Sample Paper-02
SUMMATIVE ASSESSMENT –I
Class – IX MATHEMATICS

Time allowed: 3 hours Maximum Marks: 90

General Instructions:

a) All questions are compulsory.
b) The question paper comprises of 31 questions divided into four sections A, B, C and D. You are to attempt all the four sections.
c) Questions 1 to 4 in section A are one mark questions.
d) Questions 5 to 10 in section B are two marks questions.
e) Questions 11 to 20 in section C are three marks questions.
f) Questions 21 to 31 in section D are four marks questions.
g) There is no overall choice in the question paper. Use of calculators is not permitted.

Section A

1. Find the value of $2^{\frac{1}{3}} \times 2^{-\frac{4}{3}}$.

2. Is $x^2 + \frac{4x^{3/2}}{\sqrt{x}}$ a polynomial? Justify your answer.

3. An angle is 14° more than its complement. Find its measure.

4. Where will the Point $(-2,0)$ lies?

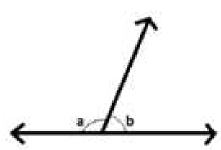
Section B

5. Find the value of $125^{\frac{1}{3}}$.

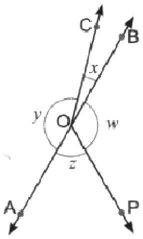
6. If $x + \frac{1}{x} = 4$, then find the value of $x^2 + \frac{1}{x^2}$.

7. If 'm, n' are lines in the same plane such that 'p' intersects m and $n \parallel m$, show that 'p' intersects 'n' also.

8. In the given figure, 'a' is greater than 'b' by one-third of a right angle. Find the value of 'a' and 'b'.



9. In Fig. 4.11, if x + y =w + z, then prove that AOB is a line.



10. Show that all the line segments drawn from a given point not on it, the perpendicular line segment is the shortest.

Section C

11. Express $2.4\overline{178}$ in the from $\frac{p}{q}$

12. Rationalise: $\frac{1}{\sqrt{7}+\sqrt{3}-\sqrt{2}}$

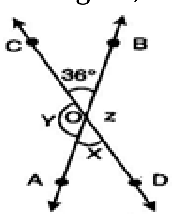
13. Show that 2x + 1 is a factor of polynomial $2x^3 - 11x^2 - 4x + 1$.

14. By actual division, find the quotient and remainder when $3x^4 - 4x^3 - 3x - 1$ is divided by x + 1.

15. What must be added to $\left(x^3 - 3x^2 + 4x - 13\right)$ to obtain a polynomial which is exactly divisible by $\left(x - 3\right)$?

16. If a point C lies between two points A and B such that AC = BC, then prove that AC = $\frac{1}{2}$AB. Explain by drawing the figure.

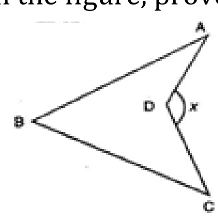
17. In the figure, line AB and CD intersect at O and $\angle$BOC = 36°. Find $\angle$X, $\angle$Y and $\angle$Z.



18. Prove that two lines which are parallel to the same line are parallel to one another.

19. The sum and difference of two angles of a triangle are $128°$ and $22°$ respectively. Find all the angles of the triangle.

20. In the figure, prove that $\angle x = \angle A + \angle B + \angle C$



Section D

21. Find the values of a and b: $\frac{7+\sqrt{5}}{7-\sqrt{5}}-\frac{7-\sqrt{5}}{7+\sqrt{5}}=a+\frac{7}{11}\sqrt{5}b.$

22. Simplify: $\frac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}}-\frac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}}-\frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}}$

23. Factorise: $2x^3-3x^2-17x+30$

24. If both (x – 2) and are factors of px^2+5x+r, show that p = r.

25. Check whether $7+3x$ is a factor of $3x^3+7x$.

26. Factorise: $\frac{1}{27}(2x+5y)^3+\left(\frac{-5}{3}y+\frac{3}{4}z\right)^3-\left(\frac{3}{4}z+\frac{2}{3}x\right)^3$

27. State and prove RHS congruence criterion.

28. Show that EFGH is a ||gram and its area is half of the area of ||gram ABCD. If E, F, G, H are respectively the mid points of the sides AB, BC, CD and DA.

29. If two parallel lines are intersected by a transversal, prove that the bisectors of the two pairs of interior angles enclose a rectangle.

30. It is given that $\angle XYZ=64^0$ and XY is produced to point P. Draw a figure from the given information. If ray YQ bisects $\angle ZYP$. Find $\angle XYQ$ and reflex $\angle QYP$.

31. A field is in the shape of a trapezium whose parallel sides are 50m and 15m. the non- parallel sides are 20m and 25m. find the area of the trapezium.

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ANSWER KEY

1. $2^{\frac{1}{3}} \times 2^{\frac{-4}{3}} = 2^{\frac{1}{3}-\frac{4}{3}} = 2^{\frac{1-4}{3}} = 2^{\frac{-3}{3}}$

$= 2^{-1} = \frac{1}{2}$

2. Yes,

$$x^2 + \frac{4x^{3/2}}{\sqrt{x}} = x^2 + 4x^{\frac{3}{2}} \times x^{-\frac{1}{2}}$$

$$= x^2 + 4x^{\frac{3}{2}-\frac{1}{2}} = x^2 + 4x$$

3. An angle is 14° more than its complement. Find its measure.

4. On the negative part of the x - axis.

5. We know that $a^{\frac{1}{n}} = \sqrt[n]{a}$, where $a > 0$.

We conclude that $125^{\frac{1}{3}}$ can also be written as $\sqrt[3]{125} = \sqrt[3]{5\times5\times5}$

$\sqrt[3]{125} = \sqrt[3]{5\times5\times5} = 5$

Therefore the value of $125^{\frac{1}{3}}$ will be 5.

6. $x + \frac{1}{x} = 4$

$\Rightarrow \left(x + \frac{1}{x}\right)^2 = 4^2$

$\Rightarrow x^2 + \frac{1}{x^2} + 2x \times \frac{1}{x} = 16$

$\Rightarrow x^2 + \frac{1}{x^2} = 16 - 2 = 14$

$\therefore x^2 + \frac{1}{x^2} = 14$

7. Let if possible p and n be non-intersecting lines

$\Rightarrow p \parallel n$

But $n \parallel m$

Therefore, $p \parallel m$

$\Rightarrow$ p and m are non intersecting lines

But it is given that p and m are intersecting lines.

So, our supposition is wrong.

Hence, p intersects n.

8. $a+b=180°$ (linear pair)

$a-b=30°$ (given)

$\therefore a=105°, b=75°$

9. As sum of all the angles about a point is equal to 360°.

Therefore, x + y + z + w = 360°

⇒ (x + y) + (z + w) = 360°

Also, z + w = x + y (Given)

∴ (x + y) + (x + y) = 360°

⇒ 2x + 2y = 360°

⇒ 2(x + y) = 360°

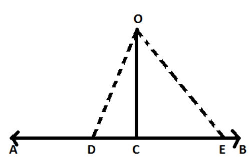
⇒ (x + y) **=** 180°

∴ AOB is a straight line.

10. **Given**: let AB is a line and O is a point outside it. OC is drawn perpendicular to AB.

To prove: OC is the shortest side for all the segments drawn from point O to line AB.

Construction: take point D and E on the line segment AB and join to O.



Proof: In $\Delta ODC, \angle OCD=90°$

$\therefore \angle ODC$ is acute angle.

$\Rightarrow \angle OCD > \angle ODC$

$OD > OC$ (greater angle has greater side opposite to it)

Hence proved.

11. Let $\frac{p}{q}=2.4\overline{178}$

$\frac{p}{q}=2.4178178178$

Multiplying by 10

$$10\frac{p}{q} = 24.178178$$

Multiplying by 1000

$$10,000\frac{p}{q} = 1000 \times 24.178178$$

$$10,000\frac{p}{q} = 24178.178178$$

$$10000\frac{p}{q} - \frac{p}{q} = 24178.178178 - 24.178178$$

$$9999\frac{p}{q} = 24154$$

$$\frac{p}{q} = \frac{24154}{9999}$$

12. $$\frac{1}{\sqrt{7}+\sqrt{3}-\sqrt{2}} = \frac{1}{\sqrt{7}+\sqrt{3}-\sqrt{2}} \times \frac{\left(\sqrt{7}+\sqrt{3}\right)+\sqrt{2}}{\left(\sqrt{7}+\sqrt{3}\right)+\sqrt{2}}$$

$$= \frac{\sqrt{7}+\sqrt{3}+\sqrt{2}}{\left(\sqrt{7}+\sqrt{3}\right)^2+\left(\sqrt{2}\right)^2} = \frac{\sqrt{7}+\sqrt{3}+\sqrt{2}}{\left(\sqrt{7}\right)^2+\left(\sqrt{3}\right)^2+2\sqrt{21}-2}$$

$$= \frac{\sqrt{7}+\sqrt{3}+\sqrt{2}}{7+3+2\sqrt{21}-2} = \frac{\sqrt{7}+\sqrt{3}+\sqrt{2}}{8+2\sqrt{21}} = \frac{\sqrt{7}+\sqrt{3}+\sqrt{2}}{2\left(4+\sqrt{21}\right)} \times \frac{4-\sqrt{21}}{4-\sqrt{21}}$$

$$= \frac{4\sqrt{7}+4\sqrt{3}+4\sqrt{2}-7\sqrt{3}-3\sqrt{7}-\sqrt{42}}{2(16-21)}$$

$$= \frac{\sqrt{7}-3\sqrt{3}+4\sqrt{2}-\sqrt{42}}{-10} = \frac{3\sqrt{3}-4\sqrt{2}+\sqrt{42}-\sqrt{7}}{10}$$

13. Let, $p(x) = 2x^3 - 11x^2 - 4x + 1$ and $g(x) = 2x+1$

By factor theorem (2x + 1) will be a factor of $p(x)$ if $p(x)\ p\left(\frac{-1}{2}\right) = 0$

Now, $p(x) = 2x^3 - 11x^2 - 4x + 1$

$$\Rightarrow \quad p\left(\frac{-1}{2}\right) = 2\left(\frac{-1}{2}\right) - 11\left(\frac{-1}{2}\right)^2 - 4\left(\frac{-1}{2}\right) + 1$$

$$= 2\left(\frac{-1}{8}\right) - 11 \times \frac{1}{4} + 4 \times \frac{1}{2} + 1 = \frac{1}{4} - \frac{11}{4} + 2 + 1$$

$$= \frac{-1-11+8+4}{4} = \frac{-12+12}{4} \quad \Rightarrow p\left(\frac{-1}{2}\right) = 0$$

As $p\left(\frac{-1}{2}\right) = 0$, therefore (2x + 1) is a factor of $2x^3 - 11x^2 - 4x + 1$.

14. By long division, we have

$$\begin{array}{r|l} & 3x^3 - 7x^2 + 7x - 10 \\ \hline x+1 & 3x^4 - 4x^3 - 3x - 1 \\ & \underline{{}_{-}3x^4 \pm 3x^3} \\ & -7x^3 - 3x - 1 \\ & \underline{\mp 7x^3 \quad \mp 7x^2} \\ & 7x^2 - 3x - 1 \\ & \underline{{}_{-}7x^2 \pm 7x} \\ & -10x - 1 \\ & \underline{\mp 10x \mp 10} \\ & 9 \end{array}$$

Quotient $= 3x^3 - 7x^2 + 7x - 10$, Remainder = 9

15. Let $f(x) = x^3 - 3x^2 + 4x - 13$ and $g(x) = x - 3$

Let k be added to $f(x)$ so that it may be exactly divisible by $(x-3)$.

$\therefore \quad p(x) = (x^3 - 3x^2 + 4x - 13) + k$

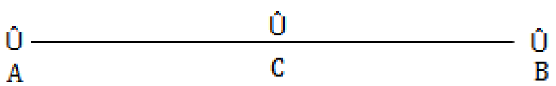
$\therefore \quad p(3) = (3)^3 - 3(3)^2 + 4(3) - 13 + k = 0$

$\Rightarrow \quad 27 - 27 + 12 - 13 + k = 0$

$\Rightarrow \quad -1 + k = 0$

$\Rightarrow \quad k = 1$

16. Given: AC = BC



So, AC + AC = AC + BC [Equals are added to equals]

$\Rightarrow$ 2AC = AB [$\because$ AC + CB concides with AB]

$\Rightarrow$ AC = $\frac{1}{2}$AB

17. $\angle$Y = $\angle$Z ....(i) [Vertically opposite angles]

And $\angle$COB + $\angle$Y = 180° [Linear pair]

$\Rightarrow$ 36° + $\angle$Y = 180°

$\Rightarrow$ $\angle$Y = $180° - 36° = 144°$

From eq. (i),

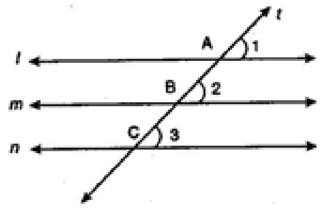
$\angle$Y = $\angle$Z = 144°

18. Given: Three lines l, m, n are such that $l \parallel m$ and $m \parallel n$.

To prove: $l \parallel n$

Construction: Draw a transversal line 't' cutting l, m and n at A, B and C respectively.

Proof : Since $l \parallel m$ and 't' intersects them at A and B.



$\Rightarrow$ $\angle 1 = \angle 2$ .....(i) [Corresponding angles]

Again $m \parallel n$ and transversal 't' intersects them B and C respectively.

$\Rightarrow$ $\angle 2 = \angle 3$ .....(ii) [Corresponding angles]

From eq. (i) and (ii), we get,

$\angle 1 = \angle 2 = \angle 3$

$\Rightarrow$ $\angle 1 = \angle 3$

But these are corresponding angles.

$\therefore l \parallel n$ [Corresponding angles axiom]

19. In a triangle ABC, given, $\angle A + \angle B = 128°$ ....(i)

And $\angle A - \angle B = 22°$ ....(ii)

On adding eq. (i) and (ii), we get,

$2\angle A = 150°$ $\Rightarrow$ $\angle A = 75°$

On subtracting eq. (ii) from (i), we get,

$2\angle B = 106°$ $\Rightarrow$ $\angle B = 53°$

In triangle ABC,

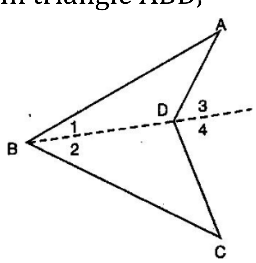
$\angle A + \angle B + \angle C = 180°$ [Sum of all the angles of a triangle = $180°$]

$\Rightarrow$ $75° + 53° + \angle C = 180°$

$\Rightarrow$ $\angle C = 52°$

20. Joined BD.

In triangle ABD,



$\angle A + \angle 1 = \angle 3$ [Exterior angles]

In triangle BCD,

$\angle C + \angle 2 = \angle 4$ [Exterior angles]

On adding, we get,

$\angle A + \angle C + \angle 1 + \angle 2 = \angle 3 + \angle 4$

$\Rightarrow \angle A + \angle B + \angle C = \angle x$

21. L.H.S. $\dfrac{7+\sqrt{5}}{7-\sqrt{5}} - \dfrac{7-\sqrt{5}}{7+\sqrt{5}}$

Rationalising the denominator, we get

$$\frac{7+\sqrt{5}}{7-\sqrt{5}}\times\frac{7+\sqrt{5}}{7+\sqrt{5}}-\frac{7-\sqrt{5}}{7+\sqrt{5}}\times\frac{7-\sqrt{5}}{7-\sqrt{5}}$$

$$=\frac{\left(7+\sqrt{5}\right)^2}{7^2-\left(\sqrt{5}\right)^2}-\frac{\left(7-\sqrt{5}\right)^2}{7^2+\left(\sqrt{5}\right)^2}$$

$$=\frac{7^2+\left(\sqrt{5}\right)^2+2\times7\times\sqrt{5}}{49-5}-\frac{7^2+\left(\sqrt{5}\right)^2-2\times7\times\sqrt{5}}{49-5}$$

$$=\frac{49+5+14\sqrt{5}}{44}-\frac{49+5-14\sqrt{5}}{44}=\frac{54+14\sqrt{5}}{44}-\frac{54-14\sqrt{5}}{44}$$

$$=\frac{54+14\sqrt{5}-54+14\sqrt{5}}{44}=\frac{28\sqrt{5}}{44}=\frac{7\sqrt{5}}{11}=0+\frac{7\sqrt{5}}{11}$$

Hence, $0+\frac{7\sqrt{5}}{11}=a+\frac{7\sqrt{5}\,b}{11}$

$$\Rightarrow a=0, b=1$$

22. $$\frac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}}-\frac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}}-\frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}}$$

$$\frac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}}=\frac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}}\times\frac{\sqrt{10}-\sqrt{3}}{\sqrt{10}-\sqrt{3}}=\frac{7\sqrt{3}\left(\sqrt{10}-\sqrt{3}\right)}{\left(\sqrt{10}\right)^2+\left(\sqrt{3}\right)^2}=\frac{7\left(\sqrt{30}-3\right)}{10-3}$$

$$\therefore\ \frac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}}=\frac{7\left(\sqrt{30}-\sqrt{3}\right)}{7}=\sqrt{30}-3$$

$$\frac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}}=\frac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}}\times\frac{\sqrt{6}-\sqrt{5}}{\sqrt{6}-\sqrt{5}}=\frac{2\sqrt{30}-2\times5}{\left(\sqrt{6}\right)^2-\left(\sqrt{5}\right)^2}$$

$$\therefore\ \frac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}}=\frac{2\sqrt{30}-10}{6-5}=2\sqrt{30}-10$$

$$\frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}}=\frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}}\times\frac{\sqrt{15}-3\sqrt{2}}{\sqrt{15}-3\sqrt{2}}=\frac{3\sqrt{30}-18}{15-18}=\frac{3\sqrt{30}-18}{-3}$$

$$\therefore\ \frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}}=\frac{3\left(\sqrt{30}-6\right)}{-3}=-\left(\sqrt{30}-6\right)=6-\sqrt{30}$$

Therefore, $\dfrac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}}-\dfrac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}}-\dfrac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}}$

$=\sqrt{30}-3-\left(2\sqrt{30}-10\right)-\left(6-\sqrt{30}\right)$

$=\sqrt{30}-3-2\sqrt{30}+10-6+\sqrt{30}$

$=10-9+2\sqrt{30}-2\sqrt{30}=1$

23. Let $p(x)=2x^3-3x^2-17x+30$

$\therefore \quad p(2)=2\times 2^3-3\times 2^2-17\times 2+30=16-12-34+30$

$p(2)=46-46=0$

As p(2) = 0, therefore (x – 2) is factor of p(x).

Let us divide p(x) by (x – 2) by long division method as given below:

$$
\begin{array}{r|l}
 & 2x^2+x-15 \\
\hline
x-2 & 2x^3-3x^2-17x+30 \\
 & \underline{-2x^3 \mp 4x^2} \\
 & \quad x^2-17x+30 \\
 & \quad \underline{-x^2 \mp 2x} \\
 & \qquad -15x+30 \\
 & \qquad \underline{\mp 15x \pm 30} \\
 & \qquad\qquad 0
\end{array}
$$

$\therefore \; p(x)=2x^3-3x^2-17x+30=(x-2)(2x^2+x-15)$

$=(x-2)(2x^2+6x-5x-15)=(x-2)\left[2x(x+3)-5(x+3)\right]$

$=(x-2)\left[(x+3)(2x-5)\right]=(x-2)(x+3)(2x-5)$

24. Let f(x) = px^2+5x+r .

As (x – 2) is a factor of *f*(*x*), so *f*(2) = 0

$p\times 2^2+5\times 2+r=0$

$\Rightarrow 4p+10+r=0 \qquad (i)$

Also $\left(x-\dfrac{1}{2}\right)$ is a factor of *f*(x), so $f\left(\dfrac{1}{2}\right)=0$

$p\left(\dfrac{1}{2}\right)^2+5.\dfrac{1}{2}+r=0$

$\Rightarrow \quad \dfrac{p}{4}+\dfrac{5}{2}+r=0$

$p+10+4r=0 \qquad ...(ii)$

From equations (i) and (ii), we have

$4p+10+r=p+10+4r$

$4p-p=10+4r-10-r$

$\Rightarrow 3p=3r$

$\Rightarrow p=r$

25. We know that if the polynomial $7+3x$ is a factor of $3x^3+7x$, then on dividing the polynomial $3x^3+7x$ by $7+3x$, we must get the remainder as 0.

We need to find the zero of the polynomial $7+3x$.

$7+3x=0 \qquad \Rightarrow \qquad x=-\frac{7}{3}$

While applying the remainder theorem, we need to put the zero of the polynomial $7+3x$ in the polynomial $3x^3+7x$, to get

$p(x)=3x^3+7x$

$=3\left(-\frac{7}{3}\right)^3+7\left(-\frac{7}{3}\right)=3\left(-\frac{343}{27}\right)-\frac{49}{3}$

$=-\frac{343}{9}-\frac{49}{3}=\frac{-343-147}{9}$

$=\frac{-490}{9}.$

We conclude that on dividing the polynomial $3x^3+7x$ by $7+3x$, we will get the remainder as $\frac{-490}{9}$, which is not 0.

Therefore, we conclude that $7+3x$ is not a factor of $3x^3+7x$.

26. Given expression can be written as

$\left[\frac{1}{3}(2x+5y)\right]^3+\left[-\frac{5}{3}y+\frac{3}{4}z\right]^3+\left[-\frac{3}{4}z-\frac{2}{3}x\right]^3$

Let $\frac{1}{3}(2x+5y)=a, \quad \frac{-5}{3}y+\frac{3}{4}z=b$

and $\quad \frac{-3}{4}z-\frac{2}{3}x=C$

a + b + c = $\frac{2}{3}x+\frac{5}{3}y-\frac{5}{3}y+\frac{5}{4}z-\frac{3}{4}z-\frac{2}{3}x$ =0

$\therefore a^3+b^3+c^3=3abc$

Thus,

$$\frac{1}{27}(2x+5y)^3+\left(\frac{-5}{3}y+\frac{3}{4}z\right)^3-\left(\frac{3}{4}z+\frac{2}{3}x\right)^3$$

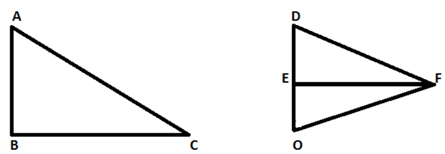
$$= \; 3\left[\frac{1}{3}(2x+5y)\left(\frac{-5}{3}y+\frac{3}{4}z\right)\left(\frac{-3}{4}z-\frac{2}{3}x\right)\right]$$

$$= \; -(2x+5y)\left(\frac{-5}{3}y+\frac{3}{4}z\right)\left(\frac{3}{4}z+\frac{2}{3}x\right)$$

$$= \; -(2x+5y)\left(\frac{-20y+9z}{12}\right)\left(\frac{9z+8x}{12}\right)$$

$$= \; \frac{1}{144}(2x+5y)\;(20y-9z)\;(9z+8x)$$

27. RHS congruence criterion: two right triangles are congruent if the hypotenuse and one side of one triangle are respectively equal to the hypotenuse and one side of the other triangle.



Given: Two right triangles ABC and DEF in which $\angle B = \angle E = 90°, AC = DF, BC = EF$

To Prove: $\Delta ABC \cong \Delta DEF$

Construction: Produce DE to O so that $EO = AB$. Join OF

Proof: In ΔABC and ΔOEF , we have

$AB = OE$ (By construction)

$\angle B = \angle FEO = 90°$

$BC = EF$

$\Delta ABC \cong \Delta OEF$ (By SAS)

$\Rightarrow \angle A = \angle O$ ....(1)

$AC = OF$ (By CPCT)............(2)

Also, $AC = DF$ (Given)

$\therefore DF = OF$

$\Rightarrow \angle D = \angle O$ (Angles opposite to equal sides in ΔDOF are equal)

From (1) and (3), we get

$\angle A = \angle D$ ....(4)

Thus, in ΔABC and ΔDEF , we have

$\angle A = \angle D$ (from (4))

$\angle B = \angle E$ (given)

$\Rightarrow \angle A + \angle B = \angle D + \angle E$

$\Rightarrow 180° - \angle C = 180° - \angle F$ $(\because \angle A + \angle B + \angle C = 180°$ and $\angle D + \angle E + \angle F = 180°)$

$\Rightarrow \angle C = \angle F$

Now, in ΔABC and ΔDEF, we have

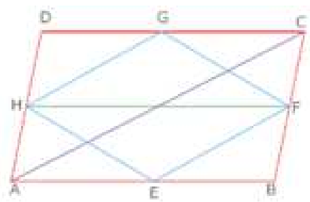
$BC = EF$ (given)

$\angle C = \angle F$ (proved above)

$AC = DF$

$\therefore \Delta ABC \cong \Delta DEF$ (By SAS)

28. Join AC and HF



E and F are the mid-points of AB and BC

$\therefore$ EF= $\frac{1}{2}$ AC and EF||AC..........(i)

Similarly ,GH = $\frac{1}{2}$ AC and GH||AC.........(ii)

From (i) and (ii)

GH = EF and GH||EF

$\therefore$ EFGH is a ||gram

$ar(\Delta HGF) = \frac{1}{2} ar(\|\ gram\ HDFC) ....(iii)$

$ar(\Delta HEF) = \frac{1}{2} ar(\|\ gram\ HABF) ....(iv)$

Adding (iii) and (iv),

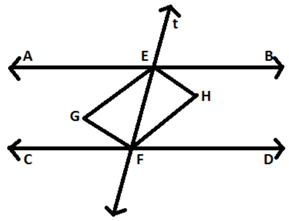
$ar(\Delta HGF) + ar(\Delta HEF) = \frac{1}{2} ar(\|\ gram\ HDCF) + ar(\|\ gram\ HABF)$

$\Rightarrow ar(\|\ gram\ EFGH) \frac{1}{2}$ are$(\|\ gram\ ABCD)$

29. **Given**: $AB \parallel CD$ and a transversal t cuts them at E and F respectively. EF, FG, EH, FH are the bisectors of the interior angles $\angle AEF, \angle CFE, \angle BEF, \angle EFD$ respectively.

To prove: $EGFH$ is a rectangle.

Proof: $AB \parallel CD$ and the transversal t cuts them at E and F respectively.



Proof: $AB \parallel CD$ and the transversal t cuts them at E and F respectively.

$\therefore \angle AEF = \angle EFD$ (alternate interior angles)

$\Rightarrow \frac{1}{2}\angle AEF = \frac{1}{2}\angle EFD \Rightarrow \angle GEF = \angle EFH$

$\therefore EG \parallel FH$ ($\because$ alternate interior angles formed above when transversal EF cuts EG and FH)

Similarly, $EG \parallel FH$

$\therefore EGFH$ is a parallelogram.

$\therefore \angle AEF + \angle BEF = 180°$ (linear pair)

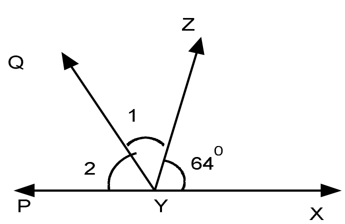
$\Rightarrow \frac{1}{2}\angle AEF + \frac{1}{2}\angle BEF = 90° \Rightarrow \angle GEF + \angle HEF = 90°$

$\Rightarrow \angle GEH = 90° (\because \angle GEF + \angle HEF = \angle GEH)$

Thus, $EGFH$ is a parallelogram one of whose angles is $90°$.

$\therefore EGFH$ is a rectangle.

30.



YQ *bisects* $\angle ZYP$

$\therefore \angle 1 = \angle 2$

$\angle 1 + \angle 2 + \angle 64^0 = 180^0$ [YX is a line]

$\angle 1 + \angle 1 + 64^0 = 180^0$

$2\angle 1 = 180^0 - 64^0$

$2\angle 1 = 116^0$

$\angle 1 = 58^0$

$\therefore \angle XYQ = 64^0 + 58^0 = 122^0$

$\angle 2 + \angle XYQ = 180^0$ $\quad\quad \angle 1 = \angle 2 = \angle QYP = 58^0$

$\angle 2 + 122^0 = 180^0$

$\angle 2 = 180^0 - 122^0$

$\angle QYP = \angle 2 = 58^0$

Re $flex$ $\angle QYP = 360^0 - \angle QYP$

$= 360^0 - 58^0$

$= 302^0$

31. BC and AD are non-parallel sides of the trapezium. Through the vertex C, we draw $CE \parallel DA$ and CE meets AB and E. Here, $AECD$ becomes a parallelogram. Then $AE = 15m, BE = 35m, CE = 20m.$

Semiperimeter of $\Delta CEB = 40m$

Area of $\Delta CEB = 100\sqrt{6} sq.m$

Also, we have $CE \perp BE$ and let $CL = h$. So, area of $\Delta CEB = \frac{1}{2} \times BE \times h = 100\sqrt{6}$

$\Rightarrow h = \frac{40\sqrt{6}}{7} m$

Area of trapezium $ABCD = \frac{1}{2}(AB + CD) \times h = \frac{1300\sqrt{6}}{7} sq.m$