MENSURATION

UNIT - 6

Let us know the history of mensuration.....

Usually mathematical ideas originates from daily activities and experiences. In ancient times people used measure of “Bitta”, “Hath” (hand span, palm length) etc. for measuring the land, height of walls, depth of well etc. Using only these measures many big palaces, buildings, castles, ponds, roads, canals, drains etc. were constructed.

The field area was measured on the basis of the amount of seed sown in it, weight was measured by taking pebbles and other natural objects as units, volume was measured in lottas, glasses, tumblers, pots etc. Since these measures varied a lot, gradually the practice of measuring things in a standard unit evolved. In Indus valley of India fine standardized system of measuring length and weights (बाट), existed as early as 5 centuries before Christ. There were standard weights for different measures. Small weights for expensive things and big weights for things which were exchanged in large quantity. Most of the weights were cubical. Weighing balances (तराजू) with two sides were also made here. Similarly there were standard measures for measuring lengths which could even measure up to 1/16 of an inch. Many methods of measurements were adopted in Iran and Central Asia from India and vice versa. Mensuration as we know today has evolved, based on all these. Many situations are involved where these are needed. Some of them are-

1. Cost of fencing with wire around any field.
2. Cost of bricks or stones used in making parapets of wells.
3. Area of any room.
4. Volume of any tank.
5. The number and cost of tiles used to make a floor.
6. Estimate of the cost of ploughing or cutting of the crop.

For these we will need to find perimeter and area of closed two dimensional shapes and surface area and volume of solid shapes. In mensuration we will learn to find perimeter, area of triangle, quadrilateral, circle etc and surface area, volume of geometrical shapes like cube, cuboid, cylinder, cone and sphere etc.

Indian mathematicians gave many formulas for geometrical shapes and figures which are similar to formulae that we use today. For example- Aryabhatt gave following formula of area of circle:-

‘समपरिणाहस्यार्थं विष्कंभार्धहतमेव वृत्तफलम्’

i.e. $$\text{Area of circle} = \frac{1}{2}(\text{Circumference}) \times \frac{1}{2}(\text{Diameter})$$

$$= \pi r^2$$

Where *r* is radius of circle.

The information presented here is collected from various sources. Teachers and students can get more information about mensuration from other sources also.

Sector of a Circle & Length of Arc

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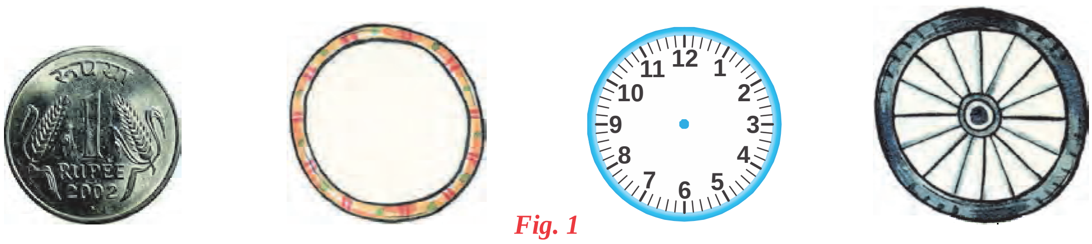

Fig. 1

We see many circular shapes like coins, bicycle wheels, dial of a clock etc. You may find many other objects around you that are circular. Can you think of few more such objects? In this chapter, we will read about circle and its properties.

Diameter of a Circle

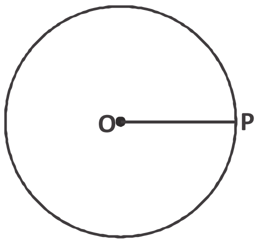

Fig. 2

You know about a circle. *Fig.*2, there is a circle with centre O and radius OP. In *Fig.*3, you can see the line segment AOB that passes through centre of the circle 'O' and has its end points at the circumference. AOB is a diameter of the circle.

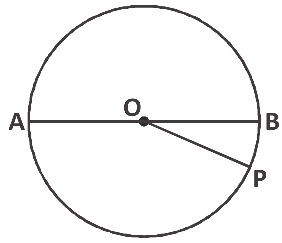

Fig. 3

Diameter of the Circle = 2 × Radius

Circumference of a Circle

In any circle drawn with any radius, the ratio of the circumference to its diameter is always fixed. This ratio is represented using a Greek letter π (pi).

$$\text{Therefore, } \frac{\text{Circumference of Circle}}{\text{Diameter}} = \pi$$

Circumference of the Circle $= \pi \times$ diameter

$= \pi \times (2 \times \text{radius})$ (Since, diameter = 2 × radius)

If radius of a circle is r, then

Its circumference $= \pi \times (2 \times r)$

Circumference of Circle $= 2\pi r$

Try This

Take circular objects from your surroundings, find out the ratio of their circumferences to the respective diameters. Is the ratio constant? If yes, what is the value?

Area of a Circle

Draw a circle with centre 'O' and radius 'r'. Inscribe a regular polygon with 'n' sides in the circle (as shown in *Fig.*4). Now, make triangles by joining vertices of the regular polygon with the centre of the circle. You can see ΔOPQ as one of the triangles.

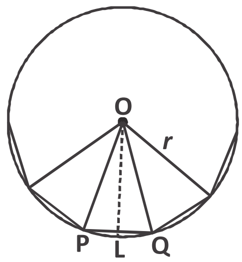

Fig. 4

Area of Triangle OPQ $= \frac{1}{2}$ PQ $\times$ OL

$= \frac{1}{2} \times$ side $\times$ length of perpendicular from centre to the side.

Since, the lengths of perpendicular from centre 'O' on each side of regular polygon are the same. Therefore, area of each triangle will be the same.

We know, that there are n such triangles.

So, area of n triangles

$= n \times \frac{1}{2}$ side × perpendicular from centre to the side

Now, what will happen if number of sides become infinite? In such situation, perimeter of the polygon will become same as circumference of the circle and area of the polygon will become equal to the area of circle. The length OL will become equal to 'r'.

Therefore, Area of Circle $= \frac{1}{2} \times$ Circumference × Radius

$= \frac{1}{2} \times 2\pi r \times r$

$= \pi r^2$

Sector of a Circle

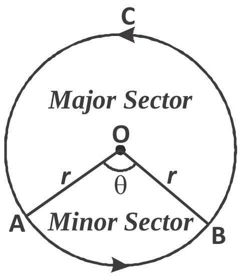

Fig. 5

Sector of circle is the portion of the circle enclosed by two radii and an arc. In *Fig.*5, you can see a circle with centre O and radius '*r*'. A, B and C are any three points on the circumference of the circle. Join center 'O' with point A and B. Radii OA and OB divide the circle in two parts OAB and OBCA. These two are the sector of the circle.

Sector OAB is enclosed by radii OA, OB and an arc $\overset{\frown}{AB}$ and sector OBCA is enclosed by radii OA, OB and an arc $\overset{\frown}{BCA}$.

Let the arc $\overset{\frown}{AB}$ subtend an angle θ at the centre 'O' then the length of the arc is proportional to angle subtended by the arc at the centre.

$$\frac{\text{Length of an arc}}{\text{Circumference of circle}} = \frac{\text{Angle subtended by the arc at centre}}{\text{Angle subtended by circle at centre}}$$

$$\frac{\text{Lenth of an arc}}{\text{Circumference of circle}} = \frac{\theta}{360^\circ}$$

$$\text{Length of an arc} = \frac{\theta}{360^\circ} \times \text{Circumference of circle}$$

$$\text{Length of an arc of sector} = \frac{\theta}{360^\circ} \times 2\pi r$$

In the similar manner, area of the sector is proportional to the interior angle subtended at the centre by the arc enclosing it.

$$\therefore \frac{\text{Area of sector}}{\text{Area of circle}} = \frac{\text{Angle subtended at centre by the enclosing arc}}{\text{Angle subtended by circle at centre}}$$

$$\frac{\text{Area of Sector}}{\text{Area of circle}} = \frac{\theta}{360^\circ}$$

$$\text{Area of sector} = \frac{\theta}{360^\circ} \times \text{Area of circle}$$

$$\text{Area of sector} = \frac{\theta}{360^\circ} \times \pi r^2$$

Area of a Circular Path

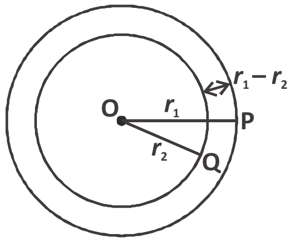

Fig. 6

Circular path is the region between two concentric circles. If the radii of the outer circle and inner circle are r_1 and r_2 respectively,

then, width of the circular path = Outer radius – Inner radius

$$= r_1 - r_2$$

Area of circular path = Area of outer circle – Area of inner circle

$$= \pi r_1^2 - \pi r_2^2$$

$$= \pi(r_1^2 - r_2^2)$$

Area of circular path = $\pi(r_1^2 - r_2^2)$

EXAMPLE-1. Diameter of a circle is 14 *cm* Find the circumference and the area of the circle.

SOLUTION : Given, diameter of circle = $2r$ = 14 *cm.*

$$\therefore \text{ radius of circle, } r = \frac{14}{2} = 7\ cm$$

We know, circumference of circle = $2\pi r$

$$= 2 \times \frac{22}{7} \times 7 = 44\ cm$$

And area of the circle $= \pi r^2$

$$= \frac{22}{7} \times 7^2 = 154\ sq\ cm$$

EXAMPLE-2. Find area of the circle, whose circumference is 176 *cm.*

SOLUTION : Here, circumference of the circle = 176 *cm*

$$\therefore\ 2\pi r = 176$$

$$2 \times \frac{22}{7} \times r = 176$$

$$r = \frac{176 \times 7}{2 \times 22} = 28\ cm$$

Therefore, area of the circle = πr^2

$$= \frac{22}{7} \times (28)^2 = 2464\ sq\ cm$$

EXAMPLE-3. Radii of two circles are 8 *cm* and 6 *cm* respectively. Find radius of the circle whose area equals the sum of the areas of the two given circles.

SOLUTION : Here, radius of the first circle $r_1 = 8\ cm$

radius of the second circle $r_2 = 6\ cm$

And radius of the required circle R = ?

We know area of required circle = Area of first circle + Area of second circle.

$$\pi R^2 = \pi r_1^2 + \pi r_2^2$$
$$\pi R^2 = \pi(r_1^2 + r_2^2)$$
$$R^2 = r_1^2 + r_2^2$$
$$R^2 = 8^2 + 6^2$$
$$R^2 = 64 + 36$$
$$R^2 = 100$$
$$R = 10\ cm$$

EXAMPLE-4. Radius of a circular ground is 35 *cm*. How long will it take for a boy to complete 10 rounds of the grounds at a speed of 5 *km* per hour.

SOLUTION : Radius of the circular ground $r = 35\ m$

Distance covered by the boy in one round (circumference) = $2\pi r$

Hence, distance covered in 10 rounds $= 10 \times 2\pi r$

$$= 10 \times 2 \times \frac{22}{7} \times 35$$

$$= 2200\ m = 2.2\ km$$

Time taken by the boy to cover 5 *km* = 60 minute

Therefore, 2.2 *km* will be covered in time $= \dfrac{60 \times 2.2}{5} = 26.4$ minute

= 26 minute and 24 second.

EXAMPLE-5. Find width of the circular path whose outer and inner circumference are 110 meters and 88 meters respectively.

SOLUTION : Let, outer radius of the circular path = r_1 meter

and, inner radius of the circular path = r_2 meter

we know that, outer circumference of circular path = 110 meter

$$2\pi r_1 = 110$$

$$2 \times \frac{22}{7} \times r_1 = 110$$

$$r_1 = \frac{110 \times 7}{2 \times 22} = 17.5 \text{ meter}$$

Inner circumference of the circular path = 88 meter

$$2\pi r_2 = 88$$

$$2 \times \frac{22}{7} \times r_2 = 88$$

$$r_2 = \frac{88 \times 7}{2 \times 22} = 14 \text{ meter}$$

Width of the circular path = $r_1 - r_2$

$= 17.5 - 14 = 3.5$ meter

EXAMPLE-6. A circular garden is surrounded by a 7 meter wide road. Circumference of the garden is 352 meter. Find area of the road.

SOLUTION : Let, the outer radius of the road = r_1

And, inner radius of the road (radius of garden) = r_2

We know that, circumference of the garden = 352 meter.

$$2\pi r_2 = 352$$

$$2 \times \frac{22}{7} \times r_2 = 352$$

$$r_2 = \frac{352 \times 7}{2 \times 22} = 56 \text{ meter}$$

Therefore, outer radius of circular path (road) $r_1 = 56 + 7 = 63$ meter

$\therefore$ Area of the circular path = $\pi(r_1^2 - r_2^2)$

$$= \frac{22}{7} \times \left[(63)^2 - (56)^2\right]$$

$$= \frac{22}{7} \times (63 + 56)\,(63 - 56)$$

$$= \frac{22}{7} \times 119 \times 7 = 2618 \; sq\ m$$

EXAMPLE-7. There is a circle of radius 21 *cm*. A sector that subtends an angle 120° at the center is cut from the circle. Find length of the arc of sector cut. Also find area of the sector.

SOLUTION : Given, radius of the circle $r = 21$ *cm*

Angle subtended by the sector $\theta = 120^{\circ}$

Therefore, length of the arc of sector $= \frac{\theta}{360^{\circ}} \times 2\pi r$

$$= \frac{120^{\circ}}{360^{\circ}} \times 2 \times \frac{22}{7} \times 21$$

= 44 *cm*

And area of the sector $= \frac{\theta}{360^{\circ}} \times \pi r^2$

$$= \frac{120^{\circ}}{360^{\circ}} \times \frac{22}{7} \times (21)^2$$

= 462 *sq cm*

EXAMPLE-8. Find the area of the shaded portion in given figure. (*Fig.*7)

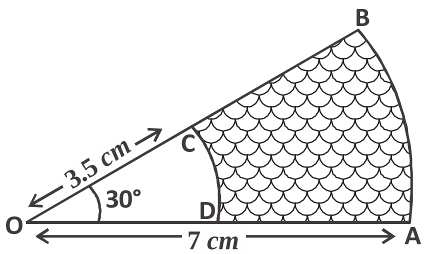

Fig. 7

SOLUTION : Area of the shaded portion ABCD

= Area of sector OAB – Area of sector OCD

$$= \frac{\theta}{360^{\circ}} \times \pi (OA)^2 - \frac{\theta}{360^{\circ}} \times \pi (OD)^2$$

$$= \frac{\theta}{360^{\circ}} \times \pi \left[(OA)^2 - (OD)^2\right]$$

$$= \frac{30^{\circ}}{360^{\circ}} \times \frac{22}{7} \times \left[(7)^2 - (3.5)^2\right]$$

$$= \frac{1}{12} \times \frac{22}{7} \times (7 + 3.5) \times (7 - 3.5)$$

$$= \frac{11}{6 \times 7} \times 10.5 \times 3.5$$

$= 9.625\ sq\ cm$ or $9.625\ cm^2$

Area of the shaded portion is 9.625 *sq cm*

Exercise-14.1

1. Find the circumference of the circle, whose radius is 17.5 *cm.*
2. Find the area of the circle, whose radius is 4.2 *cm.*
3. A horse is tied by a 14 meter long rope in a ground. What is the area of the ground, that the horse can graze, if he can move up to the full length of the rope?
4. Radius of a bicycle wheel is 35 *cm.* How much distance will it cover in 500 complete rotations?
5. Radius of a circle is 3 meter, what would be the radius of a circle whose area is 9 times the area of the first circle?
6. Inner circumference of a circular path is 440 *m.* Width of the path is 14 *m.* Find diameter of the outer circle of the circular path.
7. A circular ground of radius 50 meter is sorrounded by a 5 meter wide road. You want to cover the road with tile. Find the total cost of tiling if the rate of tiling per square meter is 30 rupees.
8. You are given a sector which subtends an angle of 70° at the centre of the circle with radius 21 *cm* Find the length of the arc and area of the given sector.
9. Area of a sector of the circle is $\frac{1}{6}$ times the area of the circle. Find the angle subtended at the centre by the given sector.
10. Area of a sector is 1540 *sq cm*, it subtends an angle of 50° at the centre of the circle. Find the radius of the circle.

What Have We Learnt

1. Diameter of the circle (2 × radius), circumference of the circle ($2\pi r$), area of the circle (πr^2). We also learnt about the sector of a circle.
2. Area of the circular path = $\pi(r_1^2 - r_2^2)$; where r_1 and r_2 are outer and inner radii respectively.