Properties of Triangles

Question1

For a triangle ABC, the value of $\cos 2A + \cos 2B + \cos 2C$ is least. If its inradius is 3 and incentre is M, then which of the following is NOT correct?
[1-Feb-2023 Shift 1]

Options:

A. Perimeter of $\triangle ABC$ is $18\sqrt{3}$

B. $\sin 2A + \sin 2B + \sin 2C = \sin A + \sin B + \sin C$

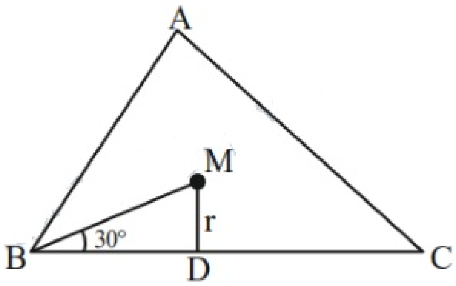
C. $\overrightarrow{MA} \cdot \overrightarrow{MB} = -18$

D. area of $\triangle ABC$ is $\frac{27\sqrt{3}}{2}$

Answer: D

Solution:

Solution:



If $\cos 2A + \cos 2B + \cos 2C$ is minimum then $A = B = C = 60^\circ$
So $\triangle ABC$ is equilateral
Now in-radius $r = 3$
So in $\triangle MBD$ we have

$\operatorname{Tan} 30^\circ = \frac{MD}{BD} = \frac{r}{a/2} = \frac{6}{a}$

$1/\sqrt{3} = \frac{1}{a} = a = 6\sqrt{3}$

Perimeter of $\triangle ABC = 18\sqrt{3}$

Area of $\triangle ABC = \frac{\sqrt{3}}{4}a^2 = 27\sqrt{3}$

Question2

Let $C(\alpha, \beta)$ be the circumcenter of the triangle formed by the lines
$4x + 3y = 69$
$4y - 3x = 17$ and
$x + 7y = 61$

Then $(\alpha - \beta)^2 + \alpha + \beta$ is equal to
[8-Apr-2023 shift 1]

Options:

A. 18

B. 15

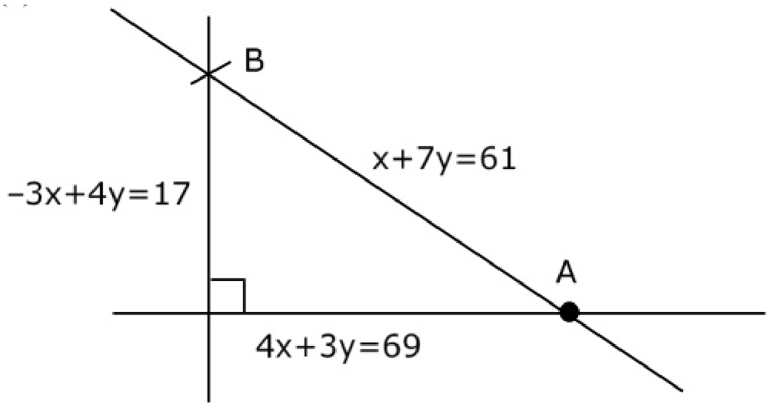
C. 16

D. 17

Answer: D

Solution:

Solution:



$$\begin{aligned} 4x + 28y &= 244 \\ 4x + 3y &= 69 \\ - \quad - \quad &- \\ \hline 25y &= 175 \end{aligned}$$

$y = 7, x = 12$

$A(12, 7)$

$-3x + 4y = 17$

$3x + 21y = 183$

$25y = 200$

$y = 8, x = 5$

$B(5, 8)$

$\therefore$ Circumcenter

$\alpha = \frac{17}{2} \beta = \frac{15}{2}$

$\left(\frac{17}{2}, \frac{15}{2}\right)$

$(\alpha - \beta)^2 + \alpha + \beta$

$1 + 16 = 17$

Question3

The ratio of intensities at two points P and Q on the screen in a Young's double slit experiment where phase difference between two waves of same amplitude are $\pi / 3$ and $\pi / 2$, respectively are
[10-Apr-2023 shift 2]

Options:

A. 3 : 2

B. 3 : 1

C. 2 : 3

D. 1 : 3

Answer: A

Solution:

Solution:

$I_{res} = 4I_o \cos^2\left(\frac{\theta}{2}\right)$

If $\theta = \frac{\pi}{3}, I_{res} = 4I_o \cdot \cos^2\left[\frac{\pi}{6}\right]$

$= 4I_0 \cdot \left(\frac{\sqrt{3}}{2}\right)^2$

$I_1 = (4I_0)\left(\frac{3}{4}\right) = 3I_0$

If $\theta = \frac{\pi}{2}, I_{res} = 4I_0 \cdot \cos^2\left(\frac{\pi}{2}\right)$

$= 4I_0\left(\frac{1}{\sqrt{2}}\right)^2$

$= (4I_o)\left(\frac{1}{2}\right)$

$I_2 = 2I_o$

$\frac{I_1}{I_2} = \frac{3}{2}$

Question4

In a triangle ABC, if $\cos A + 2\cos B + \cos C = 2$ and the lengths of the sides opposite to the angles A and C are 3 and 7 respectively, then $\cos A - \cos C$ is equal to [12-Apr-2023 shift 1]

Options:

A. $\frac{5}{7}$

B. $\frac{9}{7}$

C. $\frac{3}{7}$

D. $\frac{10}{7}$

Answer: D

Solution:

$\cos A + \cos C = 2(1 - \cos B)$

$2\cos\frac{A+C}{2}\cos\frac{A-C}{2} = 4\sin^2 B/2$

as $\cos\left(\frac{A+C}{2}\right) = \sin\frac{B}{2}$

so $\cos\frac{A-C}{2} = 2\sin\frac{B}{2}$

$2\cos B/2\cos\frac{A-C}{2} = 4\sin B/2\cos B/2$

$2\sin\left(\frac{A+C}{2}\right)\cos\left(\frac{A-C}{2}\right) = 4\sin B/2\cos B/2$

$\sin A + \sin C = 2\sin B$

$a + c = 2b \Rightarrow a = 3, c = 7, b = 5$

$\cos A - \cos C = \frac{b^2 + c^2 - a^2}{2bc} - \frac{a^2 + b^2 - c^2}{2ab}$

$= \frac{25 + 49 - 9}{70} - \frac{9 + 25 - 49}{30}$

$\frac{65}{70} + \frac{1}{2} = \frac{20}{14} = \frac{10}{7}$

Question5

Let the plane $x + 3y - 2z + 6 = 0$ meet the co-ordinate axes at the points A, B, C. If the orthocenter of the triangle ABC is $\left(\alpha, \beta, \frac{6}{7}\right)$, then $98(\alpha + \beta)^2$ is equal to _______.
[12-Apr-2023 shift 1]

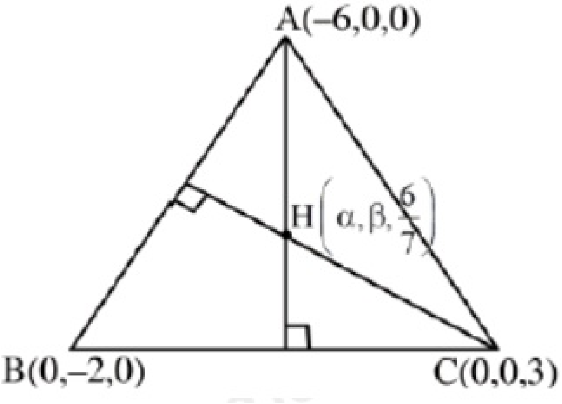
Answer: 288

Solution:

$A(-6, 0, 0) B(0, -2, 0) C = (0, 0, 3)$

$\overrightarrow{AB} = 6\hat{i} - 2\hat{j}, \overrightarrow{BC} = 2\hat{j} + 3\hat{k}$

$\overrightarrow{AC} = 6\hat{i} + 3\hat{k}$



$\overrightarrow{AH} \cdot \overrightarrow{BC} = 0$

$\left(\alpha + 6, \beta, \frac{6}{7}\right) \cdot (0, 2, 3) = 0$

$\beta = \frac{-9}{7}$

$\overrightarrow{CH} \cdot \overrightarrow{AB} = 0$

$\left(\alpha, \beta, \frac{-15}{7}\right) \cdot (6, -2, 0) = 0$

$6\alpha - 2\beta = 0$

$\alpha = \frac{-3}{7}$

$98(\alpha + \beta)^2 = (98)\frac{(144)}{49} = 288$

Question6

Let for a triangle ABC,

$\vec{AB} = -2\hat{i} + \hat{j} + 3\hat{k}$

$\vec{CB} = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$

$\vec{CA} = 4\hat{i} + 3\hat{j} + \delta\hat{k}$

If $\delta > 0$ and the area of the triangle ABC is $5\sqrt{6}$, then $\vec{CB} \cdot \vec{CA}$ is equal to [13-Apr-2023 shift 2]

Options:

A. 108

B. 60

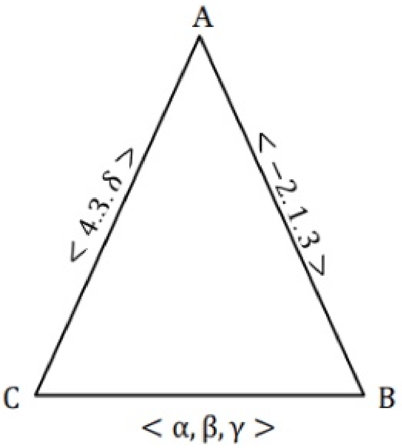
C. 54

D. 120

Answer: B

Solution:

Solution:



$\vec{CA} + \vec{AB} = \vec{CB}$

$\langle 4, 3, \delta\rangle \cdot + \langle -2, 1, 3\rangle = \vec{CB}$

$\Rightarrow \vec{CB} = \langle 2, 4, 3 + \delta\rangle$

$$\vec{AB} \times \vec{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 1 & 3 \\ -4 & -3 & -\delta \end{vmatrix}$$

$= \hat{\ell}(9 - \delta) - \hat{j}(2\delta + 12) + k(10)$

$|AB \times AC|^2 = (9 - \delta)^2 + (2\delta + 12)^2 + (10)^2$

$= 5\delta^2 + 30\delta + 325$

Area of $\Delta ABC = 5\sqrt{6}$

$\Rightarrow \frac{1}{2}\left|\vec{AB} \times \vec{AC}\right| = 5\sqrt{6}$

$\Rightarrow |\overrightarrow{AB} \times \overrightarrow{AC}|^2 = 600$
$\Rightarrow 5\delta^2 + 30\delta - 275 = 0$
$\Rightarrow S^2 + 6\delta - 55 = 0$
$\Rightarrow (\delta + 11)(\delta - 5) = 0$
$\delta = 5$
$\overrightarrow{CB} = \langle 2, 3, 8 \rangle$
$\overrightarrow{CB} \cdot \overrightarrow{CA} = \langle 2, 4, 8 \rangle . \langle 4, 3, 5 \rangle$
$= 8 + 12 + 40 = 60$

Question7

Let a, b and c be the length of sides of a triangle ABC such that $\frac{a+b}{7} = \frac{b+c}{8} = \frac{c+a}{9}$. If r and R are the radius of incircle and radius of circumcircle of the triangle ABC, respectively, then the value of $\frac{R}{r}$ is equal to:
[25-Jun-2022-Shift-1]

Options:

A. $\frac{5}{2}$

B. 2

C. $\frac{3}{2}$

D. 1

Answer: A

Solution:

$\frac{a+b}{7} = \frac{b+c}{8} = \frac{c+a}{9} = \lambda$

$a + b = 7\lambda$

$b + c = 8\lambda$

$c + a = 9\lambda$

$a + b + c = 12\lambda$

$\therefore a = 4\lambda, b = 3\lambda, c = 5\lambda$

$S = \frac{4\lambda + 3\lambda + 5\lambda}{2} = 6\lambda$

$\Delta = \sqrt{S(s-a)(s-b)(s-c)} = \sqrt{(6\lambda)(2\lambda)(3\lambda)(\lambda)} = 6\lambda^2$

$R = \frac{abc}{4\Delta} = \frac{(4\lambda)(3\lambda)(5\lambda)}{4(6\lambda^2)} = \frac{5}{2}\lambda$

$r = \frac{\Delta}{s} = \frac{6\lambda^2}{6\lambda} = \lambda$

$\frac{R}{r} = \frac{\frac{5}{2}\lambda}{\lambda} = \frac{5}{2}$

Question8

A tower PQ stands on a horizontal ground with base Q on the ground. The point R divides the tower in two parts such that QR = 15m. If from a point A on the ground the angle of elevation of R is 60° and the part PR of the tower subtends an angle of 15° at A, then the height of the tower is :
[25-Jul-2022-Shift-1]

Options:

A. $5(2\sqrt{3}+3)$m

B. $5(\sqrt{3}+3)$m

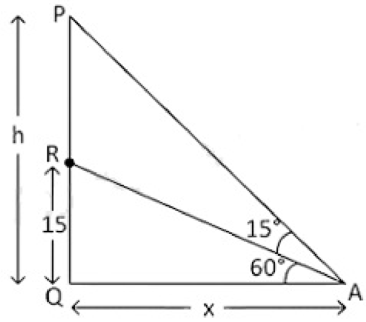
C. $10(\sqrt{3}+1)$m

D. $10(2\sqrt{3}+1)$m

Answer: A

Solution:

Solution:



For $\triangle AQR$,

$\tan 60^{\circ} = \frac{15}{x}$..... (1

From $\triangle AQP$,

$\tan 75^{\circ} = \frac{h}{x}$

$\Rightarrow (2+\sqrt{3}) = \frac{h}{x} [\because \tan 75^{\circ} = 2+\sqrt{3}]$

$\Rightarrow h = (2+\sqrt{3})x$

$= (2+\sqrt{3})\frac{15}{\sqrt{3}} [\text{ From } (1)]$

$= (2+\sqrt{3}) \times \frac{15\sqrt{3}}{3}$

$= (2+\sqrt{3}) \times 5\sqrt{3}$

$= 5(2\sqrt{3}+3)$m

Question9

Let a vertical tower AB of height 2h stands on a horizontal ground. Let from a point P on the ground a man can see upto height h of the tower with an angle of elevation 2α. When from P, he moves a distance d in the direction of $\overrightarrow{AP}$, he can see the top B of the tower with an angle of

elevation α. If d $= \sqrt{7}h$, then $\tan\alpha$ is equal to
[27-Jul-2022-Shift-1]

Options:

A. $\sqrt{5}-2$

B. $\sqrt{3}-1$

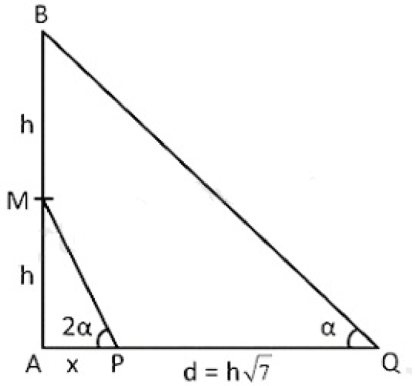
C. $\sqrt{7}-2$

D. $\sqrt{7}-\sqrt{3}$

Answer: C

Solution:

Solution:



$\triangle APM$ gives

$\tan 2\alpha = \frac{h}{x}$..... (i)

$\triangle AQB$ gives

$\tan 2\alpha = \frac{2h}{x+d} = \frac{2h}{x+h\sqrt{7}}$...... (ii)

From (i) and (ii)

$\tan 2\alpha = \frac{2\cdot\tan 2\alpha}{1+\sqrt{7}\cdot\tan 2\alpha}$

Let $t = \tan\alpha$

$\Rightarrow t = \frac{2\frac{2t}{1-t^2}}{1+\sqrt{7}\cdot\frac{2t}{1-t^2}}$

$\Rightarrow t^2 - 2\sqrt{7}t + 3 = 0$

$t = \sqrt{7}-2$

Question10

The angle of elevation of the top P of a vertical tower PQ of height 10 from a point A on the horizontal ground is 45°. Let R be a point on AQ and from a point B, vertically above R, the angle of elevation of P is 60°. If $\angle BAQ = 30^\circ$, AB = d and the area of the trapezium PQRB is α, then the ordered pair (d, α) is :
[27-Jul-2022-Shift-2]

Options:

A. $(10(\sqrt{3}-1), 25)$

B. $\left(10(\sqrt{3}-1), \frac{25}{2}\right)$

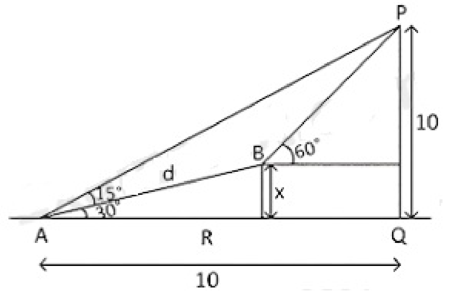
C. $(10(\sqrt{3}+1), 25)$

D. $\left(10(\sqrt{3}+1), \frac{25}{2}\right)$

Answer: A

Solution:

Solution:



Let BR = x

$\frac{x}{d} = \frac{1}{2} \Rightarrow x = \frac{d}{2}$

$\frac{10-x}{10-x\sqrt{3}} = \sqrt{3} \Rightarrow 10 - x = 10\sqrt{3} - 3x$

$2x = 10(\sqrt{3}-1)$

$x = 5(\sqrt{3}-1)$

$d = 2x = 10(\sqrt{3}-1)$

$\alpha = \frac{1}{2}(x+10)(10-x\sqrt{3}) = \text{Area}(PQRB)$

$= \frac{1}{2}(5\sqrt{3}-5+10)(10-5\sqrt{3}(\sqrt{3}-1))$

$= \frac{1}{2}(5\sqrt{3}+5)(10-15+5\sqrt{3}) - \frac{1}{2}(75-25) = 25$

Question11

A horizontal park is in the shape of a triangle OAB with AB = 16. A vertical lamp post OP is erected at the point O such that $\angle PAO = \angle PBO = 15^\circ$ and $\angle PCO = 45^\circ$, where C is the midpoint of AB. Then $(OP)^2$ is equal to
[28-Jul-2022-Shift-2]

Options:

A. $\frac{32}{\sqrt{3}}(\sqrt{3}-1)$

B. $\frac{32}{\sqrt{3}}(2-\sqrt{3})$

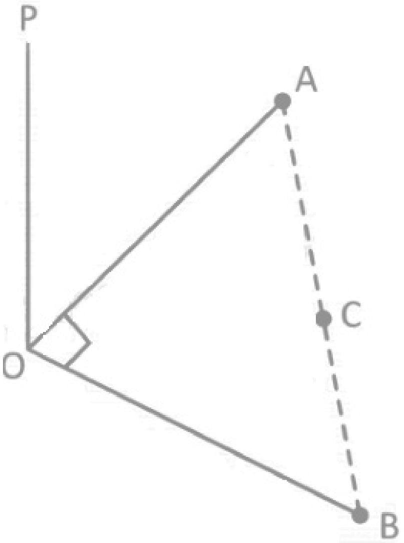
C. $\frac{16}{\sqrt{3}}(\sqrt{3}-1)$

D. $\frac{16}{\sqrt{3}}(2-\sqrt{3})$

Answer: B

Solution:

Solution:



$OP = OA \tan 15 = OB \tan 15 \ldots\ldots. \text{ (i)}$
$OP = OC \tan 45 \Rightarrow OP = OC \ldots\ldots. \text{ (ii)}$
$OA = OB \ldots\ldots \text{ (iii)}$
$OC^2 + 8^2 = OA^2$

$OP^2 + 64 = OP^2 \left(\frac{\sqrt{3}+1}{\sqrt{3}-1} \right)^2$

$64 = OP^2 \left[\frac{(\sqrt{3}+1)^2 - (\sqrt{3}-1)^2}{(\sqrt{3}-1)^2} \right]$

$= OP^2 \left(\frac{4\sqrt{3}}{(\sqrt{3}-1)^2} \right)$

$OP^2 = \frac{64(\sqrt{3}-1)^2}{4\sqrt{3}} = \frac{32}{\sqrt{3}}(2-\sqrt{3})$

Question12

The angle of elevation of the top of a tower from a point A due north of it is α and from a point B at a distance of 9 units due west of A is $\cos^{-1}\left(\frac{3}{\sqrt{13}}\right)$. If the distance of the point B from the tower is 15 units, then $\cot \alpha$ is equal to :
[29-Jul-2022-Shift-1]

Options:

A. $\frac{6}{5}$

B. $\frac{9}{5}$

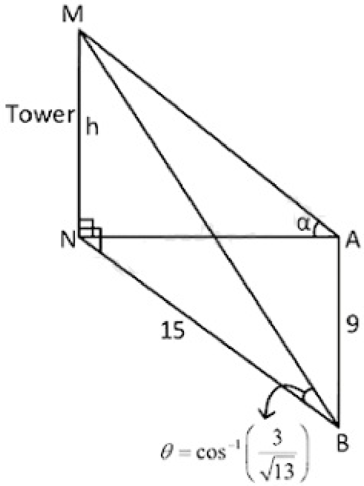
C. $\frac{4}{3}$

D. $\frac{7}{3}$

Answer: A

Solution:

Solution:



$NA = \sqrt{15^2 - 9^2} = 12$

$\frac{h}{15} = \tan\theta = \frac{2}{3}$

h = 10 units

$\cot\alpha = \frac{12}{10} = \frac{6}{5}$

Question13

A man is observing, from the top of a tower, a boat speeding towards the tower from a certain point A, with uniform speed. At that point, angle of depression of the boat with the man's eye is 30° (ignore man's height). After sailing for 20s, towards the base of the tower (which is at the level of water), the boat has reached a point B, where the angle of depression is 45°. Then, the time taken (in seconds) by the boat from B to reach the base of the tower is
[25 Feb 2021 Shift 1]

Options:

A. 10

B. $10\sqrt{3}$

C. $10(\sqrt{3} + 1)$

D. $10(\sqrt{3} - 1)$

Answer: C

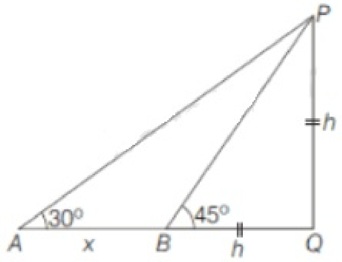
Solution:

Solution:

$\ln \Delta PQB, \tan 45^\circ = \frac{PQ}{BQ}$

$1 = \frac{PQ}{BQ}$

$PQ = BQ = h(\text{ let })$



In $\triangle PAQ$, $\tan 30^\circ = \frac{PQ}{AQ}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x + h} \Rightarrow x + h = \sqrt{3}h$

$\Rightarrow x = (\sqrt{3} - 1)h$

$\because$ Speed $= \frac{\text{Distance}}{\text{Time}}$

$\therefore$ Speed from A to B $= \frac{AB}{20} = \frac{x}{20}$

Also, distance from B to Q = h

$\therefore$ Time taken to reach Q from B $= \frac{BQ}{\text{Speed}}$

$= \frac{h}{x / 20} = \frac{h \times 20}{x}$ $[\because x = (\sqrt{3} - 1)h]$

$= \frac{h}{(\sqrt{3} - 1)h} \times 20$

$= \frac{(\sqrt{3} + 1) \times 20}{(\sqrt{3} - 1)(\sqrt{3} + 1)} = \frac{(\sqrt{3} + 1) \times 20}{2}$

$= 10(\sqrt{3} + 1)$

Question14

The angle of elevation of a jet plane from a point A on the ground is 60°. After a flight of 20s at the speed of 432km / h, the angle of elevation changes to 30°. If the jet plane is flying at a constant height, then its height is [24 Feb 2021 Shift 2]

Options:

A. $1800\sqrt{3}$m

B. $3600\sqrt{3}$m

C. $2400\sqrt{3}$m

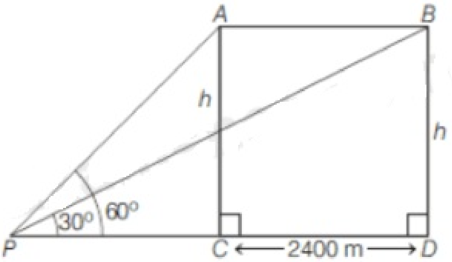
D. $1200\sqrt{3}$m

Answer: D

Solution:

Solution:

Given, angle of elevation are 60° and then 30°.



Also, in 20 sec plane covers the distance from A to B with speed $432 \mathrm{km} / \mathrm{h}$.

$\because\ 432 \times \frac{5}{18} \mathrm{m} / \mathrm{sec} = 120 \mathrm{m} / \mathrm{sec}$

$\therefore\ AB = \text{distance} = S \times T = 120 \times 20$

$= 2400 \mathrm{m}$

In $\triangle PBD$

$\tan 30^{\circ} = \frac{BD}{PD} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{PD}$

$\Rightarrow PD = h\sqrt{3}$

In $\triangle PAC$

$\tan 60^{\circ} = \frac{AC}{PC} \Rightarrow \sqrt{3} = \frac{h}{PC}$

$\Rightarrow PC = h / \sqrt{3}$

Now, $CD = PD - PC$

$2400 = h\sqrt{3} - \frac{h}{\sqrt{3}}$

$\Rightarrow 2400 = h\left(\frac{3-1}{\sqrt{3}}\right)$

$\therefore\ h = \frac{2400 \times \sqrt{3}}{2} = 1200 \times \sqrt{3}$

$\therefore$ Required height $= 1200\sqrt{3} \mathrm{m}$

Question15

Two vertical poles are 150m apart and the height of one is three times that of the other. If from the middle point of the line joining their feet, an observer finds the angles of elevation of their tops to be complementary, then the height of the shorter pole (in meters) is :
24 Feb 2021 Shift 1

Options:

A. $20\sqrt{3}$

B. $25\sqrt{3}$

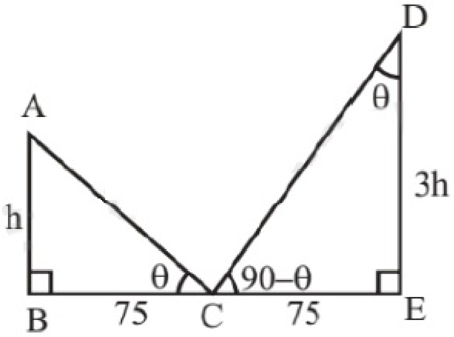
C. 30

D. 25

Answer: B

Solution:

Solution:



In right angle ABC and right △CDE

$\tan\theta = \frac{h}{75}$ and $\cot(90-\theta)$

$= \tan\theta = \frac{75}{3h}$

$\Rightarrow h^2 = \frac{(75)^2}{3}$

$h = 25\sqrt{3}m$

Question16

Let the centroid of an equilateral triangle △ABC be at the origin. Let one of the sides of the equilateral triangle be along the straight line x + y = 3. If R and r be the radius of circumcircle and incircle, respectively of △ABC, then (R + r) is equal to [18 Mar 2021 Shift 2]

Options:

A. $\frac{9}{\sqrt{2}}$

B. $7\sqrt{2}$

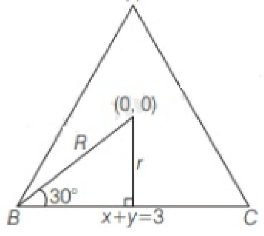
C. $2\sqrt{2}$

D. $3\sqrt{2}$

Answer: A

Solution:

Solution:



Let equation of BC is $x + y = 3$

$\therefore r = \frac{|0+0-3|}{\sqrt{2}} = \frac{3}{\sqrt{2}}$

In equilateral triangle,

$\sin 30^\circ = \frac{r}{R} \Rightarrow \frac{1}{2} = \frac{r}{R}$

$\Rightarrow R = 2r$

$\therefore r + R = r + 2r$

$= 3r = 3 \cdot \frac{3}{\sqrt{2}} = \frac{9}{\sqrt{2}}$

Question17

A pole stands vertically inside a triangular park ABC. Let the angle of elevation of the top of the pole from each corner of the park be $\frac{\pi}{3}$. If the radius of the circumcircle of $\triangle ABC$ is 2 , then the height of the pole is equal to
[18 Mar 2021 Shift 2]

Options:

A. $\frac{2\sqrt{3}}{3}$

B. $2\sqrt{3}$

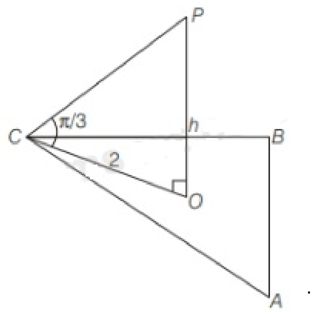
C. $\sqrt{3}$

D. $\frac{1}{\sqrt{3}}$

Answer: B

Solution:

Solution:
Let $PO = h$, $CO = 2 =$ radius and $\angle PCO = \frac{\pi}{3}$



$\tan\frac{\pi}{3} = \frac{PO}{OC} \Rightarrow \sqrt{3} = \frac{h}{2} \Rightarrow h = 2\sqrt{3}$

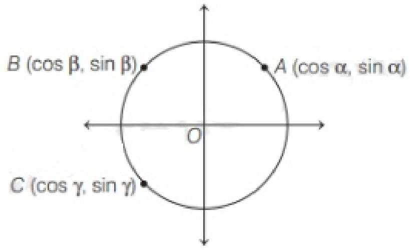
Question18

Let $\tan\alpha$, $\tan\beta$ and $\tan\gamma$, $\alpha, \beta, \gamma \neq \frac{(2n-1)\pi}{2}$, $n \in N$ be the slopes of three line segments OA, OB and OC, respectively, where O is origin. If circumcentre of $\triangle ABC$ coincides with origin and its orthocentre lies on y-axis, then the value of $\left(\frac{\cos 3\alpha + \cos 3\beta + \cos 3\gamma}{\cos\alpha\cos\beta\cos\gamma}\right)^2$ is equal to ....
[17 Mar 2021 Shift 2]

Answer: 144

Solution:

Given, slope of OA $= \tan\alpha = \dfrac{\sin\alpha}{\cos\alpha} = \dfrac{\sin\alpha - 0}{\cos\alpha - 0}$



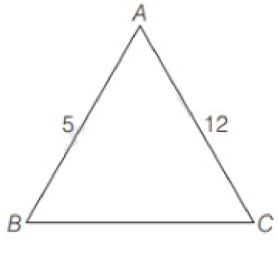
$\therefore$ Coordinates of A must be $(\cos\alpha, \sin\alpha)$
Similarly, $B = (\cos\beta, \sin\beta)$ and $C = (\cos\gamma, \sin\gamma)$
Given, circumcentre coincides with origin and orthocentre lies on Y -axis.
So, centroid must lie on Y -axis because circumcentre, orthocentre and centroid of any triangle always lies on the same line.
$\because \cos\alpha + \cos\beta + \cos\gamma = 0$ ...(i)
(because x-coordinate on Y -axis is zero)
$\Rightarrow \cos^3\alpha + \cos^3\beta + \cos^3\gamma = 3\cos\alpha \cdot \cos\beta \cdot \cos\gamma$..(ii)
Now,
$\cos 3\alpha + \cos 3\beta + \cos 3\gamma = (4\cos^3\alpha - 3\cos\alpha) + (4\cos^3\beta - 3\cos\beta) + (4\cos^3\gamma - 3\cos\gamma)$
$\Rightarrow \cos 3\alpha + \cos 3\beta + \cos 3\gamma = 4(\cos^3\alpha + \cos^3\beta + \cos^3\gamma) - 3(\cos\alpha + \cos\beta + \cos\gamma)$
$= 4(\cos^3\alpha + \cos^3\beta + \cos^3\gamma) - 3 \times 0$ (from Eq. (i))
$= 12\cos\alpha \cdot \cos\beta \cdot \cos\gamma$ (from Eq. (ii))
$\therefore \dfrac{\cos 3\alpha + \cos 3\beta + \cos 3\gamma}{\cos\alpha \cdot \cos\beta \cdot \cos\gamma} = \dfrac{12 \cdot \cos\alpha \cdot \cos\beta \cdot \cos\gamma}{\cos\alpha \cdot \cos\beta \cdot \cos\gamma}$
$\Rightarrow \left(\dfrac{\cos 3\alpha + \cos 3\beta + \cos 3\gamma}{\cos\alpha \cdot \cos\beta \cdot \cos\gamma}\right)^2 = 144$

Question19

In $\triangle ABC$, the lengths of sides AC and AB are 12cm and 5cm, respectively. If the area of $\triangle ABC$ is $30 cm^2$ and R and r are respectively the radii of circumcircle and incircle of $\triangle ABC$, then the value of $2R + r$ (in cm) is equal to
[16 Mar 2021 Shift 2]

Answer: 15

Solution:

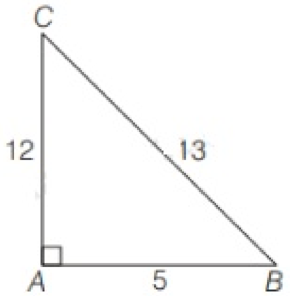


Area of $\triangle ABC = 30cm^2$

$\frac{1}{2} \cdot 5 \cdot 12 \sin A = 30cm^2$

$\Rightarrow \sin A = 1$

$\Rightarrow A = 90^{\circ}$



Here, $AB^2 + AC^2 = BC^2$

$\Rightarrow 12^2 + 5^2 = BC^2$

$\Rightarrow BC = 13$

$\therefore R = \frac{a}{2 \sin A} = \frac{13}{2 \cdot \sin 90} = \frac{13}{2}$

$r = \frac{\Delta}{s} = \frac{30}{\left(\frac{12+5+13}{2}\right)} = \frac{30}{15} = 2$

$\therefore 2R + r = 2 \cdot \frac{13}{2} + 2 = 15$

Question20

A spherical gas balloon of radius 16 meter subtends an angle 60° at the eye of the observer A while the angle of elevation of its center from the eye of A is 75°. Then the height (in meter) of the top most point of the balloon from the level of the observer's eye is :
[25 Jul 2021 Shift 1]

Options:

A. $8(2 + 2\sqrt{3} + \sqrt{2})$

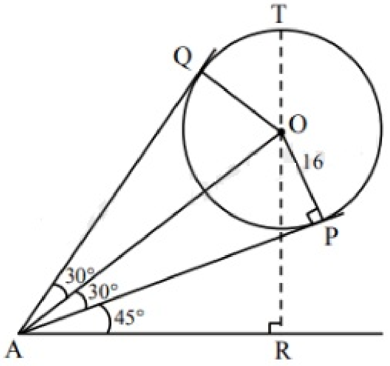
B. $8(\sqrt{6} + \sqrt{2} + 2)$

C. $8(\sqrt{2} + 2 + \sqrt{3})$

D. $8(\sqrt{6} - \sqrt{2} + 2)$

Answer: B

Solution:



O → centre of sphere
P,Q → point of contact of tangents from A
Let T be top most point of balloon & R be foot of perpendicular from O to ground.
From triangle OAP, $OA = 16\operatorname{cosec} 30° = 32$
From triangle ABO, $OR = OA \sin 75° = 32\frac{(\sqrt{3}+1)}{2\sqrt{2}}$
So level of top most point $= OR + OT$
$= 8(\sqrt{6}+\sqrt{2}+2)$

Question21

If in a triangle ABC, AB = 5 units, $\angle B = \cos^{-1}\left(\frac{3}{5}\right)$ and radius of circumcircle of ΔABC is 5 units,then the area (in sq. units) of ΔABC is : [20 Jul 2021 Shift 1]

Options:

A. $10 + 6\sqrt{2}$

B. $8 + 2\sqrt{2}$

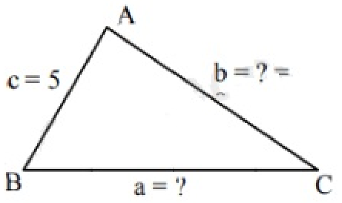
C. $6 + 8\sqrt{3}$

D. $4 + 2\sqrt{3}$

Answer: C

Solution:

Solution:



As, $\cos B = \frac{3}{5} \Rightarrow B = 53°$

As, $R = 5 \Rightarrow \frac{c}{\sin c} = 2R$

$\Rightarrow \frac{5}{10} = \sin c \Rightarrow C = 30°$

Now, $\frac{b}{\sin B} = 2R \Rightarrow b = 2(5)\left(\frac{4}{5}\right) = 8$

Now, by cosine formula

$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$

$\Rightarrow \frac{3}{5} = \frac{a^2 + 25 - 64}{2(5)a}$
$\Rightarrow a^2 - 6a - 3g = 0$
$\therefore a = \frac{6 \pm \sqrt{192}}{2} = \frac{6 \pm 8\sqrt{3}}{2}$
$\Rightarrow 3 + 4\sqrt{3}$(Reject $a = 3 - 4\sqrt{3}$)
Now, $\Delta = \frac{abc}{4R} = \frac{(3 + 4\sqrt{3})(8)(5)}{4(5)} = 2(3 + 4\sqrt{3})$
$\Rightarrow \Delta = (6 + 8\sqrt{3})$
⇒ Option (3) is correct.

Question22

Let in a right angled triangle, the smallest angle beθ. If a triangle formed by taking the reciprocal of its sides is also a right angled triangle, then $\sin\theta$ is equal to:
[20 Jul 2021 Shift 2]

Options:

A. $\frac{\sqrt{5} + 1}{4}$

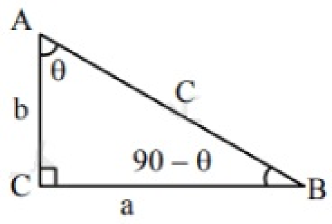
B. $\frac{\sqrt{5} - 1}{2}$

C. $\frac{\sqrt{2} - 1}{2}$

D. $\frac{\sqrt{5} - 1}{4}$

Answer: B

Solution:



$\angle A = \theta$
$\angle B = 90 - \theta$
$a =$ smallest side
$c^2 = a^2 + b^2$
$\frac{1}{a^2} = \frac{1}{b^2} + \frac{1}{c^2}$
$\frac{b^2c^2}{a^2} = b^2 + c^2$
Use $a = 2R\sin A = 2R\sin\theta$
$b = 2R\sin B = 2R\sin(90 - \theta) = 2R\cos\theta$
$c = 2R\sin C = 2\sin 90° = 2R$
$\frac{4R^2\cos^2\theta}{4R^2\sin^2\theta} = 4R^2\cos^2\theta + 4R^2$
$\cos^2\theta = \sin^2\theta\cos^2\theta + \sin^2\theta$
$1 - \sin^2\theta = \sin^2\theta(1 - \sin^2\theta) + \sin^2\theta$
$\sin^2\theta = \frac{3 - \sqrt{5}}{2}$

$\Rightarrow \sin\theta = \frac{\sqrt{5}-1}{2}$

Question23

A vertical pole fixed to the horizontal ground is divided in the ratio 3 : 7 by a mark on it with lower part shorter than the upper part. If the two parts subtend equal angles at a point on the ground 18m away from the base of the pole, then the height of the pole (in m) is
[31 Aug 2021 Shift 1]

Options:

A. $12\sqrt{15}$

B. $12\sqrt{10}$

C. $8\sqrt{10}$

D. $6\sqrt{10}$

Answer: B

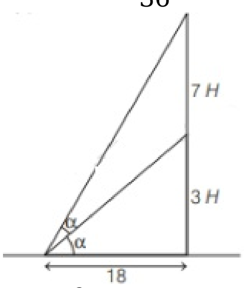
Solution:

Solution:
Let height of pole = 10h

$\tan\alpha = \frac{3H}{18}$

$\tan 2\alpha = \frac{10H}{18} = \frac{2\tan\alpha}{1-\tan^2\alpha}$

$\frac{5H}{9} = \frac{2 \cdot \left(\frac{H}{6}\right)}{1-\frac{H^2}{36}}$



$1-\frac{H^2}{36} = \frac{3}{5}$

$\frac{H^2}{36} = \frac{2}{5}$

$H^2 = \frac{72}{5}$

$H = \sqrt{\frac{72}{5}}$

$10H = 12\sqrt{10}$

Question24

Let $\frac{\sin A}{\sin B} = \frac{\sin(A - C)}{\sin(C - B)}$, where A, B and C are angles of a $\triangle ABC$. If the lengths of the sides opposite these angles are a, b and c respectively, then
[27 Aug 2021 Shift 1]

Options:

A. $b^2 - a^2 = a^2 + c^2$

B. b^2, c^2 and a^2 are in AP

C. c^2, a^2 and b^2 are in AP

D. a^2, b^2 and c^2 are in AP

Answer: B

Solution:

Solution:

Given, $\frac{\sin A}{\sin B} = \frac{\sin(A - C)}{\sin(C - B)}$

$\Rightarrow \frac{\sin(B + C)}{\sin(A + C)} = \frac{\sin(A - C)}{\sin(C - B)}$ $[\because A + B + C = \pi]$

$\Rightarrow \sin(C + B) . \sin(C - B) = \sin(A + C) . \sin(A - C)$

$\Rightarrow \sin^2 C - \sin^2 B = \sin^2 A - \sin^2 C$

$\Rightarrow 2\sin^2 C = \sin^2 A + \sin^2 B$

$\Rightarrow 2(2R\sin C)^2 = (2R\sin A)^2 + (2R\sin B)^2$

$\Rightarrow 2c^2 = a^2 + b^2 \left[\because \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R\right]$

$\Rightarrow a^2, c^2, b^2$ are in AP.

$\Rightarrow b^2, c^2, a^2$ are in AP

Question25

Two poles, AB of length a m and CD of length $a + b (b \neq a)$ metres are erected at the same horizontal level with bases at B and D.
If BD = x and $\tan \angle ACB! = \frac{1}{2}$, then
[27 Aug 2021 Shift 2]

Options:

A. $x^2 + 2(a + 2b)x - b(a + b) = 0$

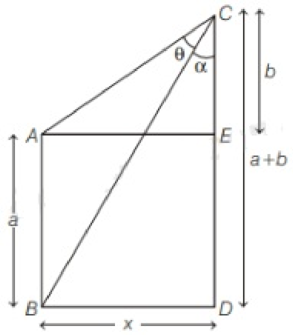
B. $x^2 + 2(a + 2b)x + a(a + b) = 0$

C. $x^2 - 2ax + b(a + b) = 0$

D. $x^2 - 2ax + a(a + b) = 0$

Answer: C

Solution:



In ΔACE

$\tan(\theta + \alpha) = \frac{x}{b}$

In ΔACD

$\tan \alpha = \frac{x}{a + b}$ and $\tan \theta = \frac{1}{2}$

Then,

$\tan(\theta + \alpha) = \frac{\tan \theta + \tan \alpha}{1 - \tan \theta . \tan \alpha}$

$\Rightarrow \frac{\frac{1}{2} + \frac{x}{a + b}}{1 - \frac{1}{2} . \frac{x}{a + b}} = \frac{x}{b}$

$\Rightarrow \frac{a + b + 2x}{2(a + b) - x} = \frac{x}{b}$

$\Rightarrow ab + b^2 + 2bx = 2ax + 2bx - x^2$

$\Rightarrow ^2 - 2ax + ab + b^2 = 0$

$\Rightarrow x^2 - 2ax + b(a + b) = 0$

Question26

A ray of light coming from the point $(2, 2\sqrt{3})$ is incident at an angle 30° on the line x = 1 at the point A. The ray gets reflected on the line x = 1 and meets x -axis at the point B. Then, the line AB passes through the point:
[Sep. 06, 2020 (I)]

Options:

A. $\left(3, -\frac{1}{\sqrt{3}}\right)$

B. $\left(4, -\frac{\sqrt{3}}{2}\right)$

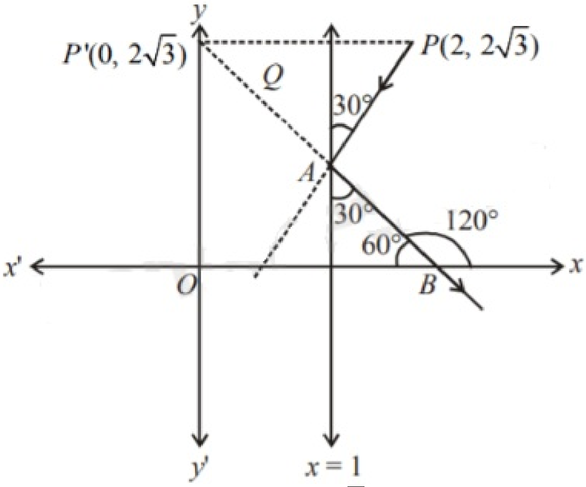
C. $(3, -\sqrt{3})$

D. $(4, -\sqrt{3})$

Answer: C

Solution:

Solution:



Slope of AB = $\tan 120° = -\sqrt{3}$
therefore Equation of line AB (i.e. BP′)
$y - 2\sqrt{3} = -\sqrt{3}(x - 0)$
$\Rightarrow \sqrt{3}x + y = 2\sqrt{3}$
∴ Point $(3, -\sqrt{3})$ lies on line AB.

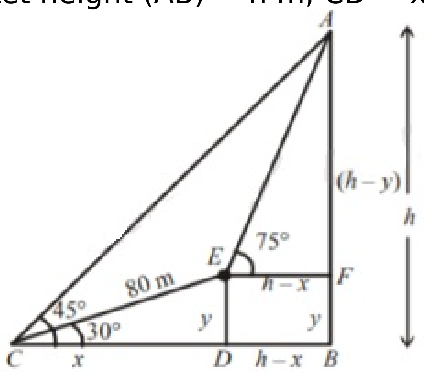
Question27

The angle of elevation of the top of a hill from a point on the horizontal plane passing through the foot of the hill is found to be 45°. After walking a distance of 80 meters towards the top, up a slope inclined at an angle of 30° to the horizontal plane, the angle of elevation of the top of the hill becomes 75°. Then the height of the hill (in meters) is ______. [Sep. 06, 2020 (I)]

Answer: 80

Solution:

Let height (AB) = h m, CD = x m and ED = y m



In rt. ΔCDE,

$\sin 30° = \frac{y}{80} \Rightarrow y = 40$

$\cos 30° = \frac{x}{80} \Rightarrow x = 40\sqrt{3}$

Now, in Δ AEF,

$\tan 75° = \frac{h - y}{h - x}$

$\Rightarrow (2 + \sqrt{3}) = \frac{h - 40}{h - 40\sqrt{3}}$

$\Rightarrow(2+\sqrt{3})(h-40\sqrt{3})=h-40$
$\Rightarrow 2h-80\sqrt{3}+\sqrt{3}h-120=h-40$
$\Rightarrow h+\sqrt{3}h=80+80\sqrt{3}$
$\Rightarrow(\sqrt{3}+1)h=80(\sqrt{3}+1)$
$\therefore h=80m$

Question28

The angle of elevation of the summit of a mountain from a point on the ground is 45°. After climbing up on km towards the summit at an inclination of 30° from the ground, the angle of elevation of the summit is found to be 60°. Then the height (in km) of the summit from the ground is:
[Sep. 06, 2020 (II)]

Options:

A. $\frac{\sqrt{3}-1}{\sqrt{3}+1}$

B. $\frac{\sqrt{3}+1}{\sqrt{3}-1}$

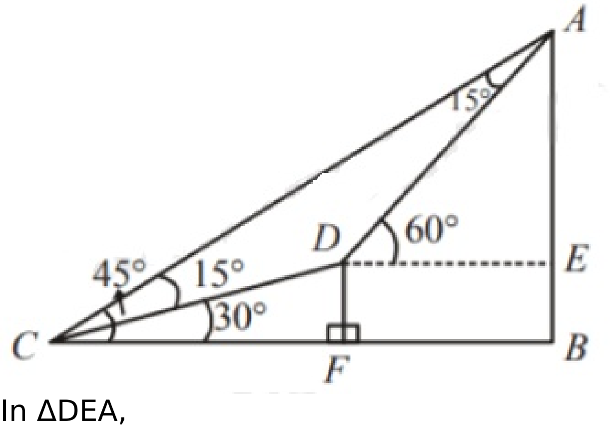
C. $\frac{1}{\sqrt{3}-1}$

D. $\frac{1}{\sqrt{3}+1}$

Answer: C

Solution:

$\because \angle DCA = \angle DAC = 30°$
$\therefore AD = DC = 1km$



In ΔDEA,
$\frac{AE}{AD}=\sin 60° \Rightarrow AE=\frac{\sqrt{3}}{2}km$
In ΔCDF, $\sin 30°=\frac{DF}{CD}\Rightarrow DF=\frac{1}{2}km$
$\therefore EB=DF=\frac{1}{2}km$
$\therefore$ Height of mountain $=AE+EB$
$=\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\right)=\left(\frac{\sqrt{3}+1}{2}\right)km$
$=\frac{1}{\sqrt{3}-1}km$

Question29

Two vertical poles AB = 15 m and CD = 10 m are standing apart on a horizontal ground with points A and C on the ground. If P is the point of intersection of BC and AD, then the height of P (in m) above the line AC is :
[Sep. 04, 2020 (I)]

Options:

A. $\frac{20}{3}$

B. 5

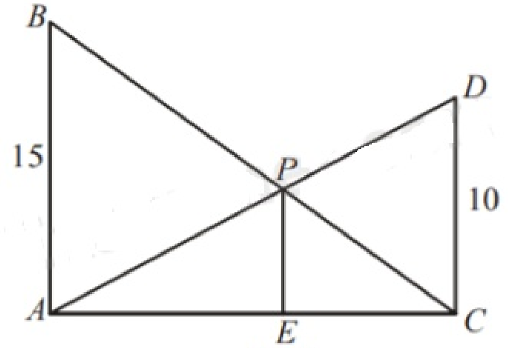
C. $\frac{10}{3}$

D. 6

Answer: D

Solution:

Solution:



$m : n$

Let PE $\perp$ AC and $\frac{AE}{EC} = \frac{m}{n}$

$\because \Delta AEP \sim \Delta ACD, \frac{m}{PE} = \frac{m+n}{10}$

$\Rightarrow PE = \frac{10m}{m+n}$ ...(i)

$\because \Delta CEP \sim \Delta CAB, \frac{n}{PE} = \frac{m+n}{15}$

$\Rightarrow PE = \frac{15n}{m+n}$ ...(ii)

From (i) and (ii),

$10m = 15n \Rightarrow m = \frac{3}{2}n$

So, PE = 6

Question30

The angle of elevation of a cloud C from a point P, 200 m above a still lake is 30°. If the angle of depression of the image of C in the lake from the point P is 60°, then PC (in m) is equal to :
[Sep. 04, 2020 (II)]

Options:

A. 100

B. $200\sqrt{3}$

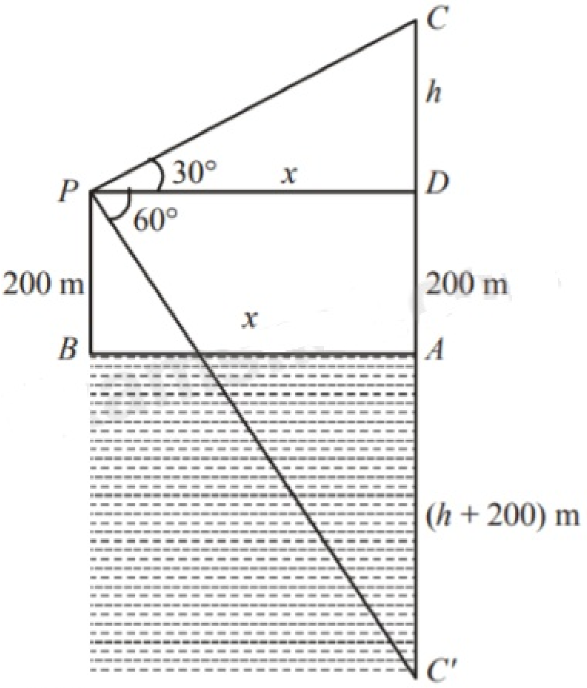
C. 400

D. $400\sqrt{3}$

Answer: C

Solution:

Solution:



Here in ΔPCD,

$\sin 30° = \frac{h}{PC} \Rightarrow PC = 2h$ ...(i)

$\tan 30° = \frac{h}{x} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x}$

$\Rightarrow x = \sqrt{3}h$ ...(ii)

Now, in right $\Delta PC'D$

$\tan 60° = \frac{h + 400}{x}$

$\Rightarrow \sqrt{3}x = h + 400 \Rightarrow 3h = h + 400$ [From (ii)]

$\Rightarrow h = 200$

So, $PC = 400m$ [From (i)]

Question31

In a triangle, the sum of lengths of two sides is x and the product of the lengths of the same two sides is y. if $x^2 - c^2 = y$, where c is the length of the third side of the triangle, then the circumradius of the triangle is : [Jan. 11,2019(I)]

Options:

A. $\frac{3}{2}y$

B. $\frac{c}{\sqrt{3}}$

C. $\frac{c}{3}$

D. $\frac{y}{\sqrt{3}}$

Answer: B

Solution:

Let two sides of triangle are a and b.
$a + b = x$
$ab = y$
$x^2 - c^2 = y \Rightarrow (a + b)^2 - c^2 = ab$
$\Rightarrow (a + b - c)(a + b + c) = ab$
$\Rightarrow 2(s - c)(2s) = ab$
$\Rightarrow 4s(s - c) = ab$
$\Rightarrow \frac{s(s - c)}{ab} = \frac{1}{4}$
$\Rightarrow \cos^2 \frac{c}{2} = \frac{1}{4}$
$\Rightarrow \cos c = -\frac{1}{2} \Rightarrow c = 120°$
∴ Area of triangle is,
$\Delta = \frac{1}{2}ab(\sin 120°) = \frac{\sqrt{3}}{4}ab$
$\because R = \frac{abc}{4\Delta}$
$\because R = \frac{abc}{\sqrt{3}ab} = \frac{c}{\sqrt{3}}$

Question32

Given $\frac{b+c}{11} = \frac{c+a}{12} = \frac{a+b}{13}$ for a ΔABC with usualnotation. If $\frac{\cos A}{\alpha} = \frac{\cos B}{\beta} = \frac{\cos C}{\gamma}$, then the ordered triad$(\alpha, \beta, \gamma)$ has a value :
[Jan. 11, 2019 (II)]

Options:

A. (7, 19, 25)

B. (3, 4, 5)

C. (5, 12, 13)

D. (19, 7, 25)

Answer: A

Solution:

Let $\frac{b+c}{11} = \frac{c+a}{12} = \frac{a+b}{13} = k(\text{say})$
$\therefore b + c = 11k, c + a = 12k, a + b = 13k$
$\therefore a + b + c = 18k$

$\therefore a = 7k, b = 6k$ and $c = 5k$

$\therefore \cos A = \frac{36k^2 + 25k^2 - 49k^2}{2.30k^2} = \frac{1}{5}$

and $\cos B = \frac{49k^2 + 25k^2 - 36k^2}{2.35k^2} = \frac{19}{35}$

and $\cos C = \frac{49k^2 + 36k^2 - 25k^2}{2.42k^2} = \frac{5}{7}$

$\therefore \cos A : \cos B : \cos C = 7 : 19 : 25$

$\therefore \frac{\cos A}{7} = \frac{\cos B}{19} = \frac{\cos C}{25}$

Hence, required ordered triplet is (7,19,25) .

Question33

With the usual notation, in ΔABC, if $\angle A + \angle B = 120°$, $a = \sqrt{3} + 1$ and $b = \sqrt{3} - 1$ then the ratio $\angle A : \angle B$, is:
[Jan. 10, 2019 (II)]

Options:

A. 7 : 1

B. 5 : 3

C. 9 : 7

D. 3 : 1

Answer: A

Solution:

$\angle A + \angle B = 120°$ ...(1)

$\Rightarrow \angle C = 180° - 120° = 60°$

$\tan\left(\frac{A - B}{2}\right) = \frac{a - b}{a + b}\cot\left(\frac{C}{2}\right)$

$= \frac{2}{2\sqrt{3}}(\cot 30°) = \frac{1}{\sqrt{3}} \times \sqrt{3} = 1$

$\Rightarrow \frac{\angle A - \angle B}{2} = \frac{\pi}{4}$ ($\angle$ is angle)

$\Rightarrow \angle A - \angle B = 90°$

From eqn (1) and (2)

$\angle A = 105°, \angle B = 15°$

Then, $\angle A : \angle B = 7 : 1$

Question34

If the angle of elevation of a cloud from a point P which is 25 m above a lake be 30° and the angle of depression of reflection of the cloud in the lake from P be 60°, then the height of the cloud (in meters) from the surface of the lake is:
[Jan. 12, 2019 (II)]

Options:

A. 60

B. 50

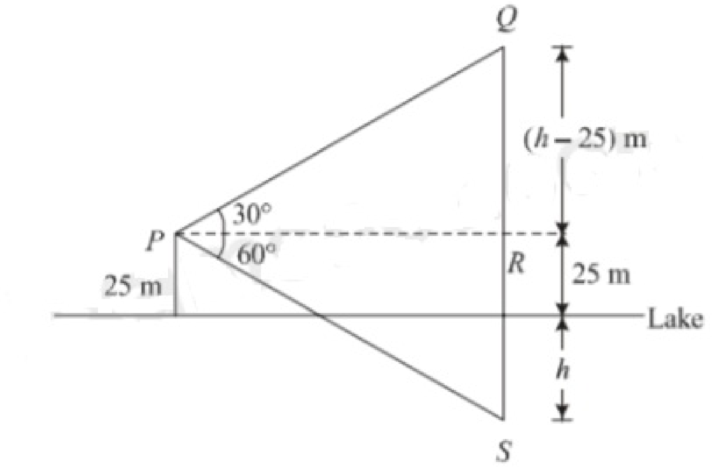
C. 45

D. 42

Answer: B

Solution:

Let height of the cloud from the surface of the lake be h meters.



∴ In ΔPRQ

$\tan 30° = \frac{h-25}{PR}$

$\therefore PR = (h-25)\sqrt{3}$ ...(i)

and in ΔPRS : $\tan 60° = \frac{h+25}{PR}$

$PR = \frac{h+25}{\sqrt{3}}$ ...(ii)

Then, from eq. (i) and (ii),

$(h-25)\sqrt{3} = \frac{h+25}{\sqrt{3}}$

∴ h = 50m

Question35

Consider a triangular plot ABC with sides AB = 7 m, BC = 5 m and CA = 6 m. A vertical lamp-post at the mid point D of AC subtends an angle 30° at B. The height (in m) of the lamp-post is:
[Jan. 10, 2019 (I)]

Options:

A. $\frac{3}{2}\sqrt{21}$

B. $\frac{2}{3}\sqrt{21}$

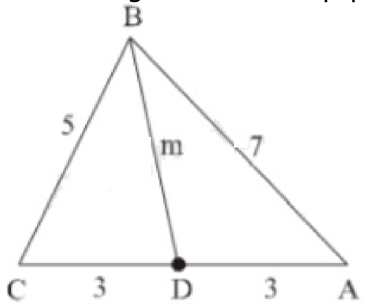
C. $2\sqrt{21}$

D. $7\sqrt{3}$

Answer: B

Solution:

Let the height of the lamp-post is h.



By Apollonius Theorem,

$$2\left(BD^2+\left(\frac{AC}{2}\right)^2\right) = BC^2 + AB^2$$

$\Rightarrow 2(m^2+3^2) = 25+49 \Rightarrow m = 2\sqrt{7}$

$\tan 30° = hBD$

$\Rightarrow h = 2\sqrt{7} \times \frac{1}{\sqrt{3}} = \frac{2\sqrt{21}}{3}$

Question36

Let A(3, 0, –1), B(2, 10, 6) and C (1, 2, 1) be the vertices of a triangle and M be the midpoint of AC. If G divides BM in the ratio, 2 : 1, then cos (∠GOA) (O being the origin) is equal to : [April 10, 2019 (I)]

Options:

A. $\frac{1}{2\sqrt{15}}$

B. $\frac{1}{\sqrt{15}}$

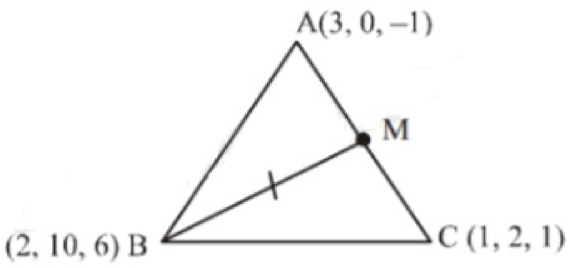
C. $\frac{1}{6\sqrt{10}}$

D. $\frac{1}{\sqrt{30}}$

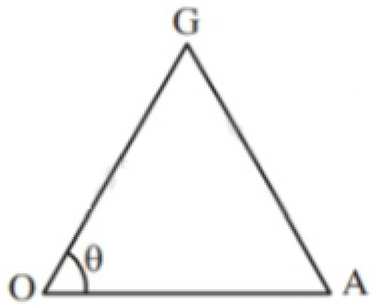
Answer: B

Solution:

G is the centroid of DABC.



$\Rightarrow G \equiv \left(\frac{3+2+1}{3}, \frac{0+10+2}{3}, \frac{-1+6+1}{3}\right) \equiv (2, 4, 2)$



$OG = \sqrt{4+16+4}$, $OA = \sqrt{9+1}$, $AG = \sqrt{1+16+9}$

$\therefore \cos\theta = \frac{(OG)^2 + (OA)^2 - (AG)^2}{2(OG)(OA)} = \frac{24+10-26}{2\sqrt{24}\sqrt{10}}$

$= \frac{8}{2\sqrt{8\times 3\times 2\times 5}} = \frac{4}{4\sqrt{15}} = \frac{1}{\sqrt{15}}$

Question37

The angles A, B and C of a triangle ABC are in A.P. and a : b = 1 : $\sqrt{3}$. If c = 4 cm, then the area (in sq.cm) of this triangle is :
[April 10, 2019(II)]

Options:

A. $\frac{2}{\sqrt{3}}$

B. $4\sqrt{3}$

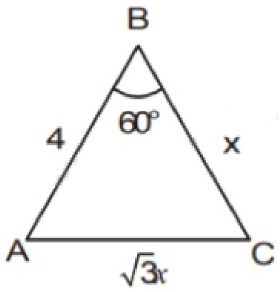
C. $2\sqrt{3}$

D. $\frac{4}{\sqrt{3}}$

Answer: C

Solution:

Solution:
Given that A, B, C, are in A.P. ⇒ 2B = A + C
Now, A + B + C = π ⇒ B $= \frac{\pi}{3}$
Area = $\frac{1}{2}(4x)\sin 60° = \sqrt{3}x$



Now $\cos 60° = \frac{16 + x^2 - 3x^2}{8x}$
$\Rightarrow 4x = 16 - 2x^2 \Rightarrow x^2 + 2x - 8 = 0$
$\Rightarrow x = 2$ [∵ x can't be negative]
Hence, area $= 2\sqrt{3}$ sq. cm

Question38

If the lengths of the sides of a triangle are in A.P. and the greatest angle is double the smallest, then a ratio of lengths of the sides of this triangle is :
[April 08, 2019 (II)]

Options:

A. 5 : 9 : 13

B. 4 : 5 : 6

C. 3 : 4 : 5

D. 5 : 6 : 7

Answer: B

Solution:

Let the sides of triangle are a > b > c where
Given A = 2C
∵ A + B + C = π and A = 2C
⇒ B = π – 3C ...(i)
∵ a, b, c are in A.P. ⇒ a + c = 2b
⇒ sin A + sin C = 2 sin B ...(ii)
⇒ sin A = sin (2C) and sin B = sin 3C
From (ii),
sin 2C + sin C = 2 sin 3C
$\Rightarrow (2\cos C + 1)\sin C = 2\sin C(3 - 4\sin^2 C)$
$\Rightarrow 2\cos C + 1 = 6 - 8(1 - \cos^2 C)$
$\Rightarrow 8\cos^2 C - 2\cos C - 3 = 0$
$\Rightarrow \cos C = \frac{3}{4}$ or $\cos C = -\frac{1}{2}$
∵C is acute angle
$\Rightarrow \cos C = \frac{3}{4} \Rightarrow \sin C = \frac{\sqrt{7}}{4}$
and $\sin A = 2\sin C\cos C = 2 \times \frac{\sqrt{7}}{4} \times \frac{3}{4} = \frac{3\sqrt{7}}{8}$
$\sin B = \frac{3\sqrt{7}}{4} - \frac{4\sqrt{7}}{4} \times \frac{7}{16} = \frac{5\sqrt{7}}{16}$
$\Rightarrow \sin A : \sin B : \sin C : : a : b : c$ is 6: 5: 4

Question39

ABC is a triangular park with AB = AC = 100 metres. A vertical tower is situated at the mid-point of BC. If the angles of elevation of the top of the tower at A and B are $\cot^{-1}(3\sqrt{2})$ and $\operatorname{cosec}^{-1}(2\sqrt{2})$ respectively, then the height of the tower (in metres) is :
[April 10, 2019 (I)]

Options:

A. $\frac{100}{3\sqrt{3}}$

B. $10\sqrt{5}$

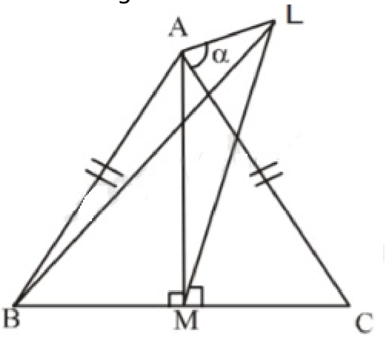
C. 20

D. 25

Answer: C

Solution:

Solution:
Let the height of the vertical tower situated at the mid point of BC be h.



In ΔALM,
$\cot A = \frac{AM}{LM}$
$\Rightarrow 3\sqrt{2} = \frac{AM}{h} \Rightarrow AM = 3\sqrt{2}h$
In ΔBLM ,
$\cot B = \frac{BM}{LM} \Rightarrow \sqrt{7} = \frac{BM}{h} \Rightarrow BM = \sqrt{7}h$
In ΔABM by Pythagoras theorem
$AM^2 + MB^2 = AB^2$
$\therefore AM^2 + MB^2 = (100)^2$
$\Rightarrow 18h^2 + 7h^2 = 100 \times 100$
$\Rightarrow h^2 = 4 \times 100 \Rightarrow h = 20$

Question40

PQR is a triangular park with PQ = PR = 200 m. A T.V. tower stands at the mid-point of QR. If the angles of elevation of the top of the tower at P, Q and R are respectively 45°, 30° and 30°, then the height of the tower (in m) is :
[2018]

Options:

A. 50

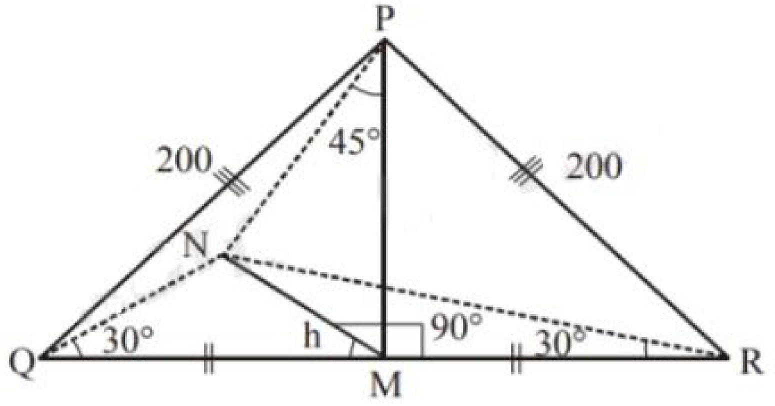
B. $100\sqrt{3}$

C. $50\sqrt{2}$

D. 100

Answer: D

Solution:



Let height of tower MN = h
In ΔQMN we have
$\tan 30° = \frac{MN}{QM}$
$\therefore QM = \sqrt{3}h = MR$ ...(1)
Now in ΔMNP
MN = PM ...(2)
In ΔPMQ we have :
$MP = \sqrt{(200)^2 - (\sqrt{3}h)^2}$
∴ From (2), we get :
$\sqrt{(200)^2 - (\sqrt{3}h)^2} = h$
$\Rightarrow h = 100m$

Question41

A man on the top of a vertical tower observes a car moving at a uniform speed towards the tower on a horizontal road. If it takes 18 min, for the angle of depression of the car to change from 30° to 45°, then after this, the time taken (in min) by the car to reach the foot of the tower, is. [Online April 16, 2018]

Options:

A. $9(1+\sqrt{3})$

B. $\frac{9}{2}(\sqrt{3}-1)$

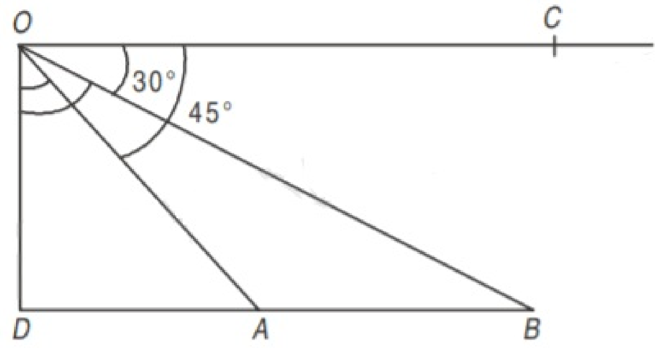
C. $18(1+\sqrt{3})$

D. $18(\sqrt{3}-1)$

Answer: A

Solution:

Here, ∠DOA = 45°, ∠DOB = 60°
Now, let height of tower = h.



In Δ DOA, $\tan(\angle DOA) = \frac{DA}{OD}$

$\Rightarrow \tan 45° = \frac{DA}{h} \Rightarrow h = DA$

Now, in ΔDOB

$\tan(\text{angle } DOB) = \frac{BD}{OD}$

$\Rightarrow \tan 60° = \frac{BD}{h} \Rightarrow BD = \sqrt{3}h$

∴ speed for the distance BA $= \frac{BD - AD}{18} = \frac{(\sqrt{3} - 1)h}{18}$

∴ required time taken $= \frac{AD}{speed} = \frac{h \times 18}{(\sqrt{3} - 1)h} = 18\sqrt{3} - 1 = 9(\sqrt{3} + 1)$

Question42

An aeroplane flying at a constant speed, parallel to the horizontal ground, 3km above it, is observed at an elevation of 60° from a point on the ground. If, after five seconds, its elevation from the same point, is 30°, then the speed (in km/hr) of the aeroplane is [Online April 15, 2018]

Options:

A. 1500

B. 750

C. 720

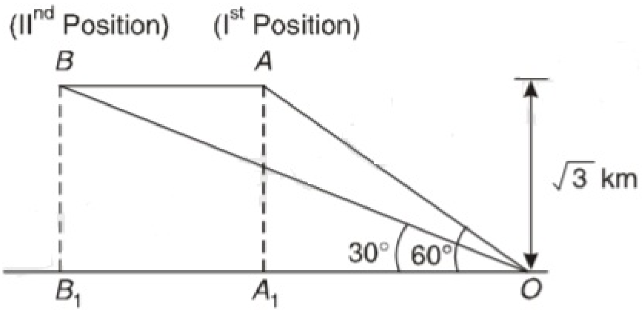
D. 1440

Answer: D

Solution:

Solution:

For ΔOA, A, $OA_1 = \frac{\sqrt{3}}{\tan 60°} = 1\,km.$



For ΔOB_1, B, $OB_1 = \sqrt{3}\tan 30° = 3\,\text{km}$
As, a distance of $3 - 1 = 2\,\text{km}$ is covered in 5 seconds.
Therefore the speed of the plane is $\frac{2 \times 3600}{5} = 1440\,\text{km / hr}$

Question43

A tower T_1 of height 60m is located exactly opposite to a tower T_2 of height 80m on a straight road. From the top of T_1, if the angle of depression of the foot of T_2 is twice the angle of elevation of the top of T_2, then the width (in m) of the road between the feet of the towers T_1 and T_2 is
[Online April 15,2018]

Options:

A. $20\sqrt{2}$

B. $10\sqrt{2}$

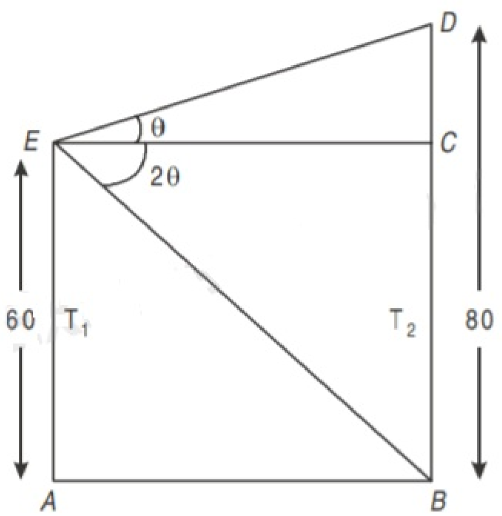
C. $10\sqrt{3}$

D. $20\sqrt{3}$

Answer: D

Solution:

Solution:
Let the distance between T_1 and T_2 be x



From the figure

EA = 60m (T $_1$) and DB = 80m (T $_2$)
$\angle DEC = \theta$ and $\angle BEC = 2\theta$
Now in ΔDEC,
$\tan\theta = \frac{DC}{AB} = \frac{20}{x}$
and in ΔBE C,
$\tan 2\theta = \frac{BC}{CE} = \frac{60}{x}$
We know that
$\tan 2\theta = \frac{2\tan\theta}{1-(\tan\theta)^2}$
$\Rightarrow \frac{60}{x} = \frac{2\left(\frac{20}{x}\right)}{1-\left(\frac{20}{x}\right)^2}$
$\Rightarrow x^2 = 1200 \Rightarrow x = 20\sqrt{3}$

Question44

Let a vertical tower AB have its end A on the level ground. Let C be the mid-point of AB and P be a point on the ground such that AP = 2AB. If ΔBPC = b, then tan b is equal to : [2017]

Options:

A. $\frac{4}{9}$

B. $\frac{6}{7}$

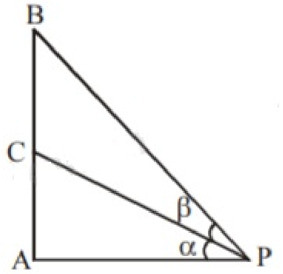
C. $\frac{1}{4}$

D. $\frac{2}{9}$

Answer: D

Solution:

Solution:
Since AP = 2AB $\Rightarrow \frac{AB}{AP} = \frac{1}{2}$
Let $\angle APC = \alpha$
$\therefore \tan\alpha = \frac{AC}{AP} = \frac{1}{2}\frac{AB}{AP} = \frac{1}{4}$
$\because$C is the mid point $\Big) \left(\therefore AC = \frac{1}{2}AB\right)$
$\Rightarrow \tan\alpha = \frac{1}{4}$



As $\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1-\tan\alpha\tan\beta}$

$\Rightarrow \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} = \frac{1}{2}$

$\left[\because \tan(\alpha + \beta) = \frac{AB}{AP} = \frac{1}{2}[\text{From}(1)]\right]$

$\Rightarrow \frac{\frac{1}{4} + \tan\beta}{1 - \frac{1}{4}\tan\beta} = \frac{1}{2}$

$\therefore \tan\beta = \frac{2}{9}$

Question45

A man is walking towards a vertical pillar in a straight path, at a uniform speed. At a certain point A on the path, he observes that the angle of elevation of the top of the pillar is 30°. After walking for 10 minutes from A in the same direction, at a point B, he observes that the angle of elevation of the top of the pillar is 60°. Then the time taken (in minutes) by him, from B to reach the pillar, is: [2016]

Options:

A. 20

B. 5

C. 6

D. 10

Answer: B

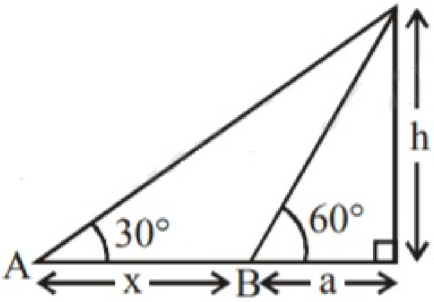
Solution:

$\tan 30° = \frac{h}{x + a}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x + a} \Rightarrow \sqrt{3}h = x + a$ ...(1)

$\tan 60° = \frac{h}{a} \Rightarrow \sqrt{3} = \frac{h}{a}$

$\Rightarrow h = \sqrt{3}a$ ...(2)



From (1) and (2)

$3a = x + a \Rightarrow x = 2a$

Here, the speed is uniform

So, time taken to cover x = 2 (time taken to cover a)

$\therefore$ Time taken to cover a $= \frac{10}{2}$ minutes = 5 minutes

----

Question46

The angle of elevation of the top of a vertical tower from a point A, due east of it is 45°. The angle of elevation of the top of the same tower from a point B, due south of A is 30°. If the distance between A and B is $54\sqrt{2}$ m, then the height of the tower (in metres), is :
[Online April 10, 2016]

Options:

A. 108

B. $36\sqrt{3}$

C. $54\sqrt{3}$

D. 54

Answer: D

Solution:

Let AP = x
BP = y
$\tan 45° = \frac{H}{x} \Rightarrow H = x$
$\tan 30° = \frac{H}{y} \Rightarrow y = \sqrt{3}H$
$x^2 + (54\sqrt{2})^2 = y^2$
$H^2 + (54\sqrt{2})^2 = 3H^2$
$(54\sqrt{2})^2 = 2H^2$
$54\sqrt{2} = \sqrt{2}H$
$54 = H$

Question47

In a ΔABC, $\frac{a}{b} = 2 + \sqrt{3}$ and ∠C = 60°. Then the ordered pair (∠A, ∠B) is equal to :
[Online April 10, 2015]

Options:

A. (45°, 75°)

B. (105°, 15°)

C. (15°, 105°)

D. (75°, 45°)

Answer: B

Solution:

$$\frac{\sin A}{\sin B} = 2 + \sqrt{3}$$
$$\frac{\sin(105°)}{\sin(15°)} = 2 + \sqrt{3}$$
$$\frac{\cos 15°}{\sin 15°} = 2 + \sqrt{3}$$

Question48

If the angles of elevation of the top of a tower from three collinear points A, B and C, on a line leading to the foot of the tower, are 30°, 45° and 60° respectively, then the ratio, AB : BC, is :
[2015]

Options:

A. $1 : \sqrt{3}$

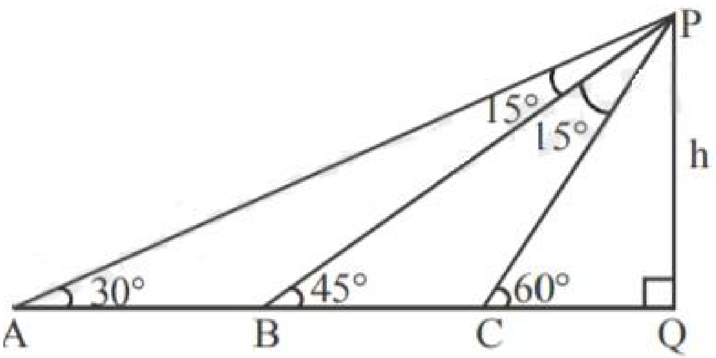
B. 2 : 3

C. $\sqrt{3} : 1$

D. $\sqrt{3} : \sqrt{2}$

Answer: C

Solution:



∵ PB bisects ∠APC, therefore
AB : BC = PA : PC
Also in ΔAPQ, $\sin 30° = \frac{h}{PA} \Rightarrow PA = 2h$
and in ΔCPQ, $\sin 60° = \frac{h}{PC} \Rightarrow PC = \frac{2h}{\sqrt{3}}$
∴ AB : BC $= 2h : \frac{2h}{\sqrt{3}} = \sqrt{3} : 1$

Question49

Let 10 vertical poles standing at equal distances on a straight line, subtend the same angle of elevation at a point O on this line and all the poles are on the same side of O. If the height of the longest pole is ‘h’ and the distance of the foot of the smallest pole from O is ‘a’; then the distance between two consecutive poles, is :

[Online April 11, 2015]

Options:

A. $\frac{h\cos\alpha - a\sin\alpha}{9\sin\alpha}$

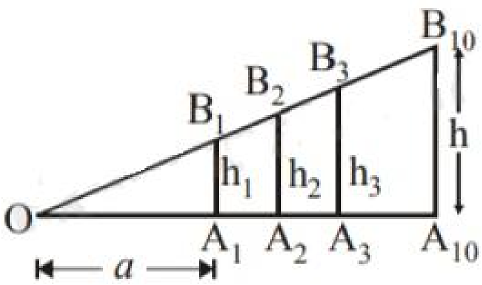
B. $\frac{h\sin\alpha + a\cos\alpha}{9\sin\alpha}$

C. $\frac{h\cos\alpha - a\sin\alpha}{9\cos\alpha}$

D. $\frac{h\sin\alpha - a\cos\alpha}{9\cos\alpha}$

Answer: A

Solution:



ΔOA_1B_1, ΔOA_2B_2, ΔOA_3B_3, ..., $\Delta OA_{10}B_{10}$ all are similar triangles.

$\Rightarrow \frac{h_1}{a_1} = \frac{h_2}{a_2} = \frac{h_3}{a_3} = \ldots.. = \frac{h_{10}}{a_{10}} = \tan\alpha$

Since, $h_{10} = h = a_{10}\tan\alpha$ ....(1)

and $a_1 = a \Rightarrow h_1 = a\tan\alpha$ ...(2)

$\Rightarrow h = (a + 9d)\tan\alpha$ where d is distance between poles ($\because a_{10} = a + 9d$)

$\Rightarrow h = a\tan\alpha + 9d\tan\alpha$

$\Rightarrow \frac{h - a\tan\alpha}{9\tan\alpha} = d \Rightarrow \frac{h - \frac{a\sin\alpha}{\cos\alpha}}{9\frac{\sin\alpha}{\cos\alpha}} = d$

$\Rightarrow d = \frac{h\cos\alpha - a\sin\alpha}{9\sin\alpha}$

Question50

A bird is sitting on the top of a vertical pole 20 m high and its elevation from a point O on the ground is 45°. It flies off horizontally straight away from the point O. After one second, the elevation of the bird from O is reduced to 30°. Then the speed (in m/s) of the bird is
[2014]

Options:

A. $20\sqrt{2}$

B. $20(\sqrt{3} - 1)$

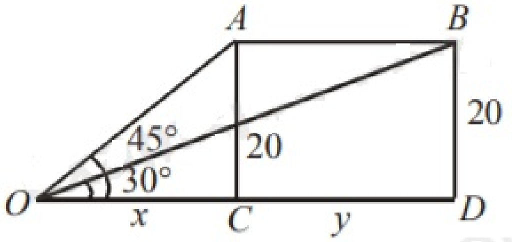
C. $40(\sqrt{2} - 1)$

D. $40(\sqrt{3} - \sqrt{2})$

Answer: B

Solution:

Given that height of pole AB = 20 m



Let O be the point on the ground such that ∠AOC = 45°
Let OC = x and CD = y

In right ΔAOC, $\tan 45° = \frac{20}{x}$ ...(i)

In right ΔBOD, $\tan 30° = \frac{20}{x+y}$ ....(ii)

From (i) and (ii), we have

$x = 20$ and $\frac{1}{\sqrt{3}} = \frac{20}{x+y}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{20}{20+y} \Rightarrow 20 + y = 20\sqrt{3}$

So, $y = 20(\sqrt{3}-1)\text{m}$ and time $= 1\text{s}$ (Given)

Hence, speed $= 20(\sqrt{3}-1)\text{m / s}$

Question51

The angle of elevation of the top of a vertical tower from a point P on the horizontal ground was observed to be a. After moving a distance 2 metres from P towards the foot of the tower, the angle of elevation changes to β. Then the height (in metres) of the tower is: [Online April 11, 2014]

Options:

A. $\frac{2\sin\alpha\sin\beta}{\sin(\beta-\alpha)}$

B. $\frac{\sin\alpha\sin\beta}{\cos(\beta-\alpha)}$

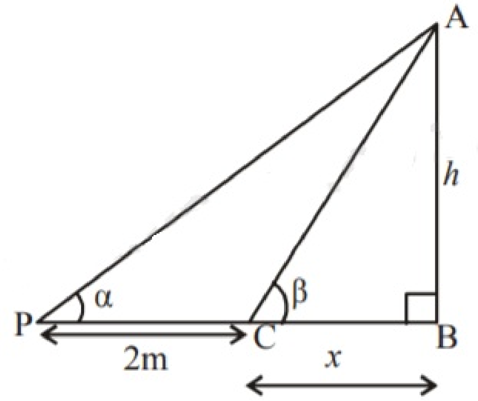
C. $\frac{2\sin(\beta-\alpha)}{\sin\alpha\sin\beta}$

D. $\frac{\cos(\beta-\alpha)}{\sin\alpha\sin\beta}$

Answer: A

Solution:

Solution:
Let AB be the tower of height 'h'.



Given : In ΔABP

$\tan\alpha = \frac{AB}{PB}$

or $\frac{\sin\alpha}{\cos\alpha} = \frac{h}{x+2}$

$\Rightarrow (x+2)\sin\alpha = h\cos\alpha$

$\Rightarrow h = \frac{x\sin\alpha + 2\sin\alpha}{\cos\alpha}$ ...(1)

Now, In ΔABC, $\tan\beta = \frac{AB}{BC}$

$\Rightarrow \frac{\sin\beta}{\cos\beta} = \frac{h}{x} \Rightarrow x = \frac{h\cos\beta}{\sin\beta}$...(2)

Putting the value of x in eq. (2) to eq. (1), we get

$$h = \frac{\frac{h\cos\beta\sin\alpha}{\sin\beta} + \frac{2\sin\alpha}{1}}{\cos\alpha}$$

$\Rightarrow h = \frac{h\cos\beta\,.\,\sin\alpha + 2\sin\alpha\sin\beta}{\sin\beta\,.\,\cos\alpha}$

$\Rightarrow h(\sin\beta\,.\,\cos\alpha - \cos\beta\,.\,\sin\alpha) = 2\sin\alpha\,.\,\sin\beta$

$\Rightarrow h[\sin(\beta-\alpha)] = 2\sin\alpha\,.\,\sin\beta$

$\Rightarrow h = \frac{2\sin\alpha\,.\,\sin\beta}{\sin(\beta-\alpha)}$

Question52

ABCD is a trapezium such that AB and CD are parallel and BC ⊥ CD. If ∠ADB = θ, BC = p and CD = q, then AB is equal to : [2013]

Options:

A. $\frac{(p^2+q^2)\sin\theta}{p\cos\theta + q\sin\theta}$

B. $\frac{p^2+q^2\cos\theta}{p\cos\theta + q\sin\theta}$

C. $\frac{p^2+q^2}{p^2\cos\theta + q^2\sin\theta}$

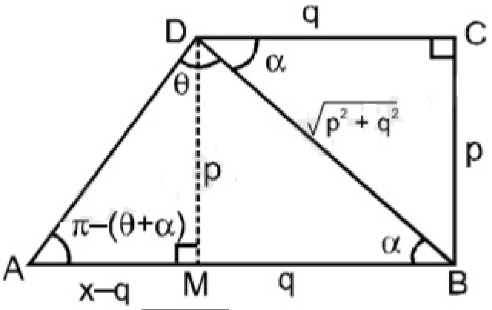
D. $\frac{(p^2+q^2)\sin\theta}{(p\cos\theta + q\sin\theta)^2}$

Answer: A

Solution:

Solution:

From Sine Rule



$$\frac{AB}{\sin\theta}=\frac{\sqrt{p^2+q^2}}{\sin(\pi-(\theta+\alpha))}$$

$$AB=\frac{\sqrt{p^2+q^2}\sin\theta}{\sin\theta\cos\alpha+\cos\theta\sin\alpha}=\frac{(p^2+q^2)\sin\theta}{q\sin\theta+p\cos\theta}$$

$$\left(\because\cos\alpha=\frac{q}{\sqrt{p^2+q^2}}\text{ and }\sin\alpha=\frac{p}{\sqrt{p^2+q^2}}\right)$$

Question53

If in a triangle ABC, $\frac{b+c}{11}=\frac{c+a}{12}=\frac{a+b}{13}$, then $\cos A$ is equal to [2012]

Options:

A. $\frac{5}{7}$

B. $\frac{1}{5}$

C. $\frac{35}{19}$

D. $\frac{19}{35}$

Answer: B

Solution:

In a triangle ABC.

$$\frac{b+c}{11}=\frac{c+a}{12}=\frac{a+b}{13}=K$$

$\Rightarrow b+c=11K,\ c+a=12K,\ a+b=13K$

On solving these equations, we get

$a=7K,\ b=6K,\ c=5K$

Now we know,

$$\cos A=\frac{b^2+c^2-a^2}{2bc}=\frac{36K^2+25K^2-49K^2}{2(6K)(5K)}=\frac{1}{5}$$

Question54

In a ΔPQR, If $3\sin P+4\cos Q=6$ and $4\sin Q+3\cos P=1$, then the angle R is equal to : [2012]

Options:

A. $\frac{5\pi}{6}$

B. $\frac{\pi}{6}$

C. $\frac{\pi}{4}$

D. $\frac{3\pi}{4}$

Answer: B

Solution:

Given that 3 sin P + 4cos Q = 6 ...(i)
4 sin Q + 3cos P = 1 ...(ii)
Squaring and adding (i) & (ii) we get
$9\sin^2 P + 16\cos^2 Q + 24\sin P\cos Q + 16\sin^2 Q + 9\cos^2 P + 24\sin Q\cos P$
$= 36 + 1 = 37$
$\Rightarrow 9(\sin^2 P + \cos^2 P) + 16(\sin^2 Q + \cos^2 Q) + 24(\sin P\cos Q + \cos P\sin Q) = 37$
$\Rightarrow 9 + 16 + 24\sin(P + Q) = 37$

Question55

For a regular polygon, let r and R be the radii of the inscribed and the circumscribed circles. A false statement among the following is [2010]

Options:

A. There is a regular polygon with $\frac{r}{R} = \frac{1}{\sqrt{2}}$

B. There is a regular polygon with $\frac{r}{R} = \frac{2}{3}$

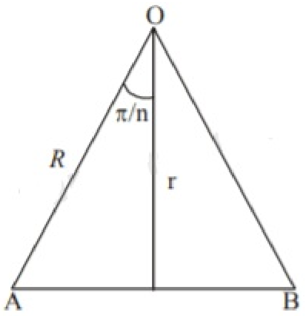
C. There is a regular polygon with $\frac{r}{R} = \frac{\sqrt{3}}{2}$

D. There is a regular polygon with $\frac{r}{R} = \frac{1}{2}$

Answer: B

Solution:

Solution:
Let O is centre of polygon of n sides and AB is one of the side, then by figure



$\cos\frac{\pi}{n} = \frac{r}{R}$

$\Rightarrow \frac{r}{R} = \frac{1}{2}, \frac{1}{\sqrt{2}}, \frac{\sqrt{3}}{2}$

for n = 3, 4, 6 respectively.

Question56

AB is a vertical pole with B at the ground level and A at the top. A man finds that the angle of elevation of the point A from a certain point C on the ground is 60°. He moves away from the pole along the line BC to a point D such that CD = 7 m. From D the angle of elevation of the point A is 45°. Then the height of the pole is
[2008]

Options:

A. $\frac{7\sqrt{3}}{2}\frac{1}{\sqrt{3}-1}$ m

B. $\frac{7\sqrt{3}}{2}(\sqrt{3}+1)$ m

C. $\frac{7\sqrt{3}}{2}(\sqrt{3}-1)$ m

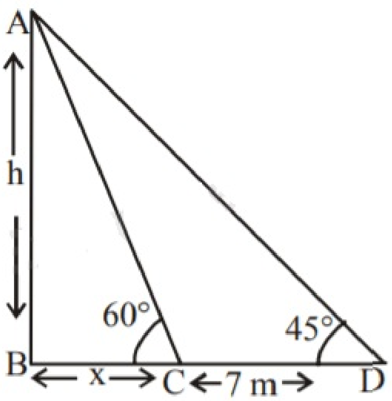
D. $\frac{7\sqrt{3}}{2}\frac{1}{\sqrt{3}-1}$ m

Answer: B

Solution:

Solution:

In right, ΔABC $\tan 60° = \frac{h}{x} = \sqrt{3}$



$\Rightarrow x = \frac{h}{\sqrt{3}}$ ...(i)

In right, $\Delta ABD = \tan 45° = \frac{h}{x+7} = 1$

$\Rightarrow h = x + 7 \Rightarrow h - \frac{h}{\sqrt{3}} = 7$ [From (i)]

$\Rightarrow h = \frac{7\sqrt{3}}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$

$\Rightarrow h = \frac{7\sqrt{3}}{2}(\sqrt{3}+1)m$

Question57

A tower stands at the centre of a circular park. A and B are two points on the boundary of the park such that AB (= a) subtends an angle of 60° at the foot of the tower, and the angle of elevation of the top of the tower from A and B is 30°. The height of the tower is [2007]

Options:

A. $\frac{a}{\sqrt{3}}$

B. $a\sqrt{3}$

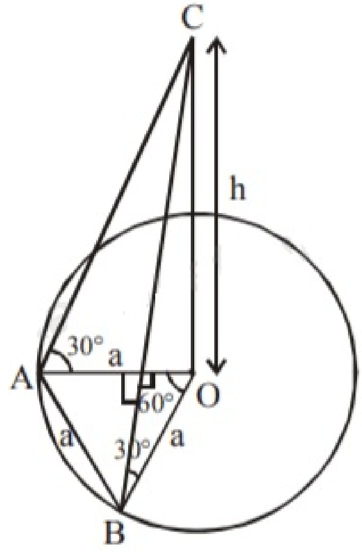
C. $\frac{2a}{\sqrt{3}}$

D. $2a\sqrt{3}$

Answer: A

Solution:

In the ΔAOB given that $\Delta AOB = 60°$
and OA = OB = radius
$\therefore \angle OBA = \angle OAB = 60°$
$\therefore \Delta$ AOB is a equilateral triangle.
$\Rightarrow$ OA = OB = AB = a
Let the height of tower is h m.



In ΔOAC, $\tan 30° = \frac{h}{a} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{a}$

$\Rightarrow h = \frac{a}{\sqrt{3}}$

Question58

If in a ΔABC , the altitudes from the vertices A, B, C on opposite sides are in H.P, then sin A, sin B, sin C are in
[2005]

Options:

A. G. P.

B. A. P.

C. A.P - G..P.

D. H. P

Answer: B

Solution:

Solution:
Let altitudes from A, B and C be p_1, p_2 and p_3 resp.
$\therefore \Delta = \frac{1}{2}p_1 a = \frac{1}{2}p_2 b = \frac{1}{2}p_3 b$
1Given that, p_1, p_2, p_3, are in H.P.
$\Rightarrow \frac{2\Delta}{a}, \frac{2\Delta}{b}, \frac{2\Delta}{c}$ are in H.P.
$\Rightarrow \frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in H.P.
$\Rightarrow$a, b, c are in A.P.
By sine formula
$\Rightarrow K \sin A, K \sin B, K \sin C$ are in AP
$\Rightarrow \sin A, \sin B, \sin C$ are in A.P.

Question59

In a triangle ABC, let $\angle C = \frac{\pi}{2}$. If r is the inradius and R is the circumradius of the triangle ABC, then 2 (r + R) equals
[2005]

Options:

A. b + c

B. a + b

C. a + b + c

D. c + a

Answer: B

Solution:

Solution:
We know that for the circle circumscribing a right triangle, hypotenutse is the diameter
$\because \angle C = 90°$

$\therefore 2R = c \Rightarrow R = \frac{c}{2}$

also $r = \frac{\Delta}{s} = \frac{\frac{1}{2} \times a \times b}{\frac{a+b+c}{2}}$

$\Rightarrow r = \frac{ab}{a+b+c}$

$\therefore 2r + 2R = \frac{2ab}{a+b+c} + c = \frac{2ab + ac + bc + c^2}{a+b+c}$

$= \frac{2ab + ac + bc + a^2 + b^2}{a+b+c}$ $(\because c^2 = a^2 + b^2)$

$= \frac{(a+b)^2 + (a+b)c}{a+b+c} = (a+b)$

Question60

The sides of a triangle are $\sin\alpha$, $\cos\alpha$ and $\sqrt{1 + \sin\alpha\cos\alpha}$ for some $0 < \alpha < \frac{\pi}{2}$. Then the greatest angle of the triangle is
[2004]

Options:

A. 150°

B. 90°

C. 120°

D. 60°

Answer: C

Solution:

Let $a = \sin\alpha$, $b = \cos\alpha$ and $c = \sqrt{1 + \sin\alpha\cos\alpha}$
Clearly a and $b < 1$ but $c > 1$ as $\sin\alpha > 0$ and $\cos\alpha > 0$
$\therefore c$ is the greatest side and greatest angle is C.

We know that, $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

$= \frac{\sin^2\alpha + \cos^2\alpha - 1 - \sin\alpha\cos\alpha}{2\sin\alpha\cos\alpha} = -\frac{1}{2}$

$\therefore C = 120°$

Question61

A person standing on the bank of a river observes that the angle of elevation of the top of a tree on the opposite bank of the river is 60° and when he retires 40 meters away from the tree the angle of elevation becomes 30°. The breadth of the river is
[2004]

Options:

A. 60 m

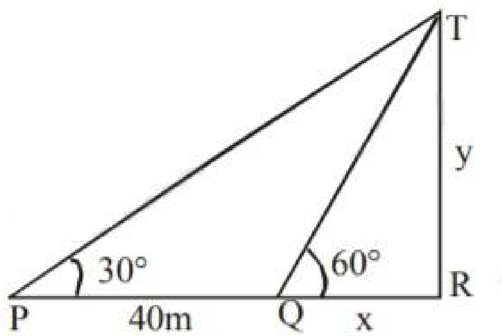
B. 30 m

C. 40 m

D. 20 m

Answer: D

Solution:



In right ΔQTR

$\tan 60° = \frac{y}{x} \Rightarrow y = \sqrt{3}x$ ...(1)

In right ΔPTR

$\tan 30° = \frac{y}{x+40} \Rightarrow y = \frac{x+40}{\sqrt{3}}$ ...(2)

From (1) and (2),

$\sqrt{3}x = \frac{x+40}{\sqrt{3}}$

$\Rightarrow x = 20m$

Question62

If in a ΔABC $a\cos^2\left(\frac{C}{2}\right) + c\cos^2\left(\frac{A}{2}\right) = \frac{3b}{2}$, then the sides a, b and c [2003]

Options:

A. satisfy a + b = c

B. are in A.P

C. are in G..P

D. are in H.P

Answer: B

Solution:

Given that, $a\cos^2\left(\frac{C}{2}\right) + c\cos^2\left(\frac{A}{2}\right) = \frac{3b}{2}$

$a[\cos C + 1] + c[\cos A + 1] = 3b$

$(a + c) + (a\cos C + c\cos B) = 3b$

We know that, $b = a\cos C + c\cos B$

a + c + b = 3b or a + c = 2b
or a, b, c are in A.P.

Question63

In a triangle ABC, medians AD and BE are drawn. If AD = 4 , ∠DAB = $\frac{\pi}{6}$ and ∠ABE = $\frac{\pi}{3}$, then the area of the ΔABC is
[2003]

Options:

A. $\frac{64}{3}$

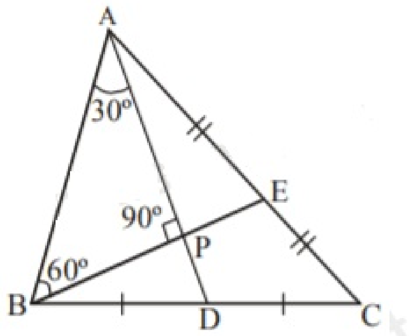
B. $\frac{8}{3}$

C. $\frac{16}{3}$

D. $\frac{32}{\sqrt{3}}$

Answer: D

Solution:



We know that median divides each other in ratio 2 : 1

$AP = \frac{2}{3}AD = \frac{8}{3}$; $PD = \frac{4}{3}$;

Let $PB = x$

$\tan 60° = \frac{\frac{8}{3}}{x}$ or $x = \frac{8}{3\sqrt{3}}$

Area of $\Delta ABD = \frac{1}{2} \times 4 \times \frac{8}{3\sqrt{3}} = \frac{16}{3\sqrt{3}}$

∴ Area of $\Delta ABC = 2 \times \frac{16}{3\sqrt{3}} = \frac{32}{3\sqrt{3}}$

[∵ Median of a D divides it into two Δ's of equal area.]

Question64

The sum of the radii of inscribed and circumscribed circles for an n sided regular polygon of side a , is
[2003]

Options:

A. $\frac{a}{4}\cot\left(\frac{\pi}{2n}\right)$

B. $a\cot\left(\frac{\pi}{n}\right)$

C. $\frac{a}{2}\cot\left(\frac{\pi}{2n}\right)$

D. $a\cot\left(\frac{\pi}{2n}\right)$

Answer: C

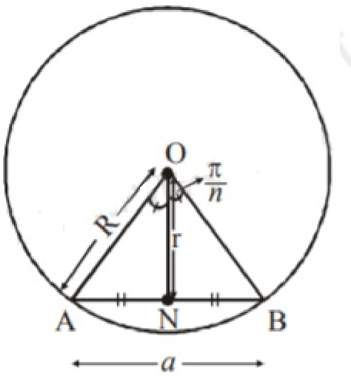
Solution:

Solution:

We know that, $\tan\left(\frac{\pi}{n}\right) = \frac{a}{2r}$; $\sin\left(\frac{\pi}{n}\right) = \frac{a}{2R}$

$r = \frac{a}{2}\cot\frac{\pi}{n}$; $R = \frac{a}{2}\operatorname{cosec}\frac{\pi}{n}$

$r + R = \frac{a}{2}\left[\cot\frac{\pi}{n} + \operatorname{cosec}\frac{\pi}{n}\right]$



$$= \frac{a}{2}\left[\cos\frac{\pi}{n} + 1\sin\frac{\pi}{n}\right] = \frac{a}{2}\left[\frac{2\cos^2\frac{\pi}{2n}}{2\sin\frac{\pi}{2n}\cos\frac{\pi}{2n}}\right] = \frac{a}{2}\cot\frac{\pi}{2\pi}$$

Question65

The upper $\frac{3}{4}$th portion of a vertical pole subtends an angle $\tan^{-1}\frac{3}{5}$ at a point in the horizontal plane through its foot and at a distance 40m from the foot. A possible height of the vertical pole is [2003]

Options:

A. 80m

B. 20m

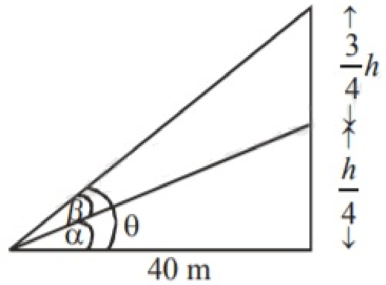
C. 40m

D. 60m

Answer: C

Solution:

Solution:



$\theta = \alpha + \beta, \beta = \tan^{-1}\left(\frac{3}{5}\right)$

or $\beta = \theta - \alpha$

$\Rightarrow \tan\beta = \frac{\tan\theta - \tan\alpha}{1 + \tan\theta \cdot \tan\alpha}$

or $\frac{3}{5} = \frac{\frac{h}{40} - \frac{h}{160}}{1 + \frac{h}{40} \cdot \frac{h}{160}}$

$h^2 - 200h + 6400 = 0 \Rightarrow h = 40$ or 160 metre

$\therefore$ possible height $= 40$ metre

Question66

In a triangle with sides a, b, c, $r_1 > r_2 > r_3$ (which are the ex-radii) then [2002]

Options:

A. a >b > c

B. a < b < c

C. a > b and b < c

D. a < b and b > c

Answer: A

Solution:

Solution:

We know that, $r_1 = \frac{\Delta}{s-a}$, $r_2 = \frac{\Delta}{s-b}$ and $r_3 = \frac{\Delta}{s-c}$

Given that,

$r_1 > r_2 > r_3 \Rightarrow \frac{\Delta}{s-a} > \frac{\Delta}{s-b} > \frac{\Delta}{s-c}$

$\Rightarrow s - a < s - b < s - c$

$\Rightarrow -a < -b < -c \Rightarrow a > b > c$

Question67

The sides of a triangle are $3x + 4y$, $4x + 3y$ and $5x + 5y$ where $x, y > 0$ then the triangle is
[2002]

Options:

A. right angled

B. obtuse angled

C. equilateral

D. none of these

Answer: B

Solution:

Let $a = 3x + 4y$, $b = 4x + 3y$ and $c = 5x + 5y$
as $x, y > 0$, $c = 5x + 5y$ is the largest side
$\therefore$C is the largest angle . Now

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$\cos C = \frac{(3x + 4y)^2 + (4x + 3y)^3 - (5x + 5y)^2}{2(3x + 4y)(4x + 3y)}$$

$$= \frac{-2xy}{2(3x + 4y)(4x + 3y)} < 0$$

$\therefore$ C is obtuse angle $\Rightarrow$ ΔABC is obtuse angled

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