Solved Paper 2016
AMU
Engineering Entrance Exam

Physics

1. The velocity of a transverse wave in a string is directly proportional to $\sqrt{T}$ and inversely proportional to $\sqrt{\mu}$. In a measurement, the mass applied at the end of string is 3.0 g, length of string is 1 m and mass of string is 5 g. If possible error in measuring mass is 0.1 g and that of length is 1mm, the percentage error in measurement of velocity is

(a) 4.5% (b) 2.7%
(c) 2.1% (d) 3.7%

2. A particle moving with an initial velocity $u\,\text{ms}^{-1}$ is retarded by a force at the rate of $a = -k\sqrt{v}$, where k is a positive constant and v is the instantaneous velocity. The particle comes to rest in a time given by

(a) $\frac{2\sqrt{u}}{k}$ (b) $k\sqrt{u}$ (c) $\frac{\sqrt{2u}}{k}$ (d) $\frac{\sqrt{u}}{2k}$

3. The expression of the trajectory of a projectile is given as $y = px - qx^2$,

where y and x are respectively the vertical and horizontal displacements and p and q are constants. The time of flight of the projectile is

(a) $\frac{p^2}{4q}$ (b) $\frac{p^2}{2q}$ (c) $\sqrt{\frac{2p}{qg}}$ (d) $p\sqrt{\frac{2}{qg}}$

4. The displacement x of a body varies with time as

$$x = -\frac{1}{3}t^2 + 16t + 3,$$

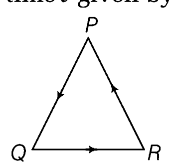
where x is in metres and t is in seconds. The time taken by the body to come to rest is

(a) 12 s (b) 24 s (c) 30 s (d) 36 s

5. A drunkard walking in a narrow lane takes 5 steps forward and 3 steps backward followed again by 5 steps forward and 3 steps backward and so on. Each step is 1m long and requires 1s. Determine how long the drunkard takes to fall in a pit 13 m away from the starting point?

(a) 37 s (b) 13 s (c) 49 s (d) 18 s

6. Three particles P, Q and R are at rest at the vertices of an equilateral triangle of side s. Each of the particles starts moving with constant speed v ms^{-1}. P is moving along PQ, Q along QR and R along RP. The particles will meet each other at time t given by



(a) $\frac{s}{v}$ (b) $\frac{3s}{v}$ (c) $\frac{3s}{2v}$ (d) $\frac{2s}{3v}$

7. Two cars are in a race. The white car passed the finishing point with a velocity v ms^{-1} more and took time t s less than the red car. If both the cars start from rest and travel with constant accelerations a_w and a_r respectively, $\frac{v}{t}$ is given

(a) $a_w a_r$ (b) $\sqrt{\frac{a_w}{a_r}}$ (c) $\sqrt{a_w a_r}$ (d) $\sqrt{\frac{a_r}{a_w}}$

8. Starting from origin, a body moves along x-axis. Its velocity at any time is given by
$v = 4t^3 - 2t$ m/s

Acceleration of the particle when it is 2 m away from the origin is

(a) 28 ms^{-2} (b) 12 ms^{-2} (c) 22 ms^{-2} (d) 14 ms^{-2}

9. The kinetic energy of a particle of mass m kg is half of that of another particle of mass $m/2$ kg. If the speed of heavier particle is increased by 3 ms^{-1}, its kinetic energy becomes equal to the original kinetic energy of the lighter particle. The original speeds of the heavier and lighter particles are

(a) 3 ms^{-1}, 6 ms^{-1} (b) 2 ms^{-1}, 4 ms^{-1}
(c) 2 ms^{-1}, 6 ms^{-1} (d) 4 ms^{-1}, 8 ms^{-1}

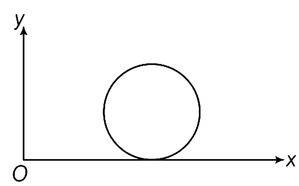
10. A man who weighs 670 N runs the first 7.0 m in 1.6 s, starting from rest and accelerating uniformly. What is the average power does the man generate during the 1.6 s time interval?

(a) 3.2 kW (b) 16 kW
(c) 0.9 kW (d) None of these

11. If the moment of inertia of a disc about an axis tangential and parallel to its surface be I, then the moment of inertia about an axis tangential but perpendicular to the surface will be

(a) $\frac{6}{5}I$ (b) $\frac{3}{4}I$ (c) $\frac{3}{2}I$ (d) $\frac{5}{4}I$

12. A disc of mass M and radius R is rolling with angular speed ω on a horizontal surface as shown in figure. The magnitude of angular momentum of the disc about the origin O is (here v is the linear velocity of the disc)

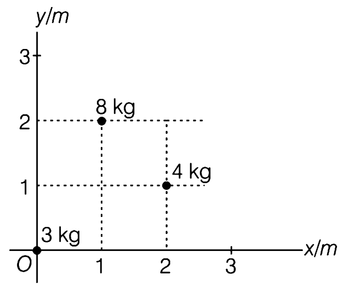


(a) $\frac{3}{2}MR^2\omega^2$ (b) $MR^2\omega$ (c) MRv (d) $\frac{3}{2}MRv$

13. The torque acting on a body about a given point is given by $\tau = \hat{\mathbf{A}} \times \hat{\mathbf{L}}$, where $\hat{\mathbf{A}}$ is a constant vector and $\hat{\mathbf{L}}$ is the angular momentum of the body about this point. It follows that

(a) the magnitude of $\hat{\mathbf{L}}$ does not change with time
(b) the component of $\hat{\mathbf{L}}$ in the direction of $\hat{\mathbf{A}}$ does not change with time
(c) $\frac{d\hat{\mathbf{L}}}{dt}$ is perpendicular to $\hat{\mathbf{L}}$ at all instants of time
(d) All of the above choices are correct

14. Find the x and y-coordinates of the centre of mass of the three particle system (as shown).



(a) 1.0 m, 1.0 m (b) 1.3 m, 0.9 m
(c) 1.1 m, 1.3 m (d) 1.3, 1.1 m

15. The density of a newly discovered planet is twice that of earth. The acceleration due to gravity at the surface of the planet is equal to that at the surface of the earth. If the radius of the earth is R, the radius of the planet would be

(a) $4R$ (b) $2R$ (c) $R/2$ (d) $R/4$

16. An asteroid of mass 2×10^{-4} M_e, where M_e is the mass of the earth, revolves in a circular orbit around the sun at a distance that is twice earth's distance from the sun. Find the ratio of the kinetic energy of the asteroid to that of earth.

(a) 0.9×10^{-6} (b) 1.6×10^{-5}
(c) 3.6×10^{-5} (d) 1.0×10^{-4}

17. Two wires of equal cross-section but one made of steel and the other of copper, are joined end to end. When the combination is kept under tension, the elongations in the two wires are found to be equal. (Y for steel $= 2 \times 10^{11}$ N/m^2 and Y for copper $= 1.1 \times 10^{11}$ N/m^2). The ratio of the lengths of the two wires is

(a) 20 : 11 (b) 2 : 1
(c) 1 : 2 (d) 1 : 1

18. The density of air in atmosphere decreases with height and can be expressed by the relation $\rho = \rho_0 e^{-\alpha h}$, where ρ_0 is the density at sea level, α is a constant and h is the height. The atmospheric pressure at the sea level is

(a) $\frac{\rho_0 g}{\alpha}$ (b) $\frac{\rho_0 \alpha h}{g}$
(c) $\frac{\alpha h}{\rho_0 g}$ (d) $\frac{h}{\rho_0 \alpha}$

19. A cylinder of radius R made of a material of thermal conductivity K_1 is surrounded by a cylindrical shell of inner radius R and outer radius $2R$ and made of a material of thermal conductivity K_2. The two ends of the combined system are maintained at two different temperatures. There is no loss of heat across the cylindrical surface and the system is in steady state. The effective thermal conductivity of the system is

(a) $\frac{3K_1 + K_2}{4}$ (b) $\frac{K_1 + 3K_2}{4}$

(c) $K_1 + K_2$ (d) $\frac{K_1 K_2}{K_1 + K_2}$

20. Work done in increasing the size of a soap bubble from a radius of 3 cm to 5 cm is nearly (surface tension of soap solution = 0.03 Nm^{-1}).

(a) 4π mJ (b) 0.4π mJ

(c) 0.2π mJ (d) 2π mJ

21. A gas under constant pressure of 4.5×10^5 p_a when subjected to 800 kJ of heat, changes the volume from 0.5 m^3 to 2.0 m^3. The change in internal energy of the gas is

(a) 6.75×10^5 J (b) 5.25×10^5 J

(c) 3.25×10^5 J (d) 1.25×10^5 J

22. A gas at pressure p_0 is contained in a vessel. If the masses of all the molecules are halved and their speeds doubled, the resulting pressure would be

(a) $4\ p_0$ (b) $2\ p_0$ (c) p_0 (d) $\frac{p_0}{2}$

23. The red shift observed for stars due to the natural expanding of universe is given by the expression

(a) $(\lambda' - \lambda) = \left(\frac{c + v}{c}\right)\lambda$ (b) $(\lambda' - \lambda) = \left(\frac{c - v}{c}\right)\lambda$

(c) $(\lambda' - \lambda) = \left(\frac{v\lambda}{c}\right)$ (d) $(\lambda' - \lambda) = \left(\frac{c\lambda}{v}\right)$

24. An ideal gas at pressure p is adiabatically compressed so that its density becomes n times the initial value. If $\gamma = C_p / C_V$, the final pressure of the gas will be

(a) $n^{(1-\gamma)p}$ (b) $n^{(r-\gamma)p}$

(c) $n^{(-\gamma)p}$ (d $n^{(\gamma)p}$

25. A quantity of a substance in a closed system is made to undergo a reversible process from an initial volume of 3 m^3 and initial pressure 10^5 N/m^2 to a final volume of 5m^3. If the pressure is proportional to the square of the volume (i.e. $p = AV^2$), the work done by the substance will be

(a) 3.6×10^2 J (b) 7.4×10^3 J

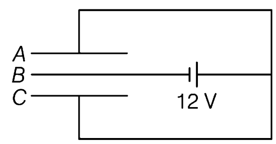
(c) 2.2×10^4 J (d) 3.6×10^5 J

26. A photographic flash unit consists of a xenon filled tube. It gives a flash of average power 2000 W for 0.04 s. The flash is due to discharge of a fully charged capacitor of 40 μF. The voltage to which it is charged before a flash is given by the unit is

(a) 1.5×10^3 V (b) 2×10^3 V

(c) 2.5×10^3 V (d) 3×10^3 V

27. Three plates A, B and C each of area 50 cm^2 have separation 3 mm between A and B and 6 mm between B and C. The energy stored when the plates are fully charged by a 12 v battery is



(a) 2 μJ (b) 1.6 nJ

(c) 5 μJ (d) 3.2 nJ

28. A point charge $+q$ is placed at a distance $d/2$ directly above the centre of a square of side d. The magnitude of electric flux through the square is

(a) $\frac{Q}{6d}$ (b) $\frac{Q}{6\varepsilon_0}$

(c) $\frac{Qd}{6\varepsilon_0}$ (d) $\frac{Q\varepsilon_0}{6d}$

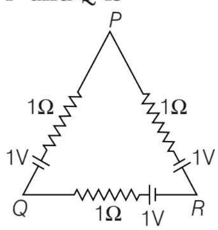
29. Three resistances P, Q and R, each of 2 Ω and an unknown resistance S form the four arms of a Wheatstone bridge circuit. When a resistance of 6 Ω is connected in parallel to S, the bridge gets balanced. The value of S is

(a) 3 Ω (b) 6 Ω (c) 1 Ω (d) 2 Ω

30. Two bulbs consume the same power when operated at 200 V and 300 V, respectively. When these bulbs are connected in series across a DC source of 500 V, then the ratio of potential difference across them is

(a) $\frac{2}{3}$ (b) $\frac{4}{9}$ (c) $\frac{6}{27}$ (d) $\frac{8}{24}$

31. Three batteries of emf 1 V and internal resistance 1Ω each are connected as shown. Effective emf of the combination between the points P and Q is



(a) zero (b) 1 V

(c) 2 V (d) $\frac{2}{3}$ V

32. A wire is being drawn to make it thinner such that the length of the wire l increases and radius r decreases. Its resistance R will finally be proportional to

(a) $\frac{1}{r}$ (b) $\frac{1}{r^2}$

(c) $\frac{1}{r^3}$ (d) $\frac{1}{r^4}$

33. Two particles X and Y having equal charges, after being accelerated through the same potential difference enter a region of uniform magnetic field and describe circular paths of radii R_1 and R_2, respectively. The ratio of masses of X and Y is

(a) $\left(\frac{R_1}{R_2}\right)^{1/2}$ (b) $\left(\frac{R_2}{R_1}\right)$

(c) $\left(\frac{R_1}{R_2}\right)^2$ (d) $\left(\frac{R_1}{R_2}\right)$

34. An ions with a charge of $+3.2\times10^{-19}$C is in a region where a uniform electric field of 5×10^4 V/m is perpendicular to a uniform magnetic field of 0.8 T. If its acceleration is zero, then its speed must be

(a) 0 (b) 1.6×10^4 m/s

(c) 4.0×10^4 m/s (d) 6.3×10^4 m/s

35. A long straight wire of radius R carries a steady current I. The current is uniformly distributed across its cross-section. The ratio of magnetic field at $R/2$ and $2R$ is

(a) $\frac{1}{2}$ (b) 2

(c) $\frac{1}{4}$ (d) $\frac{1}{1}$

36. Two circular coils 1 and 2 are made from the same wire but the radius of the first coil is twice that of the second coil. What potential difference ratio should be applied across them so that the magnetic field at their centres is the same?

(a) 2 (b) 3 (c) 4 (d) 6

37. A solenoid of inductance 50 mH and resistance 10 Ω is connected to a battery of 6V. The time elapsed before the current acquires half of its steady state value is

(a) 2 ms (b) 3.5 ms (c) 5 ms (d) 5.5 ms

38. A flat rectangular coil is placed in a uniform magnetic field and rotated about an axis passing through its centre, parallel to its shorter edges and perpendicular to the field. The maximum flux linked and maximum induced emf are ϕ and E, respectively. If the axis is shifted to coincide with one of the shorter edges, then

(a) Maximum flux and induced emf are $\phi/2$ and $E/2$

(b) Maximum flux and induced emf are $\phi/3$ and $E/3$

(c) Maximum flux and induced emf are $\phi/4$ and $E/4$

(d) Maximum flux and induced emf remain ϕ and E

39. In a series L-C-R circuit, the voltages across resistance, capacitance and inductance are 20 V each. If the capacitance is short-circuited, the voltage across the inductance will be

(a) $\frac{20}{\sqrt{2}}$ V (b) 20 V

(c) $20\sqrt{2}$ V (d) 40 V

40. Electromagnetic waves travel in a medium with a speed of 2×10^8 m/s. If the relative permeability of the medium is 1, the relative permittvity will be

(a) 1.5 (b) 2.25 (c) 3.3 (d) 1.0

41. Two lenses have 10 D power each and they are separated by a distance. Beyond which distance does the power of combination changes from positive to negative?

(a) 5 cm (b) 10 cm

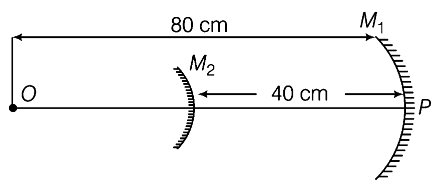
(c) 20 cm (d) 50 cm

42. A vessel of depth d is half filled with a liquid of refractive index n_1 and the upper half is occupied by immiscible liquid of refractive index n_2. Viewing it from an eye in the upper liquid, the apparent depth of the lower liquid is

(a) $\frac{d}{2n_2}$ (b) $\frac{dn_1}{2n_2}$

(c) $\frac{d}{2}\left(\frac{n_2}{n_1}\right)$ (d) $\frac{d}{2}\left(\frac{n_1+n_2}{n_1 n_2}\right)$

43. Consider an optical system consisting of a concave mirror M_1 and convex mirror M_2 of radii of curvature 60 cm and 20 cm, respectively. Two mirrors are separated by a distance of 40 cm. An object O is placed at a distance 80 cm from P. The final image is formed at a distance



(a) 40 cm on the right of M_2
(b) 40 cm on the left of M_2
(c) 48 cm on the right of M_1
(d) 48 cm on the left of M_2

44. The spherical aberration is minimized in a reflecting telescope using

(a) a concave mirror as objective
(b) a convex mirror as objective
(c) a parabolic mirror as objective
(d) an elliptical mirror as objective

45. A proton (p) and an α-particle are accelerated through the same potential difference V volt. The de-Broglie wavelengths associated with the proton and the α-particle, λ_p and λ_α respectively are in the ratio

(a) 2 : 1 (b) $2\sqrt{2}$: 1 (c) 4 : 1 (d) $\sqrt{2}$: 1

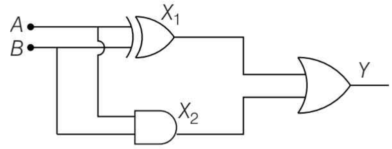
46. Lines of Balmer series are emitted by the hydrogen atom when the electron jumps from the

(a) first ($n = 1$) orbit to any higher orbit
(b) second orbit ($n = 2$) to any higher orbit
(c) higher orbits to the first orbit
(d) higher orbits to the second orbit

47. The half-life of radioactive nucleus is 100 years. The time interval between 20% and 80% decay of the parent nucleus is

(a) 100 years (b) 200 years
(c) 300 years (d) 400 years

48. The diagram given below is equivalent to a logic function of



(a) OR (b) AND
(c) NAND (d) XOR

49. In an *n-p-n* transistor circuit, the collector current is 10 mA. If 90% of the electrons emitted reach the collector, then

(a) the emitter current will be nearly 9 mA
(b) the emitter current will be nearly 11.1 mA
(c) the base current will be nearly 0.9 mA
(d) the base current will be nearly 0.3 mA

50. If the modulation index of an AM wave is changed from 0 to 1, the transmitted power is

(a) unchanged
(b) doubled
(c) increased by 50%
(d) zero

Chemistry

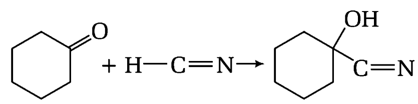
1. The antibiotic that contains arsenic is

(a) prontosil (b) ofloxacin
(c) biothionol (d) salvarsan

2. Pick out the electrophiles from the following BF_3, NH_3, $Me_3C^{\oplus}$, HCl

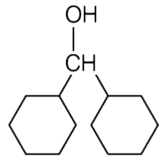
(a) BF_3, and NH_3
(b) $Me_3C^{\oplus}$ and HCl
(c) BF_3 and $Me_3C^{\oplus}$
(d) NH_3 and HCl

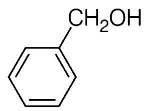
3. Classify the following reactions

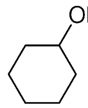


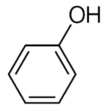
(a) Substitution
(b) Addition
(c) Elimination
(d) Rearrangement

4. Which of the following compounds has the most acidic nature?

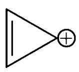
(a) 

(b) 

(c) 

(d) 

5. The most stable carbocation is

(a) 

(b) $CH_2 = CH—\overset{\oplus}{C}H_2$

(c) $CH_3\overset{\oplus}{C}HCH_3$

(d) $CH_3\overset{\oplus}{C}H = CH_2$

6. The acid strength of the following compounds

$CH \equiv CCOOH$ (I) $CH_2 = CHCOOH$ (II) CH_3CH_2COOH (III)

is in the order

(a) II > I > III (b) III > II > I
(c) I > III > II (d) I > II > III

7. In the reaction sequence

$$CH_3CH_2—CH_2Br \xrightarrow[\text{Dry ether}]{Mg} A \xrightarrow{CH_3CHO} B \xrightarrow{H_3\overset{\oplus}{O}} C$$

the product '*C*' is

(a) 1-propanol (b) 2-butanol
(c) 2-butanol (d) 2-pentanol

8. *p*-hydroxyazobenzene is

(a) an orange dye (b) a yellow dye
(c) a red dye (d) an orange-red dye

9. Which one of the following compounds can exists in Zwitter ionic form?

(a) Amino acid (b) Fat
(c) Carbohydrate (d) Alcohol

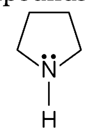
10. The following reaction

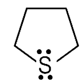
$$R - Cl + NaI \xrightarrow{\text{Acetone}} R - I + NaCl$$

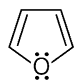
is known as

(a) Frankland reaction (b) Swarts reaction
(c) Etard reaction (d) Finkelstein reaction

11. Which of the following is aromatic compounds?

(a) 

(b) 

(c) 

(d) All of these

12. Nylon 6 is obtained by the condensation of

(a) Terepthalic acid and ethylene glycol
(b) Adipic acid and styrene
(c) Caprolactum with water at high temperature
(d) Phenol and formaldehyde

13. Mention the catalyst and reaction condition in the given reaction

$$R—CH_2—COOH \xrightarrow{?} R—\underset{\underset{X}{|}}{CH}—COOH$$

(where X = Cl, Br)

(a) X_2/grey phosphorus, H_2O
(b) X_2/red phosphorus, H_2O
(c) X_2/white phosphorus, H_2O
(d) X_2/blue phosphorus, H_2O

14. Nucleotides are joined together by between 5′ and 3′ carbon atoms of pentose sugar

(a) glycosidic linkage (b) peptide linkage
(c) ether linkage. (d) phosphodiester linkage

15. DDT is

(a) 2, 2-di (*p*-chlorophenyl) - 1, 1, 1-trichloroethane
(b)2, 2-di (*m*-chlorophenyl) - 1, 1, 1-trichloroethane
(c) 2, 2-di (*o*-chlorophenyl) - 1, 1, 1-trichloroethane
(d) 2, 2-di (*p*-chlorophenyl) - 1, 1-dichloroethane

16. The carbocation formed in S_N1 reaction of alkyl halide in the slow step is:

(a) sp^3-hybridised (b) sp^2-hybridised
(c) sp-hybridised (d) sp^3d- hybridised

17. Which of the followings is invert sugar?

(a) Sucrose (b) Cellulose (c) Glucose (d) Fructose

18. Select the correct ground state electronic configuration

	Cr	Eu	Ti^{2+}
(a)	[Ar] $3d^5\ 4s^1$	[Xe] $4f^7\ 5d^0\ 6s^2$	[Ar] $3d^2 4s^0$
(b)	[Ar] $3d^4\ 4s^2$	[Xe] $4f^7\ 5d^0\ 6s^2$	[Ar] $3d^2 4s^2$
(c)	[Ar] $3d^4\ 4s^2$	[Xe] $4f^6\ 5d^1\ 6s^2$	[Ar] $4s^2 4d^0$
(d)	[Ar] $3d^5\ 4s^1$	[Xe] $3f^6\ 5d^2\ 6s^1$	[Ar] $4s^1 3d^1$

19. Which of the following complexes can also represent facial (fac) and meridional (mer) isomers?

(a) $[Co(NH_3)_3NO_2Cl]$ (b) $[Co(NH_3)_2(NO_2)_2Cl_2]$
(c) $[Co(NH_3)_2(NO_2)_2Cl_2]$ (d) $[Co(NH_3)_3(NO_2)_3]$

20. For complexes $[NiCl_4]^{2-}$ and $Ni(CO)_4$ which one of the following statement is true

(a) $[NiCl_4]^{2-}$ is diamagnetic while $[Ni(CO)_4]$ is paramagnetic and both the complexes have square planar geometry.
(b) $[NiCl_4]^{2-}$ is paramagnetic while $[Ni(CO)_4]$ is diamagnetic and both the complexes have tetrahedral geometry.
(c) $[NiCl_4]^{2-}$ is paramagnetic while $[Ni(CO)_4]$ is diamagnetic and both the complexes have square planar geometry.
(d) $[NiCl_4]^{2-}$ is diamagnetic while $[Ni(CO)_4]$ is paramagnetic and both the complexes have tetrahedral geometry.

21. The purple colour of $KMnO_4$ can be attributed to

(a) $d-d$ transitions (b) charge transfer transition
(c) $n-\pi$ transitions (d) None of these

22. The spin only magnetic moment (μ_s) of a complex $[Mn(Br_4)]^{4-}$ is 5.9 BM. The geometry of the complex will be

(a) tetrahedral (b) square planar
(c) square pyramidal (d) tetragonal

23. Which of the following complexes would give white precipitate with excess of $AgNO_3$ sol?

(a) $[CO(NH_3)_2Cl_2]NO_3$ (b) $[CO(NH_3)_5SO_4]Cl$
(c) $[CO(NH_3)_4Cl_2]$ (d) $[CO(NH_3)_5NO_3]NO_3$

24. Which of the following complexes does not show geometrical isomerism?

(a) $[Pt(NH_3)_2Cl_2]$ (b) $[CO(NH_3)_4Cl_2]$
(c) $[COCl_2(en)_2]$ (d) $[Ni(CO)_4]$

25. The molecule which is linear is

(a) N_2O (b) NO_2 (c) SO_2 (d) H_2O

26. Which of the following statements is wrong about the oxides of nitrogen?

(a) N_2O_5 is an anhydride of HNO_3
(b) NO is in acidic oxide
(c) N_2O_3 is an anhydride of HNO_2
(d) NO is not an anhydride of an acid

27. Chemical formula for 'inorganic benzene' is

(a) $B_3N_3H_3Cl_3$ (b) $(BN)_x$ (c) $B_3N_3H_6$ (d) $B_3P_3H_6$

28. Among LiCl, RbCl, $BeCl_2$, $MgCl_2$ the compounds which greater and least ionic character respectively are

(a) LiCl and RbCl (b) RbCl and $BeCl_2$
(c) RbCl and $MgCl_2$ (d) $MgCl_2$ and $BeCl_2$

29. Which of the following statements is false for alkali metals?

(a) Lithium is the strongest reducing agent
(b) Na is amphoteric in nature
(c) Li^+ is exceptionally small
(d) All alkali metals give blue solution in liq. Ammonia

30. The correct order of bond angles (smallest first) in H_2S, NH_3, BF_3 and SiH_4 is

(a) $H_2S < SiH_4 < NH_3 < BF_3$
(b) $NH_3 < H_2S < SiH_4 < BF_3$
(c) $H_2S < NH_3 < SiH_4 < BF_3$
(d) $H_2S < NH_3 < BF_3 < SiH_4$

31. The number of P—O—P bonds in cyclic metaphosphoric acid is

(a) zero (b) two (c) three (d) four

32. Among the trihalides of nitrogen which one is the least basic

(a) NF_3 (b) NCl_3
(c) NBr_3 (d) NI_3

33. Among the following, the pair in which the two species are not isostructural is

(a) SiF_4 and SF_4 (b) $IO_3^{\ominus}$ and XeO_3
(c) $BH_4^{\ominus}$ and $NH_4^{\oplus}$ (d) $PF_6^{\ominus}$ and SF_6

34. The hybridisation and geometry of B and N in $[H_3B \leftarrow NH_3]$ are, respectively

(a) sp^3, tetrahedral and sp^3 pyramidal
(b) sp^3, pyramidal and sp^3 tetrahedral
(c) sp^3, pyramidal and sp^3 pyramidal
(d) sp^3, tetrahedral and sp^3 tetrahedral

35. Permanganate ions are

(a) tetrahedral and paramagnetic
(b) tetrahedral and diamagnetic
(c) octahedral and paramagnetic
(d) octahedral and diamagnetic

36. The root means square velocity of hydrogen at STP is 1.83×10^5 cm sec^{-1} and its means free path is 1.78×10^{-5}cm. What will be the collision number at STP?

(a) $9.476 \times 10^9 \, sec^{-1}$
(b) $9.746 \times 10^{-9} \, sec^{-1}$
(c) $9.746 \times 10^9 \, sec^{-1}$
(d) $9.647 \times 10^9 \, sec^{-1}$

37. There are certain properties related to adsorption:

I. reversible .

II. formation of unimolecular layer

III. low heat of adsorption

IV. occurs at low temperature and decreases with increasing temperature

Which of the above properties are for physical adsorption?

(a) I, II, III
(b) I, III, IV
(c) II, III, IV
(d) I, III

38. Which or the following FCC structures contains cations in the' alternate tetrahedral voids

(a) Na_2O
(b) ZnS
(c) CAF_2
(d) CaO

39. One liter of water (molecular weight 18.06) weighs 0.9970 kg. The degree of ionisation of water is .... , if $K_w = 1.10 \times 10^{-14}$ at 25°C

(a) 1.05×10^{-7}
(b) 1.9×10^{-9}
(c) 1.01×10^{-11}
(d) 4.52×10^{-7}

40. The specific conductance of 0.01 M solution of acetic acid was found to be 0.0163 Sm^{-1} at 25°C. Molar conductance of acetic acid at infinite dilution is $390.7 \times 10^{-4} Sm^2\, mol^{-1}$ at 25°C. What will be the degree of dissociation of CH_3COOH?

(a) 0.4072
(b) 0.7402
(c) 0.2720
(d) 0.0472

41. For the cell $Ag(s) | Ag^+(aq) || Cu^{2+}(aq) | Cu(s)$, the reduction potentials of the left and right hand electrodes are 0.337 and 0.799 volts, the cell emf is

(a) – 1.136 volt
(b) 1.136 volt
(c) – 0.462 volt
(d) 0.462 volt

42. 50% of a first order reaction is complete in 23 minutes. Calculate the time required to complete 90% of the reaction

(a) 70.4 minutes
(b) 76.4 minutes
(c) 38.7 minutes
(d) 35.2 minutes

43. The first order gaseous decomposition of N_2O_4 into NO_2 has a k value of $4.5 \times 10^3 s^{-1}$ at 1°C and an energy of activation of 58 kJ $mole^{-1}$. At what temperature would be $1.00 \times 10^4 s^{-1}$?

(a) 274 K
(b) 283 K
(c) 273 K
(d) 293 K

44. 30.4 kJ is required to melt one mole of NaCl. The entropy change during melting is 28.4 J mol^{-1} K^{-1}. What is the melting point of sodium chloride?

(a) 1070.4 K
(b) 535.2 K
(c) 273.1 K
(d) 1007.4 K

45. What weight of HCl is present in 155 ml of a 0.54 M solution?

(a) 3.06 g (b) 6.12 g (c) 1.53 g (d) 0.30 g

46. When PCl_5 is heated it gasifies and dissociates into PCl_3 and Cl_2. The density of the gas mixture at 200°C is 70.2. What is the degree of dissociation of PCl_5 at 200°C.

(a) 0.485 (b) 0.242 (c) 0.845 (d) 0.542

47. What is the value of K_{sp} for bismuth sulphide (Bi_2S_3) which has a solubility of 1.0×10^{-15} mol / L at 25°C?

(a) 1.08×10^{-73}
(b) 1.08×10^{-74}
(c) 1.08×10^{-72}
(d) 1.08×10^{-75}

48. At 20°C the solubility of N_2 gas in water is 0.015 g/L when the partial pressure of N_2 is 580 torr. What is the solubility of N_2 in H_2O at 20°C when its partial pressure is 800 torr?

(a) 0.207 g/L
(b) 0.0207 g/L
(c) 0.414 g/L
(d) 0.0414 g/L

49. Which of the following is incorrect?

(a) Chemisorption is caused by bond formation
(b) Chemisorption is reversible process
(c) Chemisorption is specific in nature
(d) Chemisorption increases with increase in temperature

50. In a suspension the diameter of the dispersed particles is of the order

(a) 10 Å
(b) 100 Å
(c) 1000 Å
(d) 2000 Å

Mathematics

1. The area bounded by the circle $x^2 + y^2 = 4$ and the line $x = y\sqrt{3}$ in the first quadrant (in sq units) is

(a) π (b) $\frac{\pi}{2}$

(c) $\frac{\pi}{3}$ (d) None of these

2. The value of the integral $\int_0^1 \frac{e^{5\log_e x} - e^{4\log_e x}}{e^{\log_e x^3} - e^{\log_e x^2}}\,dx$ is

(a) $\frac{1}{3}$ (b) 1

(c) $-\frac{1}{3}$ (d) -1

3. $\int \frac{x^3 dx}{1 + x^4}$ equals

(a) $\log(x^4 + 1) + C$ (b) $\frac{1}{4}\log(x^4 + 1) + C$

(c) $\frac{1}{2}\log(x^4 + 1) + C$ (d) None of these

4. The value of the integral $\int_{-2}^{0} \frac{dx}{\sqrt{12 - x^2 - 4x}}$ is

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{6}$

(c) $\frac{\pi}{3}$ (d) $-\frac{\pi}{6}$

5. The integral $\int \sqrt{16 - 9x^2}\,dx$ equals

(a) $\frac{x}{2}\sqrt{16 - 9x^2} + \frac{8}{3}\sin^{-1}\left(\frac{3x}{4}\right) + C$

(b) $\frac{3x}{2}\sqrt{16 - 9x^2} + 16\sin^{-1}\left(\frac{3x}{4}\right) + C$

(c) $\frac{\pi}{2}\sin^{-1}\left(\frac{3x}{4}\right) + \frac{9x}{2} + C$

(d) None of the above

6. The sum of two numbers is 10. Their product will be maximum when they are

(a) 3, 7 (b) 4, 6

(c) 5, 5 (d) 8, 2

7. The maximum value of $\frac{\log x}{x}$ is

(a) 1 (b) $\frac{2}{e}$

(c) e (d) $\frac{1}{e}$

8. The function $f(x) = \cos^2 x$ is strictly decreasing on

(a) $\left[0, \frac{\pi}{2}\right]$ (b) $\left[0, \frac{\pi}{2}\right)$

(c) $\left(0, \frac{\pi}{2}\right)$ (d) $\left(0, \frac{\pi}{2}\right]$

9. Consider the following propositions :

p : I take medicine

q : I can sleep

Then, the compound statement $\sim p \rightarrow \sim q$ means

(a) If I do not take medicine, then I cannot sleep

(b) I take medicine if I can sleep

(c) If I do not take medicine, then I can sleep

(d) I take medicine iff I can sleep.

10. If $\begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$ is to be square root of the two rowed unit matrix, then α, β and γ should statisfy the relation

(a) $1 + \alpha^2 + \beta\gamma = 0$ (b) $1 - \alpha^2 - \beta\gamma = 0$

(c) $1 - \alpha^2 + \beta\gamma = 0$ (d) $1 + \alpha^2 - \beta\gamma = 0$

11. If A and B are two square matrices such that $AB = A$ and $BA = B$, then

(a) A and B are idempotent

(b) only A is idempotent

(c) only B is idempotent

(d) None of the above

12. The only integral root of the equation

$$\begin{vmatrix} 2 - y & 2 & 3 \\ 2 & 5 - y & 6 \\ 3 & 4 & 10 - y \end{vmatrix} = 0 \text{ is}$$

(a) $y = 3$ (b) $y = 2$

(c) $y = 1$ (d) None of these

13. The determinant

$$\begin{vmatrix} xp + y & x & y \\ yp + z & y & z \\ 0 & xp + y & yp + z \end{vmatrix} = 0 \text{ if}$$

(a) x, y, z are in AP

(b) x, y, z are in GP

(c) x, y, z are in HP

(d) xy, yz, zx are in AP

14. Out of $(2n+1)$ consecutively numbered tickets, three tickets are drawn at random. The probability that the numbers on them are in arithmetic progression is

(a) $\frac{n}{4n^2-1}$ (b) $\frac{n^2}{4n^2-1}$

(c) $\frac{3n}{4n^2-1}$ (d) $\frac{3n^2}{4n^2-1}$

15. A certain item is manufactured by machine M_1 and M_2. It is known that machine M_1 turns out twice as many items as machine M_2. It is also known that 4% of the items produced by machine M_1 and 3% of the items produced by machine M_2 are defective. All the items produced are put into one stock pile and then one item is selected at random. The probability that the selected item is defective is equal to

(a) $\frac{10}{300}$ (b) $\frac{11}{300}$

(c) $\frac{10}{200}$ (d) $\frac{11}{200}$

16. If A and B are independent events such that $P(B)=\frac{2}{7}, P(A\cup\bar{B})=$`0.8, then $P(A)=$

(a) 0.4 (b) 0.3

(c) 0.2 (d) 0.1

17. If A and B are independent events associated to some experiment E such that $P(A^C\cap B)=\frac{2}{15}$ and $P(A\cap B^C)=\frac{1}{6}$, then $P(B)$ is equal to

(a) $\frac{1}{6},\frac{1}{5}$ (b) $\frac{1}{6},\frac{4}{5}$

(c) $\frac{4}{5},\frac{1}{5}$ (d) $\frac{4}{5},\frac{5}{6}$

18. The value of λ so that the vectors $\mathbf{a}=2\hat{\mathbf{i}}-\hat{\mathbf{j}}+\hat{\mathbf{k}}$, $\mathbf{b}=\hat{\mathbf{i}}+2\hat{\mathbf{j}}-3\hat{\mathbf{k}}$ and $\mathbf{c}=3\hat{\mathbf{i}}+\lambda\hat{\mathbf{j}}+5\hat{\mathbf{k}}$ are coplanar is

(a) – 1 (b) – 2

(c) – 3 (d) – 4

19. The differential equation of all parabolas whose axis of symmetry is parallel to X-axis is of order

(a) 2 (b) 3

(c) 1 (d) 4

20. If p and q are the order and degree of the differential equation

$$y\frac{dy}{dx}+x^3\left(\frac{d^2y}{dx^2}\right)^3+xy=\cos x, \text{ then}$$

(a) $p<q$ (b) $p=q$

(c) $p>q$ (d) None of these

21. The equation $x^2+y^2+4x+6y+13=0$ represents

(a) a pair of coincident lines

(b) a pair of concurrent straight lines

(c) a parabola

(d) a point circle

22. The two lines $ty=x+t^2$ and $y+tx=2t+t^3$ intersect at the point lies on the curve whose equation is

(a) $y^2=4x$ (b) $y^2=-4x$ (c) $x^2=4y$ (d) $x^2=-4y$

23. The directrix of the parabola $4y^2+12x-12y+39=0$ is

(a) $x=\frac{3}{4}$ (b) $x=\frac{-7}{4}$

(c) $x=\frac{-5}{2}$ (d) $x=\frac{3}{2}$

24. A line perpendicular to the line segment joining the points (1, 0) and (2, 3) divides it in the ratio 1 : n. The equation of the line is

(a) $3y+x=\frac{n+11}{n+1}$ (b) $3y-x=\frac{n+11}{n+1}$

(c) $3y+x=\frac{n-11}{n+1}$ (d) $3y-x=\frac{n+11}{n-1}$

25. If origin is the centroid of a ΔPQR with vertices $P(2a,2,6), Q(-4,3b,-10)$ and $R(8,14,2c)$, then the value of a, b and c are respectively.

(a) –2, 2, 2 (b) $-2, 2, -\frac{16}{3}$

(c) $-2, -\frac{16}{3}, 2$ (d) $-\frac{16}{3}, -2, 2$

26. The angle between the lines with direction ratios 4, – 3, 5 and 3, 4, 5 is

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{2}$

27. Given that the points $P(3,2,-4)$, $Q(5,4,-6)$ and $R(9,8,-10)$ are collinear, the ratio in which Q divides PR externally is

(a) 1 : 2 (b) 2 : 1 (c) 1 : 1 (d) 2 : 2

28. The minimum value of $9^x + 9^{1-x}$, $x \in R$ is

(a) 2 (b) 3 (c) 6 (d) 9

29. The sum of n terms of the series

$1^2 + (1^2 + 2^2) + (1^2 + 2^2 + 3^2) + \ldots\ldots\ldots\ldots$ is

(a) $\frac{n(n+1)(n+2)}{12}$ (b) $\frac{n(n+1)(n+2)^2}{12}$

(c) $\frac{n^2(n+1)(n+2)}{12}$ (d) $\frac{n(n+1)^2(n+2)}{12}$

30. If $x^y = e^{x-y}$, then $\frac{dy}{dx}$ is equal to

(a) $\frac{1}{1+\log x}$ (b) $\frac{1}{(1+\log x)^2}$

(c) $\frac{\log x}{1+\log x}$ (d) $\frac{\log x}{(1+\log x)^2}$

31. If $x^2 + y^2 = t - \frac{1}{t}$ and $x^4 + y^4 = t^2 + \frac{1}{t^2}$, then $x^3 y \frac{dy}{dx}$ equals

(a) 0 (b) 1

(c) – 1 (d) None of these

32. If $\lim_{x \to 0} \frac{\log(3+x) - \log(3-x)}{x} = K$, then K is equal to

(a) $\frac{2}{5}$ (b) $\frac{2}{3}$ (c) $\frac{1}{2}$ (d) $\frac{5}{2}$

33. If $f(x) = \frac{x}{2} - 1$, then on the interval $[0, \pi]$

(a) $\tan[f(x)]$ and $\frac{1}{f(x)}$ are both continuous

(b) $\tan[f(x)]$ and $\frac{1}{f(x)}$ are both discontinuous

(c) $\tan[f(x)]$ is continuous but $\frac{1}{f(x)}$ is not continuous

(d) $\tan[f(x)]$ is not continuous but $\frac{1}{f(x)}$ is continuous

34. If $y = \tan^{-1}(\sqrt{1+x^2} - x)$, then $\frac{dy}{dx}$ equals

(a) $\frac{1}{2(1+x^2)}$ (b) $\frac{-1}{(1+x^2)}$

(c) $\frac{-1}{2(1+x^2)}$ (d) $\frac{2}{(1+x^2)}$

35. If $y = \sec(\tan^{-1} x)$, then $\frac{dy}{dx}$ at $x = 1$ is

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\sqrt{2}$ (d) 1

36. If $f(x) = x^\alpha \log x$ and $f(0) = 0$, then the value of α for which Rolle's theorem can be applied in $[0, 1]$ is

(a) – 1 (b) $\frac{1}{2}$

(c) $\frac{-1}{2}$ (d) 0

37. An n-tuple $(x_1, x_2, x_3, \ldots\ldots\ldots, x_n)$ which satisfies all the constraints of a linear programming problem and for which the objective function is maximum (compared to all n-tuples which satisfy all the constraints) is called

(a) a solution (b) a feasible solution

(c) an optimal solution (d) an actual solution

38. Given the LPP :

Minimize $f = 2x_1 - x_2$

$$x_1 \geq 0, x_2 \geq 0$$
$$x_1 + x_2 \geq 5$$
$$-x_1 + x_2 \leq 1$$
$$5x_1 + 4x_2 \leq 40$$

The solution is

(a) 1 (b) – 1 (c) 2 (d) – 2

39. If $49^n + 16n + \lambda$ is divisible by 64 for all $n \in N$, then the least negative integral value of λ is

(a) – 1 (b) – 2

(c) – 3 (d) – 4

40. A polygon has 44 diagonals. The number of its sides are

(a) 9 (b) 8

(c) 11 (d) 7

41. The greatest value of the term independent of x, as α varies over R, in the expansion of $\left(x\cos\alpha + \frac{\sin\alpha}{x}\right)^{10}$ is

(a) ${}^{10}C_5$ (b) $\left(\frac{1}{2}\right)^5 {}^{10}C_5$

(c) $\left(\frac{1}{2}\right)^4 {}^{10}C_5$ (d) $\left(\frac{1}{2}\right)^3 {}^{10}C_5$

42. The value of $\left(\frac{1+i}{1-i}\right)^{100}$ is equal to

(a) 1 (b) – 1 (c) i (d) – i

43. The value of $\sum_{n=1}^{13}(i^n + i^{n+1})$, where $i = \sqrt{-1}$ equals

(a) 0 (b) i (c) $-i$ (d) $i-1$

44. An electrician can be paid under two schemes as follows :

I. ₹ 600 and ₹ 50 per hour

II. ₹ 170 per hour

If the job take n hours, for which values of n does the scheme I given the electrician better wages

(a) $n > 5$ (b) $n > 4$
(c) $n < 5$ (d) $n < 4$

45. If $X = \{4^n - 3n - 1 | n \in N\}$ and

$Y = \{9(n-1) | n \in N\}$, then

(a) $X \subset Y$ (b) $Y \subset X$
(c) $X = Y$ (d) None of these

46. The value of $\frac{\sin^3 3\theta}{\sin^2 \theta} - \frac{\cos^2 3\theta}{\cos^2 \theta}$ is equal to

(a) $8\cos 2\theta$ (b) $3\sin 2\theta$
(c) $\frac{1}{8}\cos 2\theta$ (d) None of these

47. The value of the expression

$1 - \frac{\sin^2 y}{1+\cos y} + \frac{1+\cos y}{\sin y} - \frac{\sin y}{1-\cos y}$ is equal to

(a) $\sin y$ (b) $\cos y$
(c) 0 (d) 1

48. The solution set of the equation $\sin^{-1} x = 2\tan^{-1} x$ is

(a) {1, 2} (b) {– 1, 2}
(c) {– 1, 1, 0} (d) {1, 1/2, 0}

49. A roots of the equation

$17x^2 + 17x\tan\left(2\tan^{-1}\frac{1}{5} - \frac{\pi}{4}\right) - 10 = 0$ is

(a) $\frac{10}{17}$ (b) -1
(c) $\frac{-7}{17}$ (d) 1

50. Let R be a reflexive relation on a finite set A having n elements and let there be m ordered pairs in R, then

(a) $m \geq n$ (b) $m \leq n$
(c) $m = n$ (d) None of these

Answers

Physics

1. (b)	**2.** (a)	**3.** (d)	**4.** (b)	**5.** (a)	**6.** (d)	**7.** (c)	**8.** (c)	**9.** (a)	**10.** (b)
11. (a)	**12.** (d)	**13.** (d)	**14.** (c)	**15.** (c)	**16.** (d)	**17.** (a)	**18.** (a)	**19.** (b)	**20.** (b)
21. (d)	**22.** (b)	**23.** (c)	**24.** (d)	**25.** (d)	**26.** (b)	**27.** (b)	**28.** (b)	**29.** (a)	**30.** (b)
31. (a)	**32.** (d)	**33.** (c)	**34.** (d)	**35.** (d)	**36.** (c)	**37.** (b)	**38.** (d)	**39.** (a)	**40.** (b)
41. (c)	**42.** (c)	**43.** (d)	**44.** (c)	**45.** (b)	**46.** (d)	**47.** (b)	**48.** (a)	**49.** (b)	**50.** (c)

Chemistry

1. (d)	**2.** (c)	**3.** (b)	**4.** (d)	**5.** (a)	**6.** (d)	**7.** (d)	**8.** (a)	**9.** (a)	**10.** (d)
11. (d)	**12.** (c)	**13.** (b)	**14.** (d)	**15.** (a)	**16.** (b)	**17.** (a)	**18.** (a)	**19.** (d)	**20.** (b)
21. (b)	**22.** (a)	**23.** (b)	**24.** (d)	**25.** (a)	**26.** (c)	**27.** (c)	**28.** (b)	**29.** (b)	**30.** (c)
31. (c)	**32.** (a)	**33.** (a)	**34.** (c)	**35.** (b)	**36.** (a)	**37.** (b)	**38.** (b)	**39.** (b)	**40.** (d)
41. (d)	**42.** (b)	**43.** (c)	**44.** (a)	**45.** (a)	**46.** (a)	**47.** (a)	**48.** (b)	**49.** (b)	**50.** (d)

Mathematics

1. (c)	**2.** (a)	**3.** (b)	**4.** (b)	**5.** (a)	**6.** (c)	**7.** (d)	**8.** (c)	**9.** (a)	**10.** (b)
11. (a)	**12.** (c)	**13.** (b)	**14.** (c)	**15.** (b)	**16.** (b)	**17.** (b)	**18.** (d)	**19.** (b)	**20.** (a)
21. (d)	**22.** (a)	**23.** (b)	**24.** (a)	**25.** (c)	**26.** (a)	**27.** (a)	**28.** (c)	**29.** (d)	**30.** (d)
31. (b)	**32.** (b)	**33.** (c)	**34.** (c)	**35.** (b)	**36.** (b)	**37.** (c)	**38.** (a)	**39.** (a)	**40.** (c)
41. (b)	**42.** (a)	**43.** (d)	**44.** (c)	**45.** (a)	**46.** (a)	**47.** (b)	**48.** (c)	**49.** (d)	**50.** (a)

Solutions

Physics

1. According to the question,

$$v \propto \sqrt{\frac{T}{\mu}} = k\sqrt{\frac{T}{\mu}}$$

As $\mu = \frac{M}{L}$ and $T = m'g$

$\Rightarrow \quad v = k\sqrt{\frac{TL}{M}} = k\sqrt{\frac{m'gL}{M}}$

$\Rightarrow \quad \frac{\Delta v}{v} = \frac{1}{2}\frac{\Delta m'}{m'} + \frac{1}{2}\frac{\Delta L}{L} + \frac{1}{2}\frac{\Delta M}{M}$

$= \frac{1}{2} \times \frac{0.1}{5} + \frac{1}{2} \times \frac{1 \times 10^{-3}}{1} + \frac{1}{2} \times \frac{0.1}{3}$

$= 0.01 + 0.0005 + 0.016$

$= 0.0271 = 2.7\%$

2. Given, $a = -k\sqrt{v}$

or $\quad a = \frac{dv}{dt} = -k\sqrt{v} \qquad \left[\because \frac{dx}{dt} = v\right]$

or $\quad dv = -k\sqrt{v}\, dt$

or $\quad \frac{dv}{\sqrt{v}} = -kdt$

Integration will yield

$$\int_u^v \frac{dv}{\sqrt{v}} = \int_0^t -kdt$$

$$[2\sqrt{v}]_u^v = -k[t]_0^t$$

$$2(\sqrt{v} - \sqrt{u}) = -kt$$

$\because$ Particle comes to rest, so final velocity $v = 0$

and $\quad -2\sqrt{u} = -kt$

$\Rightarrow \quad t = \frac{2\sqrt{u}}{k}$

3. Given, $y = px - qx^2$

$y_{\max}$ when $\frac{dy}{dx} = 0$

$\Rightarrow \quad p - 2qx = 0$

$\Rightarrow \quad y_{\max}$ or max height $(H) = \frac{p^2}{4q}$

Now, $H = y_{\max} = \frac{u_y^2}{2g}$

$\Rightarrow \quad \frac{u_y^2}{2g} = \frac{p^2}{4q}$

$\Rightarrow \quad u_y = \sqrt{\frac{gp^2}{2q}}$

Also, T = time of flight

$$= \frac{2u \sin \theta}{g} = \frac{2u_y}{g} = p\sqrt{\frac{2}{gq}}$$

4. Displacement of a body,

$$x = -\frac{1}{3}t^2 + 16t + 3 \qquad \left[\because v = \frac{dx}{dt}\right]$$

From the question, $v = 0$

$$\frac{dx}{dt} = 0$$

or $\quad \frac{d}{dt}\left(-\frac{1}{3}t^2 + 16t + 3\right) = 0$

or $\quad -\frac{2}{3}t + 16 + 0 = 0$

or $\quad -\frac{2}{3}t = -16$

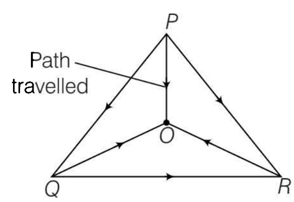
or $\quad t = \frac{16 \times 3}{2} = 24\,\text{s}$

So, time taken by the body to come to rest is 24 s.

5. According to the question, the drunkard moves 5 m forward and 3 m in backward direction and takes 8 s in the movement. In such a way, the total distantance covered by drunkard

$8 + 8 + 8 + 8 + 5 = 37$ s.

6.



The velocity component towards centroid O of triangle $= v\cos\theta$

$$= v \cos 30° = \frac{\sqrt{3}}{2} v$$

Distance $PO = \frac{2}{3}$ of altitude

$$= \frac{2}{3} \times \frac{\sqrt{3}}{2}\text{s} = \frac{1}{\sqrt{3}}\text{s}$$

$\Rightarrow \quad \text{Time} = \frac{\text{Distance}}{\text{Speed}}$

$$= \frac{\frac{1}{\sqrt{3}}s}{\frac{\sqrt{3}}{2}v} = \frac{2}{3} \cdot \frac{s}{v}$$

7. We have, $v = u + at$

For white car, $v = a_w t$

$\Rightarrow \quad \frac{v}{t} = a_w \qquad [u = 0] \ldots\text{(i)}$

For red car, $v_R = a_r t_r \quad \frac{v_r}{t_r} = a_r \qquad \ldots\text{(ii)}$

So, $\frac{v}{t} \cdot \frac{v_r}{t_r} = a_w \cdot a_r$

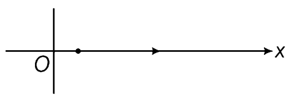
Suppose $v \approx v_R$ and $t \approx t_r$

$$\frac{v^2}{t^2} = a_w \cdot a_r$$

$$\frac{v}{t} = \sqrt{a_w \cdot a_r}$$

8. The velocity at any time

$$v = 4t^3 - 2t$$



$$v = \frac{dx}{dt} = 4t^3 - 2t$$

$\Rightarrow \quad x = \frac{4t^4}{4} - \frac{2t^2}{2}$

$\Rightarrow \quad t^4 - t^2 = 2$

$\Rightarrow \quad t^4 - t^2 - 2 = 0 \qquad [\because x = 2]$

$\Rightarrow \quad t^2 = \frac{1 \pm \sqrt{1+8}}{2} = \frac{1 \pm 3}{2}$

$$t = \sqrt{2}\text{ s}$$

Now, $a = \frac{dv}{dt} = 12t^2 - 2$

At $t = 2$s, acceleration of the particle

$$a\Big|_{x=2 \text{ or } t=\sqrt{2}} = 12 \times 2 - 2 = 22 \text{ m/s}^2$$

9. According to the question,

$$K_m = \frac{1}{2} K_{m/2}$$

$\Rightarrow \quad \frac{1}{2} m v_1^2 = \frac{1}{2} \times \frac{1}{2}\left(\frac{m}{2}\right) v_2^2$

$\Rightarrow \quad \frac{v_1^2}{v_2^2} = \frac{1}{4}$ initially

Speed of heavier particle is increased by 3 m/s.

Then, $\frac{1}{2} m(v_1 + 3)^2 = \frac{1}{2}\left(\frac{m}{2}\right) v_2^2$

$\Rightarrow \quad \frac{1}{2} m(v_1 + 3)^2 = \frac{1}{4} m \cdot 4v_1^2$

$$(v_1 + 3)^2 = 2v_1^2$$

Solving these equations, we get

$$v_2 = 3 \text{ m/s}$$

$$v_1 = 6 \text{ m/s}$$

10. From the equations of motion,

$$s = ut + \frac{1}{2} at^2$$

$$7 = \frac{1}{2} a(1.6)^2$$

$\Rightarrow \quad a = \frac{7 \times 2}{(1.6)^2} \qquad \ldots\text{(i)}$

Again $\quad v = u + at = \frac{7 \times 2}{(1.6)^2} \times 1.6$

Velocity $v = \frac{7 \times 2}{1.6}$

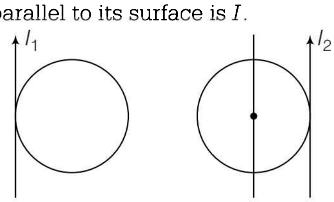
Work done = Change in KE $= \frac{1}{2} \times 670 \times \left(\frac{7 \times 2}{1.6}\right)^2$

$\therefore \quad$ Power $= \frac{\text{Work done}}{\text{Time}}$

$$= \frac{670 \times 49 \times 4}{2 \times (1.6) \times (1.6)^2} \text{ watt}$$

$$= 16030.27 \text{ watt} \simeq 16 \text{ kW}$$

11. Given that MI of a disc about an axis tangential and parallel to its surface is I.



$$I_1 = \frac{5}{4} MR^2$$

$$I_2 = \frac{3}{2} MR^2$$

Given, $I_1 = I$

So, $\quad I_2 = \frac{3}{2} \times \frac{4}{5} \times \left(\frac{5}{4} MR^2\right)$

$$I_2 = \frac{6}{5} I_1 \text{ or } I_2 = \frac{6}{5} I$$

12. The angular momentum about the origin

$$= L_{\text{translational}} + L_{\text{Rotational}}$$

$$= MvR + I.\omega$$

where, $\quad v$ = linear velocity

$$I = \frac{1}{2} mR^2$$

$$|L| = MvR + \frac{1}{2} MR^2\omega$$

$$= M(\omega R) \cdot R + \frac{1}{2} MR^2\omega$$

$$= MR^2\omega + \frac{1}{2} MR^2\omega$$

$$= \frac{3}{2} MR^2\omega = \frac{3}{2} MR^2 \frac{v}{R} = \frac{3}{2} MRv$$

13. $\tau = \mathbf{A} \times \mathbf{L}$

$\mathbf{A}$ = constant vector

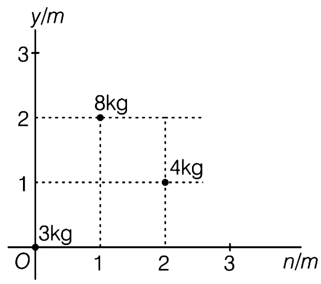
$\mathbf{L}$ = angular momentum

In $\mathbf{A}$ and $\mathbf{L}$ there will be no change.

So, all the above choices are correct.

14. Coordinates of centre of mass,

$$X_{CM} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}$$



$$X_{CM} = \frac{0 + 8 \times 1 + 4 \times 2}{3 + 4 + 8}$$

$$= \frac{16}{15}$$

$$= 1.1$$

$$Y_{CM} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3}$$

$$Y_{CM} = \frac{0 + 8 \times 2 + 4 \times 1}{3 + 4 + 8}$$

$$= \frac{20}{15}$$

$$= \frac{4}{3} = 1.3$$

$(X_{CM}, Y_{CM}) = 1.1 \text{ m}, 1.3 \text{ m}$

15. Let the density of earth $= d$

Density of planet $= 2d$

$$g_E = \frac{GM_e}{R^2} \quad \text{...(i)}$$

$$g_P = \frac{GM_P}{R_P^2} \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$g_P = g_E \quad \text{[Given]}$$

$$R_P^2 = \frac{M_P}{M_E} \times R^2 = \frac{\frac{4}{3}\pi \frac{R_P^3}{2d}}{\frac{4}{3}\pi \frac{R^3}{d}} \times R^2$$

$$R_P = \frac{R}{2}$$

16. The KE of planet $K_{as} = \frac{GM_S\ M_{as}}{2\ r_{as}}$

$$\text{KE of earth} = \frac{GM_S\ M_E}{2r_E}$$

$$\therefore \quad \frac{K_{as}}{K_E} = \frac{GM_S \frac{M_{as}}{2\ r_{as}}}{\frac{GM_S\ M_E}{2r_E}}$$

$$= \frac{r_E}{2r_E} \times \frac{M_E \times 2 \times 10^{-4}}{M_E}$$

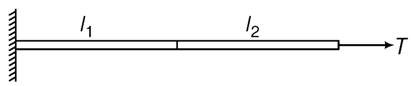
$$= \frac{2 \times 10^{-4}}{2}$$

$$= 1 \times 10^{-4}$$

17. Given, $Y_{steel} = 2 \times 10^{11} \text{ N/m}^2$

$$Y_{copper} = 1.1 \times 10^{11} \text{ N/m}^2$$

The wires are joined end to end.



$$\Delta l_1 = \Delta l_2$$

Also $\quad T_1 = T_2$

The formula of Young's modulus

$$Y = \frac{Tl}{A\Delta l}$$

$$\Rightarrow \quad l = \frac{YA\Delta l}{T}$$

$$\Rightarrow \quad \frac{l_1}{l_2} = \frac{Y_1 A_1 \Delta l_1}{T_1} / \frac{Y_2 A_2 \Delta l_2}{T_2} = \frac{Y_1 A}{Y_2 A} \quad [\because A_1 = A_2]$$

$$\Rightarrow \quad \frac{l_1}{l_2} = \frac{Y_1}{Y_2} = \frac{2 \times 10^{11}}{1.1 \times 10^{11}} = \frac{20}{11}$$

18. Because variation of density with respect to height is given by the relation,

$$\rho = \rho_0 e^{-\alpha h}$$

where, ρ_0 = density at sea level

α = a constant

h = height

Hence, the pressure due to a small air column of length dh at height h is given by

$$dp = (dh) \cdot \rho g$$

$$dp = (dh)\rho_0 e^{-\alpha h} \cdot g$$

$$= \rho_0 g \cdot e^{-\alpha h}\ dh$$

$$p = \int_0^\infty (\rho_0 g)\ e^{-\alpha h}\ dh$$

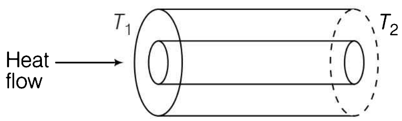
$$= \rho_0 g \int_0^\infty e^{-\alpha h}\ dh$$

$$= \rho_0 g \left(\frac{e^{-\alpha h}}{-\alpha} \right)$$

Atmospheric pressure at the sea level

$$p = \frac{\rho_0 g}{\alpha}$$

19. According to question,



Let the heat flow rate of combination is H.

As the cylinders are in parallel,

$$H = H_1 + H_2$$

From formula of heat flow,

$$KA\left(\frac{\Delta T}{\Delta l}\right) = K_1 A_1 \left(\frac{\Delta T}{\Delta l}\right) + K_2 A_2 \left(\frac{\Delta T}{\Delta l}\right)$$

$$\Rightarrow \quad K = \frac{K_1 A_1 + K_2 A_2}{A}$$

$$= \frac{K_1 \pi R^2 + K_2 \{\pi(2R)^2 - \pi R^2\}}{\pi (2R)^2}$$

$$= \frac{K_1 + 3K_2}{4}$$

20. Given, $R_1 = 3 \text{ cm} = 3 \times 10^{-2}$ m

$R_2 = 5 \text{ cm} = 5 \times 10^{-2}$ m

and $T = 0.03$ N/m

Original surface area $= 2 \times 4\pi R_1$

For second bubble $= 2 \times 4\pi R_2$

Work done = Surface tension × extension in area

$$= T \times \Delta A$$

$$= 0.03 \times 2[4\pi R_2^2 - 4\pi R_1^2]$$

$$= 0.03 \times 8\pi[(5)^2 - (3)^2] \times 10^{-4}$$

$$= 0.03 \times 8\pi \times 16 \times 10^{-4}$$

$$= 0.384\pi \times 10^{-3} \text{ J}$$

$$\simeq 0.4\pi \text{ mJ}$$

21. Given, $p = 4.5 \times 10^5$ Pa

$dQ = 800$ kJ

$= 800 \times 10^3$ J

Change in volume $= (2.0 - 0.5)\text{ m}^3$

$= 1.5 \text{ m}^3$

From first law of thermodynamics,

$$dQ = dU + p \cdot dV$$

$$dU = dQ - p \cdot dV$$

$$= 800 \times 10^3 - 4.5 \times 10^5 \times 1.5$$

$$= -1.25 \times 10^5 \text{ J}$$

Change in internal energy $(V) = 1.25 \times 10^5$ J.

22. The pressure p exterted by a perfect gas on the walls of the containing vessel is given by

$$p = \frac{1}{3} \frac{mnc^2}{v}$$

According to the question,

$$p = p_0$$

$$m = m_0$$

where, n = number of molecules

c = speed of molecules

According to question,

$$m' = \frac{m_0}{2}$$

$$c' = 2c$$

$$p' = p'_0$$

$$p'_0 = \frac{1}{3}\left(\frac{m_0}{2}\right) \times n \times (2c)^2$$

$$= \frac{1}{3} m_0 nc^2 \cdot 2 = p_0 \times 2 = 2p_0$$

23. The real shift observed for stars is due to the natural expanding of universe.

$$\Delta\lambda = (\lambda' - \lambda) = \frac{v\lambda}{c}$$

24. For adiabatic process,

$$pV^\gamma = \text{constant} \qquad \left(\because d = \frac{m}{V}\right)$$

$$p\left(\frac{m}{d}\right)^\gamma = \text{constant}$$

We have, $p\left(\frac{m}{d}\right)^\gamma = p'\left(\frac{m}{n \cdot d}\right)^\gamma$

$$p' = \frac{p\left(\frac{m}{d}\right)^\gamma}{\left(\frac{m}{nd}\right)^\gamma} = \frac{p \cdot m^\gamma \times n^\gamma \cdot d^\gamma}{d^\gamma \cdot m^\gamma}$$

$$p' = n^\gamma \cdot p$$

25. Given, $p = AV^2$

For a closed system, $\Delta Q = 0$

$\therefore \quad \Delta W = \Delta U$

Also, work done by the substance

$$W = \int_{V_i}^{V_f} p \cdot dV$$

$$= \int_{V_i}^{V_f} AV^2 dV$$

$$= A\left(\frac{V^3}{3}\right)_{V_i}^{V_f} \qquad \left(\because A = \frac{p_i}{V_i^2}\right)$$

where, A is constant.

$$= \frac{A}{3}[V_f^3 - V_i^3]$$

$$= \frac{p_i}{3V_i}(V_f^3 - V_i^3)$$

$$= \frac{10^5}{3 \times 9}(125 - 27)$$

$$= \frac{10^5}{3 \times 9} \times 98 = 3.6 \times 10^5 \text{ J}$$

26. The energy released by the capacitor

$$\frac{1}{2}CV^2 = P \times t$$

$$\Rightarrow \quad V^2 = \frac{2 \times P \times t}{C}$$

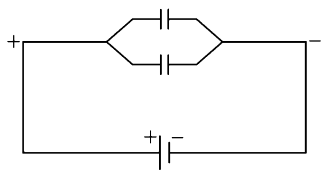
$$= \frac{2 \times 2000 \times 0.04}{40 \times 10^{-6}}$$

$$= 4 \times 10^6$$

Voltage, $V = \sqrt{4 \times 10^6}$ volt

$$= 2 \times 10^3 \text{ volt}$$

27. Equivalent circuit of the given figure



Capacitance of capacitor AB,

$$C = \frac{\varepsilon_0 A}{d}$$

$$= \frac{8.854 \times 10^{-12} \times 50 \times 10^{-4}}{3 \times 10^{-3}}$$

$$= 14.75 \times 10^{-12}$$

$$C = \frac{8.854 \times 10^{-12} \times 50 \times 10^{-4}}{6 \times 10^{-3}}$$

$$= 7.36 \times 10^{-12}$$

Energy stored $= \left(\frac{1}{2}CV_1^2\right) + \frac{1}{2}CV_2^2$

$$= \frac{1}{2}[14.75 + 7.36] \times 144 \times 10^{-12}$$

$$= 22.11 \times 72 \times 10^{-12}$$

$$= 1.59 \times 10^{-9} = 1.6 \text{ nJ}$$

28. According to Gauss's law, the position of charge inside the closed surface is immaterial. So, the magnitude of that emerging out flux will be $\frac{Q}{6\varepsilon_0}$.

As the their 6 faces of cube.

29. In the balanced position of Wheatstone bridge, $\frac{P}{Q} = \frac{R}{S}$

When the resistance of 6 Ω is connected in parallel to S, the effective resistance

$$\frac{1}{S} = \frac{1}{S'} + \frac{1}{6}$$

For the balanced position of Wheatstone bridge,

$$\frac{P}{Q} = \frac{R}{S}$$

$$\frac{2}{2} = \frac{2}{S'}$$

$$S' = 2$$

$$\therefore \quad \frac{1}{S'} = \frac{1}{S} + \frac{1}{6}$$

$$\frac{1}{S} = \frac{1}{S'} - \frac{1}{6} = \frac{1}{2} - \frac{1}{6}$$

$$S = 3\Omega$$

30. According to question,

$$P_1 = P_2$$

Power, $\quad P = \frac{V^2}{R}$

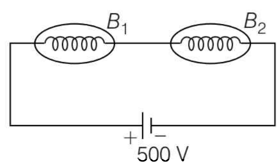
$$\therefore \quad \frac{V_1^2}{R_1} = \frac{V_2^2}{R_2} \quad [\because P_1 = P_2]$$

$$\Rightarrow \quad \frac{R_1}{R_2} = \frac{V_1^2}{V_2^2}$$

$$\frac{R_1}{R_2} = \frac{(200)^2}{(300)^2}$$

$$= \frac{200 \times 200}{300 \times 300} = \frac{4}{9}$$

Now, bulbs are connected in series, then current will be same in both bulbs.



or

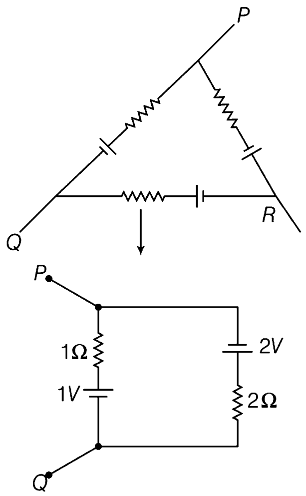
$$\Rightarrow \quad I_1 = I_2$$

$$\Rightarrow \quad \frac{V_1}{R_1} = \frac{V_2}{R_2}$$

The ratio of potential difference

$$\Rightarrow \quad \frac{V_1}{V_2} = \frac{R_1}{R_2} = \frac{4}{9}$$

31. The given circuit is



Equivalent emf is given by $\varepsilon_{eq} = \frac{2 \times 1}{2+1} = \frac{2}{3}$

We have,

$$\frac{E_{eq}}{\varepsilon_{eq}} = \frac{E_1}{\varepsilon_1} - \frac{E_2}{\varepsilon_2}$$

$$\frac{\varepsilon_{eq}}{\left(\frac{2}{3}\right)} = \frac{2}{2} - \frac{1}{1}$$

$\Rightarrow \qquad E_{eq} = 0$ volt

32. Resistance, $R = \frac{\rho l}{A}$

If a given mass of wire is recast to increase its length or decrease its radius, then we have

m = volume × density = Ald ...(i)

where, d is the density

So, $\qquad A = \frac{m}{ld}$

$\therefore \qquad R = \frac{\rho l}{\left(\frac{m}{ld}\right)} = \frac{\rho d}{m} \cdot l^2$

ρ, d and m are constant,

$$R \propto l^2 \quad \text{or} \quad \frac{R_2}{R_1} = \left(\frac{l_2}{l_1}\right)^2$$

From above Eq. (i), we have

$$l = \frac{m}{A \cdot d}$$

$\therefore \qquad R = \frac{\rho m}{A \cdot dA} = \frac{m\rho}{d} \cdot \frac{1}{A^2}$

$$R \propto \frac{1}{A^2} \qquad [\because A = \pi r^2]$$

$\Rightarrow \qquad R \propto \frac{1}{r^4}$

33. The radius of charged particle in magnetic field,

$$r = \frac{mv}{qB} = \frac{\sqrt{2mE_k}}{qB}$$

According to question, let m_x and m_y are the masses, q are charges and B magnetic field and E_K, kinetic energy. When the charged particles are accelerated at same potential, then KE will be same.

So, $\qquad$ radius, $R_1 = \frac{\sqrt{2m_x \times E_K}}{qB}$

and $\qquad R_2 = \frac{\sqrt{2\, m_y E_K}}{qB}$

$\therefore \qquad \frac{m_x}{m_y} = \left(\frac{R_1}{R_2}\right)^2$

34. The force experienced by the charge $q = 3.2 \times 10^{-19}$ C in electric field $F_e = qE$

The force experienced by the charge in magnetic field,

$$F_m = q(\mathbf{v} \cdot \mathbf{B})$$
$$= qvB \sin\theta$$

Here, $\qquad \theta = 90^\circ$

$$F_m = qvB$$

As the charge particle is not accelerated, so the resultant forces

$$F_e = F_m$$
$$qE = qvB$$

$\Rightarrow \qquad v = \frac{E}{B}$

We have $\qquad E = 5 \times 10^4$ V/m

and $\qquad B = 0.8$ T

$\therefore$ speed $v = \frac{5 \times 10^4}{0.8} = \frac{50}{8} \times 10^4 = 6.3 \times 10^4$ m/s

35. The magnetic field inside the current carrying wire B at $\frac{R}{2}$ distance,

$$B_{R/2} = \frac{\mu_0 i}{2\pi R^2} \times \frac{R}{2} = \frac{\mu_0 i}{4\pi R}$$

At distance, $2R$

$$B_{2R} = \frac{\mu_0 i}{2\pi(2R)} = \frac{\mu_0 i}{4\pi R}$$

$\therefore \qquad \frac{B_{R/2}}{B_{2R}} = 1:1$

36. The magnetic field at the centre of a circular coil,

$$B_{\text{centre}} = \frac{\mu_0 Ni}{2r}$$

$\because \quad i = \frac{V}{R},$

and $\quad R = \frac{VA}{\rho \cdot 2\pi r}$

$\therefore \quad B_{\text{centre}} = \frac{\mu_0 NVA}{4\rho \cdot \pi r^2}$

Where, A = area of cross-section of the wire

According to the question,

$$B_1 = B_2 = \frac{V_1}{r_1^2} = \frac{V_2}{r_2^2}$$

$\Rightarrow \quad \frac{V_1}{(2r_2)^2} = \frac{V_2}{r_2^2}$

$\Rightarrow \quad V_1 = 4V_2$

Ratio of potential difference $\frac{V_1}{V_2} = 4$

37. Inductance, $L = 50$ mH

$$= 50 \times 10^{-3} \text{ H}$$

For steady state,

$$i = i_0 \,(1 - e^{-t^{R/L}})$$

Substitution of value gives

$$\frac{i_0}{2} = i_0 \left[1 - e^{t\left(\frac{10}{50 \times 10^{-3}}\right)}\right]$$

$\Rightarrow \quad e^{200t} = 2$

$\Rightarrow \quad 200t = \log 2 \quad \Rightarrow \quad t = \frac{\log_e 2}{200}$

$$\log_e 2 = 0.693$$

$$t = \frac{0.693}{200}$$

$$= 0.00346 \text{ s}$$

$$= 3.46 \text{ ms} = 3.5 \text{ ms}$$

38. If the rotation axis is shifted to one of the shorter edge of the rectangular coil, there will be no change in the flux linked with the coil, so the emf will remain steady.

So, ϕ and E will remain same.

39. In series L-C-R circuit, the supply voltage

$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

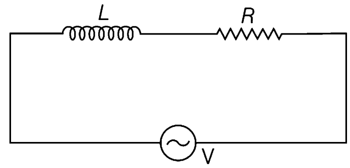
$$= \sqrt{(20)^2 + (20 - 20)^2} = 20 \text{ V}$$

When capacitance is short-circuited, then

$$V_R^2 + V_L^2 = (20)^2$$

$$I \times L = IR$$

$$X_L = R$$



$\therefore \quad 2V_L^2 = (20)^2$

$$V_L^2 = \frac{400}{2} = 200$$

Voltage, $V_L = \sqrt{200}$

$$= 10\sqrt{2} = \frac{20}{\sqrt{2}} \text{ V}$$

40. Given, speed of EM wave $= 2 \times 10^8$ m/s-

$$\mu_r = 1$$

$$\varepsilon_r = ?$$

The velocity or speed of EM wave in the medium,

$$v = \frac{1}{\sqrt{\mu_r \varepsilon_r}} \times \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

We have, $\quad c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$

where μ_0 and ε_0 are permeability and permittivity of vacuum respectively.

$\therefore \quad v = \frac{1}{\sqrt{\mu_r \varepsilon_r}} \cdot c$

$$\frac{2 \times 10^8}{3 \times 10^8} = \frac{1}{\sqrt{\mu_r \varepsilon_r}} \qquad [\because C = 3 \times 10^8 \text{ m/s}]$$

$$\frac{2}{3} = \frac{1}{\sqrt{\mu_r \varepsilon_r}}$$

$$\left(\frac{2}{3}\right)^2 = \frac{1}{\mu_r \varepsilon_r} = \frac{1}{1 \times \varepsilon_r}$$

Permittivity, $\quad \varepsilon_r = \frac{9}{4} = 2.25$

41. Let the power of lenses are P_1 and P_2 respectively.

When they are separated by distance d, the equivalent power of the combination,

$$P_{\text{eq}} = P_1 + P_2 - 2dP_1P_2$$

$$= 10 + 10 - 2d\,(10 \times 10)$$

$$= 20 - 100\,d$$

Now, $\quad 20 - 100d < 0$

$\Rightarrow \quad 100d < 20$

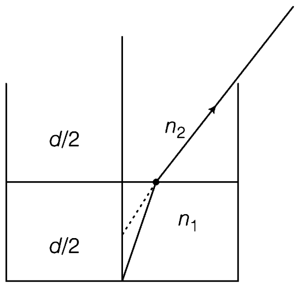
$$d > \frac{1}{5} \text{ m}$$

$\Rightarrow \quad d > \frac{100}{5}$ cm or 20 cm

42. The depth of the vessel = d

Half of the depth $\frac{d}{2}$ of the liquid is filled by liquid of refractive index of n_1.

Upper half, i.e. $\frac{d}{2}$ distance is filled by the liquid of refractive index n_2.



Apparent depth is given by the formula

$$= \frac{d(n_1 + n_2)}{2(n_1 \times n_2)}$$

Apparent depth of lower half

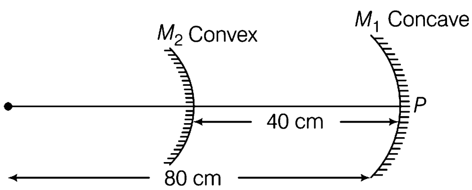
$$= \frac{d}{2}\left(\frac{n_2}{n_1}\right)$$

43. Concave mirror M_1 with radius $r_1 = 60$ cm

Convex mirror M_2 with radius $r_2 = 20$ cm

Focus, $f_1 = \frac{60}{2} = 30$ cm

Focus, $f_2 = \frac{20}{2} = 10$ cm



$$OP = 80 = u$$

From mirror formula,

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

From concave mirror, the image formed by mirror M_1 will be

$$f_1 = -30 \text{ cm}$$
$$u = -80 \text{ cm}$$
$$v = ?$$
$$\frac{1}{-30} = \frac{1}{v} + \frac{1}{-80}$$

$$\frac{1}{v} = \frac{1}{80} - \frac{1}{30} = \frac{30 - 80}{80 \times 30}$$

$$\frac{1}{v} = -\frac{50}{80 \times 30}$$

$$v = -48 \text{ cm}$$

The image will be out of reflecting surface of mirror M_2. So, this will be the final image and it will form on the left of M_1.

44. To minimize the spherical aberration in a reflecting telescope, a parabolic mirror is used as objective.

45. The de-Broglie wavelength associated with proton,

$$\lambda_p = \frac{h}{\sqrt{2\, m_p q_p \cdot V}}$$

where, h = Planck constant

m_p = mass of proton

q_p = charge on proton

V = potential difference

$$\lambda_\alpha = \frac{h}{\sqrt{2 m_\alpha q_\alpha \cdot V}}$$

$$m_\alpha = 4 m_p$$

$$q_\alpha = 2 q_p$$

$$\therefore \quad \frac{\lambda_p}{\lambda_\alpha} = \frac{\frac{h}{\sqrt{2 m_p q_p V}}}{\frac{h}{\sqrt{2 m_\alpha q_\alpha V}}}$$

$$= \sqrt{\frac{2\, m_\alpha q_\alpha V}{2 m_p \cdot q_p V}}$$

$$= \sqrt{\frac{2 \times 4 \cdot m_p \times 2 q_p}{2\, m_p q_p}}$$

$$= \sqrt{4 \times 2} = 2\sqrt{2} : 1$$

46. Lines of Balmer series are emitted by hydrogen atom when electrons jump from the higher orbit to the second orbit $n_1 = 2, n_2 = 3, 4, 5, \ldots$

47. Half-life, $T = 100$ years

We have, $\frac{M}{M_0} = \left(\frac{1}{2}\right)^{t/T}$

$$\left(\frac{M_0 \times \frac{20}{100}}{M_0}\right) = \left(\frac{1}{2}\right)^{t_1/T} \quad \ldots\text{(i)}$$

and

$$\left(\frac{M_0 \times \frac{80}{100}}{M_0}\right) = \left(\frac{1}{2}\right)^{t_2/T} \quad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{\frac{1}{5}}{\frac{4}{5}} = \frac{\left(\frac{1}{2}\right)^{t_1/T}}{\left(\frac{1}{2}\right)^{t_2/T}}$$

$$\frac{1}{4} = \left(\frac{1}{2}\right)^{(t-t_2)/T}$$

$$\left(\frac{1}{2}\right)^2 = \left(\frac{1}{2}\right)^{\Delta t/T}$$

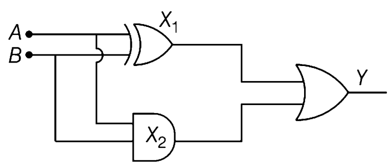
where, $\Delta t = t_1 - t_2$

$$\therefore \quad 2 = \frac{\Delta t}{T}$$

Time, $\Delta t = 2 \times T$

$= 2 \times 100 = 200$ years

48.



Output of $X_1 = A\overline{B} + \overline{A}B$

Output of $X_2 = AB$

Then, $Y = X_1 + X_2$

$Y = A\overline{B} + \overline{A}B + AB$

$Y = A(B + \overline{B}) + \overline{A}B$

$Y = A + \overline{A}B$

$Y = A + B$ $\quad [\because A + \overline{A}B = A + B]$

So, the circuit is equivalent to OR-gate function.

49. Let emitter current is x,

So, from equation we have,

$$\frac{90}{100}x = 10 \text{ mA}$$

$$x = \frac{10 \times 100}{90} \text{ mA}$$

$$= 11.1 \text{ mA}$$

50. We know that transmitted power is given by

$$P_T = P_C\left(1 + \frac{\mu^2}{2}\right)$$

at, $\mu = 0$, $P_{T_0} = P_C$

at $\mu = 1$, $P_{T_1} = \frac{3}{2}P_C$

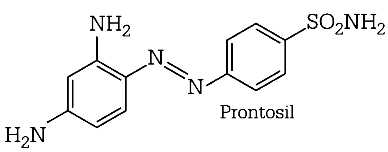
$$\text{Increase in power} = \frac{P_{T_1} - P_{T_0}}{P_{10}} \times 100$$

$$= \frac{\frac{3}{2}P_C - P_C}{P_C} \times 100 = 50\%$$

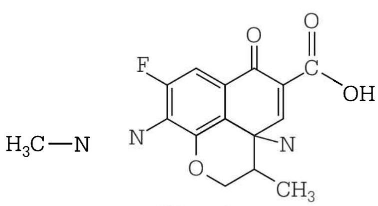
Therefore, the transmitted power is increased by 50%.

Chemistry

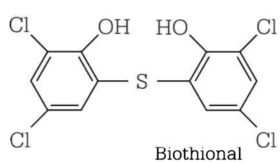
1. The structures of the given antibiotics are:



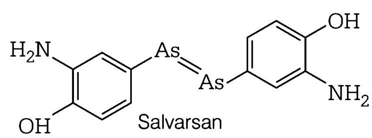
Prontosil



Ofloxacin



Biothional

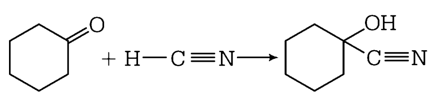


Salvarsan

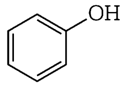
Hence, amongst the given antibiotics only salvarsan contains arsenic.

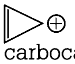
2. Electrophiles are electron defficient species. They may be Lewis acids (in which central atom has incomplete octet), cations or carbocations. Amongst the given species, BF_3 is a Lewis acid (as boron has incomplete octet) and Me_3C^+ is a carbocation. Thus, both acts as electrophiles.

3.



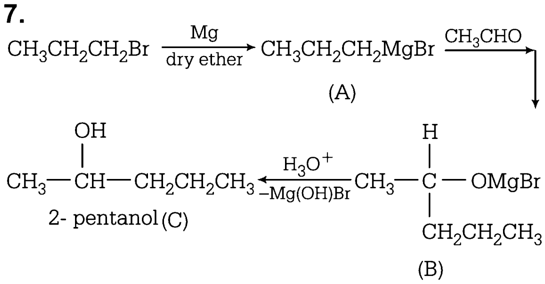
The reaction show the nucleophilio addition of hydrogen cyanide on the carboxyl group. Hence, it is an addition reaction.

4.  is the most acidic nature among the following compounds.

5.  Cyclopropenylcation is most stable ($n = 0$) carbocation as it is aromatic in nature.

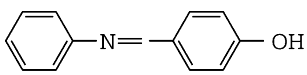
6. Acidic strength increases as the electronegativity of the carbon atom directly attached to —COOH group increases or the hybridisation of the carbon atom directly attached to —COOH changes from $sp^3 \rightarrow sp^2 \rightarrow sp$. Thus, the acedic strength of the given compounds is in the order of

$$\underset{\text{I}}{CH \equiv \overset{sp}{C}COOH} > \underset{\text{II}}{CH_2 = \overset{sp^2}{C}HCOOH} > \underset{\text{III}}{CH_3\overset{sp^3}{C}H_2COOH}$$

7. 

Hence, the product '*C*' is 2-pentanol

8. *p*-hydroxyazobenzene is an orange coloured dye.



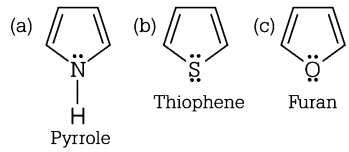
p-hydroxy azobenzene
(Orange dye)

9. Amino acids contain amino ($—NH_2$) and carboxyl ($—COOH$) as functional groups. In aqueous solution, the carboxyl group can lose a proton and amino group can accept a proton, giving rise to a dipolar ion known as Zwitter ion. This is neutral but contains both negative and positive charges.

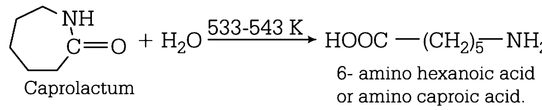
10. $R - X + NaI \xrightarrow{\text{Acetone}} R - I + NaX$
where $X = Cl, Br$

Alkyl iodides are often prepared by alkyl chlorides/bromides by heating with NaI in dry acetone. This reaction is known as Finkelstein reaction. NaCl or NaBr thus formed is precipitated in dry acetone.

11. According to Huckel's rule for aromaticity, planar monocyclic rings containing $(4n + 2)\,\pi$ electrons (where $n = 0, 1, 2, 3$ and so on) should be aromatic. Another requirement for aromaticity is planarity of the ring. In the given compounds, all contains 6π electrons, therefore for them, n comes out be 1, hence, pyrrole, thiophene and furan are all aromatic.



12. Nylon-6 is obtained from the condensation polymerisation of monomer caprolactum on heating with water at high temperature.

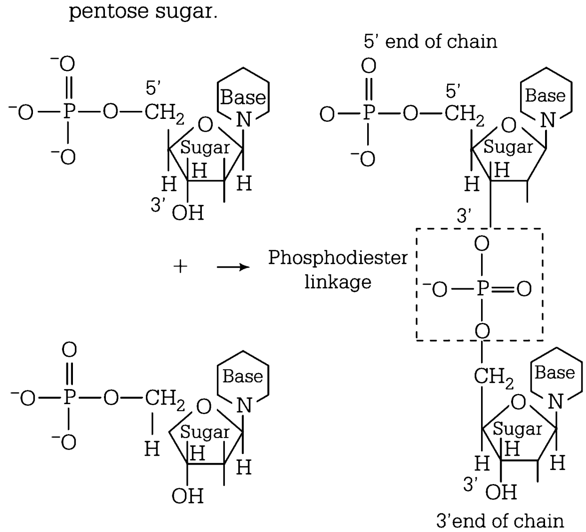


$$HOOC(CH_2)_5NH_2 \xrightarrow[\text{Polymerisation}]{\Delta} \underset{\text{Nylon 6}}{[OC—(CH_2)_5—NH]_n}$$

13. The given reaction is Hell-Volhard Zelinsky (H.V.Z.) reaction. In this reaction, carboxylic acids (except formic acid) react with chlorine or bromine in presence of small quantities of red phosphorus to give α-chloro or α-bromo acids.

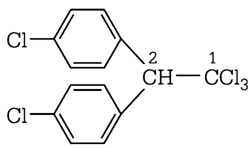
$$R—CH_2—COOH \xrightarrow[(ii)\ H_2O]{(i)\ X_2,\ \text{red P}} \underset{\alpha - \text{halo carboxylic acid}}{R—\underset{X}{\underset{|}{C}H}—COOH} \quad [X = Cl, Br]$$

14. Nucleotides are joined together by phosphodiester linkage between 5′ and 3′ carbon atoms of the pentose sugar.



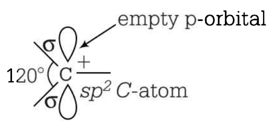
Formation of a dinucletide

15. DDT is a organochlorine compound whose IUPAC is 2,2-di (*p*-chlorophenyl) 1,1,1-trichloroethane.



DDT

16. In S_N1 reaction, the carbocation formation is a slow step. The carbocation formed is a planar, species containing sp^2-hybridised carbon atom. electron deficient and an empty *p*-orbital.



17. Sucrose is dextrorotatory but on hydrolysis it gives an equimolar mixture of dextrorotatory [D−(+)]-glucose and laevorotatory [D−(−)−] fructose. As the laevorotation of fructose (−92°4°) is much more than dextrorotation of glucose (+52.5°), therefore, the resulting mixture becomes laevorotatory. Therefore, the hydrolysis of sucrose is accompanied by a change in sign of optical rotation from dextrorotatory to laevorotatory. Therefore, sucrose is called invert sugar.

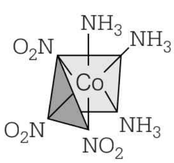
18. The correct electronic configuration is :

$Cr(24) = [Ar]^{18} 3d^5 4s^1$

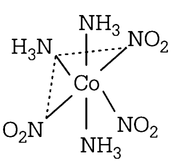
$Eu(63) = [Xe]^{54} 4f^7 6s^2$

$Ti^{2+}(22) = [Ar]^{18} 3d^2 4s^0$

19. $[Co(NH_3)_3(NO_2)_3]$ complexes can also represent (fac) and meridional (mer) isomers. If three donor atoms of the same ligands occupy adjacent positions at the corners of an octahedral face, we have the facial (fac) isomer. When the positions are around the meridian of the octahedron, are get meridional (mer) isomer.

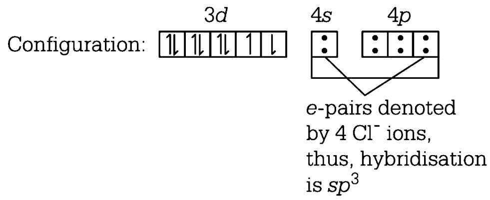


fac-isomer



mer-isomer

20. $[NiCl_4]^{2-}$



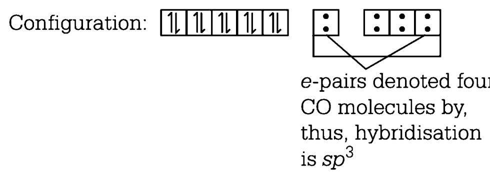
Oxidation state of metal = + 2

Type of hybridisation = sp^3 (tetrahedral)

Number of unpaired electrons = 2

Magnetic nature = Paramagnetic

$[Ni(CO)_4]$



Oxidation state of metal = 0
Type of hybridisation = sp^3 (tetrahedral)

Number of unpaired electrons = 0
Magnetic nature = Diamagnetic
Hence, $[NiCl_4]^{2-}$ is paramagnetic while $[Ni(CO)_4]$ is diamagnetic and both the complexes have tetrahedral geometry.

21. The purple colour of $KMnO_4$ can be attributed to charge transfer transition from O to Mn.

22. Since, the coordination number of Mn^{2+} ion in the complex ion is 4, it will be either tetrahedral (sp^3-hybridisation) or square planar (dsp^2 hybridisation). But the fact that the magnetic moment of the complex ion is 5.9 BM

$$\mu_s = \sqrt{n(n+2)} \Rightarrow 5.9 = \sqrt{n(n+2)} \Rightarrow n = 5$$

∴ The complex has 5 unpaired e^- suggests that it should be tetrahedral in shape rather the square planar because of the presence of five unpaired electrons in the *d*-orbitals.

23. $[CO(NH_3)_5SO_4]Cl$ gives a white precipitate with excess of $AgNO_3$ solution. This is because, when $[CO(NH_3)_5SO_4]Cl$ is dissolved in water, it will give chloride, Cl^- ions in the solution. On addition of $AgNO_3$, a white precipitate of AgCl is obtained.

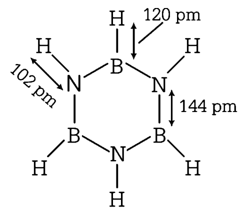
24. Geometrical isomerism is not possible for the complexes with coordination number 2 and 3 and tetrahedral complexes with coordination number 4 because in this case, all the four positions are equivalent. Thus, amongst them, $[Ni(CO_4)]$ will not show geometrical isomerism.

25. The molecule which is linear is N_2O (nitrous oxide).

It is an oxide of nitrogen $N \equiv \overset{+}{N} — O^-$. While others have bent-shape.

26. NO is not an acidic oxide. It is one of the several oxides of nitrogen. It is a colourless gas under standard conditions. Due to the presence of odd electron on N-atoms, NO is very reactive and reacts instantly with O_2 to form NO_2 and with Cl_2, form NOCl.

27. Borazine is a compound of boron, nitrogen and hydrogen i.e. $B_3N_3H_6$ and is called an inorganic benzene, because of the similarity of its structure with benzene.



It has a six membered ring of alternating boron and nitrogen atoms, each of which is further linked to a H-atom.

28. RbCl and $BeCl_2$ have greatest and least ionic character respectively. This is because, larger the cation, more is the ionic character or smaller the cation, greater is the covalent character (Fajan rules).

29. Na is amphoteric in nature. This statement is false. In chemistry amphoteric is a molecule or ion that can react both with an acid as well as a base. But it only reacts with acid to give hydrogen gas. It does not react with base.

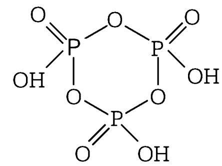
$$2Na + 2HCl \longrightarrow 2NaCl + H_2$$

30. Presence of lone pairs of electrons greatly affects the bond angle. As the number of lone pairs of electrons increases, bond angle decreases.

Thus, the correct order of bond angles,

$$\underset{92.5°}{H_2\ddot{\underset{\cdot\cdot}{S}}} < \underset{107°}{\ddot{N}H_3} < \underset{109°25'}{SiH_4} < \underset{120°}{BF_3}$$

31. The number of P—O—P bonds in cyclic metaphosphoric acid is three.

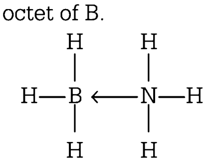


Cyclometa phosphoric acid, $(HPO_3)_3$

32. Due to the presence of lone pair of electrons, trihalides of nitrogen behaves as Lewis bases. However, NF_3 has a little tendency to donate a pair of electrons because of high electronegativity of F lone pairs of N are attracted largely towards F and it would not be available for donatism. The Lewis base strength of other trihalides increases as the electronegativity of halogen decreases i.e. $NF_3 < NCl_3 < NBr_3 < NI_3$. Hence, NF_3 is least basic.

33. As SiF_4 is tetrahedral in shape while SF_4 has a *see-saw* structure so, these pairs is not isostructural species.

34. Since, BF_3 is electron deficient and needs two electrons to complete its octet. It acts as a Lewis acid and reacts with NH_3 which is a Lewis base due to the presence of lone pair of *e* - on N-atom. Acceptance of a pair of electrons from NH_3 completes the octet of B.



Central atom : Boron
Hybridisation : sp^2
Geometry : tetrahedral
In $BF_3 \rightarrow NH_3$. Both N and B contains atoms with sp^3-hybridisation with tetrahedral geometry.

35. Mn in permanganate, MnO_4^- ions, undergose sp^3 hybridisation and hence four oxygen atoms are arranged tetrahedrally around manganese.

Further, Mn in MnO_4^- is in +7 state with $1s^2, 2s^2 2p^6, 3s^2 3p^6 3d^0, 4s^0$ configuration and hence, all electrons are paired, therefore is diamagnetic.

Hence, permanganate ions are tetrahedral and diamagnetic.

36. $Z = \dfrac{V_{rms}}{\lambda}$

$$= \frac{1.83 \times 10^5}{1.783 \times 10^{-5}} = 1.026 \times 10^{10}$$

$$\approx 9.476 \times 10^9 \text{ sec}^{-1}$$

37. Physical adsorption or van der Waals' adsorption involves

(I) Reversible

(III)Low temperature of adsorption, i.e. appreciable only at low temperature below the boiling point of adsorbate.

(IV)Occurs at low temperature

38. In ZnS, anion form FCC, cation occupy alternate tetrahedral voids. Hence, number of ions (or atoms) in a closed packed structure.

39. Given $K_w = 1.10 \times 10^{-14}$

The value of K_w is temperature dependent as it is an equilibrium constant.

The molarity of pure H_2O

$$= \frac{\text{Density}}{M}$$

$$= \left(\frac{1000\ \text{gL}^{-1}}{18.0\ \text{g mol}^{-1}}\right)$$

$$= 55.55 M$$

$$\alpha = \frac{10^{-7}}{C} = \frac{10^{-7}}{55.6}$$

$$= 1.9 \times 10^{-9}$$

40. Degree of dissociation at 25°C

$$\alpha = \frac{\mu^c}{\mu^\infty}$$

$$= \frac{0.0163\ \text{Sm}^{-1}}{390.7 \times 10^{-4}\ \text{Sm}^2}\ \text{mol}^{-1}$$

$$= 0.0472$$

41. $E^\circ_{cell} = E^\circ_{\text{(right hand electrode)}} - E^\circ_{\text{(Left hand electrodes)}}$

$Ag(s) | Ag^+(aq) || Cu^{2+}(aq) | Cu$

E°/V

LHS RHS

Electrode

$Ag^+ + 2e^- \rightleftharpoons Ag$ 0.337

$Cu^{2+} + 2e^- \rightleftharpoons Cu$ 0.799

$E^\circ_{cell} = \text{RHS} - \text{LHS}$

$= 0.799 - 0.337 = 0.462$ Volt

42. $t_{1/2} = 23$ min

$\therefore \quad k = \frac{0.693}{t_{1/2}} = \frac{0.693}{23}$

Further, for a reaction to complete 90% of reaction,

$$t = \frac{2.303}{k}\log\frac{a}{a-0.9a}$$

$$= \frac{2.303}{0.693} \times 23 \log\frac{a}{0.1a}$$

$$= \frac{2.303}{0.693} \times 23 \log 10$$

$$= 76.4 \text{ minutes}$$

43. According to Arrhenius equation

$$\log\frac{K_2}{K_1} = \frac{E_a}{2.303R}\left(\frac{T_2 - T_1}{T_1T_2}\right)$$

$$\log\frac{1\times10^4}{4.5\times10^3} = \frac{58\times1000\,\text{J mol}^{-1}}{2.303\times8.314\,\text{J K}^{-1}\text{mol}^{-1}} \times \left(\frac{T_2 - 274}{274T_2}\right)$$

$\Rightarrow \quad T_2 = 282.7 \approx 283\,\text{K}$

44. $NaCl(s) \longrightarrow NaCl(l)$

$$\Delta S = \frac{\Delta H}{\text{Temp}}$$

$$T = \frac{\Delta H}{\Delta S}$$

$$= \frac{30.4 \times 10^3}{28.4}$$

$$= 1070.4\text{K}$$

45. HCl; molecular weight $= 36.4\text{g/mol}$

$M = (g/mw)/\text{L}$

$M \cdot L = g/mw$

$mw \times m \times L = g$

$g = (36.4\ \text{g/mol})(0.54\ \text{mol/L})$ (155 mL) (1L/1000 mL)

$g = 3.04\ \text{g} \simeq 3.06\ \text{g}$

46. $PCl_5(g) \longrightarrow PCl_3(g) + Cl_2(g)$

The initial vapour density will be same, it would be $M_{PCl_5}/2$

$\therefore$ Initial vapour density

$= (31 + 5 \times 35.5)/2 = 104.25$

Vapour density at equilibrium at 200°C = 70.2

$\therefore$ Total moles at equilibrium

Total moles initial $= 1 + \alpha$

$$= \frac{\text{Vapour density initial}}{\text{Vapour density at equilibrium}}$$

$$= \frac{104 \cdot 25}{70 \cdot 2} = 1 \cdot 485$$

$1 + \alpha = 1 \cdot 485$

$\alpha = 0 \cdot 485$

47. The system initially contains H_2O and solid Bi_2S_3, which dissolves as follows

$$Bi_2S_3(s) \rightleftharpoons 2Bi^{3+}(aq) + 3S^2(aq)$$

Therefore, $K_{sp} = [Bi^{3+}]^2[S^{2-}]^3$

Since, no Bi^{3+} and S^{2-} were present in solution before the Bi_2S_3 dissolved,

$$[Bi^{3+}]_0 = [S^{2-}]_0 = 0$$

Thus, the equilibrium concentration of these ions will be determined by the amount of salt that dissolves to reach equilibrium

$$1 \cdot 0 \times 10^{-15} \text{ mol / L } Bi_2S_3(s) \longrightarrow 2(1 \cdot 0 \times 10^{-15} \text{ mol / L}) \quad Bi^{3+}(aq) + 3(1 \cdot 0 \times 10^{-15} \text{ mol / L}) S^2(aq)$$

The equilibrium reaction are

$$[Bi^{3+}] = [Bi^{3+}]_0 + \text{change} = 0 + 2 \cdot 0 \times 10^{-15} \text{ mol / L}$$

$$[S^{2-}] = [S^{2-}] + \text{change} = 0 + 3 \cdot 0 \times 10^{-15} \text{ mol / L}$$

$$K_{sp} = [Bi^{3-}]^2[S^{2-}]^3 = (2 \cdot 0 \times 10^{-15})^2 (3 \cdot 0 \times 10^{-15})^2$$

$$= 1 \cdot 1 \times 10^{-73}$$

48. At 20° solubility of N_2 in water is 0.015 g/L.

When partial pressure is 580 torr.

According to Henry's law

$$m \propto p$$

$$m = K_H p$$

$$0 \cdot 015 = K_H \times 580$$

$$K_H = \frac{580}{0 \cdot 015}$$

For the solubility of N_2 in H_2O at 20°C

$$= \frac{800}{K_H} = \frac{800}{580} \times 0 \cdot 015$$

$$= 0 \cdot 0207 \text{ g / L}$$

49. If the chemical forces hold the gas molecules to the surface of the adsorbent, the adsorption is termed as chemical adsorption (chemisorption). Chemisorption is irreversible.

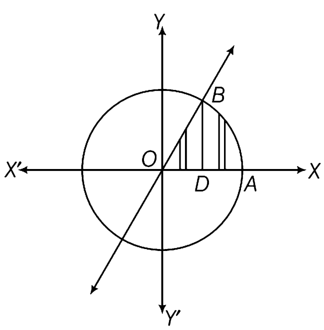
50. In a suspension (e.g. sand stirred into water), the dispersed particles are aggregates of millions of molecules. The diametre of these particles is of the order of 2000 Å $(0 \cdot 2\ \mu)$

Mathematics

1. We have, circle $x^2 + y^2 = 4$ ...(i)

and line, $x = y\sqrt{3}$

$\Rightarrow$ $$y = \frac{x}{\sqrt{3}}$$ ...(ii)



On putting the value of y from Eq. (ii) to Eq. (i), we get

$$x^2 + \left(\frac{x}{\sqrt{3}}\right)^2 = 4$$

$\Rightarrow$ $$x^2 + \frac{x^2}{3} = 4$$

$\Rightarrow$ $$3x^2 + x^2 = 12$$

$\Rightarrow$ $$4x^2 = 12$$

$\Rightarrow$ $$x^2 = 3$$

$\Rightarrow$ $$x = \pm\sqrt{3}$$

So, coordinates of B are $(\sqrt{3}, 1)$. Also, circle cut the X-axis at $A(2, 0)$ and Y-axis at $(0, 2)$.

[$\because$ 2 is radius of circle]

Required area = Area of region $ODBO$ + Area of region $DABD$

$$= \int_0^{\sqrt{3}} y\,(\text{line})dx + \int_{\sqrt{3}}^2 y\,(\text{circle})dx$$

$$= \int_0^{\sqrt{3}} \frac{x}{\sqrt{3}} dx + \int_{\sqrt{3}}^2 \sqrt{4 - x^2}\ dx$$

$$= \frac{1}{\sqrt{3}}\left[\frac{x^2}{2}\right]_0^{\sqrt{3}} + \int_{\sqrt{3}}^2 \sqrt{2^2 - x^2}\ dx$$

$$= \frac{1}{\sqrt{3}}\left[\frac{(\sqrt{3})^2 - 0}{2}\right] + \left[\frac{x}{2}\sqrt{4 - x^2} + \frac{4}{2}\sin^{-1}\left(\frac{x}{2}\right)\right]_{\sqrt{3}}^2$$

$$= \frac{1}{\sqrt{3}} \times \frac{3}{2} + \left[\frac{2}{2}\sqrt{4 - 4} + 2\sin^{-1}\left(\frac{2}{2}\right) - \frac{\sqrt{3}}{2}\sqrt{4 - 3} - 2\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right]$$

$$= \frac{\sqrt{3}}{2} + 0 + 2\sin^{-1}(1) - \frac{\sqrt{3}}{2} - 2\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

$$= 2 \times \frac{\pi}{2} - 2 \times \frac{\pi}{3} = \frac{\pi}{3}$$

2. Let $I = \int_0^1 \frac{e^{5\log_e x} - e^{4\log_e x}}{e^{\log_e x^3} - e^{\log_e x^2}}\,dx$

$$= \int_0^1 \frac{e^{\log_e x^5} - e^{\log_e x^4}}{e^{\log_e x^3} - e^{\log_e x^2}}\,dx$$

$$= \int_0^1 \frac{x^5 - x^4}{x^3 - x^2}\,dx \qquad [\because e^{\log_e x} = x]$$

$$= \int_0^1 \frac{x^4\,(x-1)}{x^2\,(x-1)}\,dx$$

$$= \int_0^1 x^2\,dx$$

$$= \left[\frac{x^3}{3}\right]_0^1 = \frac{1^3}{3} - \frac{0^3}{3}$$

$$\Rightarrow \quad I = \frac{1}{3}$$

3. Let $I = \int \frac{x^3}{1+x^4}\,dx \qquad \ldots\text{(i)}$

Again, let $1 + x^4 = t$

$\Rightarrow \quad 4x^3\,dx = dt$

$\Rightarrow \quad x^3\,dx = \frac{1}{4}dt$

On putting these values in Eq. (i), we get

$$I = \frac{1}{4}\int \frac{dt}{t}$$

$$= \frac{1}{4}\log t + C$$

$$\Rightarrow \quad I = \frac{1}{4}\log(1+x^4) + C \qquad \text{[From Eq. (i)]}$$

4. Let $I = \int_{-2}^{0} \frac{dx}{\sqrt{12 - x^2 - 4x}}$

$$= \int_{-2}^{0} \frac{dx}{\sqrt{12 - (x^2 + 4x + 4) + 4}}$$

$$= \int_{-2}^{0} \frac{dx}{\sqrt{16 - (x+2)^2}}$$

$$= \int_{-2}^{0} \frac{dx}{\sqrt{4^2 - (x+2)^2}}$$

$$= \left[\sin^{-1}\left(\frac{x+2}{4}\right)\right]_{-2}^{0}$$

$$= \sin^{-1}\left(\frac{0+2}{4}\right) - \sin^{-1}\left(\frac{-2+2}{4}\right)$$

$$= \sin^{-1}\left(\frac{1}{2}\right) - \sin^{-1}(0)$$

$$= \frac{\pi}{6} - 0$$

$$= \frac{\pi}{6}$$

5. Let $I = \int \sqrt{16 - 9x^2}\,dx$

$$= \int \sqrt{9\left(\frac{16}{9} - x^2\right)}\,dx$$

$$= 3\int \sqrt{\left(\frac{4}{3}\right)^2 - x^2}\,dx$$

$$= 3\left[\frac{1}{2}x\frac{\sqrt{16-9x^2}}{3} + \frac{\left(\frac{4}{3}\right)^2}{2}\sin^{-1}\left(\frac{x}{\frac{4}{3}}\right)\right] + C$$

$$= 3\left[\frac{1}{2}x\frac{\sqrt{16-9x^2}}{3} + \frac{16}{9\times 2}\sin^{-1}\left(\frac{3x}{4}\right)\right] + C$$

$$= \frac{x}{2}\sqrt{16 - 9x^2} + \frac{8}{3}\sin^{-1}\left(\frac{3x}{4}\right) + C$$

6. Let one number be x and second number be $(10 - x)$.

According to the question,

$P = x(10 - x)$, where P is the product of numbers on differentiating both sides w.r.t. 'x', we get

$$\frac{dP}{dx} = x\,\frac{d}{dx}(10 - x) + (10 - x) \times \frac{d}{dx}(x)$$

$$= x \times (-1) + (10 - x) \times 1$$

$$= -x - x + 10$$

$$\frac{dP}{dx} = -2x + 10 \qquad \ldots\text{(i)}$$

For maximum or minimum value,

$$\frac{dP}{dx} = 0$$

$\Rightarrow \quad -2x + 10 = 0$

$\Rightarrow \quad x = 5$

Now, on differentiating Eq. (i) w.r.t. 'x', we get

$$\frac{d^2P}{dx^2} = -2 < 0$$

$\therefore$ P is maximum at $x = 5$

Hence, numbers are 5, 5.

7. Let $y = \frac{\log x}{x}$

On differentiating both sides w.r.t, x, we get

$$\frac{dy}{dx} = \frac{x\,\frac{d}{dx}\log x - \log x\,\frac{d}{dx}\,x}{x^2}$$

$$= \frac{x \cdot \frac{1}{x} - \log x \cdot 1}{x^2}$$

$$\therefore \quad \frac{dy}{dx} = \frac{1 - \log x}{x^2} \qquad \ldots\text{(i)}$$

For maximum or minimum values,

$$\frac{dy}{dx} = 0$$

$\Rightarrow \quad \frac{1 - \log x}{x^2} = 0$

$\Rightarrow \quad 1 - \log x = 0$

$\Rightarrow \quad \log x = 1$

$\Rightarrow \quad x = e$

Now, differentiate Eq. (i) w.r.t. 'x', we get

$$\frac{d^2 y}{dx^2} = \frac{x^2\left(-\frac{1}{x}\right) - (1 - \log x)\cdot 2x}{x^4}$$

$$= \frac{-x - 2x + 2x \log x}{x^4}$$

$$= \frac{-3x + 2x \log x}{x^4}$$

$$\left(\frac{d^2 y}{dx^2}\right)_{x=e} = \frac{-3e + 2e \log e}{e^4}$$

$$= \frac{-3e + 2e}{e^4}$$

$$= \frac{-e}{e^4} = -\frac{1}{e^3} < 0$$

$\therefore y = \frac{\log x}{x}$ is maximum at $x = e$

$\Rightarrow \quad y_{\max} = \frac{\log e}{e} = \frac{1}{e}$

8. We have, $f(x) = \cos^2 x$

On differentiating both sides w.r.t. 'x', we get

$f'(x) = -2\cos x \sin x = -\sin 2x$

$\because \quad -1 \le \sin 2x \le 1$

$\therefore \quad f'(x) > 0$ for $x \in \left(\frac{-\pi}{2}, 0\right)$

and $\quad f'(x) < 0$ for $x \in \left(0, \frac{\pi}{2}\right)$

$\therefore \quad f(x) = \cos^2 x$ is strictly decreasing on $\left(0, \frac{\pi}{2}\right)$.

9. $\sim p$: I do not take medicine.

$\sim q$: I cannot sleep.

$\sim p \to \sim q$: If I do not take medicine, then I cannot sleep.

10. We have,

$$\begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}^{1/2}$$

$\Rightarrow \quad \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ [Taking square both sides]

$\Rightarrow \quad \begin{bmatrix} \alpha^2 + \beta\gamma & \alpha\beta - \alpha\beta \\ \alpha\gamma - \alpha\gamma & \beta\gamma + \alpha^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \quad \begin{bmatrix} \alpha^2 + \beta\gamma & 0 \\ 0 & \alpha^2 + \beta\gamma \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

On comparing both sides, we get

$\alpha^2 + \beta\gamma = 1$

$\Rightarrow \quad 1 - \alpha^2 - \beta\gamma = 0$

11. Since, $AB = A$ ...(i)

$\Rightarrow \quad (AB)A = A \cdot A = A^2$

$\Rightarrow \quad A(BA) = A^2$

[By associativity of matrix multiplication]

$\Rightarrow \quad AB = A^2$ [$\because BA = B$]

$\Rightarrow \quad A = A^2$ [From Eq. (i)]

Since, $BA = B$ ...(ii)

$\Rightarrow \quad (BA)B = B \cdot B = B^2$

$\Rightarrow \quad B(AB) = B^2$

[By associativity of matrix multiplication]

$\Rightarrow \quad BA = B^2$ [$\because AB = A$]

$\Rightarrow \quad B = B^2$ [From Eq. (ii)]

Thus, AA is equal to A and BB is equal to B.

Hence, A and B are idempotent.

12. We have, $\begin{vmatrix} 2-y & 2 & 3 \\ 2 & 5-y & 6 \\ 3 & 4 & 10-y \end{vmatrix} = 0$

$\Rightarrow (2-y)[(5-y)(10-y) - 24] - 2[2(10-y) - 18] + 3[8 - 3(5-y)] = 0$

$\Rightarrow (2-y)[50 - 15y + y^2 - 24] - 2[20 - 2y - 18] + 3[8 - 15 + 3y] = 0$

$\Rightarrow (2-y)[y^2 - 15y + 26] - 2[2 - 2y] + 3[3y - 7] = 0$

$\Rightarrow 2y^2 - 30y + 52 - y^3 + 15y^2 - 26y - 4 + 4y + 9y - 21 = 0$

$\Rightarrow \quad -y^3 + 17y^2 - 43y + 27 = 0$

$\Rightarrow \quad y^3 - 17y^2 + 43y - 27 = 0$

$\Rightarrow \quad (y-1)(y^2 - 16y + 27) = 0$

$\Rightarrow y - 1 = 0$ or $y^2 - 16y + 27 = 0 \quad \Rightarrow \quad y = 1$

13. We have, $\begin{vmatrix} xp+y & x & y \\ yp+z & y & z \\ 0 & xp+y & yp+z \end{vmatrix} = 0$

$\Rightarrow \quad \begin{vmatrix} xp & x & y \\ yp & y & z \\ -yp-z & xp+y & yp+z \end{vmatrix} = 0$

[Applying $C_1 \to C_1 - C_3$]

$$\Rightarrow \begin{vmatrix} 0 & x & y \\ 0 & y & z \\ -z-xp^2 & xp+y & yp+z \end{vmatrix} = 0$$

[Applying $C_1 \to C_1 - pC_2$]

$$\Rightarrow \quad 0 - 0 + (-z - xp^2)[xz - y^2] = 0$$

[Expanding along C_1]

$$\Rightarrow \quad -(z - xp^2)[xz - y^2] = 0$$

$$\Rightarrow \quad z - xp^2 = 0 \text{ or } xz - y^2 = 0$$

$$\therefore \quad y^2 = xz$$

Hence, x, y, z are in GP.

14. Let S be the sample space and E be that event of favourable cases.

$$\therefore \quad n(S) = {}^{2n+1}C_3$$

$$= \frac{(2n+1)2n(2n-1)}{1 \cdot 2 \cdot 3} = \frac{n(4n^2-1)}{3}$$

Let three numbers a, b, c be drawn where $a < b < c$ and given a, b, c be in AP.

$$\therefore \quad 2b + a + c \quad \ldots\text{(i)}$$

It is clear from Eq. (i), a and c both are even or odd.

Out of $(2n+1)$ tickets consecutively numbers either $(n+1)$ of them will be odd and n of them will be even, or $(n+1)$ of them will be even and n of them will be odd.

$$\therefore \quad n(E) = {}^{n+1}C_2 + {}^nC_2$$

$$= \frac{(n+1)n}{2} + \frac{n(n-1)}{2} = n^2$$

$\therefore$ Required probability $P(E) = \frac{n(E)}{n(S)}$

$$= \frac{n^2}{\frac{n(4n^2-1)}{3}} = \frac{3n}{4n^2-1}$$

15. Let number of item manufactured by machine $M_1 = 200$

According to the question, number of items manufactured by M_1 is twice of number of item manufactured by machine M_2.

$\therefore$ Number of item manufactured by machine $M_2 = \frac{200}{2} = 100$

$\because$ 4% of the items produced by machine M_1 and 3% of the items produced by machine M_2 are defective.

$\therefore$ Number of defective items produced by machine $M_1 = 200 \times 4\% = 200 \times \frac{4}{100} = 8$

and number of defective items produced by machine

$M_2 = 100 \times 3\% = 100 \times \frac{3}{100} = 3$

$\therefore$ Total number of items produced by machine M_1 and $M_2 = 200 + 100 = 300$

and total number of defective items produced by M_1 and $M_2 = 8 + 3 = 11$

$\therefore$ Probability that the selected item is defective

$$= \frac{\text{Total number of defective items produced by machine } M_1 \text{ and } M_2}{\text{Total number of items produced by machine } M_1 \text{ and } M_2}$$

$$= \frac{11}{300}$$

16. We know that,

$$P(A \cup \overline{B}) = P(A) + P(\overline{B}) - P(A \cap \overline{B})$$

$$= P(A) + [1 - P(B)] - P(A) \cdot P(\overline{B})$$

$$= P(A) + [1 - P(B)] - P(A)[1 - P(B)]$$

$$\Rightarrow \quad 0.8 = P(A) + \left(1 - \frac{2}{7}\right) - P(A)\left(1 - \frac{2}{7}\right) \quad \text{[Given]}$$

$$\Rightarrow \quad \frac{4}{5} = P(A)\left[1 - 1 + \frac{2}{7}\right] + \frac{5}{7}$$

$$\Rightarrow \quad \frac{4}{5} - \frac{5}{7} = P(A) \cdot \frac{2}{7}$$

$$\Rightarrow \quad \frac{28-25}{35} = P(A) \cdot \frac{2}{7}$$

$$\Rightarrow \quad P(A) = \frac{3}{35} \cdot \frac{7}{2}$$

$$= \frac{3}{5 \times 2}$$

$$= 0.3$$

17. We have, $P(A^C \cap B) = \frac{2}{15}$ and $P(A \cap B^C) = \frac{1}{6}$

$$\Rightarrow P(A^C) \cdot P(B) = \frac{2}{15} \text{ and } P(A) \cdot P(B^C) = \frac{1}{6}$$

$$\Rightarrow [1 - P(A)] \cdot P(B) = \frac{2}{15} \text{ and } P(A) \cdot [1 - P(B)] = \frac{1}{6}$$

$$\Rightarrow [1 - P(A)] \cdot P(B) = \frac{2}{15} \text{ and } P(A) = \frac{1}{6[1 - P(B)]}$$

$$\therefore \quad \left[1 - \frac{1}{6[1 - P(B)]}\right] \cdot P(B) = \frac{2}{15}$$

$$\Rightarrow \quad \frac{[6 - 6P(B) - 1]}{6[1 - P(B)]} \cdot P(B) = \frac{2}{15}$$

$$\Rightarrow \quad 15[5 - 6P(B)]P(B) = 2 \times 6[1 - P(B)]$$

$$\Rightarrow \quad 75P(B) - 90P(B)^2 = 12 - 12P(B)$$

Let $P(B) = x$

$$\therefore \quad 75x - 90x^2 = 12 - 12x$$

$$\Rightarrow \quad 90x^2 - 75x - 12x + 12 = 0$$
$$\Rightarrow \quad 90x^2 - 87x + 12 = 0$$
$$\Rightarrow \quad 90x^2 - 15x - 72x + 12 = 0$$
$$\Rightarrow \quad 15x\,(6x - 1) - 12(6x - 1) = 0$$
$$\Rightarrow \quad (6x - 1)\,(15x - 12) = 0$$
$$\Rightarrow \quad 6x - 1 = 0 \text{ or } 15x - 12 = 0$$
$$\Rightarrow \quad x = \frac{1}{6} \text{ or } x = \frac{12}{15}$$
$$\Rightarrow \quad x = \frac{1}{6} \text{ or } x = \frac{4}{5}$$

18. For coplanarity $[a, b, c] = 0$

$$\therefore \quad \begin{vmatrix} 2 & -1 & 1 \\ 1 & 2 & -3 \\ 3 & \lambda & 5 \end{vmatrix} = 0$$
$$\Rightarrow \quad 2[10 + 3\lambda] + 1[5 + 9] + 1[\lambda - 6] = 0$$
$$\Rightarrow \quad 20 + 6\lambda + 14 - 6 + \lambda = 0$$
$$\Rightarrow \quad 28 + 7\lambda = 0$$
$$\Rightarrow \quad \lambda = -4$$

19. Equation of all parabolas whose axis of symmetry is parallel to X-axis is $(y - y_0)^2 = \pm 4a(x - x_0)$, where a, x_0 and y_0 are constant ...(i)

From Eq. (i) has 3 constant.

$\therefore$ Order of its differential equation is 3.

20. We have, $y\dfrac{dy}{dx} + x^3\left(\dfrac{d^2y}{dx^2}\right)^3 + xy = \cos x$

Order, $p = 2$ and degree, $q = 3$

$\therefore \quad p < q \qquad [\because\ 2 < 3]$

21. We have,

$$x^2 + y^2 + 4x + 6y + 13 = 0$$
$$\Rightarrow \quad (x^2 + 4x + 4) - 4 + (y^2 + 6y + 9) - 9 + 13 = 0$$
$$\Rightarrow \quad (x + 2)^2 + (y + 3)^2 - 13 + 13 = 0$$
$$\Rightarrow \quad (x + 2)^2 + (y + 3)^2 = 0$$

which is equation of a point circle.

22. We have, $ty = x + t^2$

$$\Rightarrow \quad ty - x = t^2 \qquad \text{...(i)}$$
$$\text{and} \quad y + tx = 2t + t^3 \qquad \text{...(ii)}$$

On solving Eqs. (i) and (ii), we get

$$y = 2t \text{ and } x = t^2$$
$$\Rightarrow \quad t = \pm\sqrt{x}$$
$$\text{Now,} \quad y = \pm 2\sqrt{x}$$
$$y^2 = 4x \qquad \text{[Squaring both sides]}$$

23. We have,

$$4y^2 + 12x - 12y + 39 = 0$$
$$\Rightarrow [(2y)^2 - 2 \times 2y \times 3 + 9] - 9 + 12x + 39 = 0$$
$$\Rightarrow \quad (2y - 3)^2 + 12x + 30 = 0$$
$$\Rightarrow \quad (2y - 3)^2 = -12x - 30$$
$$\Rightarrow \quad 4\left(y - \frac{3}{2}\right)^2 = -12\left(x + \frac{5}{2}\right)$$

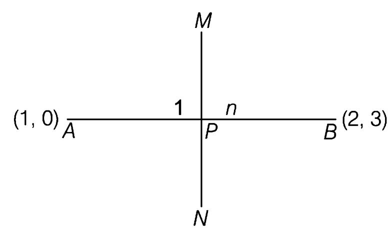
On comparing this equation with $Y_n^2 = -4aX$, we get

$$a = \frac{3}{4}$$

Now, equation of directrix is $X = a$

$$\therefore \quad x + \frac{5}{2} = \frac{3}{4}$$
$$\Rightarrow \quad x = \frac{3}{4} - \frac{5}{2}$$
$$\Rightarrow \quad x = \frac{3 - 10}{4}$$
$$\Rightarrow \quad x = -\frac{7}{4}$$

24. Slope of $AB = \dfrac{3 - 0}{2 - 1} = \dfrac{3}{1}$



$\therefore$ Slope of $MN = -\dfrac{1}{3}$

Now, coordinates of

$$P \equiv \left(\frac{2 \times 1 + 1 \times n}{n + 1}, \frac{3 \times 1 + 0 \times n}{n + 1}\right)$$

[By section formula]

$$\equiv \left(\frac{2 + n}{n + 1}, \frac{3}{n + 1}\right)$$

$\therefore$ Required equation is given as

$$\left(y - \frac{3}{n + 1}\right) = -\frac{1}{3}\left(x - \frac{2 + n}{n + 1}\right)$$
$$\Rightarrow \quad 3y - \frac{9}{n + 1} = -x + \frac{2 + n}{n + 1}$$
$$\Rightarrow \quad 3y + x = \frac{11 + n}{n + 1}$$

25. Coordinates of centroid of ΔPQR

$$\equiv\left(\frac{2a-4+8}{3}, \frac{2+3b+14}{3}, \frac{6-10+2c}{3}\right)$$

But it is given that origin is the centroid of ΔPQR

$$\therefore \quad (0, 0, 0) = \left(\frac{2a+4}{3}, \frac{3b+16}{3}, \frac{2c-4}{3}\right)$$

On comparing both sides, we get

$$\frac{2a+4}{3} = 0, \frac{3b+16}{3} = 0 \text{ and } \frac{2c-4}{3} = 0$$

$$\Rightarrow \quad 2a = -4, 3b = -16 \text{ and } 2c = 4$$

$$\Rightarrow \quad a = -2, b = -\frac{16}{3} \text{ and } c = 2$$

$$\therefore \quad (a, b, c) \equiv (-2, -\frac{16}{3}, 2)$$

26. Let θ be the angle between the lines with direction ratios 4, − 3, 5 and 3, 4, 5.

$$\therefore \cos\theta = \frac{4\times 3 - 3\times 4 + 5\times 5}{(\sqrt{4^2+(-3)^2+5^2})(\sqrt{3^2+4^2+5^2})}$$

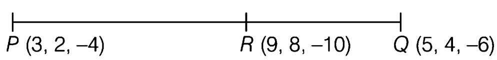
$$= \frac{12-12+25}{\sqrt{16+9+25}\sqrt{9+16+25}}$$

$$= \frac{25}{(\sqrt{50})^2} = \frac{25}{50} = \frac{1}{2}$$

$$\Rightarrow \quad \cos\theta = \cos\frac{\pi}{3}$$

$$\Rightarrow \quad \theta = \frac{\pi}{3}$$

27. Let Q divides PR externally in $K : 1$.



By external section formula, we get

$$5 = \frac{9\times K - 1\times 3}{K-1}$$

$$\Rightarrow \quad 5K - 5 = 9K - 3$$

$$\Rightarrow \quad 5K - 9K = -3 + 5 \quad \Rightarrow \quad -4K = 2$$

$$\Rightarrow \quad K = -\frac{2}{4}$$

$$\Rightarrow \quad K = -\frac{1}{2}$$

Hence, Q divides PR in the ratio 1:2 (externally).

28. We know that, AM $\geq$ GM

$$\frac{9^x + 9^{1-x}}{2} \geq \sqrt{9^x \cdot 9^{1-x}}$$

$$9^x + 9^{1-x} \geq 2\sqrt{9} = 6$$

29. We have,

$$1^2 + (1^2 + 2^2) + (1^2 + 2^2 + 3^2) + \ldots\ldots \qquad \ldots\text{(i)}$$

Let T_n be the nth term of Eq. (i), then

$$T_n = 1^2 + 2^2 + 3^2 + \ldots\ldots + n^2 = \Sigma n^2$$

$$= \frac{n(n+1)(2n+1)}{6}$$

$$= \frac{n(2n^2+3n+1)}{6}$$

$$= \frac{2n^3}{6} + \frac{3n^2}{6} + \frac{n}{6}$$

$$\Rightarrow \quad T_n = \frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}$$

Sum of n terms, $S_n = \Sigma T_n$

$$\therefore S_n = \frac{1}{3}\Sigma n^3 + \frac{1}{2}\Sigma n^2 + \frac{1}{6}\Sigma n$$

$$= \frac{1}{3}\left[\frac{n(n+1)}{2}\right]^2 + \frac{1}{2}\frac{n(n+1)(2n+1)}{6} + \frac{1}{6}\frac{n(n+1)}{2}$$

$$= \frac{1}{12}[n^2(n+1)^2 + n(n+1)(2n+1) + n(n+1)]$$

$$= \frac{1}{12}n(n+1)[n(n+1) + 2n + 1 + 1]$$

$$= \frac{1}{12}n(n+1)[n^2 + n + 2n + 2]$$

$$= \frac{1}{12}n(n+1)[n(n+1) + 2(n+1)]$$

$$= \frac{1}{12}n(n+1)[(n+1)(n+2)]$$

$$= \frac{1}{12}n(n+1)^2(n+2)$$

30. We have, $x^y = e^{x-y}$

On taking log both sides, we get

$$\log x^y = \log e^{x-y}$$

$$\Rightarrow \quad y\log x = x - y \qquad \ldots\text{(i)}$$

$$[\because \log m^n = n\log m \text{ and } \log e^m = m]$$

On differentiating both sides w.r.t. 'x', we get

$$y\cdot\frac{1}{x} + \log x\left(\frac{dy}{dx}\right) = 1 - \frac{dy}{dx}$$

$$\Rightarrow \quad (\log x)\frac{dy}{dx} + \frac{dy}{dx} = 1 - \frac{y}{x}$$

$$\Rightarrow \quad \frac{dy}{dx}(\log x + 1) = \frac{x-y}{x}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{y\log x}{x(\log x + 1)} \qquad \text{[From Eq. (i)]}$$

$$= \frac{\log x}{(\log x + 1)^2}$$

$$\left[\because \text{From Eq. (i) } \frac{y}{x} = \frac{1}{\log x + 1}\right]$$

31. We have,

$$x^2 + y^2 = t - \frac{1}{t} \text{ and } x^4 + y^4 = t^2 + \frac{1}{t^2} \quad \dots\text{(i)}$$

Now, $\left(t - \frac{1}{t}\right)^2 = t^2 + \frac{1}{t^2} - 2t \times \frac{1}{t}$

$[\because (a-b)^2 = a^2 + b^2 - 2ab]$

$\Rightarrow \quad (x^2 + y^2)^2 = x^4 + y^4 - 2$ [From Eq. (i)]

$\Rightarrow \quad x^4 + y^4 + 2x^2y^2 = x^4 + y^4 - 2$

$[\because (a+b)^2 = a^2 + b^2 + 2ab]$

$\Rightarrow \quad 2x^2y^2 = -2$

$\Rightarrow \quad x^2y^2 = -1 \quad \dots\text{(ii)}$

On differentiating both sides in Eq. (ii), we get

$$2xy^2 + x^2 \times 2y\frac{dy}{dx} = 0$$

$\Rightarrow \quad 2\left[xy^2 + x^2y\frac{dy}{dx}\right] = 0$

$\Rightarrow \quad x^2y\frac{dy}{dx} = -xy^2$

$\Rightarrow \quad x^3y\frac{dy}{dx} = -x^2y^2$

[Multiplying both sides by x]

$\Rightarrow \quad x^3y\frac{dy}{dx} = -(-1)$ [From Eq. (ii)]

$\Rightarrow \quad x^3y\frac{dy}{dx} = 1$

32. We have,

$$\lim_{x \to 0} \frac{\log(3+x) - \log(3-x)}{x} = K \quad \left[\frac{0}{0} \text{ form}\right]$$

$\Rightarrow \quad \lim_{x \to 0} \frac{\frac{1}{3+x} - \frac{-1}{3-x}}{1} = K$ [By L' Hospital rule]

$\Rightarrow \quad \lim_{x \to 0} \frac{3 - x + 3 + x}{(3+x)(3-x)} = K$

$\Rightarrow \quad \lim_{x \to 0} \frac{6-0}{9-x^2} = K$

$\Rightarrow \quad \frac{6}{9-0} = K$

$\Rightarrow \quad K = \frac{6}{9}$

$\Rightarrow \quad K = \frac{2}{3}$

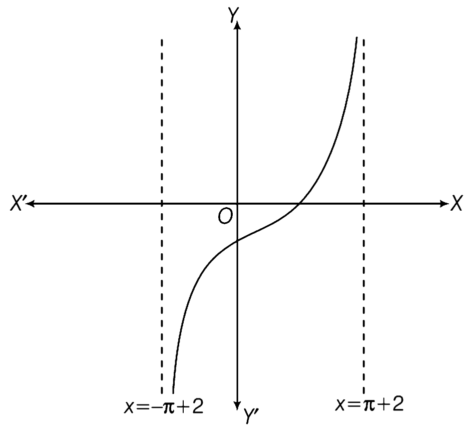
33. Given, $f(x) = \frac{x}{2} - 1$

$\therefore \quad \tan(f(x)) = \tan\left(\frac{x}{2} - 1\right)$

and $\quad \frac{1}{f(x)} = \frac{1}{\frac{x}{2} - 1} = \frac{2}{x-2}$

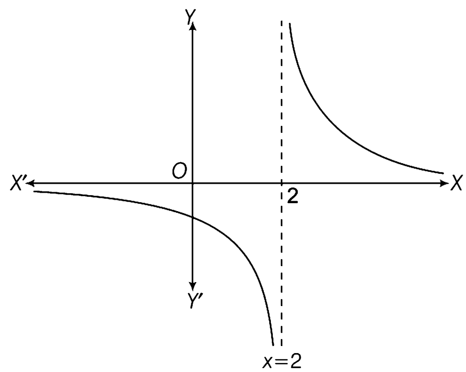
By using graph transformation method we can draw the graph of $\tan(f(x))$ and $\frac{1}{f(x)}$ as follows :

Graph of tan f(x)



Graph at $\tan[f(x)]$ in $x \in (-\pi + 2, \pi + 2)$

Graph of $\frac{1}{f(x)}$



From the above graphs of $\tan(f(x))$ and $\frac{1}{f(x)}$, we can easily observe that $\tan(f(x))$ is continuous in $x \in [0, \pi]$ but $\frac{1}{f(x)}$ is not continuous in $x \in [0, \pi]$.

34. We have, $\quad y = \tan^{-1}(\sqrt{1+x^2} - x)$

$\Rightarrow \quad \tan y = \sqrt{1+x^2} - x \quad \dots\text{(i)}$

On differentiating both sides w.r.t. 'x', we get

$$\sec^2 y\frac{dy}{dx} = \frac{1}{2}(1+x^2)^{\frac{1}{2}-1} \times 2x - 1$$

$\Rightarrow \quad (1 + \tan^2 y)\frac{dy}{dx} = \frac{1}{2}(1+x^2)^{-1/2} \cdot 2x - 1$

$[\because \sec^2\theta = 1 + \tan^2\theta]$

$$\Rightarrow [1 + (\sqrt{1+x^2} - x)^2] \frac{dy}{dx} = \frac{x}{\sqrt{1+x^2}} - 1$$

$$\Rightarrow [1 + 1 + x^2 + x^2 - 2x\sqrt{1+x^2}] \frac{dy}{dx} = \left(\frac{x - \sqrt{1+x^2}}{\sqrt{1+x^2}} \right)$$

$$\Rightarrow [2 + 2x^2 - 2x\sqrt{1+x^2}] \frac{dy}{dx} = \frac{x - \sqrt{1+x^2}}{\sqrt{1+x^2}}$$

$$\Rightarrow 2[1 + x^2 - x\sqrt{1+x^2}] \frac{dy}{dx} = \frac{x - \sqrt{1+x^2}}{\sqrt{1+x^2}}$$

$$\Rightarrow 2[(\sqrt{1+x^2})^2 - x\sqrt{1+x^2}] \frac{dy}{dx} = \frac{x - \sqrt{1+x^2}}{\sqrt{1+x^2}}$$

$$\Rightarrow 2\sqrt{1+x^2}\,[\sqrt{1+x^2} - x] \frac{dy}{dx} = -\frac{(\sqrt{1+x^2} - x)}{\sqrt{1+x^2}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-(\sqrt{1+x^2} - x)}{\sqrt{1+x^2} \cdot 2\sqrt{1+x^2}\,(\sqrt{1+x^2} - x)}$$

$$= \frac{-1}{2(1+x^2)}$$

35. We have, $y = \sec(\tan^{-1} x)$

$$= \sec[\sec^{-1}(\sqrt{1+x^2})]$$

$$[\because \tan^{-1}\theta = \sec^{-1}\sqrt{(1+\theta^2)}]$$

$$\Rightarrow \quad y = \sqrt{1+x^2}$$

On differentiating both sides w.r.t 'x', we get

$$\frac{dy}{dx} = \frac{1}{2}(1+x^2)^{-1/2} \frac{d}{dx}(1+x^2)$$

$$= \frac{1}{2} \frac{1}{\sqrt{1+x^2}} \cdot 2x$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{x}{\sqrt{(1+x^2)}}$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{x=1} = \frac{1}{\sqrt{1+1^2}} = \frac{1}{\sqrt{2}}$$

36. We have, $f(x) = x^{\alpha} \log x$ and $f(x) = 0$

For Rolle's theorem, in [0, 1]

$$\Rightarrow \quad f(0) = f(1) = 0$$

$\because$ The function has to be continuous in [0, 1].

$$\Rightarrow \quad f(0) = \lim_{x \to 0^+} f(x) = 0$$

$$= \lim_{x \to 0} x^{\alpha} \log x = 0$$

$$= \lim_{x \to 0} \frac{\log x}{x^{-\alpha}} = 0$$

Applying L'Hospital rule, we get

$$\lim_{x \to 0} \frac{\frac{1}{x}}{-\alpha x^{-\alpha-1}} = 0 \quad \Rightarrow \quad \lim_{x \to 0} \frac{-x^{\alpha}}{\alpha} = 0$$

Here, x is very near to zero.

$\therefore$ From given option,

For $\alpha = \frac{1}{2}$, i.e. $(x)^{1/2}$ is near to zero.

37. An n-tuple $(x_1, x_2, x_3, \ldots\ldots, x_n)$ which statisfies all the constraints of a linear programming problem and for which the objective function is maximum is called an optimal solution.

38. Given, minimize $f = 2x_1 - x_2$

$$x_1 \geq 0, x_2 \geq 0$$
$$x_1 + x_2 \geq 5$$
$$-x_1 + x_2 \leq 1$$

and $\quad x_1 + x_2 \leq 1$

Graph of $x_1 + x_2 = 5$

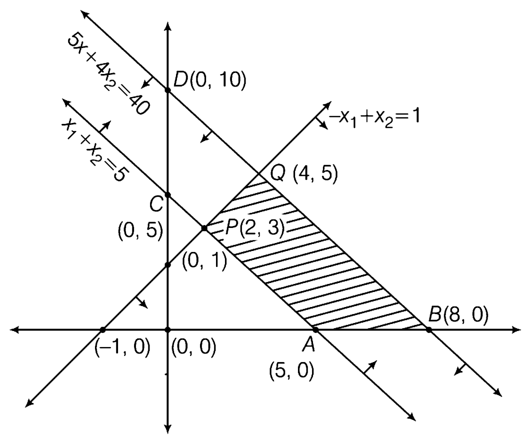
x_1	0	5
x_2	5	0

Putting (0, 0) in the inequality $x_1 + x_2 \geq 5$

$\Rightarrow \quad 0 + 0 \geq 5$

$\Rightarrow \quad 0 \geq 5 \quad$ (which is false)

$\Rightarrow$ Half plane away from origin.



Graph of $-x_1 + x_2 = 1$

$\Rightarrow \quad x_1 - x_2 = 1$

x_1	0	-1
x_2	1	0

Putting (0, 0) in the inequality $-x_1 + x_2 \leq 1$

$\Rightarrow 0 + 0 \leq 1 \Rightarrow 0 \leq 1 \quad$ (which is true)

Half plane towards the origin.

and graph of $5x_1 + 4x_2 = 40$

x_1	0	8
x_2	10	0

Putting (0, 0) in the inequality $5x_1 + 4x_2 \le 40$

$\Rightarrow \quad 0 + 0 \le 40$

$\Rightarrow \quad 0 \le 40$ (which is true)

$\Rightarrow$ Half plane towards the origin.

Here, we obtain feasible region $ABPQ$s Coordinates of Q.

Solving $x_1 - x_2 = -1$

$5x_1 + 4x_2 = 40$

We get $x_1 = 4, x_2 = 5, Q \equiv (4, 5)$,

Coordinates of P.

Solving $x_1 - x_2 = -1$

and $x_1 + x_2 = 5$

We get $x_1 = 2, x_2 = 3$

$P \equiv (2, 3)$

Corner point	$Z = 2x_1 - x_2$
$A(5, 0)$	10
$B(8, 0)$	16
$P(2, 3)$	1 (minimum)
$Q(4, 5)$	3

39. For least negative integral value of λ, $n = 1$

$$49^1 + 16 \times 1 + \lambda = 64$$

$\Rightarrow \quad 49 + 16 + \lambda = 64$

$\Rightarrow \quad 65 + \lambda = 64$

$\Rightarrow \quad \lambda = -1$

40. Let n be the number of sides of given polygon.

$\therefore$ Polygon with n sides $= \frac{n(n-3)}{2}$ diagonal

$\Rightarrow \quad \frac{n(n-3)}{2} = 44$ [given]

$\Rightarrow \quad n^2 - 3n = 88$

$\Rightarrow \quad n^2 - 3n - 88 = 0$

$\Rightarrow \quad n^2 - 11n + 8n - 88 = 0$

$\Rightarrow \quad n(n-11) + 8(n-11) = 0$

$\Rightarrow \quad (n+8)(n-11) = 0$

$\Rightarrow \quad n + 8 = 0$ or $n - 11 = 0$

$\Rightarrow \quad n = -8$ or $n = 11$

But side cannot be negative.

$\therefore \quad n = 11$

41. Given, $\left(x\cos\alpha + \frac{\sin\alpha}{x}\right)^{10}$

$$= \frac{1}{x^{10}}(x^2\cos\alpha + \sin\alpha)^{10}$$

General term of the expansion

$$T_{r+1} = \frac{1}{x^{10}} \times {}^{10}C_r(x^2\cos\alpha)^{10-r}(\sin\alpha)^r$$

$$= \frac{1}{x^{10}} \times {}^{10}C_r(x^2)^{10-r}(\cos\alpha)^{10-r}(\sin\alpha)^r$$

For this term to be independent of x,

$10 - r = 5$

$\Rightarrow \quad r = 5$

$\therefore \quad T_{5+1} = \frac{1}{x^{10}} \times {}^{10}C_r x^{10} \cdot \cos^5\alpha \cdot \sin^5\alpha$

$$= {}^{10}C_r(\cos\alpha \cdot \sin\alpha)^5$$

$\therefore \quad = {}^{10}C_r\left(\frac{2\sin\alpha \cdot \cos\alpha}{2}\right)^5$

$$= \frac{1}{2^5}\,{}^{10}C_r(\sin 2\alpha)^5$$

$\therefore$ Greatest value of T_6 (term independent of x) will be $\frac{1}{2^5}\,{}^{10}C_5 \times (1) = \left(\frac{1}{2}\right)^5 \times {}^{10}C_r$

42. We have,

$$\left(\frac{1+i}{1-i}\right)^{100} = \left[\frac{(1+i)(1+i)}{(1-i)(1+i)}\right]^{100}$$

$$= \left[\frac{(1+i)^2}{1^2 - i^2}\right]^{100}$$

$[\because (a-b)(a+b) = a^2 - b^2]$

$$= \left[\frac{(1+i)^2}{1+1}\right]^{100} \quad [\because i^2 = -1]$$

$$= \left[\frac{1 + i^2 + 2i}{2}\right]^{100}$$

$[\because (a+b)^2 = a^2 + b^2 + 2ab]$

$$= \left[\frac{1 - 1 + 2i}{2}\right]^{100} \quad [\because i^2 = -1]$$

$$= \left[\frac{2i}{2}\right]^{100}$$

$$= (i)^{100} = (i^2)^{50}$$

$$= (-1)^{50} \quad [\because i^2 = -1]$$

$$= 1 \quad [\because (-1)^{\text{even}} = 1]$$

43. We have, $\sum_{n=1}^{13}[i^n + i^{n+1}]$

$$= (i^1 + i^2) + (i^2 + i^3) + (i^3 + i^4) + \ldots\ldots + (i^{13} + i^{14})$$

$$= (i + i^2 + i^3 + ...+ i^{13}) + (i^2 + i^3 + i^4 + ...+ i^{14})$$
$$= i(1 + i + i^2 + ...+ i^{12}) + i^2(1 + i + i^2 + ...+ i^{12})$$
$$= (i + i^2)(1 + i + i^2 + ... + i^{12})$$
$$= (i - 1)\left[1\frac{(i^{13} - 1)}{(i - 1)}\right] \qquad [\because i^2 = -1]$$
$$= i^{12+1} - 1$$
$$= i^{12} \cdot i - 1$$
$$= (i^2)^6 \cdot i - 1$$
$$= (-1)^6 \cdot i - 1 = i - 1 \qquad [\because i^2 = -1]\ [\because (-1)^{\text{even}} = 1]$$

44. Here, number of hours $= n$

For scheme I give the electrician better wages

$$600 + 50n > 170n$$
$$\Rightarrow \quad 600 > 170n - 50n$$
$$\Rightarrow \quad 600 > 120n$$
$$\Rightarrow \quad \frac{600}{120} > n$$
$$\Rightarrow \quad 5 > n$$
$$\Rightarrow \quad n < 5$$

$\therefore$ For $n < 5$, scheme I will give better wages to electrician.

45. We have, $X = \{4^n - 3n - 1 | x \in N\}$

Now, consider

$$4^n - 3n - 1 = (1 + 3)^n - 3n - 1$$
$$= 3^n + {}^nC_{n-1}3^{n-1} + ... + {}^nC_2 3^2 + {}^nC_1 3 + {}^nC_0 - 3n - 1$$

(Using Binomial expansion)

$$= 3^2(3^{n-2} + {}^nC_{n-1}3^{n-3} + ... + {}^nC_2)$$
$$= 9(3^{n-2} + {}^nC_{n-1}3^{n-3} + ... + {}^nC_2)$$

$\Rightarrow$ Set X has natural numbers which are multiples of 9 (but not all)

Clearly, set Y has all multiples of 9.

$$\Rightarrow \quad X \subset Y$$

46. We have, $\dfrac{\sin^2 3\theta}{\sin^2 \theta} - \dfrac{\cos^2 3\theta}{\cos^2 \theta}$

$$= \left[\frac{\sin 3\theta}{\sin\theta}\right]^2 - \left[\frac{\cos 3\theta}{\cos\theta}\right]^2$$
$$= \left[\frac{3\sin\theta - 4\sin^3\theta}{\sin\theta}\right]^2 - \left[\frac{4\cos^3\theta - 3\cos\theta}{\cos\theta}\right]^2$$

$[\because \sin 3A = 3\sin A - 4\sin^3 A$ and $\cos 3A = 4\cos^3 A - 3\cos A]$

$$= [3 - 4\sin^2\theta]^2 - [4\cos^2\theta - 3]^2$$
$$= [3 - 4(1 - \cos^2\theta)]^2 - [4\cos^2\theta - 3]^2$$
$$= [3 - 4 + 4\cos^2\theta]^2 - [(4\cos^2\theta - 1) - 2]^2$$
$$= (4\cos^2\theta - 1)^2 - [(4\cos^2\theta - 1) - 2]^2$$
$$= (4\cos^2\theta - 1)^2 - [(4\cos^2\theta - 1)^2 + 2^2 - 2 \times 2(4\cos^2\theta - 1)]$$

$[\because (a - b)^2 = a^2 + b^2 - 2ab]$

$$= (4\cos^2\theta - 1)^2 - (4\cos^2\theta - 1)^2 - 4 + 4(4\cos^2\theta - 1)$$
$$= -4 + 16\cos^2\theta - 4 = 16\cos^2\theta - 8$$
$$= 8[2\cos^2\theta - 1]$$
$$= 8\cos 2\theta \qquad [\because \cos 2A = 2\cos^2 A - 1]$$

47. We have,

$$1 - \frac{\sin^2 y}{1 + \cos y} + \frac{1 + \cos y}{\sin y} - \frac{\sin y}{1 - \cos y}$$
$$= 1 - \frac{(1 - \cos^2 y)}{1 + \cos y} + \frac{(1 + \cos y)(1 - \cos y) - \sin^2 y}{\sin y(1 - \cos y)}$$
$$= 1 - \frac{(1 - \cos y)(1 + \cos y)}{(1 + \cos y)} + \frac{(1 - \cos^2 y) - \sin^2 y}{\sin y(1 - \cos y)}$$

$[\because a^2 - b^2 = (a - b)(a + b)]$

$$= 1 - (1 - \cos y) + \frac{\sin^2 y - \sin^2 y}{\sin y(1 - \cos y)}$$
$$= 1 - 1 + \cos y + 0$$
$$= \cos y$$

48. We have,

$$\sin^{-1} x = 2\tan^{-1} x$$
$$\sin^{-1} x = \sin^{-1}\left(\frac{2x}{1 + x^2}\right)$$
$$\left[\because 2\tan^{-1} A = \sin^{-1}\left(\frac{2A}{1 + A^2}\right)\right]$$
$$\Rightarrow \quad x = \frac{2x}{1 + x^2}$$
$$\Rightarrow \quad x(1 + x^2) = 2x$$
$$\Rightarrow \quad x + x^3 - 2x = 0$$
$$\Rightarrow \quad x^3 - x = 0$$
$$\Rightarrow \quad x(x^2 - 1) = 0$$
$$\Rightarrow \quad x = 0 \text{ or } x^2 - 1 = 0$$
$$\Rightarrow \quad x = 0 \text{ or } x^2 - 1 = 0$$
$$\Rightarrow \quad x = 0 \text{ or } x^2 = 1$$
$$\Rightarrow \quad x = 0 \text{ or } x = \pm 1$$
$$\therefore \quad x \in \{0, -1, 1\}$$

49. We have,

$$17x^2 + 17x\tan\left(2\tan^{-1}\left(\frac{1}{5}\right) - \frac{\pi}{4}\right) - 10 = 0$$

$$\Rightarrow 17x^2 + 17x\tan\left[\tan^{-1}\left\{\frac{2\times\frac{1}{5}}{1-\left(\frac{1}{5}\right)^2}\right\} - \tan^{-1}(1)\right] - 10 = 0$$

$$\left[\because 2\tan^{-1}A = \tan^{-1}\left(\frac{2A}{1-A^2}\right) \text{and } \frac{\pi}{4} = \tan^{-1}(1)\right]$$

$$\Rightarrow 17x^2 + 17x\tan\left[\tan^{-1}\left(\frac{\frac{2}{5}}{\frac{25-1}{25}}\right) - \tan^{-1}(1)\right] - 10 = 0$$

$$\Rightarrow 17x^2 + 17x\tan\left[\tan^{-1}\frac{2\times 5}{24} - \tan^{-1}(1)\right] - 10 = 0$$

$$\Rightarrow 17x^2 + 17x\tan\left[\tan^{-1}\frac{5}{12} - \tan^{-1}(1)\right] - 10 = 0$$

$$\Rightarrow 17x^2 + 17x\tan\left[\tan^{-1}\frac{\frac{5}{12}-1}{1+\frac{5}{12}\times 1}\right] - 10 = 0$$

$$\Rightarrow 17x^2 + 17x\left(\frac{5-12}{12+5}\right) - 10 = 0$$

$$\Rightarrow 17x^2 + 17x\left(\frac{-7}{17}\right) - 10 = 0$$

$$\Rightarrow 17x^2 - 7x - 10 = 0$$

$$\Rightarrow 17x^2 - (17x - 10x) - 10 = 0$$

$$\Rightarrow 17x^2 - 17x + 10x - 10 = 0$$

$$\Rightarrow 17x(x-1) + 10(x-1) = 0$$

$$\Rightarrow (17x+10)(x-1) = 0$$

$$\Rightarrow x = \frac{-10}{17} \text{ or } x = 1$$

50. Given, number of elements in set $A = n$

and number of ordered pair $= m$

For relation to be reflexive, it must have atleast n ordered pairs.

$\therefore \quad m \geq n$