarithmic form , $\log_2\left(\frac{1}{32}\right) = -5$.

(iii) $(81)^{\frac{1}{4}} = 3$

Logarithmic form, $\log_{81} 3 = \frac{1}{4}$.

Example 2 : Convert the following Logarithmic form into Exponential form :

(i) $\log_7 1 = 0$ (ii) $\log_{128} 2 = \frac{1}{7}$ (iii) $\log_{10} 1000 = 3$

Solution : (i) $\log_7 1 = 0$ Exponential form, $7^0 = 1$.

(ii) $\log_{128} 2 = \frac{1}{7}$, Exponential form $128^{1/7} = 2$.

(iii) $\log_{10} 1000 = 3$, Exponential form, $10^3 = 1000$.

Note : $\log_m 1 = \log_n 1 = 0 \left(m \neq n\right)$

Exercise 9.1

Write the following in Logarithmic form [Q.1 to Q.6]

1. $2^6 = 64$
2. $10^4 = 10000$
3. $2^{10} = 1024$
4. $5^{-2} = \frac{1}{25}$
5. $10^{-3} = 0 \cdot 001$
6. $4^{3/2} = 8$

Write the following in Exponential form [Q.7 to Q.12]

7. $\log_5 25 = 2$
8. $\log_3 729 = 6$
9. $\log_{10} \left(0 \cdot 001\right) = -3$
10. $\log_{10} \left(0 \cdot 1\right) = -1$
11. $\log_3 \left(\frac{1}{27}\right) = -3$
12. $\log_{\sqrt{2}} 4 = 4$
13. If $\log_{81} x = \frac{3}{2}$, then find the value of x
14. If $\log_{125} p = 1/6$, then find the value of p
15. If $\log_4 m = 1 \cdot 5$, then find the value of m
16. Prove that : $\log_4 \left[\log_2 \left\{\log_2 \left(\log_3 81\right)\right\}\right] = 0$

9.03 Fundamental laws of logarithms :

Rule 1 : Logarithm of Product of two terms is equal to the sum of Logarithm of individual terms, that means, $\log_a \left(MN\right) = \log_a M + \log_a N$.

Proof : Let $M = a^x$

$\therefore$ $\log_a M = x$ (i)

and $N = a^y$

$\therefore$ $\log_a N = y$ (ii)

Now, $MN = a^x \times a^y$

or, $MN = a^{(x+y)}$ (iii)

(iii) Using definition of Logarithm

$\log_a \left(MN\right) = x + y$

or $\log_a \left(MN\right) = \log_a M + \log_a N$ [from (i) & (ii)]

Rule 1 : Can be generalised as :

$$\log_a (A \times B \times C \times D \times \cdots \times Z) = \log_a A + \log_a B + \log_a C + \cdots + \log_a Z.$$

Note : $\log_a (M + N + P) \neq \log_a M + \log_a N + \log_a P$

But, if $M + N + P = MNP$, then

$$\log_a (M + N + P) = \log_a M + \log_a N + \log_a P$$

Rule 2 : Logarithm of quotient of two numbers is equal to the difference of Logarithm of numerator and logrithm of denominator, that means,

$$\log_a \left(\frac{M}{N}\right) = \log_a M - \log_a N$$

Proof : Let

$M = a^x$ $\therefore$ $\log_a M = x$ (i)

and $N = a^y$ $\therefore$ $\log_a N = y$ (ii)

Now, $\dfrac{M}{N} = \dfrac{a^x}{a^y} = a^{(x-y)}$ (iii)

Using definition of logarithm

$$\log_a (M / N) = (x - y)$$

$$\log_a (M / N) = \log_a M - \log_a N, \quad \text{[from (i) and (ii)]}$$

Note : $\log_a (M - N) \neq \log_a M - \log_a N$

Rule 3 : Logarithm of an exponential is equal to the product of index and logarithm of that number that means,

$$\log_a M^N = N \log_a M$$

Proof: Let $M = a^x$ $\therefore$ $\log_a M = x$ ..... (i)

Now, $M^N = (a^x)^N$

or $M^N = a^{Nx}$ .....(ii)

By definition of logarithm

$$\log_a M^N = Nx$$

$$\Rightarrow \log_a M^N = N \log_a M, \text{ [from (i)]}$$

If $N = \dfrac{p}{q}$,

then $\log_a M^{(p/q)} = \dfrac{p}{q} \log_a M$

Rule 4 : If $a > 1$, $\log_a 0 = -\infty$, and if $0 < a < 1$, $\log_a 0 = +\infty$,

Proof: We know that, $3^{-\infty} = \dfrac{1}{3^{\infty}} = \dfrac{1}{\infty} = 0$

$\therefore$ If $a^x = 0$ and $a > 1$, then $x = -\infty$

i.e.. if $a > 1$, then $\qquad a^{-\infty} = 0$

$\therefore$ By definition of logarithm,

$$\log_a 0 = -\infty \qquad (\text{where } a > 1)$$

Similarly, we know that

$$\left(\frac{1}{3}\right)^{\infty} = \frac{1}{3^{\infty}} = \frac{1}{\infty} = 0$$

$\therefore$ if $a^x = 0$ and $0 < a < 1$, then $x = +\infty$

i.e., if $a < 1$ then $a^{+\infty} = 0$

$\therefore$ By definition,

$$\log_a 0 = +\infty \qquad (\text{where } 0 < a < 1)$$

Note : It can be shown graphically :

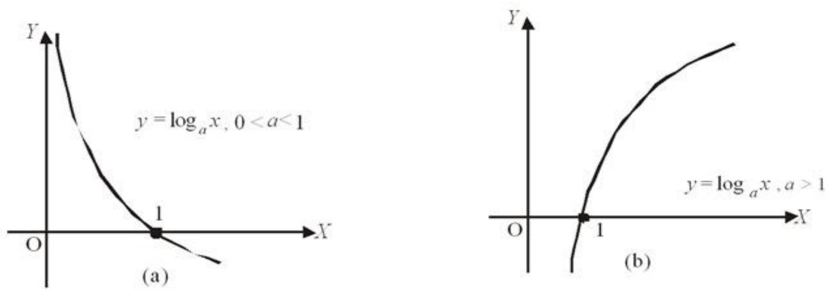

Fig. 9.01

Rule 5 : To Prove : $M = a^{\log_a M}$

Proof : Let $\log_a M = x$ ..... (i)

$a^x = M$ ..... (ii)

Using (i) in (ii), we get

$$M = a^{\log_a M}$$

In perticular, using 'e' instead of a, we get

$$M = e^{\log_e M}$$

9.04 Base changing formula :

To Prove : $\log_b M = \dfrac{\log_a M}{\log_a b}$

Proof : Let $\log_b M = x \quad \therefore \quad M = b^x$ (i)

and $\log_a b = y \quad \therefore \quad b = a^y$ (ii)

Using (ii) in (i), we get

$$M = b^x = \left(a^y\right)^x$$

$$\Rightarrow \qquad M = \left(a^y\right)^x$$

$$\Rightarrow \qquad M = a^{xy}$$

$\Rightarrow \quad \log_a M = xy$ (By definition of logarithm)

$\Rightarrow \quad \log_a M = \log_b M \times \log_a b$

$$\Rightarrow \quad \log_b M = \frac{\log_a M}{\log_a b}$$

To Prove : $\log_b a \times \log_a b = 1$

Proof : Let $\log_b a = x$ (i)

$\therefore \quad b^x = a$

$\Rightarrow \quad b = a^{1/x}$

$\Rightarrow \quad \log_a b = 1/x$ (By definition of logarithm) (ii)

from (i) and (ii) $\log_b a \times \log_a b = x \times 1/x = 1$

Note : If the base of logarithm is not mentioned, then it is taken as 10

Illustrative Examples

Example 3 : Prove that $\log 540 = 2\log 2 + 3\log 3 + \log 5.$

Solution : L.H.S. $= \log 540$

$= \log\left(2^2 \times 3^3 \times 5\right)$

$= \log 2^2 + \log 3^3 + \log 5$

$= 2\log 2 + 3\log 3 + \log 5 =$ R.H.S.

Example 4 : Prove that $\log(1+2+3) = \log 1 + \log 2 + \log 3$

Solution : L.H.S. $= \log(1+2+3)$

$= \log 6$

$= \log(1 \times 2 \times 3)$

$= \log 1 + \log 2 + \log 3 =$ R.H.S.

Example 5 : If $\log_{10} 2 = 0\cdot3010$ and $\log_{10} 3 = 0\cdot4771$, then find the value of :

(i) $\log_{10} 5$ (ii) $\log_{10} 24$ (iii) $\log_{10}(8/9)$

Solution : (i) $\log_{10} 5 = \log_{10}\left(\frac{5 \times 2}{2}\right) = \log_{10}\frac{10}{2} = \log_{10} 10 - \log_{10} 2 = 1 - 0\cdot3010 = \cdot6990$

(ii) $\log_{10} 24 = \log_{10}\left(2^3 \times 3\right) = 3\log_{10} 2 + \log_{10} 3$

$= 3(0\cdot3010) + 0\cdot4771 = 0\cdot9030 + 0\cdot4771 = 1\cdot3801$

(iii) $\log_{10}(8/9) = \log_{10} 8 - \log_{10} 9 = \log_{10} 2^3 - \log_{10} 3^2 = 3\log_{10} 2 - 2\log_{10} 3$

$= 3(0\cdot3010) - 2(0\cdot4771) = \cdot9030 - 0\cdot9542 = -0\cdot0512$

Example 6 : Prove that : $4\log\frac{24}{25} - 16\log\frac{9}{10} + 7\log\frac{81}{80} = \log 5.$

Solution : L.H.S. $= 4\log\frac{24}{25} - 16\log\frac{9}{10} + 7\log\frac{81}{80}$

$= 4(\log 24 - \log 25) - 16(\log 9 - \log 10) + 7(\log 81 - \log 80)$

$= 4\log 24 - 4\log 25 - 16\log 9 + 16\log 10 + 7\log 81 - 7\log 80$

$= 4\log\left(2^3 \times 3\right) - 4\log 5^2 - 16\log 3^2 + 16\log(2\times 5) + 7\log 3^4 - 7\log\left(2^4 \times 5\right)$

$= 4\left\{\log 2^3 + \log 3\right\} - 8\log 5 - 32\log 3 + 16\left(\log 2 + \log 5\right) + 28\log 3 - 7\left(\log 2^4 + \log 5\right)$

$= 12\log 2 + 4\log 3 - 8\log 5 - 32\log 3 + 16\log 2 + 16\log 5 + 28\log 3 - 28\log 2 - 7\log 5$

$= 28\log 2 - 28\log 2 + 32\log 3 - 32\log 3 + 16\log 5 - 15\log 5$

$= \log 5 = \text{R.H.S.}$

Example 7 : Solve the following equation :

$$\log(x+1) + \log(x-1) = \log 11 + 2\log 3.$$

Solution : $\log(x+1) + \log(x-1) = \log 11 + 2\log 3$

$\Rightarrow \quad \log\{(x+1)(x-1)\} = \log 11 + \log 3^2$

$\Rightarrow \quad \log\left(x^2 - 1\right) = \log 11 + \log 9$

$\Rightarrow \quad \log\left(x^2 - 1\right) = \log(11 \times 9)$

$\Rightarrow \quad \log\left(x^2 - 1\right) = \log 99$

$\Rightarrow \quad x^2 - 1 = 99$

$\Rightarrow \quad x^2 = 100$

$\therefore \quad x = 10 \qquad (\because x > 1)$

Example 8 : Prove that

$$\log_b a \times \log_c b \times \log_a c = 1.$$

Solution : Changing the base of all logarithm to '*e*', we get

$$\log_b a = \frac{\log_e a}{\log_e b}; \quad \log_c b = \frac{\log_e b}{\log_e c}; \quad \log_a c = \frac{\log_e c}{\log_e a}$$

$\therefore$ L.H.S. $= \dfrac{\log_e a}{\log_e b} \times \dfrac{\log_e b}{\log_e c} \times \dfrac{\log_e c}{\log_e a}$

$= 1 =$ R.H.S.

Exercise 9.2

(Assume the base 10 of all logarithm for those questions in which base is not given)

1. Prove that : $\log 630 = \log 2 + 2\log 3 + \log 5 + \log 7$.

2. Prove that : $\log \dfrac{9}{14} + \log \dfrac{35}{24} - \log \dfrac{15}{16} = 0$

3. Prove that : $\log 10 + \log 100 + \log 1000 + \log 10000 = 10$.

4. If $\log 2 = 0\cdot 3010$, $\log 3 = 0\cdot 4771$, $\log 7 = 0\cdot 8451$ and $\log 11 = 1\cdot 0414$, then evaluate the following :

   (i) $\log 36$ (ii) $\log \dfrac{42}{11}$ (iii) $\log\left(\dfrac{11}{7}\right)^5$

   (iv) $\log 70$ (v) $\log \dfrac{121}{120}$ (vi) $\log 5^{1/3}$

5. Find the value of *x* from the given equation :

   $\log_x 4 + \log_x 16 + \log_x 64 = 12.$

6. Solve : $\log(x+1) - \log(x-1) = 1$

7. Evaluate : $3^{2-\log_3 4}$.

8. Write the solution of the following questions in a single term.

(i) $\log 2+1$ (ii) $\log 2x+2\log x$

9. Prove that :

(i) $\log_5 3\cdot\log_3 4\cdot\log_2 5=2$. (ii) $\log_a x\times\log_b y=\log_b x\times\log_a y$.

9.05 System of logarithms :

Logarithms with base e and 10 are mostly used.

(i) Natural or Napierian system of logarithm :

This method was named after the mathematician Napier. In this method, the base is taken as $'e'$, where e is defined as

$$e=1+\frac{1}{1}+\frac{1}{1\times 2}+\frac{1}{1\times 2\times 3}+\cdots=2\cdot 71828183 \text{ (approx)}$$

It is used for Higher Mathematical calculations. For practical and numerical calculations, it is of no use.

(ii) Common or Brigg's system of logarithm :

In this system, base is taken as 10. When base is not given, we always take it as 10. Common logrithm is often used in all behavioural numberical calculations. Brigg had developed the first log table on base 10. So, it is called Brigg's system.

Note : All experimental calculations where base is not given, we assume it 10.

9.06 Relation between Napierian logarithm and common logarithm :

$$\log_{10} n=\log_e n\times\frac{1}{\log_e 10}, \text{ (Using base changing formula)}$$

But $\log_e 10=2\cdot 30258509$

$$\therefore \quad \frac{1}{\log_e 10}=\frac{1}{2\cdot 30258509}=0\cdot 43429448$$

$$\log_{10} n=0\cdot 43429448\times\log_e n$$

9.07 Characteristic and mantissa of the logarithms :

Logarithm of every number is the sum of two numbers in which one number is an integer and another a fraction or decimal number. For example :

$$\log 532=2+\cdot 7259=2\cdot 7259$$

The integral part is known as Characteristic and the fractional part is known as Mantissa. Here Characteristics is 2 and mantissa is 0.7259.

Note :

(i) Integral part can be positive or negative but the fractional part should be always positive. If the mantissa is negative, then it is made positive by substracting one from the characteristics (integer).

(ii) The negative characteristic is written by putting a bar on the number.

Example 9 : If $\log 142=2\cdot 1523$ then 2 is called as characteristics and .1523 is called as Mantissa.

Example 10 : If $\log M=-2\cdot 1423$, then to find its characteristics, the mantissa is made positive as shown below:

$$-2\cdot 1423=-2-\cdot 1423=(-2-1)+(1-\cdot 1423)=-3+\cdot 8577$$

Now, here characteristics is -3 and mantissa is $\cdot 8577$

Note : In the example 10 , the characteristics is negative, so it is written as $\overline{3}\cdot 8577$ and it is read as 3 bar point

8577. So it should be expressed as $\log M = \overline{3}\cdot 8577$ instead of $\log M = -2\cdot 1423$

9.08 Method of finding logarithm of a number :

(i) **Characteristic of logarithm of a number greater then 1 :** Using the laws of Indices we have

$$10^0 = 1 \quad \therefore \quad \log_{10} 1 = 0$$
$$10^1 = 10 \quad \therefore \quad \log_{10} 10 = 1$$
$$10^2 = 100 \quad \therefore \quad \log_{10} 100 = 2$$
$$10^3 = 1000 \quad \therefore \quad \log_{10} 1000 = 3$$

It is clear from the above results, that logrithm of numbers lying between 1 and 10 are in between 0 and 1. Here characteristics is 0. Logarithm of numbers lying between 10 and 100 and 100 are in between 1 and 2 means characteristic is 1. Logarithm of numbers lying between 100 and 1000 will be in between 2 and 3 and characteristic will be 2.

So, it concludes that the characteristic of logarithm of number greater than 1 is always be positive and the number of digits in characteristics of a number is always one less than the integral part i.e. if n is the integral part, then (n-1) will be characteristic.

Example : The integral part of number 42.5 is 42 which has two digits, thus the characteristic of $\log 42\cdot 5$ will be $2-1=1$. Similarly, the characteristic of $\log 425\cdot 23$ is $3-1=2$

(ii) **Characteristic of a number lying between 0 and 1 :**

We know that

$$10^0 = 1 \quad \therefore \quad \log_{10} 1 = 0$$
$$10^{-1} = \frac{1}{10} = \cdot 1 \quad \therefore \quad \log_{10} \cdot 1 = -1$$
$$10^{-2} = \frac{1}{100} = \cdot 01 \quad \therefore \quad \log_{10} \cdot 01 = -2$$
$$10^{-3} = \frac{1}{1000} = \cdot 001 \quad \therefore \quad \log_{10} \cdot 001 = -3$$

Just like that, the characteristic of logrithm of any positive number less than one is always negative and one more than the number of zeros between decimal point and first significant number i.e. if there is 4 zeros between decimal point and first significant number, the characteristic of its logrithm is $-(n+1)$ or $\left(\overline{n+1}\right)$, where n denotes number of zeros so discussed.

Example : Characteristics of $\log 0\cdot 035$ is -2 and characteristics $\log 0\cdot 00035$ is -4.

Note : Significant number : Remaining number after decimal point by removing the zero before and after non-zero number is called significant number.In .000123 or 123000, significant number is 123 and 1 is first significant number.

$$\log 0\cdot 0003 = \log \frac{3}{10000} = \log\left(3\times 10^{-4}\right)$$
$$= \log 10^{-4} + \log 3$$
$$= -4 + 0\cdot 4771 = \overline{4}\cdot 4771 \qquad [\log 3 = 0.4771 \text{ from logarithmic table}]$$

and $$\log 0\cdot 007294 = \log \frac{7\cdot 294}{1000} = \log\left(7\cdot 294\times 10^{-3}\right)$$
$$= \log 10^{-3} + \log 7\cdot 294 = -3 + 0\cdot 8629$$

$= \bar{3} \cdot 8629$ [$\log 7.294 = 0.86291$]

Working Rule to Find the Characteristics :

(i) Convert the number into a decimal form.

(ii) If $a > 1$, then Characteristic $= n - 1$ (where n is the number of digits in integral part)

(iii) If $0 < a < 1$, then Characteristic $= -(n+1)$ (where n is the number of zeros after the decimal part discussed)

9.09 Method to find the maintissa of logarithm of a number :

To find the value of mantissa, we use the Logarithm table given at the end of the book.

Properties of Log Table :

(1) It contains the value of Mantissa.

(2) It has three parts :

(i) **First Column :** It contains number from 10 to 99

(ii) **Next ten Columns :** It has values for numbers ranging from $0, 1, 2, \ldots, 9$

(iii) **Column of Mean Difference :** Last three columns are devided into three sub-columns which are leaded by numbers 1, 2, 3, 4, 5, 6, 7 ,8 9.

Working Rule to find the Msntissa :

To understand, how to find the mantissa let us consider an example of $\log 38 \cdot 56$

(i) Remove the decimal in a number if any like here the number will be 3856.

(ii) Firstly we will take the first two digits i.e. 38 and move to the column one of the log table.

(iii) Number 5 is to be seen in the next ten columns of the table.

(iv) Note the number in the 38th row and 5th column i.e. 5855.

(v) Now move to the Mean Difference column of Log Table.

(vi) Read the value of the digit 6 in the column, which is 7.

(vii) Add the numbers i.e. $5855 + 7 = 5862$ Put the decimal before the four digits, thus we get the Mantissa Therefore Mantissa of $\log 38 \cdot 56$ is $\cdot 5862$

Note :

(i) It is clear from the above result that the decimal is to be put before the given Mantissa.

(ii) If the number is a one digit number whose Log value is to be derived, then we put zero after the number i.e. To find Mantissa of log 3, we see log 30 or log 300 in the Log table.

(iii) If a number has more than four significant digits, then using log table for 4 digit.

(a) If next number is less than 5, then we let it.

(b) If next number is gteater than 5, then use add 1 to previous number.
So, we find the logrithm of obtained 4 digit number.

(iv) To find logarithm of any number :

(a) We find the characteristic by method described in section 9.08

(b) We find the mantissa by method described in section 9.00
i.e. characteristic and mantissa of 38.56 is 1 and .5862 respectively.
So, log 38.56=1.5862

(v) Mantissa of same numbers are same . For example, 2834, 28.34 and 0.002834 has characteristic 3, 1 and -3 and but their Mantissa will remain same which is .4524.

Illustrative Examples

Example 11 : Determine the characteristic of the following numbers :

(i) 5970 (ii) 125.35 (iii) 2.5795 (iv) 0.7598 (v) 0.0074

Solution :

(i) Number 5970 has 4 digits, thus the characteristic is $4-1=3$

(ii) Number has 3 digit (integral part, 125) thus the characteristic is 3 - 1 = 2

(iii) Number has 1 digit (integral part, 2) thus the characteristic is 1 - 1 = 0

(iv) Number has the characteristic - (0 + 1) = - 1 or $\overline{1}$ (as there is no zero after the decimal part)

(v) Number has the characteristic - (2 + 1) = - 3 or $\overline{3}$ (as there is two zero after the decimal part)

Example 12 : Using log table, find logarithm of the following numbers :

(i) 25794 (ii) $5\cdot3498$ (iii) $0\cdot3582$ (iv) $0\cdot003$ (v) $0\cdot000125$

Solution :

(i) Number 25794 has 5 digits, thus characteristics of log 25794 is $=5-1=4$

Thus finding the value of 25 in 7th column which is 4099 with 9 mean difference column, we have 15.

$\therefore$ $4099+15=$ 4114 (We have left the 4 because it is less than 5)

therefore, Mantissa is $=\cdot4114$

$\therefore$ $\log 25794 = 4\cdot4114$

(ii) Characteristics of $\log 5\cdot3498$ is $= 1-1 = 0$

Thus finding the value of 53 in 4th column with 9 mean difference column, we have the mantissa =.7284

$\therefore$ $\log 5\cdot3498 = 0\cdot7284$

(iii) Characteristics of $\log(0\cdot3582)$ or $=-(0+1)=-1=\overline{1}$

mantissa $=\cdot(5539+2)=\cdot5541$

$\therefore$ $\log 0\cdot3582 = \overline{1}\cdot5541$

(iv) Characteristics of $\log 0\cdot003$ or $=-(2+1)=-3=\overline{3}$

mantissa $=\cdot(4771)$

$\therefore$ $\log 0\cdot003 = \overline{3}\cdot4771$

(v) Characteristics of $\log 0\cdot000125$ or $=-(3+1)=-4=\overline{4}$

mantissa $=\cdot(0969)=\cdot0969$

$\therefore$ $\log 0\cdot000125 = \overline{4}\cdot0969$

Exercise 9.3

1. Determine the Characteristics of the logarithm of the following numbers :

(i) 1270 (ii) 20·125 (iii) 7·985 (iv) 431·5
(v) 0·02 (vi) 0·02539 (vii) 70 (viii) 0·000287
(ix) 0·005 (x) 0·00003208 (xi) 0·000485 (xii) 0·007
(xiii) 0·0005309

2. Using the log table find the logarithm of the following numbers :

(i) 2813 (ii) 400 (iii) 27·28 (iv) 9
(v) 0·678 (vi) 0·0035 (vii) 0·08403 (viii) 0·000287
(ix) 1·234 (x) 0·00003258 (xi) 0·000125 (xii) 0·00003208

9.10 Antilogarithms and their tables :

The inverse of Logarithm is Anti logarithm. Thus, a positive number n is an Antilog of number m, if $\log n = m$ i.e. n is the antilog of m $\Rightarrow$ $n = \text{anti}\log m$ $\Leftrightarrow \log n = m$.

(i) $\log 300 = 2 \cdot 4771 \Leftrightarrow \text{antilog } 2 \cdot 4771 = 300$

(ii) $\log 432 \cdot 5 = 2 \cdot 6360 \Leftrightarrow \text{antilog } 2 \cdot 6360 = 432 \cdot 5$

(iii) $\log 0 \cdot 1257 = \overline{1} \cdot 0993 \Leftrightarrow \text{antilog } (\overline{1} \cdot 0993) = 0 \cdot 1257$

(iv) $\log 0 \cdot 000425 = \overline{4} \cdot 6284 \Leftrightarrow \text{antilog } (\overline{4} \cdot 6284) = 0 \cdot 0004250$

The value of Antilog can be derived using an Antilog table given at the last of the book.

The numbers in the first column of the table range from to and rest of the columns are similar as of log table.

Working Rule to find Antilogarithm :

(i) For the given number, the integral part is left and the value of two consecutive number of decimal part is seen in the antilog table.

(ii) Firstly the first two digits after decimal is seen in the first column.

(iii) Secondly the third digit after decimal is chosen from the given nine columns followed by the fourth digit is seen in the mean difference column. The numbers obtained are added. (Same as in case of finding log)

(iv) Now we have to focus on the characteristics part. If the integral part is positive, say 'n', then the decimal is put after the $(n+1)$ digits of the number obtained in step (iii).

If the integral part is negative, say $\overline{n}$, then $(n-1)$ zeros are put on the right side of the decimal in the number obtained in step (iii)

Note : If number is negative for which we have to find antilog, then we have to make decimal part positive.

We add 1 to it and substract 1 from integral part. Thus, we find the antilog of number.

Example : The number -3.6432 has the decimal part negative. Therefore, we convert the number as shown below :

$$-3.6432 = -3 - 1 + 1 - 0.6432$$
$$= -4 + 0.3568 = \overline{4}.3568$$

Illustrative Examples

Example 13 : Determine the Antilog of $3 \cdot 7523$

Solution :

(i) The fractional part is $\cdot 7523$

(ii) In the antilog table, look for .75 in the first column followed by the third column with number 2 and 3 in the mean difference, which is 4

(iii) Add the numbers obtained in step (ii) $= 5649 + 4 = 5653$

(iv) The given mumber has integral part 3 thus the decimal to be put after four digits.

(v) Thus $\text{antilog } 3 \cdot 7523 = 5653 \cdot 0$

Example 14 : Determine the $\text{antilog } \overline{2} \cdot 0258$

Solution :

(i) The fractional part is $\cdot 0258$

(ii) In the antilog table, look for .02 in the first column followed by the column with number 5 and 8 in the mean difference which is 2

(iii) Add the numbers obtained in step (ii)

$\therefore \quad 1059 + 2 = 1061.$

(iv) The given mumber has integral part $\overline{2}$ thus $2 - 1 = 1$.

i.e. putting one zero after the right side of the and we write the number obtained in step (iii) i.e. .01061.

Example 15 : If $\log x = 0\cdot5428$, then find the value of x .

Solution : $\log x = 0\cdot5428$

or $x = \text{antilog } 0\cdot5428$

$= 3\cdot489$

Exercise 9.4

1. Find the Antilogarithm of the following numbers :

(i) $1\cdot3210$ (ii) $2\cdot4127$ (iii) $0\cdot084$

(iv) $\overline{1}\cdot301$ (v) $\overline{3}\cdot2462$ (vi) $\overline{2}\cdot0258$

2. Evaluate :

(i) antilog $3\cdot1234$ (ii) antilog $\overline{2}\cdot5821$ (iii) antilog $0\cdot3$

(iv) antilog $2\cdot466$

3. Find the value of x in the following :

(i) $\log x = \overline{2}\cdot6727$ (ii) $\log x = 0\cdot452$

Miscellaneous Examples

Example 16 : If $\text{antilog } 1\cdot4339 = 27\cdot16$, then find $\text{antilog } \overline{2}\cdot4339$

Solution : Since the fractional part of $1\cdot4339$ and $\overline{2}\cdot4339$ are same, therefore, the antilog of both the terms will be same

If integral part is $\overline{2}$, then in the antilog, there will be one zero after decimal.

Thus, required $\text{antilog } \overline{2}\cdot4339 = \cdot02716$

Example 17 : If $\log 0\cdot723 = \overline{1}\cdot8591$, then find the value of $\sqrt[3]{72\cdot3}$

Solution : Let $x = \sqrt[3]{72\cdot3}$

$\therefore$ $\log x = \log (72\cdot3)^{1/3}$ or $\log x = \frac{1}{3}\log(72\cdot3)$

or $\log x = \frac{1}{3}(1\cdot8591)$ [$\because \log 0\cdot723$ and $\log 7\cdot23$ have same mantissa]

or $\log x = 0\cdot6197$

or $x = \text{antilog } (\cdot6197)$ $\therefore$ $x = 4\cdot166$

Example 18 : Express $\log\left(\frac{2^3 \times 3}{5^2 \times 7^3}\right)$ as the sum and the difference of logarithm.

Solution : $\log\left(\frac{2^3 \times 3}{5^2 \times 7^3}\right) = \log\left(2^3 \times 3\right) - \log\left(5^2 \times 7^3\right)$

$= \log 2^3 + \log 3 - \log 5^2 - \log 7^3$

$= 3\log 2 + \log 3 - 2\log 5 - 3\log 7$

Example 1 9: If $\log_{\sqrt{8}} b = 3\frac{1}{3}$, then find the value of b.

Solution : $\log_{\sqrt{8}} b = 3\frac{1}{3}$ $\Rightarrow$ $\left(\sqrt{8}\right)^{\frac{10}{3}} = b$

$$\Rightarrow \quad b = \left(2^{\frac{3}{2}}\right)^{\frac{10}{3}} = 2^{\left(\frac{10}{3}\right)\left(\frac{3}{2}\right)} = 2^5 = 32 \quad \therefore \quad b = 32$$

Example 20 : If $\frac{\log a}{b-c} = \frac{\log b}{c-a} = \frac{\log c}{a-b}$, Prove that : $a^a \cdot b^b \cdot c^c = 1$

Solution : Let $\frac{\log a}{b-c} = \frac{\log b}{c-a} = \frac{\log c}{a-b} = k$

then, $\log a = k(b-c)$, $\log b = k(c-a)$ and $\log c = k(a-b)$

$\therefore \quad a\log a + b\log b + c\log c = a \cdot k(b-c) + b \cdot k(c-a) + ck(a-b) = 0$

$\therefore \quad \log a^a + \log b^b + \log c^c = 0$

$\log a^a b^b c^c = \log 1$ or $a^a \cdot b^b \cdot c^c = 1$ $\quad [\because \log 1 = 0]$

Miscellaneous Exercise 9

1. If $\log_{\sqrt{2}} x = 4$, then the value of x will be :

   (A) $4^{\sqrt{2}}$ (B) $\frac{1}{4}$ (C) 4 (D) $4 \times \sqrt{2}$

2. If $\log_x 243 = 2 \cdot 5$, then the value of x will be :

   (A) 9 (B) 3 (C) 1 (D) 81

3. The value of $\log(1 + 2 \times 3)$:

   (A) $2\log 3$ (B) $\log 1 \cdot \log 2 \cdot \log 3$ (C) $\log 1 + \log 2 + \log 3$ (D) $\log 7$

4. The value of $\log(m+n)$:

   (A) $\log m + \log n$ (B) $\log mn$ (C) $\log m \times \log n$ (D) None of these

5. The value of $\log_b a \cdot \log_c b \cdot \log_a c$:

   (A) 0 (B) $\log abc$ (C) 1 (D) $\log\left(b^a \cdot c^b \cdot a^c\right)$

6. If $a > 1$, then value of $\log_a 0$:

   (A) $-\infty$ (B) ∞ (C) 0 (D) 1

7. If $0 < a < 0$, then value of $\log_a 0$:

   (A) $-\infty$ (B) ∞ (C) 0 (D) 1

8. Another form of $\log_a b$ is :

   (A) a^b (B) b^a (C) $\frac{1}{\log_b a}$ (D) $\log_b a$

9. Numbers $\log_2 7$ is :

   (A) Integer (B) Rational (C) Ir-rational (D) Prime

10. If $a = \log_3 5$ and $b = \log_7 25$ then the correct option is :

(A) $a<b$ (B) $a>b$ (C) $a=b$ (D) None of these

11. If $\log_2 x + \log_2 (x-1) = 1$, then find the value of x.
12. If $\log(a-b) = \log a - \log b$, then find the value of a in terms of b will be ?
13. If $\frac{1}{\log_a x} + \frac{1}{\log_c x} = \frac{2}{\log_b x}$, then find the relation between a, b and c .
14. If $\log 2 = 0 \cdot 3010$, then find the value of $\log 200$.
15. Find the value of $\log 0 \cdot 001$
16. If $\log 7 = 0 \cdot 8451$ and $\log 3 = 0 \cdot 4771$, then find $\log (21)^5$.
17. Find the value of $\log 6 + 2\log 5 + \log 4 - \log 3 - \log 2$.
18. If $\frac{\log 144}{\log 12} = \log x$, then find x .
19. Prove that : $\log_{10} \tan 1^0 \cdot \log_{10} \tan 2^0 \ldots\ldots \log_{10} \tan 89^0 = 0$
20. Prove that : $\log_3 4 \cdot \log_4 5 \cdot \log_5 6 \cdot \log_6 7 \cdot \log_7 \cdot 8 \cdot \log_8 9 = 2$
21. If $\log 52 \cdot 04 = 1 \cdot 7163$, $\log 80 \cdot 65 = 1 \cdot 9066$ and $\log 9 \cdot 753 = 0 \cdot 9891$, then find the value of

    $\log \frac{52 \cdot 04 \times 80 \cdot 65}{9 \cdot 753}$
22. If $\log 32 \cdot 9 = 1 \cdot 5172$, $\log 568 \cdot 1 = 2 \cdot 7544$ and $\log 13 \cdot 28 = 1 \cdot 1232$, then find the value of

    $\log \frac{(13 \cdot 28)^3}{32 \cdot 9 \times 568 \cdot 1}$
23. If $\log 2 = 0 \cdot 3010$ and $\log 3 = 0 \cdot 4771$, then find the value of $\log (0 \cdot 06)^6$
24. Prove that : $\log\left(\frac{x^y y^x}{z^x x^z}\right) = x(\log y - \log z) + (y-z)\log x.$
25. Express $\log \frac{11^3}{5^7 \times 7^5}$ as a sum and difference of logarithm.
26. (a) If antilog $1 \cdot 5662 = 36 \cdot 83$, then evaluate the following :

    (i) antilog $\overline{1} \cdot 5662$ (ii) antilog $2 \cdot 5662$ (iii) antilog $\overline{2} \cdot 5662$

    (b) Find the value of antilog $(\log x)$
27. Find the value of $(17)^{\frac{1}{2}}$ if $\log 17 = 1 \cdot 2304$ and antilog $0 \cdot 6152 = 4 \cdot 123$
28. If $\log_{10} 3 = 0 \cdot 4771$, find the value of $\log_{10} 0 \cdot 027$
29. Using the log table, evaluate : $\frac{520 \cdot 4 \times 8 \cdot 065}{97 \cdot 53}$
30. If $\log x - \log(x-1) = \log 3$, then find x .

Important Points

1. If $a^x = n$, then $\log_a n = x$ $[a > 0,\ a \neq 1,\ n > 0]$
2. $\log_a MN = \log_a M + \log_a N$
3. $\log_a \frac{M}{N} = \log_a M - \log_a N$
4. $\log_a (M)^N = N \log_a M$
5. $\log_a a = 1$
6. $\log_a 1 = 0$
7. $\log_a M = \log_b M \times \log_a b$
8. $\log_b a \times \log_a b = 1$
9. $\log_a 0 = -\infty$, If $a > 1$
10. $\log_a 0 = +\infty$, $0 < a < 1$
11. $M = e^{\log_e M}$
12. $\log_{10} n = \log_e n \times 0\cdot 43429$
13. The inverse of logarithm is known as Antilogarithm
14. $m = \log n \Leftrightarrow n = \text{antilog } m$
15. $\text{antilog}(\log n) = n$

Answers

Exercise 9.1

1. $\log_2 64 = 6$	2. $\log_{10} 10000 = 4$	3. $\log_2 1024 = 10$
4. $\log_5 \left(\frac{1}{25}\right) = -2$	5. $\log_{10} 0\cdot 001 = -3$	6. $\log_4 8 = \frac{3}{2}$
7. $5^2 = 25$	8. $3^6 = 729$	9. $10^{-3} = 0\cdot 01$
10. $10^{-1} = 0\cdot 1$	11. $3^{-3} = \frac{1}{27}$	12. $\left(\sqrt{2}\right)^4 = 4$
13. 729	14. $\sqrt{5}$	15. 8

Exercise 9.2

4. (i) $1\cdot 5562$ (ii) $0\cdot 5818$ (iii) $0\cdot 9815$ (iv) $1\cdot 8451$ (v) $0\cdot 0037$ (vi) 0.2330

5. 2 6. $1\frac{2}{9}$ 7. $2\frac{1}{4}$ 8. (i) $\log 20$ (ii) $\log 2x^3$

Exercise 9.3

1. (1) 3	(ii) 1	(iii) 0	(iv) 2	(v) $\overline{2}$	(vi) $\overline{2}$
(vii) 1	(viii) $\overline{4}$	(ix) $\overline{3}$	(x) $\overline{5}$	(xi) $\overline{4}$	(xii) $\overline{3}$
(xiii)					
1. (1) 3.4492	(ii) 2.6021	(iii) 1.4359	(iv) 0.9542	(v) $\overline{1}.8312$	
(vi) $\overline{3}.5441$	(vii) $\overline{2}.9245$	(viii) $\overline{4}.4579$	(ix) 0.0913	(x) $\overline{5}.5130$	
(xi) $\overline{4}.0969$	(xii) $\overline{5}.5062$				

Exercise 9.4

1. (i) 20·94 (ii) 258·6 (iii) 1·213 (iv) 0·2000 (v) 0·001763 (vi) 0.01061

2. (i) 1328·0 (ii) 0·03820 (iii) 1·995 (iv) 2·924

3. (i) 0·04707 (ii) 2·831

Miscellaneous Exercise 9

1. (C) 2. (A) 3. (D) 4. (D) 5. (C) 6. (A) 7. (B)
8. (C) 9. (C) 10. (A)

11. 2 12. $\dfrac{b^2}{b-1}$ 13. $b^2 = ac$

14. 2·3010 15. $\overline{3}$ or -3 16. 6·611

17. 2 18. 100 21. 2·6338

22. $-0{\cdot}9020$ or $\overline{1}{\cdot}0980$ 23. $-7{\cdot}3314$ or $\overline{8}{\cdot}6686$

25. $3\log 11 - 7\log 5 - 5\log 7$

26. (a) (i) 0·3683 (ii) 368·3 (iii) 0·03683 (b) x

27. 4·123 28. $\overline{2}{\cdot}4313$ 29. 43·03 30. $\dfrac{3}{2}$