Relations and Functions

Question 1.
The domain of the function ${}^{7-x}P_{x-3}$ is
(a) {1, 2, 3}
(b) {3, 4, 5, 6}
(c) {3, 4, 5}
(d) {1, 2, 3, 4, 5}

Answer: (c) {3, 4, 5}
The function $f(x) = {}^{7-x}P_{x-3}$ is defined only if x is an integer satisfying the following inequalities:
1. $7 – x \geq 0$
2. $x – 3 \geq 0$
3. $7 – x \geq x – 3$
Now, from 1, we get $x \leq 7$ ……… 4
from 2, we get $x \geq 3$ ……………. 5
and from 2, we get $x \leq 5$ ………. 6
From 4, 5 and 6, we get
$3 \leq x \leq 5$
So, the domain is {3, 4, 5}

Question 2.
The domain of $\tan^{-1} (2x + 1)$ is
(a) R
(b) R – {1/2}
(c) R – {-1/2}
(d) None of these

Answer: (a) R
Since $\tan^{-1} x$ exists if $x \in (-\infty, \infty)$
So, $\tan^{-1} (2x + 1)$ is defined if
$-\infty < 2x + 1 < \infty$
$\Rightarrow -\infty < x < \infty$

$\Rightarrow x \in (-\infty, \infty)$
$\Rightarrow x \in R$
So, domain of $\tan^{-1}(2x + 1)$ is R.

Question 3.
Two functions f and g are said to be equal if f
(a) the domain of f = the domain of g
(b) the co-domain of f = the co-domain of g
(c) f(x) = g(x) for all x
(d) all of above

Answer: (d) all of above
Two functions f and g are said to be equal if f
1. the domain of f = the domain of g
2. the co-domain of f = the co-domain of g
3. f(x) = g(x) for all x

Question 4.
If the function $f : R \to R$ be given by $f(x) = x^2 + 2$ and $g : R \to R$ is given by $g(x) = x/(x - 1)$. The value of gof(x) is
(a) $(x^2 + 2)/(x^2 + 1)$
(b) $x^2/(x^2 + 1)$
(c) $x^2/(x^2 + 2)$
(d) none of these

Answer: (a) $(x^2 + 2)/(x^2 + 1)$
Given $f(x) = x^2 + 2$ and $g(x) = x/(x - 1)$
Now, $gof(x) = g(x^2 + 2) = (x^2 + 2)/(x^2 + 2 - 1) = (x^2 + 2)/(x^2 + 1)$

Question 5.
Given g(1) = 1 and g(2) = 3. If g(x) is described by the formula g(x) = ax + b, then the value of a and b is
(a) 2, 1
(b) -2, 1
(c) 2, -1
(d) -2, -1

Answer: (c) 2, -1
Given, g(x) = ax + b
Again, g(1) = 1

$\Rightarrow a \times 1 + b = 1$
$\Rightarrow a + b = 1$ ……… 1
and $g(2) = 3$
$\Rightarrow a \times 2 + b = 3$
$\Rightarrow 2a + b = 3$ …….. 2
Solve equation 1 and 2, we get
$a = 2, b = -1$

Question 6.
Let $f : R \rightarrow R$ be a function given by $f(x) = x^2 + 1$ then the value of $f^{-1}(26)$ is
(a) 5
(b) -5
(c) ±5
(d) None of these

Answer: (c) ±5
Let $y = f(x) = x^2 + 1$
$\Rightarrow y = x^2 + 1$
$\Rightarrow y - 1 = x^2$
$\Rightarrow x = \pm\sqrt{(y - 1)}$
$\Rightarrow f^{-1}(x) = \pm\sqrt{(x - 1)}$
Now, $f^{-1}(26) = \pm\sqrt{(26 - 1)}$
$\Rightarrow f^{-1}(26) = \pm\sqrt{(25)}$
$\Rightarrow f^{-1}(26) = \pm 5$

Question 7.
the function $f(x) = x - [x]$ has period of
(a) 0
(b) 1
(c) 2
(d) 3

Answer: (b) 1
Let T is a positive real number.
Let f(x) is periodic with period T.
Now, $f(x + T) = f(x)$, for all $x \in R$
$\Rightarrow x + T - [x + T] = x - [x]$, for all $x \in R$
$\Rightarrow [x + T] - [x] = T$, for all $x \in R$
Thus, there exist $T > 0$ such that $f(x + T) = f(x)$ for all $x \in R$

Now, the smallest value of T satisfying f(x + T) = f(x) for all $x \in R$ is 1
So, f(x) = x – [x] has period 1

Question 8.
The function f(x) = sin (πx/2) + cos (πx/2) is periodic with period
(a) 4
(b) 6
(c) 12
(d) 24

Answer: (a) 4
Period of sin (πx/2) = 2π/(π/2) = 4
Period of cos (πx/2) = 2π/(π/2) = 4
So, period of f(x) = LCM (4, 4) = 4

Question 9.
The domain of the function $f(x) = x/(1 + x^2)$ is
(a) R – {1}
(b) R – {-1}
(c) R
(d) None of these

Answer: (c) R
Given, function $f(x) = x/(1 + x^2)$
Since f(x) is defined for all real values of x.
So, domain(f) = R

Question 10.
If f : R → R is defined by $f(x) = x^2 – 3x + 2$, the f(f(y)) is
(a) $x^4 + 6x^3 + 10x^2 + 3x$
(b) $x^4 – 6x^3 + 10x^2 + 3x$
(c) $x^4 + 6x^3 + 10x^2 – 3x$
(d) $x^4 – 6x^3 + 10x^2 – 3x$

Answer: (d) $x^4 – 6x^3 + 10x^2 – 3x$
Given, $f(x) = x^2 – 3x + 2$
Now, $f(f(y)) = f(x^2 – 3x + 2)$
$= (x^2 – 3x + 2)^2 – 3(x^2 – 3x + 2) + 2$
$= x^4 – 6x^3 + 10x^2 – 3x$

Question 11.
If n is the smallest natural number such that n + 2n + 3n + …. + 99n is a perfect square, then the number of digits in square of n is
(a) 1
(b) 2
(c) 3
(d) 4

Answer: (c) 3
Given that
n + 2n + 3n + …. + 99n
= n × (1 + 2 + 3 + …….. + 99)
= (n × 99 × 100)/2
= n × 99 × 50
= n × 9 × 11 × 2 × 25
To make it perfect square we need 2 × 11
So n = 2 × 11 = 22
Now n^2 = 22 × 22 = 484
So, the number of digit in n^2 = 3

Question 12.
Let f : R – R be a function defined by f(x) = cos(5x + 2), then f is
(a) injective
(b) surjective
(c) bijective
(d) None of these

Answer: (d) None of these
Given, f(x) = cos(2x + 5)
Period of f(x) = 2π/5
Since f(x) is a periodic function with period 2π/5, so it is not injective.
The function f is not surjective also as its range [-1, 1] is a proper subset of its co-domain R

Question 13.
The function f(x) = sin (πx/2) + 2cos (πx/3) – tan (πx/4) is periodic with period
(a) 4
(b) 6
(c) 8
(d) 12

Answer: (d) 12
Period of sin $(\pi x/2) = 2\pi/(\pi/2) = 4$
Period of cos $(\pi x/3) = 2\pi/(\pi/3) = 6$
Period of tan $(\pi x/4) = \pi/(\pi/4) = 4$
So, period of f(x) = LCM (4, 6, 4) = 12

Question 14.
If the function f : R → R be given by $f(x) = x^2 + 2$ and g : R → R is given by $g(x) = x/(x - 1)$. The value of gof(x) is
(a) $(x^2 + 2)/(x^2 + 1)$
(b) $x^2/(x^2 + 1)$
(c) $x^2/(x^2 + 2)$
(d) none of these

Answer: (a) $(x^2 + 2)/(x^2 + 1)$
Given $f(x) = x^2 + 2$ and $g(x) = x/(x - 1)$
Now, $gof(x) = g(x^2 + 2) = (x^2 + 2)/(x^2 + 2 - 1) = (x^2 + 2)/(x^2 + 1)$

Question 15.
The domain of the function ${}^{7-x}P_{x-3}$ is
(a) {1, 2, 3}
(b) {3, 4, 5, 6}
(c) {3, 4, 5}
(d) {1, 2, 3, 4, 5}

Answer: (c) {3, 4, 5}
The function $f(x) = {}^{7-x}P_{x-3}$ is defined only if x is an integer satisfying the following inequalities:
1. $7 - x \geq 0$
2. $x - 3 \geq 0$
3. $7 - x \geq x - 3$
Now, from 1, we get $x \leq 7$ ………4
from 2, we get $x \geq 3$ …………….5
and from 2, we get $x \leq 5$ ……….6
From 4, 5 and 6, we get
$3 \leq x \leq 5$
So, the domain is {3, 4, 5}

Question 16.
If $f(x) = e^x$ and $g(x) = \log_e x$ then the value of fog(1) is

(a) 0
(b) 1
(c) -1
(d) None of these

Answer: (b) 1
Given, $f(x) = e^x$
and g(x) = logx
fog(x) = f(g(x))
= f (logx)
$= e^{\log x}$
= x
So, fog(1) = 1

Question 17.
A relation R is defined from the set of integers to the set of real numbers as (x, y) = R if $x^2 + y^2 = 16$ then the domain of R is
(a) (0, 4, 4)
(b) (0, -4, 4)
(c) (0, -4, -4)
(d) None of these

Answer: (b) (0, -4, 4)
Given that:
$(x, y) \in R \Leftrightarrow x^2 + y^2 = 16$
$\Leftrightarrow y = \pm\sqrt{(16 - x^2)}$
when $x = 0 \Rightarrow y = \pm 4$
$(0, 4) \in R$ and $(0, -4) \in R$
when $x = \pm 4 \Rightarrow y = 0$
$(4, 0) \in R$ and $(-4, 0) \in R$
Now for other integral values of x, y is not an integer.
Hence R = {(0, 4), (0, -4), (4, 0), (-4, 0)}
So, Domain(R) = {0, -4, 4}

Question 18.
The period of the function f(x) = sin (2πx/3) + cos (πx/3)
(a) 3
(b) 4
(c) 12
(d) None of these

Answer: (c) 12
Given, function f(x) = sin (2πx/3) + cos (πx/2)
Now, period of sin (2πx/3) = 2π/{(2π/3)} = (2π × 3)/(2π) = 3
and period of cos (πx/2) = 2π/{(π/2)} = (2π × 2)/(π) = 2 × 2 = 4
Now, period of f(x) = LCM(3, 4) = 12
Hence, period of function f(x) = sin (2πx/3) + cos (πx/2) is 12

Question 19.
If f(x) = ax + b and g(x) = cx + d and f{g(x)} = g{f(x)} then
(a) f(a) = g(c)
(b) f(b) = g(b)
(c) f(d) = g(b)
(d) f(c) = g(a)

Answer: (c) f(d) = g(b)
Given, f(x) = ax + b and g(x) = cx + d and
Now, f{g(x)} = g{f(x)}
⇒ f{cx + d} = g{ax + b}
⇒ a(cx + d) + b = c(ax + b) + d
⇒ ad + b = cb + d
⇒ f(d) = g(b)

Question 20.
The domain of the function f (x) = 1/(2 – cos 3x) is
(a) (1/3, 1)
(b) [1/3, 1)
(c) (1/3, 1]
(d) R

Answer: (d) R
Given
function is f(x) = 1/(2 – cos 3x)
Since -1 ≤ cos 3x ≤ 1 for all x ∈ R
So, -1 ≤ 2 – cos 3x ≤ 1 for all x ∈ R
⇒ f(x) is defined for all x ∈ R
So, domain of f(x) is R
