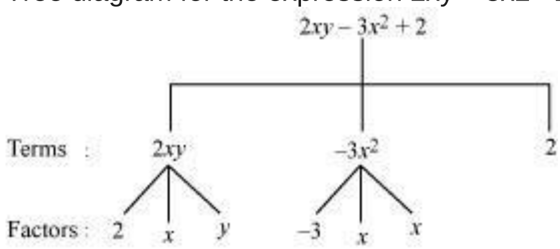
Algebraic Expressions

- Algebraic expressions are formed by combining variables with constants using operations of addition, subtraction, multiplication and division.

  For example: 4xy, 2x2 – 3, 7xy + 2x, etc.
  In an algebraic expression, say 2xy – 3x2 + 2; 2xy, (–3x2), 2 are known as the terms of the expression.
  The expression 2xy – 3x2 +2 is formed by adding the terms 2xy, (–3x2) and 2 where 2, x, y are factors of the term 2xy; (–3), x, x are factors of the term (–3x2); 2 is the factor of the term 2.
  For an expression, the terms and its factors can be represented easily and elegantly by a tree diagram.
  Tree diagram for the expression 2xy – 3x2 +2:



  Note: In an expression, 1 is not taken as separate factor.

- The numerical factor of a term is known as its coefficient. For example, for the term –3x2y, the coefficient is (–3).
- The terms having the same algebraic factors are called like terms, while the terms having different algebraic factors are called unlike terms.

  For example: 13x2y, – 23x2y are like terms; 12xy, 3x2 are unlike terms

- Expressions can be classified on the basis of the number of terms present in them.
  - Expression containing only one term is called a **monomial**.

    For example, $2x$, $-3x^2$, $2xy$ etc.
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  - Expression containing only two unlike terms is called a **binomial**.

    For example, $2x + 3$, $3x^2 - 2$, $-2xy + 3y^2$ etc.
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  - Expression with three terms, where the terms are unlike is called a **trinomial**.

    For example, $2x^2 - 3x + 1$, $-2xy + 5y + 6x$ etc.

  In general, the expression with one or more terms is called a polynomial.

- **Degree** of the polynomial is the highest exponent of the variable in the polynomial.

  For example, polynomial $2x^2 - 3x + 1$ has degree 2.

- Addition and subtraction of algebraic expressions:
  - The sum or difference of two like terms is a like term, with its numerical coefficient equal to the sum or difference of the numerical coefficients of the two like terms.
  - When algebraic expressions are added, the like terms are added and unlike terms are left as they were.

Example : Subtract (x^2-2y^2+y) from the sum of $(-2x^2+3x+2)$ and $(-2y+3x^2+5x)$

Solution:

$$\left(-2x^2+3x+2\right)+\left(-2y+3x^2+5x\right)$$

$$=\left(-2x^2+3x^2\right)+\left(3x+5x\right)-2y+2 \quad \text{[Rearranging terms]}$$

$$=x^2+8x-2y+2$$

$$\therefore \left(x^2+8x-2y+2\right)-\left(x^2-2y^2+y\right)$$

$$=x^2+8x-2y+2-x^2+2y^2-y$$

$$=\left(x^2-x^2\right)+2y^2+8x+\left(-2y-y\right)+2 \quad \text{[Rearranging terms]}$$

$$=2y^2+8x-3y+2$$

- **Value of an expression at given values of variables:**
  - The value of an expression depends on the values of the variables forming the expression.
  - The value of an expression at particular values of variables can be found by substituting the variables by the corresponding values given.

Example:

What is the value of the expression $-a^2b + 2ab + b$ at $a = -1$ and $b = 2$?

Solution:

The given expression is $-a^2b + 2ab + b$.

Substituting $a = -1$ and $b = 2$ we get,

$-a^2b + 2ab + b = -(-1)^2(2) + 2(-1)(2) + (2)$

$= -2 - 4 + 2$

$= -4$

- Geometric formulae are written in general form by using variables.

For example,

(1) Perimeter of an equilateral triangle = 3 × length of one side

Let the length of the side of the equilateral triangle be l.

∴ Perimeter of an equilateral triangle = $3l$

(2) Area of a square = $(\text{side})^2$

Let the side of square be *s.*
∴ Area of a square = s^2

- Certain number patterns can be written in general form by using expressions.
- The successor and predecessor of a natural number n is denoted by (n + 1) and (n – 1) respectively.
- Any even number is denoted by 2n and any odd number is denoted by 2n + 1, where n is as whole number.

For example, 9, 19, 29, ….
Here, 9 = 10 – 1, 19 = 10 × 2 – 1, 29 = 10 × 3 – 1; therefore, this pattern can be written in general form as (10n – 1), where n is the ordinal number of terms.