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Mental ability is the ability to distinguish between right and wrong, to judge the minutest difference and to adapt to the ever changing environment, the wit to master the situation, the capacity to learn and to put past experience to the most advantageous use in future and the ability to distinguish between important, less important and more important.

❖ EXAMPLES ❖

Completing the Given Series

- Ex.1 Which number would replace question mark in the series 7, 12, 19, ?, 39.
 - (A) 29
- (B) 28
- (C) 26
- (D) 24
- Sol. Clearly, the given sequence follows the pattern:

$$+5, +7, +9$$
 ... i.e., $7 + 5 = 12, 12 + 7 = 19, ...$

 \therefore Missing number = 19 + 9 = 28

Hence, the answer is (B).

- Ex.2Which is the number that comes next in the sequence: 0, 6, 24, 60, 120, 210?
 - (A) 240
- (B) 290 (C) 336
- (D) 504

Sol. Clearly, the given series is

$$1^3 - 1$$
, $2^3 - 2$, $3^3 - 3$, $4^3 - 4$, $5^3 - 5$, $6^3 - 6$.

$$\therefore$$
 Next number = $7^3 - 7 = 343 - 7 = 336$

Hence, the answer is (C).

Ex.3Which is the number that comes next in the following sequence?

- (A) 32
- (B) 60
- (C) 62
- (D) 64
- Sol. The given sequence is a combination of two series:

Now, the pattern followed in each of the above two series is : +8, +16, +32

So, missing number = (28 + 32) = 60

Hence, the answer is (B).

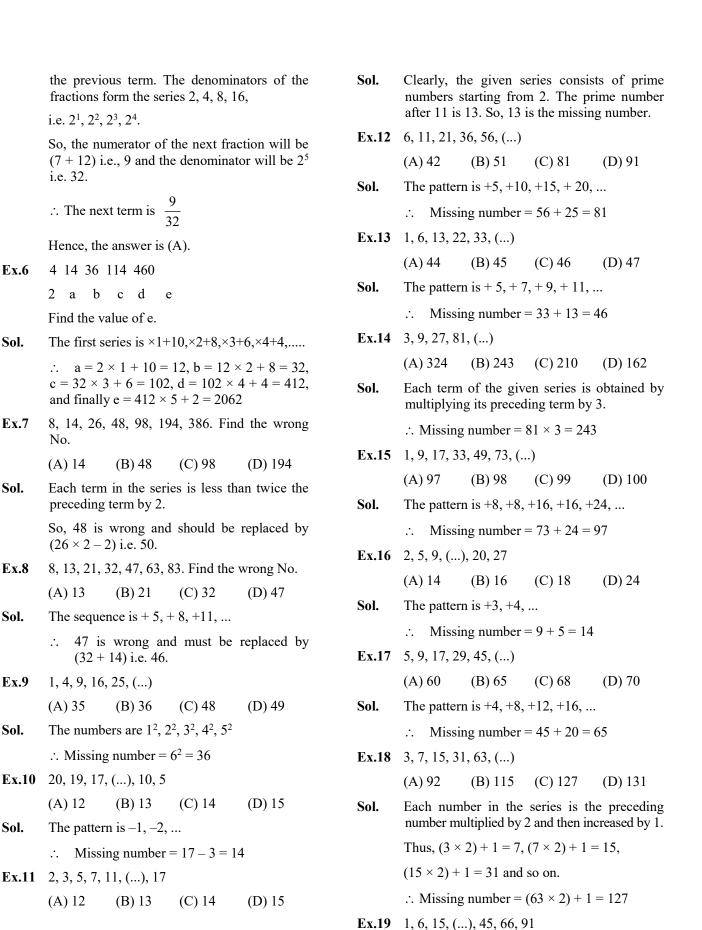
- Ex.4 Find out the missing number in the following sequence: 1, 3, 3, 6, 7, 9, ?, 12, 21.
 - (A) 10
- (B) 11
- (C) 12
- (D) 13
- Sol. Clearly, the given sequence is a combination of two series:

The pattern followed in I is +2, +4, ...; and the pattern followed in II is +3. Thus, missing number = 7 + 6 = 13.

Hence, the answer is (D).

- Which fraction comes next in the sequence Ex.5 $\frac{1}{2}, \frac{3}{4}, \frac{5}{8}, \frac{7}{16}, ?$

- (A) $\frac{9}{32}$ (B) $\frac{10}{17}$ (C) $\frac{11}{34}$ (D) $\frac{12}{35}$
- Sol. Clearly, the numerators of the fractions in the given sequence form the series 1, 3, 5, 7, in which each term is obtained by adding 2 to



(A) 25

(B) 26

(C) 27

(D) 28

Sol. The pattern is
$$+5, +9, ..., +21, +25$$

$$\therefore$$
 Missing number = $15 + 13 = 28$

Sol. Each term in the series is the sum of the preceding two terms.

Thus,
$$1 + 2 = 3$$
; $2 + 3 = 5$; $3 + 5 = 8$ and so on.

$$\therefore$$
 Missing number = $5 + 8 = 13$

Series

Series is a sequence of numbers obtained by some particular predefined rule and applying that predefined rule it is possible to find out the next term of the series.

A series can be created in many ways. Some of these are discussed below:

(i) Arithmetic Series.

For example,

are arithmetic series because in each of them the next number can be obtained by adding or subtracting a fixed number. For example in 3, 5, 7, 9, 11,..... every successive number is obtained by adding 2 to the previous number.

(ii) Geometric Series.

For example,

are geometric series because, in each of them, the next number can be obtained by multiplying (or dividing) the previous number by a fixed number. (For example, in : 3125–625, 125, –25, 5... every successive number is obtained by dividing the previous number by –5.)

(iii) Series of squares, cubes :

These series can be formed by squaring or cubing every successive number.

For example,

are such series. (In the first and second, every number is squared to get the next number while in the third it is cubed).

(iv) Mixed Series:

By a mixed series, we mean a series which is created according to any non-conventional (but logical) rule. Because there is no limitation to people's imagination, there are infinite ways in which a series can be created and naturally it is not possible to club together all of them.

Classification

(I) Two-tier Arithmetic Series:

In an arithmetic series the difference of any two successive numbers is fixed. A Two-tier Arithmetic Series shall be the one in which the differences of successive numbers themselves form an arithmetic series.

For example,

are examples of such series. In 1, 2, 5, 10, 17, 26, 37,; for example, the differences of

successive numbers are 1, 3, 5, 7, 9, 11,which is an arithmetic series.

Note:

Two-tier arithmetic series can be denoted as a quadratic function. For example, the above series

(a) is $0^2 + 1$, $1^2 + 1$, $2^2 + 1$, $3^2 + 1$,... which can be denoted as $f(x) = x^2 + 1$, where x = 0, 1, 2,... similarly example (b) can be denoted as

$$f(x) = x^2 + x + 3, x = 0, 1, 2, 3,...$$

(II) Three-tier Arithmetic Series:

This, as the name suggests, is a series in which the differences of successive numbers form a two-tier arithmetic series; whose successive term's differences, in turn, form an arithmetic series.

For example

(a) 336, 210, 120, 60, 24, 6, 0,...

is an example of three-tier arithmetic series.

[The differences of successive terms are 126, 90, 60, 36, 18, 6,.....

The differences of successive terms of this new series are 36, 30, 24, 18, 12,..... which is an arithmetic series.]

Note:

Three-tier arithmetic series can be denoted as a cubic function. For example, the above series is (from right end) $1^3 - 1$, $2^3 - 2$, $3^3 - 3$, $4^3 - 4$, which can also be denoted as $f(x) = x^3 - x$; x = 1, 2,...

(III) Twin Series:

We shall call these twin series, because they are two series packed in one.

$$1, 3, 5, 1, 9, -3, 13, -11, 17, \dots$$

is an example of twin series. (The first, third, fifth etc., terms are 1, 5, 9, 13, 17 which is an arithmetic series. The second, fourth, sixth etc. are 3, 1, -3, -11 which is a geometricoarithmetic series in which successive terms are obtained by multiplying the previous term by 2 and then subtracting 5.)

(IV) Other Series:

Besides, numerous other series are possible and it is impossible to even think of all of them.

SUMMARY OF THREE STEPS

[Very Important]

Step I:

Do a preliminary screening of the series. If it is a simple series you will be able to solve it easily.

Step II:

If you fail in preliminary screening then determine the trend of the series. Determine whether it is increasing, decreasing or alternating.

Step III (A):

Perform this step only if a series is increasing or decreasing. Use the following rules:

- (a) If the rise of a series is slow or gradual, the series is likely to have an addition-based increase; successive numbers are obtained by adding some numbers.
- (b) If the rise of a series is very sharp initially but slows down later on, the series is likely to be formed by adding squared or cubed numbers.
- (c) If the rise of a series is throughout equally sharp, the series is likely to be multiplication-based; successive terms are obtained by multiplying by some terms (and, maybe, some addition or subtraction could be there, too).
- (d) If the rise of a series is irregular, there may be two possibilities. Either there may be a mix of two series or two different kinds of operations may be going on alternately. (The first is more likely when the increase is very irregular: the second is more likely when there is a pattern, even in the irregularity of the series).

Step (III) (B):

(to be performed when the series is alternating)

[Same as (iv) of step (iii), Check two possibilities]

Some Solved Examples

Ex.21 Find the next number of the series

- (i) 8, 14, 26, 50, 98, 194
- (ii) 8, 8, 9, 9, 11, 10, 14, 11
- (iii) 325, 259, 204, 160, 127, 105
- (iv) 54, 43, 34, 27, 22, 19
- (v) 824, 408, 200, 96, 44, 18
- (vi) 16, 17, 21, 30, 46, 71
- (vii) 3, 3, 6, 18, 72, 360
- (viii) 3, 4, 8, 17, 33, 58
- (ix) 6, 16, 36, 76, 156, 316
- (x) -2, 4, 22, 58, 118, 208

Sol.

- (i) Sharp increase and terms roughly doubling every time. On checking with 2 as multiple the series is:
 - next term = previous term $\times 2$ –2. Next term = 382.
- (ii) Irregular. Very irregular. Likely to be, therefore, mixed. On checking it is a mix of two series: 8, 9, 11, 14, (+1, +2, +3 etc.) and 8, 9, 10, 11. Next term = 14 + 4 + 18.
- (iii) Gradual slow decrease. Likely to be arithmetical decrease. Check the differences of successive terms. They are 66, 55, 44, 33, 22. Hence, next decrease will be: 11.
 - Next term = 105 11 = 94.
- (iv) Gradual slow decrease. Likely to be arithmetical decrease. Check the differences. They are 11, 9, 7, 5, 3. Hence, next decrease will be 1. Next term = 19 1 = 18.
- (v) Sharp decrease and terms roughly being halved every time. Checking with 2 as divisor the series is:

Next term (previous term -8) $\div 2$.

Next term = 5.

(vi) preliminary screening tells us that each term is obtained by adding 1², 2², 3², 4², 5²..... respectively.

Next term =
$$71 + 6^2 = 107$$

- (vii) Sharp increase. The series is : \times 1, \times 2, \times 3, \times 4, \times 5,Next term = $360 \times 6 = 2160$
- (viii) Sharp increase that slows down later no. (Ratios of successive terms rise sharply from

 $4 \div 3 = 1.3$ to $8 \div 4 = 2$ to $17 \div 8 = 2$. 125 and then start falling to $33 \div 17 \approx 1.9$ and then to $58 \div 33 \div 1.8$). Hence likely to be addition of squared or cubed numbers. On checking the series is:

$$+1^2+2^2, +3^2, +4^2, +5^2, \dots$$

Next term =
$$58 + 6^2 = 94$$
.

- (ix) Sharp increase with terms roughly doubling each time. Likely to have geometrical nature with 2 as multiple. On checking the series is: $\times 2 + 4$. Next term = $316 \times 2 + 4 = 636$
- (x) Series increases sharply but then its speed of rise slows down. Likely to be addition of squared or cubed numbers. On checking, the series is: $1^3 3$, $2^3 4$, $3^3 5$, $4^3 6$... Next term = $7^3 9 = 334$.

Wrong Number in Series

In examinations, a series is more likely to be given the format of a complete series in which an incorrect number is included & it is required to find out the wrong number. On studying a given series and applying the concepts employed so far you should be able to understand and thus "decode" the formation of the series. Usually six terms are given and it means that at least five correct terms are given.

Some Unique Series

These series may be asked in examinations, so you must be aware of them.

I. Series of Date or Time

1. Which of the following doesn't fit into the series?

- Sol. Each successive date differs by 22 days. Recall that 96 is a leap year, you will find that 12–3–96 should be replaced by 11–3–96.
- **2.** Which of the following doesn't fit into the series?

- **Sol.** Each successive time differs by 2 hrs. 20 minutes. So 12.30 should be replaced by 12.40.
- **Note:** Keep in mind that the problem of series may be based on dates or times. Sometimes it

doesn't strike our mind and the question is solved wrongly.

II. Fractional series

Which of the following doesn't fit into the series?

1.
$$\frac{4}{5}$$
, $\frac{7}{15}$, $\frac{1}{15}$, $-\frac{1}{5}$, $-\frac{8}{15}$

Sol. Whenever you find that most of the fractions have the same denominators, change all the denominators to the same value. For example, in this question, the series becomes:

$$\frac{12}{15}$$
, $\frac{7}{15}$, $\frac{1}{15}$, $-\frac{3}{15}$, $-\frac{8}{15}$

Now, it is clear that numerators must decrease successively by 5. Therefore, $\frac{1}{15}$ should be replaced by $\frac{2}{15}$.

2.
$$\frac{4}{5}$$
, $\frac{23}{35}$, $\frac{18}{35}$, $\frac{12}{35}$, $\frac{8}{35}$, $\frac{3}{35}$

Sol. By the above rule if we change all the fractions with the same denominators, the series is

$$\frac{28}{35}$$
, $\frac{23}{35}$, $\frac{18}{35}$, $\frac{12}{35}$, $\frac{8}{35}$, $\frac{3}{35}$.

We see that numerators decrease by 5, thus $\frac{12}{35}$ should be replaced by $\frac{13}{35}$.

Now, we conclude that the above fractions decrease successively by $\frac{5}{35}$ or $\frac{1}{7}$.

$$\frac{118}{225}$$
, $\frac{100}{199}$, $\frac{82}{173}$, $\frac{66}{147}$, $\frac{46}{121}$, $\frac{28}{95}$

Sol. We see that all the denominators differ, so we can't use the above rule. In this case usually, the numerators and denominators change in a definite pattern. Here, numerators decrease successively by 18 whereas denominators

decrease successively by 26. Thus $\frac{66}{147}$ should

be replaced by
$$\frac{64}{147}$$
.

4.
$$\frac{12}{89}$$
, $\frac{15}{86}$, $\frac{18}{82}$, $\frac{21}{80}$, $\frac{24}{77}$, $\frac{27}{74}$

Sol. Numerators increase successively by 3whereas denominators decrease successively by 3. Thus $\frac{18}{82}$ should be replaced by $\frac{18}{83}$.

III. Some numbers followed by their LCM or HCF

- 1. 1, 2, 3, 6, 4, 5, 6, 60, 5, 6, 7,.... (Fill up the blank)
- Sol. The series can be separated in three parts. 1, 2, 3, 6/4, 5, 6, 60/5, 6, 7..... In each part fourth number is LCM of first three
- **2.** 8, 6, 24, 7, 3, 21, 5, 4, 20,....,9, 18 (1) 1 (2) 3 (3) 4 (4) 5 (5) 6
- Sol. 8, 6, 24/7, 3, 21/5, 4, 20/_, 9, 18

 Third number in each part is LCM of first two numbers. Thus, the answer should be 6.
- **3.** 8, 4, 4, 7, 8, 1, 3, 9, 3, 2, 1,.... (1) 1 (2) 2 (3) 3 (4) 5 (5) N.O.T
- **Sol.** 8, 4, 4/7, 8, 1/3, 9, 3/2, 1...

 In each part, third number is HCF of first to numbers. Thus our answer should be 1.

IV. Some numbers followed by their product

- 2, 3, 6, 18, 108, 1844
 Which of the above numbers does not fit into the series?
- **Sol.** 1844 is wrong, because $2 \times 3 = 6$, $3 \times 6 = 18$, $18 \times 6 = 108$, but $108 \times 18 = 1944$.

V. By use of digit-sum

1. 14, 19, 29, 40, 44, 51, 59, 73
Which of the above numbers doesn't fit into the series?

Sol. Next number = Previous number + Digit-sum of previous number Like,

$$19 = 14 + (4 + 1)$$

$$29 = 19 + (1 + 9)$$

$$40 = 29 + (2 + 9)$$

Thus, we see that 51 should be replaced by 52.

2. Fill up the blanks

Sol. 1st, 3rd, 5th, 7th, ... numbers follow the pattern of +4 (14 + 4 = 18, 18 + 4 = 22,...). Where as 2nd, 4th, 6th are the digit-sums of their respective previous number (5 = 1 + 4, 9 = 1 + 8),...) Thus, our answer is 34 and 7.

VI. Odd number out

Sometimes a group of numbers is written out of which one is different from others.

- 1. 22, 44, 88, 132, 165, 191, 242. Find the number which doesn't fit in the above series (or group).
- **Sol.** 191; Others are divisible by 11 or 191 is the single prime number.
- 2. Which one of the following series doesn't fit into the series?

- **Sol.** 51; All other are prime numbers.
- **Ex.22** Find the wrong number in the series:

(C) 124

(D) 215

Sol. Clearly, the correct sequence is

$$2^3 - 1, 3^3 - 1, 4^3 - 1, 5^3 - 1, 6^3 - 1, 7^3 - 1, 8^3 - 1$$

 \therefore 28 is wrong and should be replaced by $(3^3 - 1)$ i.e. 26.

Hence, the answer is (B).

Ex.23 Find the wrong number in the series:

3, 8, 15, 24, 34, 48, 63

(A) 15

(B) 24

(C) 34

(D) 48

Sol. The difference between consecutive terms of the given series are respectively 5, 7, 9, 11 and 13. Clearly, 34 is a wrong number and must be replaced by (24 + 11) i.e. 35.

Hence, the answer is (C).

Ex.24 24, 27, 31, 33, 36

(A) 24

(B) 27

(C) 31

(D) 33

Sol. Each term in the series is increased by 3 to obtain the next term.

So, 31 is wrong and must be replaced by (27+3) i.e. 30.

Ex.25 196, 169, 144, 121, 80

(A) 80 (B)

(B) 121

(C) 169

(D) 196

Sol. The sequence is $(14)^2$, $(13)^2$, $(12)^2$, $(11)^2$, $(10)^2$.

So, 80 is wrong and must be replaced by $(10)^2$ i.e. 100.

Ex.26 3, 5, 7, 9, 11, 13

(A) 3

(B) 5

(C)7

(D) 9

Sol. The series consists of consecutive prime numbers. So, 9 is wrong.

Ex.27 121, 143, 165, 186, 209

(A) 143

(B) 165

(C) 186

(D) 209

Sol. Each term of the series is increased by 22 to obtain the next term.

So, 186 is wrong and must be replaced by (165 + 22) i.e. 187.

Ex.28 1, 2, 4, 8, 16, 32, 64, 96

(A) 4

(B) 32

(C) 64

(D) 96

Sol. Each term of the series is obtained by multiplying the preceding term by 2

So, 96 is wrong and must be replaced by (64×2) i.e. 128.

EXERCISE

Q.1	462, 420, 3	80, (), 30	06										
V. -2			(C) 342	(D) 352									
Q.2	3, 8, 22, 63, 185, ()												
			(C) 295	(D) 285									
Q.3	1, 2, 5, 12,	1, 2, 5, 12, 27, 58, 121, ()											
	(A) 246	(B) 247	(C) 248	(D) 249									
Q.4	0.5, 0.55, 0	.65, 0.8, ()										
	(A) 0.9	(B) 0.82	(C) 1	(D) 0.95									
Directions:													
In each of the following questions, one term in the													
number series is wrong. Find out the wrong term.													
Q.5		2, 48, 20, 8											
	(A) 188	(B) 92	(C) 48	(D) 20									
Q.6	1, 3, 7, 15,	27, 63, 127											
	(A) 7	(B) 15	(C) 27	(D) 63									
Q.7	5, 10, 17, 2	4, 37											
	(A) 10	(B) 17	(C) 24	(D) 37									
Q.8	1, 3, 10, 21	, 64, 129, 2	56, 778										
	(A) 10	(B) 21	(C) 129	(D) 256									
Q.9	15, 16, 22, 29, 45, 70												
	(A) 16	(B) 22	(C) 45	(D) 70									
Q.10	6, 14, 30, 64, 126												
_		(B) 14	(C) 64	(D) 126									
Q.11	10, 26, 74, 218, 654, 1946, 5834												
		(B) 74		(D) 654									
Q.12	3, 7, 15, 39, 63, 127, 255, 511												
•		(B) 39		(D) 127									
Q.13	445, 221, 109, 46, 25, 11, 4												
C			(C) 109	(D) 221									
Q.14	1236 2346	, 3456, 456	6 5686										
Ų.1 4		(B) 3456		(D) 5686									
O 15				(B) 3000									
Q.15		(B) 320, 550,	, 2560 (C) 550	(D) 2560									
0.17				(D) 2300									
Q.16		3, 22, 18, 32 (B) 9		(D) 22									
0.4=	(A) 0	(D) 9	(C) 13	(D) 22									

Q.17 8, 27, 125, 343, 1331

(B) 343

(C) 1331 (D) None

(A) 8

SISE											
Q.18	10, 14, 28,	32, 64, 68,	132								
	(A) 28	(B) 32	(C) 64	(D) 132							
Q.19		7, 11, 11, 15		(D) 15							
0.00		(B) 12		(D) 15							
Q.20		3, 32, 4, 41, (B) 11		(D) 51							
Q.21		12, 40, 26, 7		(D) 31							
Q.21		(B) 20		(D) 26							
Q.22		110, 132, 1		, ,							
			(C) 110	(D) 150							
Q.23	8, 13, 21, 3	32, 47, 63, 8	3								
	(A) 13	(B) 32	(C) 47	(D) 63							
Q.24		80, 85, 82,		(D) 5 0							
 .			(C) 86								
Directi	Direction-(Q.25–40) : In the following series find the number in place of question mark?										
Q.25	23,19, ?, 12, 9.										
	(A) 14	(B) 15	(C) 16	(D) 17							
Q.26	$8\frac{4}{7}$, $9\frac{3}{3}$, $11\frac{5}{5}$, $13\frac{8}{4}$,?										
	(A) $\frac{80}{3}$	(B) $\frac{70}{3}$	(C) $\frac{50}{3}$	(D) 20							
Q.27	25, 22, 30,	<u>?</u> , 35.									
	(A) 27	(B) 26	(C) 28	(D) 29							
Q.28	8, 13, 18, 1	11, 16, 21, 1	6, 21, 26, <u>?</u>	? ?							
	(A) 28, 33	, 38	(B) 23, 28	3, 33							
	(C) 22, 27,	, 32	(D) 21, 20	, 31							
Q.29	29, 24, 26,	21, 23, ?									
	(A) 20	(B) 19	(C) 17	(D) 18							
Q.30	3, 5, 6, 8,	11, 13, ?									
	(A) 17	(B) 18	(C) 15	(D) 20							
Q.31	2, 8, 14, 26	5, 38, 56, <u>?</u>									
	(A) 78	(B) 80	(C) 74	(D) 75							
Q.32	2, 6, 8, 10,	10, 16, 20,	22, 24, 24,	30, <u>?</u>							
	(A) 34	(B) 36	(C) 32	(D) 30							
Q.33	2, 4, 12, 6,	12, 36, 18,	36, 108, <u>?</u>								

(A) 72

(B) 54

(C) 90

(D) 108

Q.34 2, 7, 15, 26, 40, 57, ?

2, 5, 11, $\frac{?}{2}$, 47.

(A) 17 (B) 18

11, 5, 20, 12, ?, 26, 74, 54.

(A) 80

Q.35

Q.36

(B) 81

(C) 75

(D) 77

(D) 26

Q.38 1, 4, 7, 10, $\underline{?}$, 16, 19, $\underline{?}$

(A) 13, 22

(B) 11, 22

(C) 13, 21

(D) 13, 23

Q.39 5, 13, $\underline{?}$, 109, 325, 973.

(A) 39

(B) 36

(C) 37

(D) 35

Q.40 1, 4, 10, ?, 46.

(4) 20

(A) 20 (B) 22

(C) 24

(D) 19

Q.37 324, 289, 256, 225, 196, ?

(A) 196

(A) 30

(B) 121

(B) 38

(C) 144

(C) 21

(C) 48

(D) 169

(D) 28

ANSWER KEY

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
С	Α	С	С	С	С	С	D	В	C	D	В	В	D	С	В	D	D	В	С
21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
С	D	C	С	С	D	A	В	D	В	С	A	В	D	Α	В	D	A	С	В