CBSE Test Paper 03

CH-05 Complex & Quadratic

1. If $x = \omega^2 - \omega - 3, \omega$ being a non real cube root of unity , then the value of $x^4 + 6x^3 + 10x^2 - 12x - 19$ is
   a. 5
   b. none of these
   c. 19
   d. 12
2. Find the Amplitude of $-i$
   a. -$\frac{\pi}{2}$
   b. $\frac{\pi}{2}$
   c. π
   d. none of these
3. The number of solutions of the equation $Im\left(z^2\right) = 0, |z| = 2$ is
   a. 1
   b. 4
   c. 2
   d. 3
4. Square roots of i are
   a. $\pm\frac{1}{\sqrt{2}}(1+i)$
   b. none of these

c. ± 1

d. $\pm\frac{1}{\sqrt{2}}(1-i)$

5. Find the Amplitude of $-1-i$

a. -3 $\pi/4$

b. 3 $\pi/4$

c. $\pi/4$

d. none of these

6. Fill in the blanks:

The complex number ($\sin 135^{0}$ - i $\sin 135^{0}$) is written in polar form as ________.

7. Fill in the blanks:

The conjugate of complex number 3 + i is ________.

8. Express $\left(\frac{1}{2}+\frac{5}{2}i\right)-\frac{3}{2}i+\left(\frac{-5}{2}-i\right)$ in the form of a + ib.

9. Express the complex number sin 50^{0} + i cos 50^{0} in the polar form.

10. Find the product of complex number (- 5 + 7i), (- 13 - 3i).

11. If z_1 = 3 + 2i and z_2 = 2 - i, then verify that $\overline{z_1 z_2} = \overline{z_1}\ \overline{z_2}$

12. Simplify the following complex number $\overline{9-i}+\overline{6+i^3}-\overline{9+i^2}$

13. Find the value of (4 + $3\sqrt{-20}$)$^{1/2}$ + (4 - $3\sqrt{-20}$)$^{1/2}$.

14. If $x+iy=\frac{(a+i)^2}{2a-i}$, show that $x^2+y^2=\frac{(a^2+1)^2}{4a^2+1}$.

15. Write the complex number z = $\frac{i-1}{\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}}$ in the polar form.

Solution

1. (a) 5

   Explanation:

   $x = \omega^2 - \omega - 3$

   $\Rightarrow x + 3 = \omega^2 - \omega$

   $\Rightarrow x^2 + 6x + 9 = \omega^4 - 2\omega^3 + \omega^2 = \omega - 2 + \omega^2 = -3$

   $\Rightarrow x^2 + 6x + 12 = 0$

   Now $x^4 + 6x^3 + 10x^2 - 12x - 19 = \left(x^2 - 2\right)\left(x^2 + 6x + 12\right) + 5$

   $= \left(x^2 - 2\right) \times 0 + 5 = 5$

2. (a)-$\frac{\pi}{2}$

   Explanation: Let $Z = -i = r\left(cos\theta + isin\theta\right)$

   Then comparing the real and imaginary parts ,we get

   $r\cos\theta = 0 \quad$ and $\quad r\sin\theta = -1$

   $\therefore r^2\left(\cos^2\theta + \sin^2\theta\right) = 1$

   $\Rightarrow r^2 = 1 \Rightarrow r = 1$

   $\therefore \quad cos\theta = 0 \quad and \quad sin\theta = -1$

   $\left[cos\left(\frac{\Pi}{2}\right) = 0 \quad , sin\left(\frac{\Pi}{2}\right) = 1\right]$

   [Format of amplitude is -θ in the fourth quadrant]

   Since θ lies in the fourth quadrant, we have the principal value of the argument (Amplitude) = -$\frac{\pi}{2}$

3. (b) 4

   Explanation:

   Let $z = x + iy$

   , then we have

   $|z| = 2 \Rightarrow \sqrt{x^2 + y^2} = 2 \Rightarrow x^2 + y^2 = 4$..........$(ii)$

$\left(x^2-y^2\right)^2=\left(x^2+y^2\right)^2-4x^2.y^2$
$\Rightarrow\left(x^2-y^2\right)^2=16$
$\Rightarrow x^2-y^2=\pm 4\ldots\ldots(iii)$

When $x^2-y^2=4$ from (ii) and (iii) we get $2x^2=8\Rightarrow x^2=4\Rightarrow x=\pm 2$
Now from (i) we get y=0
Hence $z=\pm 2$
Now when $x^2-y^2=-4$ from (ii) and (iii) we get $2y^2=8\Rightarrow y^2=4\Rightarrow y=\pm 2$
Now from (i) we get x=0
Hence $z=\pm 2i$
Therefore the solutions of the equation $Im\left(z^2\right)=0,|z|=2$ are $z=\pm 2i$ and $z=\pm 2$ and so number of solutions=4.

4. (a) $\pm\frac{1}{\sqrt{2}}(1+i)$

Explanation:
Let $\sqrt{i}=x+iy$
Squaring both sides ,we get
$i=(x+iy)^2=x^2-y^2+2xyi$
Subtracting (i) from (iii), we get
$2y^2=1\Rightarrow y^2=\frac{1}{2}\Rightarrow y=\pm\frac{1}{\sqrt{2}}$

Comparing real and imaginary parts , we get
$x^2-y^2=0\ldots\ldots\ldots.(i)\quad and\quad 2xy=1\ldots\ldots\ldots\ldots.(ii)$
We have $\left(x^2-y^2\right)^2=\left(x^2+y^2\right)^2-4x^2.y^2$
$\Rightarrow 0=\left(x^2+y^2\right)^2-(2xy)^2=\left(x^2+y^2\right)^2-1$
$\Rightarrow\left(x^2+y^2\right)^2=1$
$\Rightarrow x^2+y^2=1\ldots\ldots\ldots$ (iii)
Adding (i) and (iii) we get
$2x^2=1\Rightarrow x^2=\frac{1}{2}\Rightarrow x=\pm\frac{1}{\sqrt{2}}$
Hence $\sqrt{i}=x+iy$=$\pm\frac{1}{\sqrt{2}}(1+i)$

5. (a) -3

Explanation:

Let $Z=-1-i = r\left(cos\theta + isin\theta\right)$

Then comparing the real and imaginary parts ,we get
$r\cos\theta=-1 \quad \text{and} \quad r\sin\theta=-1$
$\therefore r^2\left(\cos^2\theta+\sin^2\theta\right)=2$
$\Rightarrow r^2=2 \Rightarrow r=\sqrt{2}$
$\therefore \quad \sqrt{2}\cos\theta=-1 \quad \text{and} \quad \sqrt{2}\sin\theta=-1$
$\Rightarrow \cos\theta=\frac{-1}{\sqrt{2}} \text{ and } \sin\theta=\frac{-1}{\sqrt{2}}$

$\left[cos\left(\frac{\Pi}{4}\right)=sin\left(\frac{\Pi}{4}\right)=\frac{1}{\sqrt{2}}\right]$

Since both $sin\theta \quad and \quad cos\theta$ are negative and we have $sin\theta \quad and \quad cos\theta$ are negative in the third quardrant ,

the principal value of the argument (Amplitude) = $-\left(\Pi-\frac{\Pi}{4}\right)=-\frac{3\Pi}{4}$

6. 1 (cos 45^o + i sin 45^o)

7. 3 - i

8. We have, $\left(\frac{1}{2}+\frac{5}{2}i\right)-\frac{3}{2}i+\left(\frac{-5}{2}-i\right)$
= $\frac{1}{2}-\frac{5}{2}+i\left(\frac{5}{2}-\frac{3}{2}-1\right)$
= $\frac{-4}{2}$ + i (1 - 1) = - 2 + i (0) = - 2 + 0i

9. Suppose, $z=sin50^o+i\ cos50^o$
$=sin(90^o\ -40^o)+i\ cos(90^o\ -40^o)$
= cos 40^o + i sin 40^o

10. $(-5+7i).(-13-3i)=(-5)(-13)+(-5)(-3i)+(7i)(-13)+(7i)(-3i)$

= 65 + 15i - 91i - 21i^2 = 65 - 76i + 21 [∵ i^2 = - 1]

$= 86 - 76i$

11. Now, z_1 z_2 = (3 + 2i) (2 - i)

    = 6 - 3i + 4i - 2i^2 = 6 + i - 2 (- 1) = 8 + i

    $\Rightarrow \overline{z_1 z_2} = \overline{8+i}$ = 8 - i ...(i)

    and $\overline{z_1}$ $\overline{z_2}$ = (3 - 2i) (2 + i) = 6 + 3i - 4i - 2i^2

    = 6 - i - 2 (- 1) = 8 - i ...(ii)

    Form Eqs. (i) and (ii), we get

    $\overline{z_1 z_2}$ = $\overline{z_1}$ $\overline{z_2}$

    Hence verified.

12. We have, $\overline{9-i} + \overline{6+i^3} - \overline{9+i^2}$

    = (9 + i) + (6 + i^3) - (9 + i^2)

    = 9 + i + 6 + i - 9 + 1 [∵ i^3 = i, i^2 = - 1]

    = 7 + 2i

13. Given that, (4 + $3\sqrt{-20}$)$^{1/2}$ + (4 - $3\sqrt{-20}$)$^{1/2}$

    = $\sqrt{4+i3\sqrt{20}} + \sqrt{4-3i\sqrt{20}}$

    = $\sqrt{4+i6\sqrt{5}} + \sqrt{4-6i\sqrt{5}}$

    Using, [$\sqrt{a+ib} + \sqrt{a-ib}$ = $\sqrt{2\left(\sqrt{a^2+b^2}+a\right)}$]

    = $\sqrt{2\left\{\sqrt{(4)^2+(6\sqrt{5})^2}+4\right\}} = \sqrt{2(\sqrt{16+180}+4)}$

    = $\sqrt{2(14+4)}$

    = $\sqrt{36}$ = $\pm$ 6

14. Here $x+iy = \frac{(a+i)^2}{2a-i}$, ... (i)

    Taking conjugate on both sides, we have

    $\overline{x+iy} = \frac{\overline{(a+i)}^2}{\overline{2a-i}}$

    $\Rightarrow x-iy = \frac{(a-i)^2}{2a+i}$ .... (ii)

    Multiplying (i) and (ii), we have

$(x+iy)(x-iy)=\frac{(a+i)^2}{2a-i}\times\frac{(a-i)^2}{2a+i}$

$\Rightarrow x^2-i^2y^2=\frac{(a+i)^2(a-i)^2}{4a^2-i^2}$

$\Rightarrow x^2+y^2=\frac{\left(a^2-i^2\right)^2}{4a^2+1}\,[\because i^2=-1]$

$=\frac{\left(a^2+1\right)^2}{4a^2+1}$

15. We have, z = $\frac{i-1}{\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}}$

Let, -1 + i = r (cosθ + i sinθ)

$\Rightarrow$ r cosθ = - 1 ...(i)

and r sinθ = 1 ...(ii)

On squaring and adding Eqs. (i) and (ii), we get

r^2 ($\cos^2\theta$ + $\sin^2\theta$) = 1 + 1

$\Rightarrow$ r^2 = 2

$\therefore$ r = $\sqrt{2}$ [taking positive square root]

On putting the value of r in Eqs. (i) and (ii), we get

cosθ = $\frac{-1}{\sqrt{2}}$ and sinθ = $\frac{1}{\sqrt{2}}$

Since, sinθ is positive and cosθ is negative.

So, θ lies in II quadrant.

$\therefore$ θ = $\left(\pi-\frac{\pi}{4}\right)=\frac{3\pi}{4}$

$\Rightarrow$ i - 1 = $\sqrt{2}\left(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right)$

$\therefore$ z = $\frac{i-1}{\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)}=\frac{\sqrt{2}\left(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right)}{\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)}$

= $\frac{\sqrt{2}\left(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right)}{\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)}\times\frac{\left(\cos\frac{\pi}{3}-i\sin\frac{\pi}{3}\right)}{\left(\cos\frac{\pi}{3}-i\sin\frac{\pi}{3}\right)}$ [multiplying numerator and denominator by $\left(\cos\frac{\pi}{3}-i\sin\frac{\pi}{3}\right)$]

$=\frac{\sqrt{2}\left[\left(\cos\frac{3\pi}{4}\cdot\cos\frac{\pi}{3}+\sin\frac{3\pi}{4}\cdot\sin\frac{\pi}{3}\right)+i\left(\sin\frac{3\pi}{4}\cdot\cos\frac{\pi}{3}-\cos\frac{3\pi}{4}\cdot\sin\frac{\pi}{3}\right)\right]}{\left(\cos^2\frac{\pi}{3}+\sin^2\frac{\pi}{3}\right)}$

= $\frac{\sqrt{2}\left[\cos\left(\frac{3\pi}{4}-\frac{\pi}{3}\right)+i\sin\left(\frac{3\pi}{4}-\frac{\pi}{3}\right)\right]}{1}$

= $\sqrt{2}\left[\cos\frac{5\pi}{12}+i\sin\frac{5\pi}{12}\right]$