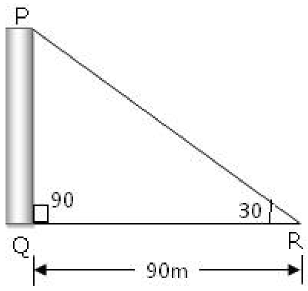
Heights and Distance

Exercise-10

Question 1:

A pole stands vertically on the ground. If the angle of elevation of the top of the pole from a point 90 m away from the pole has measure 30, find the height of the pole.

Solution :



Suppose $\overline{PQ}$ represents the pole. R is a point 90 m away from the pole.

$\therefore$ PQ = Height of a pole and QR = 90 m

$\angle Q$ is a right angle in ΔPQR and $m\angle R = 30°$.

In ΔPQR,

$$\tan 30 = \frac{PQ}{QR}$$

$$\therefore \frac{1}{\sqrt{3}} = \frac{PQ}{90}$$

$$\therefore \frac{90}{\sqrt{3}} = PQ$$

$$\therefore PQ = \frac{90}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{90}{3} \times 1.73$$

$$= 30 \times 1.73$$

$$= 51.9$$

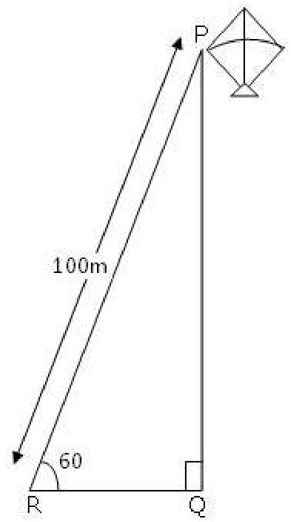
$\therefore$ PQ = 51.9m

$\therefore$ The height of the pole is 51.9 m.

Question 2:

A string of a kite is 100 m long and it makes an angle of measure 60 with the horizontal. Find the height of the kite, assuming that there is no slack in the string.

Solution :



Let $\overline{PR}$ be the string of a kite and $\overline{QR}$ be the horizontal direction.

$\therefore$ PR = 100 m, $m\angle PQR = 90^\circ$, $m\angle PRQ = 60^\circ$ and

PQ = Height of the kite.

In ΔPQR, $\sin 60 = \dfrac{PQ}{PR}$

$\therefore \dfrac{\sqrt{3}}{2} = \dfrac{PQ}{100}$

$\therefore PQ = \dfrac{\sqrt{3} \times 100}{2}$

$= 50\sqrt{3}$

$= 50 \times 1.73$

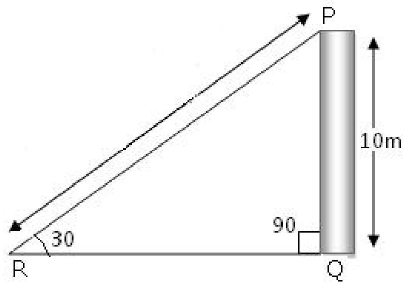
$= 86.5$ m

$\therefore$ The height of a kite is 86.5 m.

Question 3:

A circus artist is climbing from the ground along a rope stretched from the top of a vertical pole and tied at the ground. The height of pole is 10 m and the angle made by the rope with ground level has measure 30. Calculate the distance covered by the artist in climbing to the top of the pole.

Solution :



Let $\overline{PQ}$ be the pole, whose height is 10 m
and $\overline{PR}$ be the length of a rope streched from the top of a vertical pole. The artist is moving from R to P.
The angle made by the rope with the ground level measures 30°.
$\therefore$ m∠PRQ = 30°
In ΔPQR,

$$\sin 30 = \frac{PQ}{PR}$$

$$\therefore \frac{1}{2} = \frac{10}{PR}$$

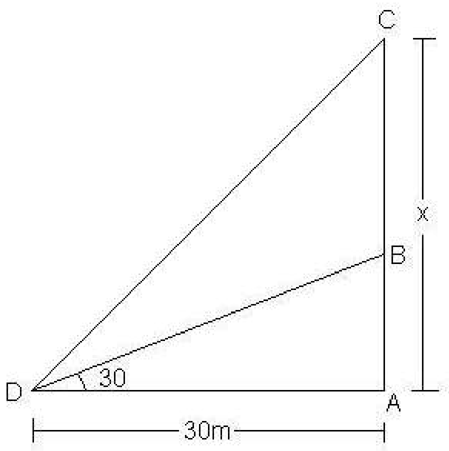
$$\therefore PR = 10 \times 2$$
$$= 20 \text{ m}$$

$\therefore$ The distance covered by the artist in climbing to the top of the pole is 20 m.

Question 4:

A tree breaks due to a storm and the broken part bends such that the top of the tree touches the ground making an angle having measure 30 with the ground. The distance from the foot of the tree to the point where the top touches the ground is 30 m. Find the height of the tree.

Solution :



Suppose, $\overline{AC}$ is the tree broken at point B such that the broken part $\overline{CB}$ takes the position $\overline{BD}$ and touches the ground at D.

Then, AD = 30 m and m$\angle$ADB = 30º.

In ΔDAB, we have

$$\tan 30 = \frac{AB}{AD}$$

$$\therefore \frac{1}{\sqrt{3}} = \frac{AB}{30}$$

$$\therefore AB = \frac{30}{\sqrt{3}}$$

$$\therefore AB = 10\sqrt{3} \qquad \ldots \ldots(1)$$

Now, $\cos 30 = \dfrac{AD}{BD}$

$$\therefore \frac{\sqrt{3}}{2} = \frac{30}{BD}$$

$$\therefore BD = \frac{30 \times 2}{\sqrt{3}}$$

$$= 20\sqrt{3} \qquad \ldots \ldots(2)$$

So, the height of the tree AC,

$$= AB + BC$$

$$= AB + BD$$

$$= 10\sqrt{3} + 20\sqrt{3}$$

$$= 30\sqrt{3}$$

$$= 30 \times 1.73$$

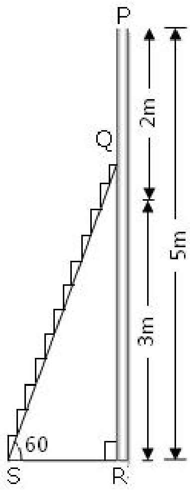
$$= 51.9$$

$\therefore$ The height of the tree is 51.9 m.

Question 5:

An electrician has to repair an electric fault on the pole of height 5 m. He needs to reach a point 2 m below the top of the pole to undertake the repair work. What should be the length of the ladder that he should use, which when inclined at an angle of measure 60 to the horizontal would enable him to reach the required position.

Solution :



Let $\overline{PR}$ represent a pole of lenght 5 m.

Q is a point 2 m below the top of the pole P to undertake the repair work.

$\therefore$ PQ = 2 m and QR = 3 m

Now, $\overline{QS}$ represents the ladder which is inclined at an angle of measure 60° to the horizontal.

$\therefore$ QS = length of the ladder

In ΔQRS,

$$\sin 60 = \frac{QR}{QS}$$

$$\therefore \frac{\sqrt{3}}{2} = \frac{3}{QS}$$

$$\therefore QS = \frac{6}{\sqrt{3}}$$

$$= 2\sqrt{3}$$

$$= 2 \times 1.73$$

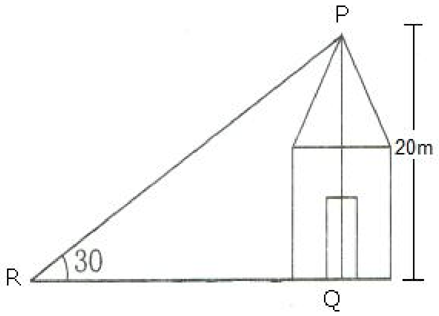
$$= 3.46$$

$\therefore$ The length of the ladder is 3.46 m

Question 6:

As observed from a fixed point on the bank of a river, the angle of elevation of a temple on the opposite bank has measure 30. If the height of the temple is 20 m, find the width of the river.

Solution :



Let $\overline{PQ}$ represent the temple and P is the top of the temple.

$\therefore$ PQ = 20 m

R is a fixed point on the bank of a river from a temple on the opposite bank and $\overline{QR}$ is the width of the river.

The angle of elevation of the top of a temple on the opposite bank from R measures 30°.

$\therefore$ $m\angle PRQ = 30°$

In ΔPQR,

$$\cot 30 = \frac{QR}{PQ}$$

$$\therefore \sqrt{3} = \frac{QR}{20}$$

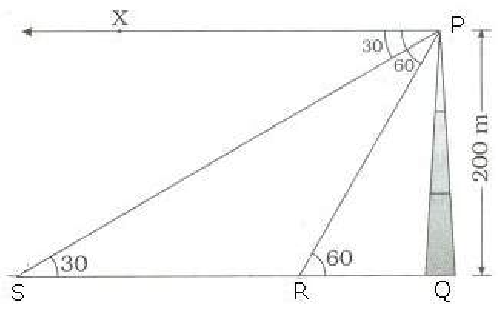
$$\therefore QR = 20\sqrt{3}$$
$$= 20 \times 1.73$$
$$= 34.6$$

$\therefore$ The width of the river is 34.6 m.

Question 7:

As observed from the top of a hill 200 m high, the angles of depression of two vehicles situated on the same side of the hill are found to have measure 30 and 60 respectively. Find the distance between the two vehicles.

Solution :



Let $\overline{PQ}$ be the hill with height $= 200$ m
R and S are two vehicles situated on the same side of the hill.
The angles of depression of vehicles R and S from P
are 60° and 30° respectively, so
$m\angle PRQ = m\angle XPR = 60°$ and
$m\angle PSQ = m\angle XPS = 30°$.
The distance between two vehicles is $\overline{SR}$.

$$\text{In } \Delta PQR,\ \tan 60 = \frac{PQ}{QR}$$

$$\therefore \sqrt{3} = \frac{200}{QR}$$

$$\therefore QR = \frac{200}{\sqrt{3}} = \frac{200\sqrt{3}}{3}$$

$$\text{In } \Delta PQS,\ \tan 30 = \frac{PQ}{QS}$$

$$\therefore \frac{1}{\sqrt{3}} = \frac{200}{QS}$$

$$\therefore QS = 200\sqrt{3}$$

The distance between the two vehicles situated on the same side of the hill

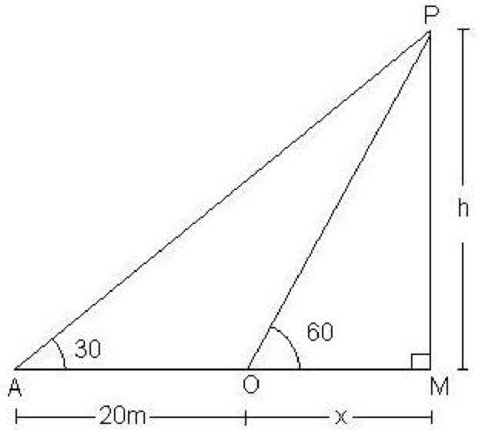
$$= RS$$
$$= QS - QR$$
$$= 200\sqrt{3} - \frac{200\sqrt{3}}{3}$$
$$= \frac{600\sqrt{3} - 200\sqrt{3}}{3}$$
$$= \frac{400\sqrt{3}}{3}$$
$$= \frac{400(1.73)}{3}$$
$$= \frac{692}{3}$$
$$= 230.6$$

$\therefore$ The distance between the two vehicles is 230.6 m.

Question 8:

A person standing on the bank of a river, observes that the angle subtended by a tree on the opposite bank has measure 60. When he retreats 20 m from the bank, he finds the angle to have measure 30. Find the height of the tree and the breadth of the river.

Solution :



Let $\overline{PM}$ be the tree. $\overline{OM}$ represents the width of the river.

So, $m\angle PMO = 90^{\circ}$ and OA = 20 m

Let, OM = Breadth of the river = x m and

PM = Height of a tree = h m

$\therefore$ AM = OM + OA = x + 20

In ΔPMO, $\tan 60 = \frac{PM}{OM}$

$\therefore \sqrt{3} = \frac{h}{x}$

$\therefore h = \sqrt{3}x \qquad \ldots\ldots(1)$

In ΔPAM, $\tan 30 = \frac{PM}{AM}$

$\therefore \frac{1}{\sqrt{3}} = \frac{h}{x+20} \qquad \ldots\ldots(2)$

From equations (1) and (2), we get

$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}x}{x+20}$

$\therefore x + 20 = \sqrt{3}x \times \sqrt{3}$

$\therefore x + 20 = 3 \times x$

$\therefore 3x - x = 20$

$\therefore x = 10$

Now, $h = \sqrt{3}x$

$= \sqrt{3} \times 10$

$= (1.73) \times 10$

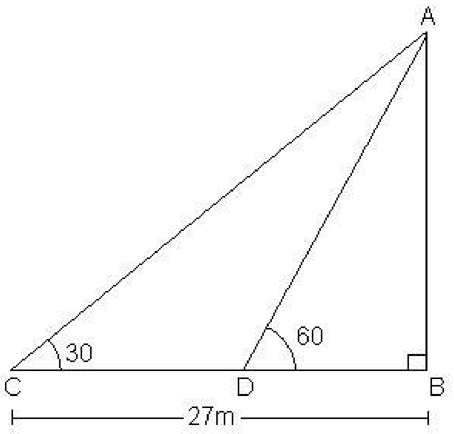
$= 17.3$

$\therefore$ The breadth of the river = OM = x = 10 m and the height of the tree = PM = h = 17.3 m.

Question 9:

The shadow of a tower is 27 m, when the angle of elevation of the sun has measure 30. When the angle of elevation of the sun has measure 60, find the length of the shadow of the tower.

Solution :



Here, $\overline{AB}$ is the tower and $\overline{CB}$ is its shadow when the angle of elevation of the Sun measures 30°.

Then, $m\angle ACB = 30°$, $m\angle B = 90°$ and $CB = 27$ m

$\overline{DB}$ is a shadow of the tower when the angle of elevation of the Sun measures 60°.

In ΔACB,

$$\tan 30 = \frac{AB}{BC}$$

$$\therefore \frac{1}{\sqrt{3}} = \frac{AB}{27}$$

$$\therefore AB = \frac{27}{\sqrt{3}} = \frac{27}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 9\sqrt{3}$$

In ΔADB,

$$\tan 60 = \frac{AB}{DB}$$

$$\therefore \sqrt{3} = \frac{9\sqrt{3}}{DB}$$

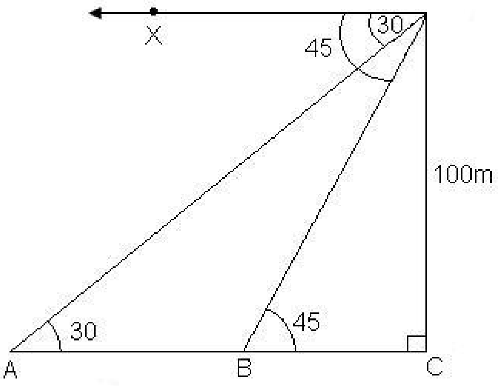
$$\therefore DB = \frac{9\sqrt{3}}{\sqrt{3}} = 9 \text{ m}$$

$\therefore$ The length of the shadow of the tower is 9 m.

Question 10:

From a point at the height 100 m above the sea level, the angles of depression of a ship in the sea is found to have measure 30. After some time the angle of depression of the ship has measure 45. Find the distance travelled by the ship during that time interval.

Solution :



Let the ship travel from A to B in given time.
The distance travelled by the ship is AB.
Suppose, the observation point is at O
OC = 100 m.
The angles of depression of A and B from O are 30° and 45° respectively.

$\therefore$ m∠OAC = m∠XOA = 30° and
m∠OBC = m∠XOB = 45°

In ΔOCB, $\tan 45 = \frac{OC}{BC} = \frac{100}{BC}$

$\therefore 1 = \frac{100}{BC}$

$\therefore BC = 100$

In ΔOCA, $\tan 30 = \frac{OC}{AC} = \frac{100}{AC}$

$\therefore \frac{1}{\sqrt{3}} = \frac{100}{BC + AB}$

$\therefore \frac{1}{1.73} = \frac{100}{100 + AB}$

$\therefore 100 + AB = 173$

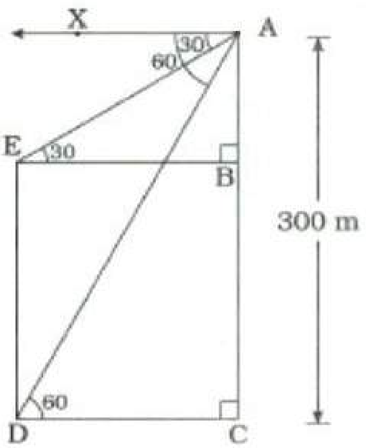
$\therefore AB = 173 - 100$
$= 73$ m

$\therefore$ The distance travelled by the ship during the given time interval is 73 m.

Question 11:

From the top of a 300 m high light-house, the angles of depression of the top and foot of a tower have measure 30 and 60. Find the height of the tower.

Solution :



Let $\overline{AC}$ be the lighthouse and $\overline{ED}$ be the tower.
Height of lighthouse = AC = 300 m
Let $\overline{ED}$ = h
Let $\overline{EB}$ be the perpendicular from E to $\overline{AC}$.
The angles of depression of the top E and the bottom D of the tower $\overline{ED}$ measures 30° and 60° respectively from A.
Then, $m\angle AEB = 30°$ and $m\angle ADC = 60°$
Now,

In ΔADC, $\tan 60 = \dfrac{AC}{DC}$

$\therefore \sqrt{3} = \dfrac{300}{DC}$

$\therefore DC = \dfrac{300}{\sqrt{3}} = EB$

Now, In ΔAEB,

$\tan 30 = \dfrac{AB}{EB}$

$\therefore \dfrac{1}{\sqrt{3}} = \dfrac{AB}{\frac{300}{\sqrt{3}}}$

$$\begin{aligned} \therefore AB &= \frac{300}{3} \\ &= \frac{300}{\sqrt{3}} \times \frac{1}{\sqrt{3}} \\ &= \frac{300}{3} \\ &= 100 \text{ m} \end{aligned}$$

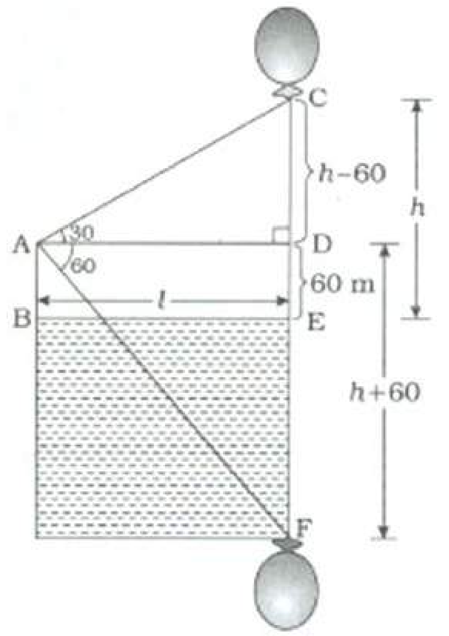
$$\begin{aligned} \text{Height of the tower } ED &= BC \\ &= AC - AB \\ &= 300 - 100 \\ &= 200 \end{aligned}$$

$\therefore$ The height of the tower is 200 m.

Question 12:

As observed from a point 60 m above a lake, the angle of elevation of an advertising ballon has measure 30 and from the same point the angle of depression of the image of the ballon in the lake has measure 60. Calculate the height of the balloon above the lake.

Solution :



Let $\overline{BE}$ be the surface of the lake
A is a point 60 m above the lake.
F is the image of the balloon C in the lake.
Horizontal line $\overline{AD}$ intersects $\overline{CE}$ in D.
$m\angle CAD = 30°$, $m\angle FAD = 60°$, AB = 60 m.
Let CE = h, BE = l
Then CD = h − 60 and DF = h + 60

In ΔADC, $\tan 30 = \frac{CD}{AD}$

$\therefore \frac{1}{\sqrt{3}} = \frac{h-60}{l}$

$\therefore l = \sqrt{3}(h-60) \quad \ldots \ldots (1)$

In ΔADF, $\tan 60 = \frac{DF}{AD}$

$\therefore \sqrt{3} = \frac{h+60}{l}$

$\therefore \sqrt{3}l = h + 60 \quad \ldots \ldots (2)$

From equation (1) and (2),

$\sqrt{3}\left[\sqrt{3}(h-60)\right] = h + 60$

$\therefore 3(h-60) = h + 60$

$\therefore 3h - 180 = h + 60$

$\therefore 2h = 240$

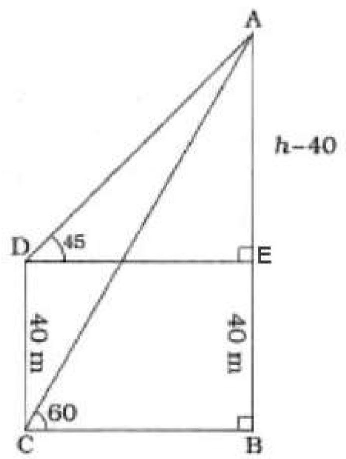
$\therefore h = 120$ m

$\therefore$ The height of the balloon above the lake is 120 m.

Question 13:

Watching from a window 40 m high of a multistoreyed building, the angle of elevation of the top of a tower is found to have measure 45. The angle of elevation of the top of the same tower from the bottom of the building is found to have measure 60. Find the height of the tower.

Solution :



Let $\overline{CD}$ be the window, $\overline{AB}$ be the tower and D be the point of observation.

$\therefore$ CD = 40 m and

Let the height of the tower = AB = h.

Horizonal line $\overline{DE}$ intersects $\overline{AB}$ in E.

$\therefore$ BE = CD = 40 m and

$AE = AB - BE = (h - 40)$.

$m\angle AED = m\angle ABC = 90^\circ$

Now, the angle of elevation of A from D is 45° and the angle of elevation of A from C is 60°.

$\therefore m\angle ADE = 45^\circ$ and $m\angle ACB = 60^\circ$

In ΔAED and in ΔAEC

$$\tan 45 = \frac{AE}{DE}$$

$$\therefore 1 = \frac{h - 40}{DE}$$

$$\therefore DE = h - 40 = BC \quad \ldots \ldots (1)$$

$$\tan 60 = \frac{AB}{BC}$$

$$\therefore \sqrt{3} = \frac{h}{h - 40}$$

$$\therefore h = (h - 40)\sqrt{3}$$

$$\therefore h = \sqrt{3}h - 40\sqrt{3}$$

$$\therefore h\left(\sqrt{3} - 1\right) = 40\sqrt{3}$$

$$\therefore h = \frac{40\sqrt{3}}{\sqrt{3} - 1}$$

$$= \frac{40\sqrt{3}}{\sqrt{3} - 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1}$$

$$= \frac{40\sqrt{3}\left(\sqrt{3} + 1\right)}{3 - 1}$$

$$= \frac{120 + 40\sqrt{3}}{2}$$

$$= 60 + 20(1.73)$$

$$= 60 + 34.6$$

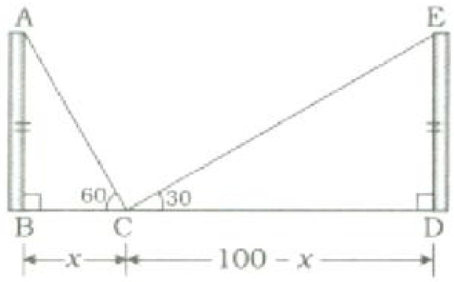
$$= 94.6 \text{ m}$$

$\therefore$ The height of the tower is 94.6 m.

Question 14:

Two pillars of equal height stand on either side of a road, which is 100 m wide. The angles of elevation of the top of the pillars have measure 60 and 30 at a point on the road between the pillars. Find the position of the point from the nearest end of a pillars and the height of pillars.

Solution :



Let $\overline{AB}$ and $\overline{DE}$ be two pillars of equal height.

$\therefore AB = DE$

The angle of elevation of A and E from C are respectively 60° and 30°.

$\therefore m\angle ACB = 60°$ and $m\angle EDC = 30°$ and BD = 100 m.

Let BC = x

$\therefore CD = BD - BC = 100 - BC = (100 - x)$

In ΔABC, $\tan 60 = \dfrac{AB}{BC}$

$\therefore \sqrt{3} = \dfrac{AB}{x}$

$\therefore x = \dfrac{AB}{\sqrt{3}} \qquad \ldots \quad \ldots(1)$

In ΔEDC, $\tan 30 = \dfrac{DE}{CD}$

$\therefore \dfrac{1}{\sqrt{3}} = \dfrac{AB}{100 - x} \qquad (\because AB = DE)$

$\therefore 100 - x = \sqrt{3}AB$

$\therefore 100 - \dfrac{AB}{\sqrt{3}} = \sqrt{3}AB$

$\therefore 100\sqrt{3} - AB = 3AB$

$\therefore 4AB = 100\sqrt{3}$

$\therefore AB = 25\sqrt{3} = 25 \times 1.73 = 43.25$ m

$\therefore$ The height of the pillar is 43.25 m.

From equation (1),

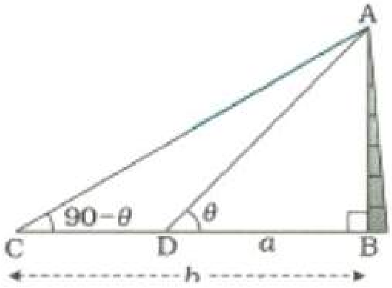
$x = \dfrac{AB}{\sqrt{3}} = \dfrac{25\sqrt{3}}{\sqrt{3}} = 25$

$\therefore$ The distance of the point from the nearest end of the pillars is 25 m and the height of each pillar is 43.25 m.

Question 15:

The angles of elevation of the top of a tower from two points at distance a and b metres from the base and in the same straight line with it are complementary. Prove that the height of the tower is $\sqrt{ab}$ metres.

Solution :



$\overline{AB}$ is a tower. D and C are two points on the same side of a tower, BD = a and BC = b.

$\angle ADB$ and $\angle ACB$ are the complementary angles.

If $m\angle ADB = \theta$, then $m\angle ACB = 90 - \theta$.

In ΔADB, $\tan\theta = \dfrac{AB}{BD}$

$\therefore \tan\theta = \dfrac{AB}{a} \qquad \ldots \ldots(1)$

In ΔABC, $\tan(90-\theta) = \dfrac{AB}{BC}$

$\therefore \cot\theta = \dfrac{AB}{b} \qquad \ldots \ldots(2)$

Multiplying (1) and (2),

$\tan\theta \cdot \cot\theta = \dfrac{AB}{a} \cdot \dfrac{AB}{b}$

$\therefore 1 = \dfrac{(AB)^2}{ab}$

$\therefore (AB)^2 = ab$

$\therefore AB = \sqrt{ab}$

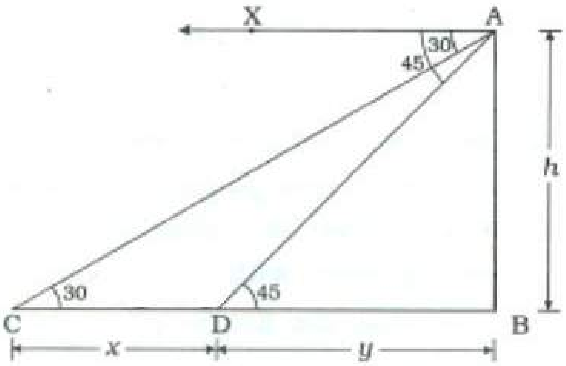
Height of the tower $= AB = \sqrt{ab}$

Hence proved.

Question 16:

A man on the top of a vertical tower observes a car moving at a uniform speed coming directly towards it. If it takes 12 minutes for the angle of depression to change its measure from 30 to 45, how soon after this, will the car reach the tower ?

Solution :



Let $\overline{AB}$ be the tower and height of the tower AB = h m.
At C the angle of depression of the car measures 30° and 12 minutes later it reaches D where the angle of depression is 45°.
Let, CD = x, DB = y
Here, AB = h m, $m\angle ACB = m\angle XAC = 30°$ and $m\angle ADB = m\angle XAD = 45°$

In ΔACB, $\tan 30 = \frac{AB}{CB}$

$\therefore \frac{1}{\sqrt{3}} = \frac{h}{x+y}$

$\therefore x + y = \sqrt{3}h$ ... ...(1)

In ΔABD, $\tan 45 = \frac{AB}{BD}$

$\therefore 1 = \frac{h}{y}$

$\therefore h = y$ ... ...(2)

From results (1) and (2),

$x + y = \sqrt{3}y$

$\therefore x = \left(\sqrt{3}-1\right)y$ ... ...(3)

The distance covered in 12 minutes by car is CD = x.
$\therefore$ Time taken to cover the distance x is 12 minutes.
So, y = Time taken to cover distance DB

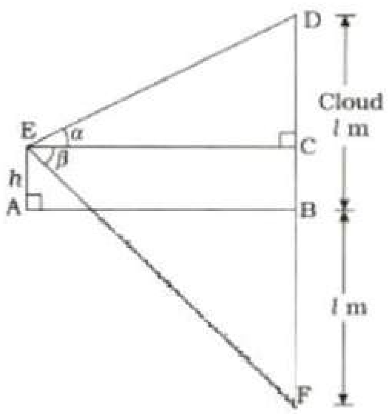
$$= \frac{y}{x} \times 12$$
$$= \frac{y}{\left(\sqrt{3}-1\right)y} \times 12$$
$$= \frac{12}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$
$$= \frac{12\left(\sqrt{3}+1\right)}{3-1}$$
$$= \frac{12(1.73+1)}{2}$$
$$= 6 \times 2.73$$
$$= 16.38 \text{ minutes}$$

$\therefore$ The time taken by car to reach the tower is 16.38 minutes.

Question 17:

If the angle of elevation of a cloud from a point h metres above a lake has measure and the angle of depression of its reflection in the lake has measure 13, prove that the height of the could is $\frac{h(\tan\beta+\tan\alpha)}{\tan\beta-\tan\alpha}m$

Solution :



Let $\overline{AB}$ be the surface of the lake. E is the point above h m from A.

$\therefore$ AE = h

Let the height of the cloud BD = l m.

Let F be the reflection of kite C.

Horizontal line $\overline{EC}$ interects $\overline{BD}$ in C.

$\therefore$ BF = l m

$m\angle DEC = \alpha$ and $m\angle CEF = \beta$.

Here, AE = BC = h

$\therefore$ CD = BD − BC = l − h and

CF = BF + BC = l + h

In ΔECD, $\tan\alpha = \frac{CD}{EC} = \frac{l-h}{EC}$

$\therefore EC = \frac{l-h}{\tan\alpha}$ ... ...(1)

In ΔECF, $\tan\beta = \frac{CF}{EC} = \frac{l+h}{EC}$

$\therefore EC = \frac{l+h}{\tan\beta}$ ... ...(2)

$\therefore$ From results (1) and (2),

$$\frac{l+h}{\tan\beta} = \frac{l-h}{\tan\alpha}$$

$$\therefore \frac{l+h}{l-h} = \frac{\tan\beta}{\tan\alpha}$$

$$\therefore \frac{l+h+l-h}{l+h-l+h} = \frac{\tan\beta+\tan\alpha}{\tan\beta-\tan\alpha}$$

(Using Dividendo − Componendo)

$$\therefore \frac{2l}{2h} = \frac{\tan\beta+\tan\alpha}{\tan\beta-\tan\alpha}$$

$$\therefore l = \frac{h(\tan\beta+\tan\alpha)}{\tan\beta-\tan\alpha}$$

$\therefore$ The height of the cloud from the surface of the lake is $\frac{h(\tan\beta+\tan\alpha)}{\tan\beta-\tan\alpha}$ m

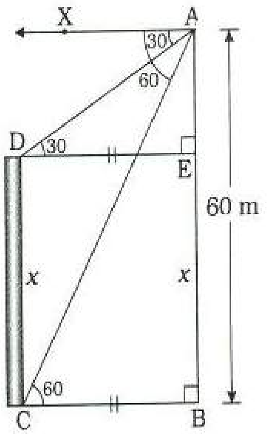
Hence proved.

Question 18:

From the top of a building $\overline{AB}$, 60 m high, the angles of depression of the top and bottom at a vertical lamp post $\overline{CD}$ are observed to have measure 30 and 60 respectively. Find,

1. the horizontal distance between building and lamp post.
2. the height of the lamp post.
3. the difference between the heights of the building and the lamp post

Solution :



Let $\overline{AB}$ be the building and $\overline{CD}$ be the lamppost

The height of the building AB = 60 m

Horizontal line $\overline{DE}$ intersects $\overline{AB}$ in E.

Let BE = CD = x m

$\therefore$ AE = AB − BE = (60 − x) m

$m\angle AED = m\angle ABC = 90^\circ$

Now, the angle of depression of the top D and the bottom C of the post $\overline{CD}$ have measures 30° and 60° respectively from A.

Then, $m\angle ADE = m\angle XAD = 30^\circ$ and

$m\angle ACB = m\angle XAC = 60^\circ$

In right angled ΔADE, $\tan 30 = \dfrac{AE}{DE}$

- -

$\therefore \sqrt{3} = \dfrac{60}{BC}$

$\therefore BC = \dfrac{60}{\sqrt{3}} = 20\sqrt{3}$ ... ...(2)

From (1) and (2),

$\sqrt{3}(60 - x) = 20\sqrt{3}$

$\therefore 60 - x = 20$

$\therefore x = 40$ ... ...(i)

1) The horizontal distance between the building and the lamppost.

= BC

$= \sqrt{3}(60 - x)$

$= \sqrt{3}(60 - 40)$

$= 20\sqrt{3}$

$= 20(1.73)$

$= 34.6$

2) The height of the lamppost = CD

= x

= 40 m

3) The difference between the heights of the building and the lamppost = AB − BE

= 60 − x

= 60 − 40

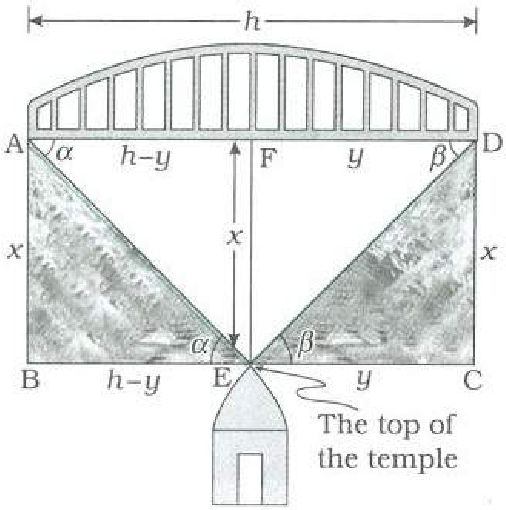
= 20 m

Question 19:

A bridge across a valley is h metres long. There is a temple in the valley directly below the bridge. The angles of depression of the top of the temple from the two ends of the bridge have measures OC and B. Prove that the height of the bridge above the top of the temple is

$\frac{h(\tan\alpha \cdot \tan\beta)}{\tan\alpha + \tan\beta}$.

Solution :



Let $\overline{AD}$ be the bridge and E be the top of the temple.
The perpendicular from E on $\overline{AD}$ is $\overline{EF}$.
$\therefore$ $\overline{EF}$ represents the height from the top of the temple to the bridge.
Let EF = x m and DF = y m.
$\therefore$ AF = h − y

In ΔABE, $\tan\alpha = \frac{x}{h-y}$

$\therefore h - y = \frac{x}{\tan\alpha}$

$\therefore y = h - \frac{x}{\tan\alpha}$ ... ...(1)

In ΔDCE, $\tan\beta = \frac{x}{y}$

$\therefore y = \frac{x}{\tan\beta}$ ... ...(2)

Now, from equations (1) and (2),

$h - \frac{x}{\tan\alpha} = \frac{x}{\tan\beta}$

$\therefore h\tan\alpha\tan\beta - x\tan\beta = x\tan\alpha$

$\therefore h\tan\alpha\tan\beta = x(\tan\alpha + \tan\beta)$

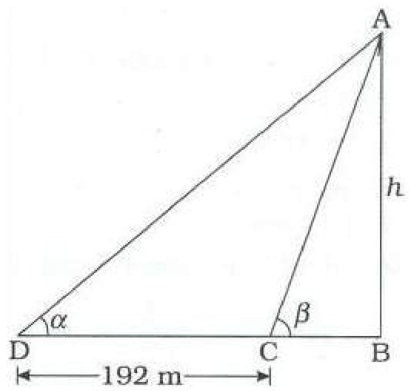
$\therefore x = \frac{h\tan\alpha\tan\beta}{\tan\alpha + \tan\beta}$

$\therefore$ The height of the bridge above the top of the temple is $\frac{h(\tan\alpha \cdot \tan\beta)}{\tan\alpha + \tan\beta}$.

Question 20:

At a point on level ground, the angle of elevation of a vertical tower is found to be such that its tangent is $\frac{5}{12}$ On walking 192 metres towards the tower, the tangent of the angle is found to be $\frac{3}{4}$. Find the height of the tower.

Solution :



Let $\overline{AB}$ be the tower.

The angle of elevation of tower from D is α and on walking 192 metres towards the tower from D to B ,

The angle of elevation of the tower from C is β.

$\therefore$ CD = 192 m and $\tan\alpha = \frac{5}{12}$, $\tan\beta = \frac{3}{4}$

Let BC = x m and AB = h m.

$m\angle ADB = \alpha$ and $m\angle ACB = \beta$

Now, in ΔABC, $\tan\beta = \frac{AB}{BC}$

$\therefore \frac{3}{4} = \frac{h}{x}$

$\therefore 4h = 3x$

$\therefore x = \frac{4h}{3} \qquad \ldots\ldots(1)$

and in ΔABD, $\tan\alpha = \frac{AB}{BD}$

$\therefore \frac{5}{12} = \frac{h}{x+192} \quad (\because BD = BC + CD = x + 192)$

$\therefore 12h = 5x + 960$

$\therefore 12h = 5\left(\frac{4h}{3}\right) + 960$

$\therefore 36h = 20h + 2880$

$\therefore 16h = 2880$

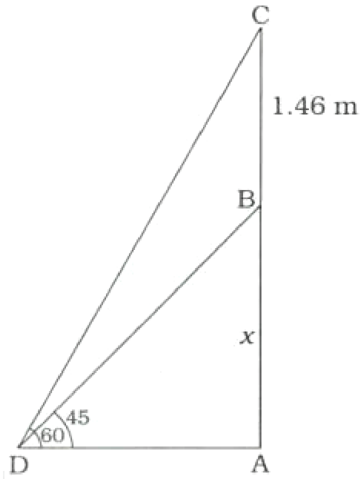
$\therefore h = 180$

$\therefore$ The height of the tower = AB = h = 180 m.

Question 21:

A statue 1.46 m tall, stands on the top of a pedestal. From the point on the ground the angle of elevation of the top of the statue has measure 60 and from the same point, the angle of elevation of the top of the pedestal has measure 45. Find the height of the pedestal.

Solution :



Let $\overline{AB}$ be the pedestal and AB = x m.
$\overline{BC}$ is a statue and BC = 1.46 m.
The angle of elevation of the top of the statue C from D is 60° and
From the same point the angle of elevation of the top
of the pedestal B is 45°.

Now, in ΔBAD, $\tan 45 = \frac{AB}{AD}$

$\therefore 1 = \frac{x}{AD}$

$\therefore AD = x$

Now, in ΔCAD,

$\tan 60 = \frac{AC}{AD}$

$\therefore \sqrt{3} = \frac{AB + BC}{x}$

$\therefore \sqrt{3}x = x + 1.46$

$\therefore \left(\sqrt{3} - 1\right)x = 1.46$

$\therefore (1.73 - 1)x = 1.46$

$\therefore 0.73x = 1.46$

$\therefore x = \frac{1.46}{0.73} = 2$

$\therefore$ The height of the pedestal is = AB = x = 2 m.

Question 22:

Select a proper option (a), (b), (c) or (d) from given options :

Question 22(1):

On walking ……….. metres on a hill making an angle of measure 30 with the ground, one can reach the height of ‘a’ metres from the ground.

Solution :

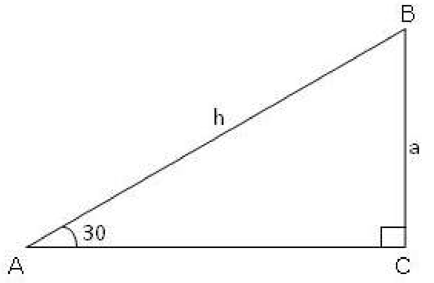
c. 2a

Suppose, on walking h metres, one can reach the height of 'a' metres from the ground.

$\sin 30 = \frac{a}{h}$

$\therefore \frac{1}{2} = \frac{a}{h}$

$\therefore h = 2a$



Question 22(2):

The angle of elevation of the top of the tower from a point P on the ground has measure 45. The distance of the tower from the point P is a and height of the tower is b. Then, ……….

Solution :

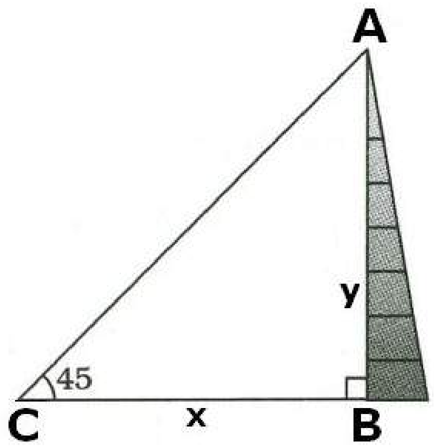
c. a = b

From the figure, we see that,

$\tan 45 = \frac{b}{a}$

$\therefore 1 = \frac{b}{a}$

$\therefore a = b$



Question 22(3):

A 3 m long ladder leans on the wall such that its lower end remains 1.5 m away from the base of the wall. Then, the ladder makes an angle of measure ………. with the ground.

Solution :

c. 60°

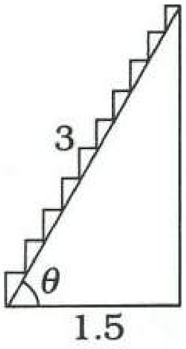
From the figure, we see that,

$\cos\theta = \frac{1.5}{3}$

$\therefore \cos\theta = \frac{1}{2}$

but, $\cos 60 = \frac{1}{2}$

$\therefore \theta = 60°$



Question 22(4):

A tower is $50\sqrt{3}$ m high. The angle of elevation of its top from a point 50 m away from its foot has measure ……..

Solution :

b. 60°

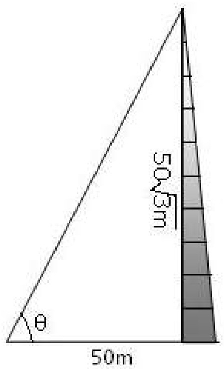
From the figure, we see that,

$\tan\theta = \frac{50\sqrt{3}}{50}$

$\therefore \tan\theta = \sqrt{3}$

but, $\tan 60 = \sqrt{3}$

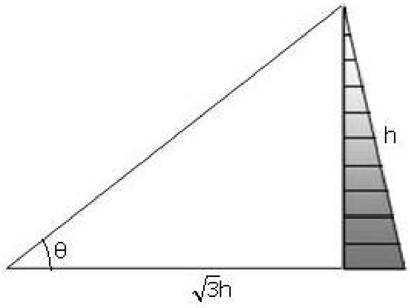
$\therefore \theta = 60°$



Question 22(5):

If the ratio of the height of a tower and the length of its shadow is 1 $\sqrt{3}$, then the angle of elevation of the sun has measure ……..

Solution :



a. 30°

Let the height of the tower be h and length of the shadow be a.

Now, $\tan\theta = \frac{h}{a} = \frac{1}{\sqrt{3}}$

From the figure, we see that,

$\tan\theta = \frac{h}{\sqrt{3}h} = \frac{1}{\sqrt{3}}$

but $\tan 30 = \frac{1}{\sqrt{3}}$

$\therefore\ \theta = 30^\circ$

Question 22(6):

If the angles of elevation of a tower from two points distance a and b (a > b) from its foot on the same side of the tower have measure 30 and 60, then the height of the tower is ……….

Solution :

b. $\sqrt{ab}$

Let the height of the tower be h.

From the figure, we see that,

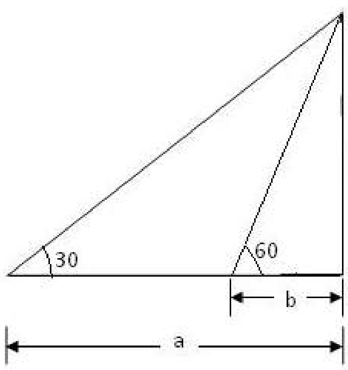
$\tan 60 = \frac{h}{b}$ and $\tan 30 = \frac{h}{a}$

Now, $\tan 60 \cdot \tan 30 = \frac{h}{b} \cdot \frac{h}{a}$

$\therefore\ \sqrt{3} \cdot \frac{1}{\sqrt{3}} = \frac{h^2}{ab}$

$\therefore\ h^2 = ab$

$\therefore\ h = \sqrt{ab}$



Question 22(7):

The tops of two poles of height 18 m and 12 m are connected by a wire. If the wire makes an angle of measure 30 with horizontal, then the length of the wire is ……..

Solution :

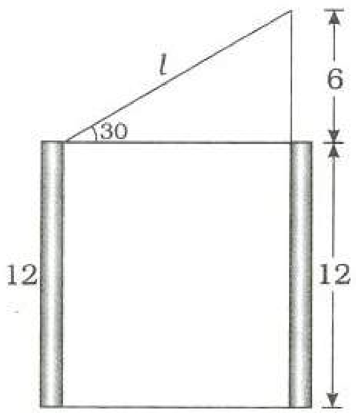
a. 12m

Let the length of the wire be l.
From the figure, we see that,

$\sin 30 = \frac{6}{l}$

$\therefore \frac{1}{2} = \frac{6}{l}$

$\therefore l = 12$ m



Question 22(8):

The angle of elevation of the top of the building A from the base of building B has measure 50. The angle of elevation of the top of the building B from the base of building

1. A has measure 70. Then,……….
2. building A is taller than building B.
3. Building B is taller than building A.
4. Building A and building B are equally tall.
5. The relation about the heights of A and B cannot be determined.

Solution :

b. building B is taller than building A.

Let the height of the building B be b and the height of the building A be a and the distance between the two building be x.
From the figure, we see that,

$\tan 70 = \frac{b}{x}$ and $\tan 50 = \frac{a}{x}$

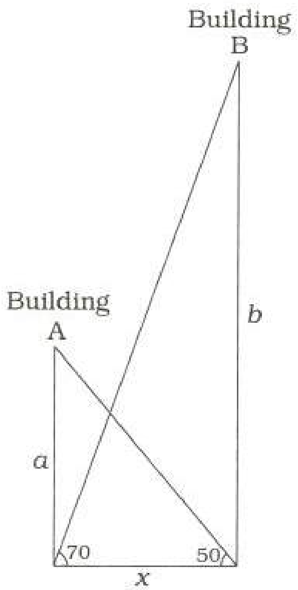
$\therefore x = \frac{b}{\tan 70}$ and $x = \frac{a}{\tan 50}$

$\therefore \frac{b}{\tan 70} = \frac{a}{\tan 50}$

$\therefore \frac{\tan 70}{\tan 50} = \frac{b}{a} > 1 \quad (\because \tan 70 > \tan 50)$

$\therefore b > a$

$\therefore$ Height of building B > Height of building A



Question 22(9):

If the angle of elevation of the top of a tower of a distance 400 m from its foot has measure 30, then the height of the tower is ……..

Solution :

b. $\frac{400}{\sqrt{3}}$

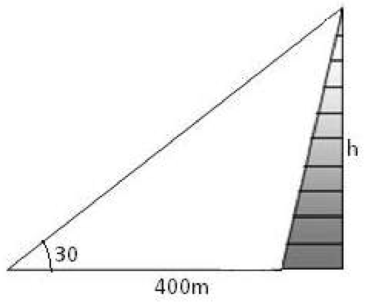
From the figure, we see that,

$$\tan 30 = \frac{h}{400}$$

$$\therefore h = 400 \tan 30$$

$$\therefore h = 400 \times \frac{1}{\sqrt{3}}$$

$$\therefore h = \frac{400}{\sqrt{3}}$$



Question 22(10):

The angle of depression of a ship from the top of a tower 30 m height has measure 60. Then, the distance of the ship from the base of the tower is ……….

Solution :

c. $10\sqrt{3}$

Let the distance of the ship from the base of the tower be h.

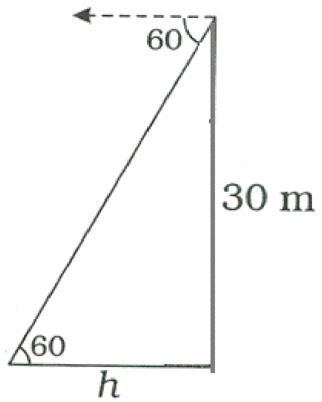
From the figure, we can see that,

$$\tan 60 = \frac{30}{h}$$

$$\therefore \sqrt{3} = \frac{30}{h}$$

$$\therefore h = \frac{3 \times 10}{\sqrt{3}}$$

$$= 10\sqrt{3} \text{ m}$$



Question 22(11):

When the length of the shadow of the pole is equal to the height of the pole, then the angle of elevation source of light has measure ………..

Solution :

a. 45°

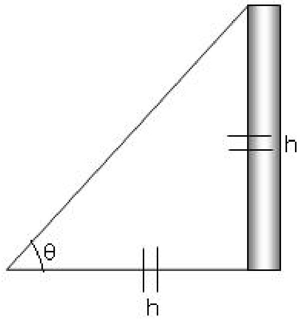
Let the length of the pole be h and so the length of the shadow become h.

Suppose, the angle of elevation of the Sun is θ.

From the figure, we can see that,

$\tan\theta = \frac{h}{h} = 1$

$\therefore \theta = 45^{\circ}$



Question 22(12):

From the top of a building h metre high, the angle of depression of an object on the ground has measure O. The distance (in metres) of the object from the foot of the building is……..

Solution :

c. h cotθ

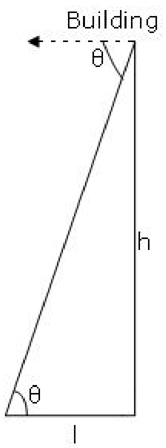
Let the distance of the object from the foot of the building be l.

From the figure, we can see that,

$\tan\theta = \frac{h}{l}$

$\therefore l = \frac{h}{\tan\theta} = h\cot\theta$

$\therefore$ The distance of a point from the building $l = h\cot\theta$

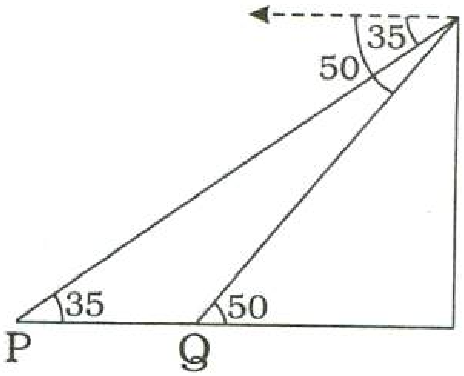


Question 22(13):

As observed from the top of the light house the angle of depression of the two ships P and Q anchored in the sea to the same side are found to have measure 35 and 50 respectively. Then from the light house….

Solution :

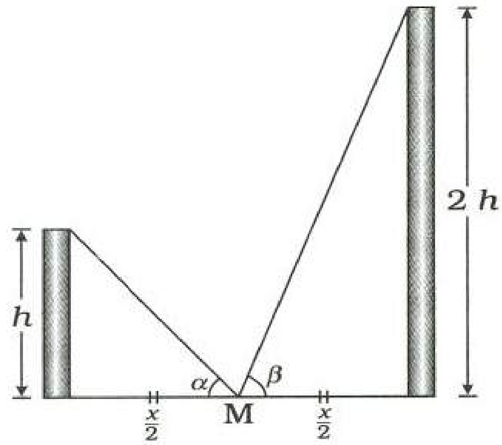
c. the distance of P is more than Q.
From the figure, we can see that,
The distance of P from the lighthouse is more than the distance of Q from the lighthouse.



Question 22(14):

Two poles are x metres apart and the height of one is double than that of the other. If from the mid-point of the line joining their feet, an observer finds the angle of elevation of their tops to be complementary, then the height of the shorter pole is ……

Solution :



d. $\frac{x}{2\sqrt{2}}$

Let the height of the poles be h and 2h respectively.
Let the angles of elevation of the top of the poles from the midpoint M of the line joining the feet of the poles be α and β.
Also, given that they are the complementary angles.

$\therefore \alpha + \beta = 90$

Here, from the figure, we can see that,

$$\tan\alpha = \frac{h}{\frac{x}{2}} \text{ and } \tan\beta = \frac{2h}{\frac{x}{2}}$$

$$\therefore \tan\alpha = \frac{2h}{x} \text{ and } \tan\beta = \frac{4h}{x}$$

Now,

$$\therefore \tan\beta = \frac{4h}{x} = 2\cdot\frac{2h}{x} = 2\tan\alpha$$

$$\therefore \tan(90 - \alpha) = 2\tan\alpha \quad (\because \alpha + \beta = 90)$$

$$\therefore \cot\alpha = 2\tan\alpha$$

$$\therefore \frac{1}{\tan\alpha} = 2\tan\alpha$$

$$\therefore \tan^2\alpha = \frac{1}{2}$$

$$\therefore \tan\alpha = \frac{1}{\sqrt{2}} \qquad (\because \alpha \text{ is the acute angle.})$$

Now, $\tan\alpha = \frac{2h}{x}$

$$\therefore \frac{1}{\sqrt{2}} = \frac{2h}{x}$$

$$\therefore h = \frac{x}{2\sqrt{2}}$$

$\therefore$ The height of the shorter pole is $h = \frac{x}{2\sqrt{2}}$.