

DAY TWENTY FIVE

Electromagnetic Waves

Learning & Revision for the Day

- Electromagnetic Waves and their Characteristics
- Maxwell's Equations
- Transverse Nature of Electromagnetic Waves
- Spectrum of Electromagnetic Radiation

Electromagnetic Waves and their Characteristics

Electromagnetic waves are those waves, in which electric and magnetic fields vary sinusoidally in space with time. The electric and magnetic fields are mutually perpendicular to each other and each field is perpendicular to the direction of propagation of the wave.

- Maxwell's theory predicted that electromagnetic waves of all frequencies (and hence all wavelengths) propagate in vacuum, with a speed given by

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}.$$

where, μ_0 is the magnetic permeability and ϵ_0 is the electric permittivity of vacuum. Now, for the vacuum, $\mu_0 = 4\pi \times 10^{-7} \text{ TmA}^{-1}$ and $\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2 \text{N}^{-1} \text{m}^{-2}$.

Substituting these values in the above relation, we have

$$c = \frac{1}{[(4\pi \times 10^{-7})(8.85 \times 10^{-12})]^{1/2}} \simeq 3.0 \times 10^8 \text{ ms}^{-1}$$

- All the electromagnetic waves are of the transverse nature whose speed depends upon the medium, but their frequency does not depend on the medium.
- Transverse waves can be polarised.
- Energy is being transported with the electromagnetic waves.

Conduction Current

It is a current in the electric circuit, which arises due to the flow of electrons in the connecting wires of the circuit, in a definite closed path.

Maxwell's Displacement Current

It is that current which comes into play in the region, whenever the electric field and hence the electric flux is changing with time.

$$i_d = \epsilon_0 \frac{d\phi_E}{dt}$$

The generalised form of the Ampere's law is

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 (i_c + i_d) = \mu_0 \left(i_c + \epsilon_0 \frac{d\phi_E}{dt} \right)$$

where, i_c is conduction current.

Maxwell's Equations

Maxwell in 1862, gave the basic laws of electricity and magnetism in the form of four fundamental equations, which are known as Maxwell's equations.

- **Gauss's law for electrostatics** This law states that the total electric flux through any closed surface is always equal to $\frac{1}{\epsilon_0}$ times the net charged enclosed by that surface.

$$\text{Mathematically, } \oint_S \mathbf{E} \cdot d\mathbf{S} = \frac{q}{\epsilon_0}$$

- **Gauss's law for magnetism** This law also predicts that the isolated magnetic monopole does not exist.
i.e. net magnetic flux through any closed surface is always zero.

$$\text{Mathematically, } \oint_S \mathbf{B} \cdot d\mathbf{S} = 0$$

- **Faraday's law of electromagnetic induction** It states that the induced e.m.f. produced in a circuit is numerically equal to time rate of change of magnetic flux through it.

$$\text{Mathematically, } \oint \mathbf{E} \cdot d\mathbf{l} = - \frac{d\phi_B}{dt}$$

- **Ampere-Maxwell's law** At an instant in a circuit, the conduction current is equal to displacement current.

$$\text{Mathematically, } \oint \mathbf{E} \cdot d\mathbf{l} = \mu_0 \left(I_c + \epsilon_0 \frac{d\phi_E}{dt} \right)$$

These equations are collectively called Maxwell's equations.

Properties of Electromagnetic Waves

- If the electromagnetic wave is travelling along the positive direction of the X-axis, the electric field is oscillating parallel to the Y-axis and the magnetic field is oscillating parallel to the Z-axis.

$$E = E_0 \sin(\omega t - kx) \Rightarrow B = B_0 \sin(\omega t - kx)$$

In this, E_0 and B_0 are the amplitudes of the fields.

Further, $c = \frac{E_0}{B_0} = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$ = speed of light in vacuum

- The rate of flow of energy in an electromagnetic wave, is described by the vector **S** called the **Poynting vector**, which is defined by the expression,

$$\mathbf{S} = \frac{1}{\mu_0} \mathbf{E} \times \mathbf{B}$$

- The time average of S over one cycle is known as the wave intensity. When the average is taken, we obtain an expression involving the time average of $\cos^2(kx - \omega t)$ which equals $\frac{1}{2}$.

$$\text{Thus, } I = S_{\text{av}} = \frac{E_0 B_0}{2\mu_0} = \frac{E_0^2}{2\mu_0 c} = \frac{c B_0^2}{2\mu_0} \text{ Wm}^{-2}$$

- The total average energy per unit volume is,

$$u = u_E + u_B = \frac{\epsilon_0 E_0^2}{2} = \frac{B^2}{2\mu_0}$$

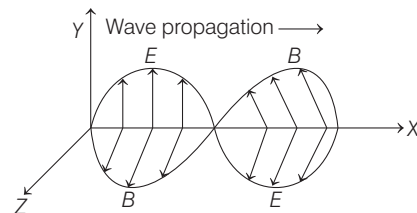
- The radiation pressure p exerted on a perfectly absorbing surface, $p = \frac{S}{c}$.
- If the surface is a perfect reflector and incidence is normal, then the momentum transported to the surface in a time t is given by, $p = \frac{2u}{c}$ and the radiation pressure will be, $p = \frac{2S}{c}$.
- Energy density of electromagnetic wave,

$$u_e = \frac{1}{2} \epsilon_0 E_{u_B}^2 = \frac{1}{2} \frac{B^2}{\mu_0}$$

- Momentum delivered, $p = \frac{u}{c}$ (absorbing surface)
 $p = \frac{2u}{c}$ (reflecting surface)
- Energy of wave = $\frac{hc}{\lambda} = hv$

Transverse Nature of Electromagnetic Waves

According to Maxwell, electromagnetic waves consist of time varying electric and magnetic fields, which are perpendicular to each other, as well as direction of wave propagation.



Spectrum of Electromagnetic Radiation

The array obtained on arranging all the electromagnetic waves in an order on the basis of their wavelength is called the electromagnetic spectrum.

The Electromagnetic Spectrum

Name	Frequency Range (Hz)	Wavelength Range (m)	Source
Radio waves	10^4 to 10^8	0.1 to 600	Oscillating electric circuits
Microwaves	10^9 to 10^{12}	10^{-3} to 0.3	Oscillating current in special vacuum tubes
Infrared	10^{11} to 5×10^{14}	10^{-6} to 5×10^{-3}	Outer electrons in atoms and molecules
Visible light	4×10^{14} to 7×10^{14}	4×10^{-7} to 8×10^{-7}	Outer electrons in atoms
Ultraviolet	10^{15} to 10^{17}	1.5×10^{-7} to 3.5×10^{-7}	Outer electrons in atoms
X-rays	10^{18} to 10^{20}	10^{-11} to 10^{-8}	Inner electrons in atoms and sudden deacceleration of high energy free electrons
Gamma rays	10^{19} to 10^{24}	10^{-16} to 10^{-13}	Nuclei of atoms and sudden deacceleration of high energy free electrons

Various Electromagnetic Radiations

- **Gamma rays** The main sources of gamma rays are the natural and artificial radioactive substances. These rays affect the photographic plate and mainly used in the treatment of cancer disease.
- **X-rays** X-rays are produced, when highly energetic cathode rays are stopped by a metal target of high melting point. They affect the photographic plate and can penetrate through the transparent materials. They are mainly used in

detecting the fracture of bones, hidden bullet, needle, costly material etc. inside the body, and also used in the study of crystal structure.

- **Ultraviolet Rays** The major part of the radiations received from sun consists of the ultraviolet radiation. Its other sources are the electric discharge tube, carbon arc, etc. These radiations are mainly used in excitation of photoelectric effect and to kill the bacteria of many diseases.
- **Visible Light** Visible light is obtained from the glowing bodies, while they are white hot. The light obtained from the electric bulbs, sodium lamp, fluorescent tube is the visible light.
- **Thermal or Infrared Waves** A body on being heated, emits out the infrared waves. These radiations have the maximum heating effect. The glass absorbs these radiations, therefore for the study of these radiations, rock salt prism is used instead of a glass prism. These waves are mainly used for therapeutic purpose by the doctors because of their heating effect.
- **Microwaves** These waves are produced by the spark discharge or magnetron valve. They are detected by the crystal or semiconductor detector. These waves are used mainly in radar and long distance communication.
- **Radio waves** They can be obtained by the flow of high frequency alternating current in an electric conductor. These waves are detected by the tank circuit in a radio receiver or transmitter.

Applications of Electromagnetic Spectrum

- Radio waves are used in radar and radio broadcasting.
- Microwaves are used in long distance wireless communications *via* satellites.
- Infrared, visible and ultraviolet radiations are used to know the structure of molecules.
- Diffraction of X-rays by crystals, gives the details of the structure of crystals.

DAY PRACTICE SESSION 1

FOUNDATION QUESTIONS EXERCISE

- 1** During the propagation of electromagnetic waves in a medium, **→ JEE Main 2014**
- electric energy density is double of the magnetic energy density
 - electric energy density is half of the magnetic energy density
 - electric energy density is equal to the magnetic energy density
 - Both electric and magnetic energy densities are zero
- 2** A perfectly reflecting mirror has an area of 1 cm^2 . Light energy is allowed to fall on it for 1 h at the rate of 10 W cm^{-2} . The force acting on the mirror is
- $6.7 \times 10^{-8} \text{ N}$
 - $2.3 \times 10^{-4} \text{ N}$
 - 10^{-3} N
 - zero
- 3** The magnetic field between the plates of radius 12 cm, separated by a distance of 4 mm of a parallel plate capacitor of capacitance 100 pF along the axis of plates having conduction current of 0.15 A, is
- zero
 - 1.5 T
 - 15 T
 - 0.15 T
- 4** Instantaneous displacement current of 1.0 A in the space between the parallel plates of a $1 \mu\text{F}$ capacitor, can be established by changing potential difference of
- 10^{-6} Vs^{-1}
 - 10^6 Vs^{-1}
 - 10^{-8} Vs^{-1}
 - 10^8 Vs^{-1}
- 5** A large parallel plate capacitor, whose plates have an area of 1 m^2 and are separated from each other by 1 mm, is being charged at a rate of 25 Vs^{-1} . If the dielectric between the plates has the dielectric constant 10, then the displacement current at this instant is
- $25 \mu\text{A}$
 - $11 \mu\text{A}$
 - $2.2 \mu\text{A}$
 - $1.1 \mu\text{A}$
- 6** A parallel plate capacitor with plate area A and separation between the plates d , is charged by a constant current I . Consider a plane surface of area $A/2$ parallel to the plates and drawn simultaneously between the plates. The displacement current through this area is
- I
 - $\frac{I}{2}$
 - $\frac{I}{4}$
 - $\frac{I}{8}$
- 7** Select the correct statement from the following **→ JEE Main (Online) 2013**
- Electromagnetic waves cannot travel in vacuum
 - Electromagnetic waves are longitudinal waves
 - Electromagnetic waves are produced by charges moving with uniform velocity
 - Electromagnetic waves carry both energy and momentum as they propagate through space
- 8** In an apparatus, the electric field was found to oscillate with an amplitude of 18 Vm^{-1} . The magnitude of the oscillating magnetic field will be
- $4 \times 10^{-6} \text{ T}$
 - $6 \times 10^{-8} \text{ T}$
 - $9 \times 10^{-9} \text{ T}$
 - $11 \times 10^{-11} \text{ T}$
- 9** The magnetic field in a travelling electromagnetic wave has a peak value of 20 nT. The peak value of electric field strength is **→ JEE Main 2013**
- 3 V/m
 - 6 V/m
 - 9 V/m
 - 12 V/m
- 10** The rms value of the electric field of the light coming from the sun is 720 NC^{-1} . The average total energy density of the electromagnetic wave is
- $4.58 \times 10^{-6} \text{ Jm}^{-3}$
 - $6.37 \times 10^{-9} \text{ Jm}^{-3}$
 - $81.35 \times 10^{-12} \text{ Jm}^{-3}$
 - $3.3 \times 10^{-3} \text{ Jm}^{-3}$
- 11** A radiation of energy E falls normally on a perfectly reflecting surface. The momentum transferred to the surface is
- $\frac{E}{c}$
 - $\frac{2E}{c}$
 - Ec
 - $\frac{E}{c^2}$
- 12** An electromagnetic wave in vacuum has the electric and magnetic fields \mathbf{E} and \mathbf{B} , which are always perpendicular to each other. The direction of polarisation is given by \mathbf{X} and that of wave propagation by $\hat{\mathbf{k}}$. Then, **→ AIEEE 2012**
- $\mathbf{X} \parallel \mathbf{B}$ and $\hat{\mathbf{k}} \parallel \mathbf{B} \times \mathbf{E}$
 - $\mathbf{X} \parallel \mathbf{E}$ and $\hat{\mathbf{k}} \parallel \mathbf{E} \times \mathbf{B}$
 - $\mathbf{X} \parallel \mathbf{B}$ and $\hat{\mathbf{k}} \parallel \mathbf{E} \times \mathbf{B}$
 - $\mathbf{X} \parallel \mathbf{B}$ and $\hat{\mathbf{k}} \parallel \mathbf{B} \times \mathbf{E}$
- 13** An electromagnetic wave travels in vacuum along z-direction $\mathbf{E} = (E_1 \hat{\mathbf{i}} + E_2 \hat{\mathbf{j}}) \cos(kz - \omega t)$. Choose the correct option from the following
- The associated magnetic field is given as $\mathbf{B} = \frac{1}{c}(E_1 \hat{\mathbf{i}} - E_2 \hat{\mathbf{j}}) \cos(kz - \omega t)$
 - The associated magnetic field is given as $\mathbf{B} = \frac{1}{c}(E_1 \hat{\mathbf{i}} + E_2 \hat{\mathbf{j}}) \cos(kz - \omega t)$
 - The given electromagnetic field is circularly polarised
 - The given electromagnetic wave is plane polarised
- 14** Match List I (Electromagnetic wave type) with List II (Its association/application) and select the correct option from the choices given below the lists. **→ JEE Main 2014**
- | List I | | List II | |
|----------------------|----|---|--|
| A. Infrared waves | 1. | To treat muscular strain | |
| B. Radio waves | 2. | For broadcasting | |
| C. X-rays | 3. | To detect fracture of bones | |
| D. Ultraviolet waves | 4. | Absorbed by the ozone layer of the atmosphere | |
- Codes**
- | | |
|-------------|-------------|
| A B C D | A B C D |
| (a) 4 3 2 1 | (b) 1 2 4 3 |
| (c) 3 2 1 4 | (d) 1 2 3 4 |

- 15** Arrange the following electromagnetic radiations per quantum in the order of increasing energy.

A. Blue light B. Yellow light
C. X-ray D. Radio wave

→ JEE Main 2016 (Offline)

(a) D, B, A, C (b) A, B, D, C
(c) C, A, B, D (d) B, A, D, C

Direction (Q. Nos. 16-20) *Each of these questions contains two statements : Statement I (Assertion) and Statement II (Reason). Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c), (d) given below*

- (a) Statement I is true; Statement II is true; Statement II is the correct explanation for Statement I
(b) Statement I is true; Statement II is true; Statement II is not the correct explanation for Statement I
(c) Statement I is true; Statement II is false
(d) Statement I is false; Statement II is true

- 16 Statement I** Ultraviolet radiation being higher frequency waves are dangerous to human being.

Statement II Ultraviolet radiations are absorbed by the atmosphere.

- 17 Statement I** If the earth did not have atmosphere, its average surface temperature would be lower than what is now.

Statement II Greenhouse effect of the atmosphere would be absent, if the earth did not have atmosphere.

- 18 Statement I** Electromagnetic waves exert radiation pressure.

Statement II Electromagnetic waves carry energy.

- 19 Statement I** Light is a transverse wave, but not an electromagnetic wave.

Statement II Maxwell showed that speed of electromagnetic waves is related to the permeability and the permittivity of the medium through which it travels.

- 20 Statement I** Out of radio waves and microwaves, the radio waves undergo more diffraction.

Statement II Radio waves have greater frequency compared to microwaves. → JEE Main (Online) 2013

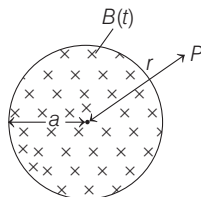
DAY PRACTICE SESSION 2

PROGRESSIVE QUESTIONS EXERCISE

- 1** You are given a $2\text{ }\mu\text{F}$ parallel plate capacitor. How would you establish an instantaneous displacement current of 1 mA in the space between its plates?

(a) By applying a varying potential difference of 500 V/s
(b) By applying a varying potential difference of 400 V/s
(c) By applying a varying potential difference of 100 V/s
(d) By applying a varying potential difference of 300 V/s

- 2** A uniform but time varying magnetic field $B(t)$ exists in a circular region of radius a and is directed into the plane of the paper as shown in the figure. The magnitude of the induced electric field at a point P , a distance r from the centre of the circular region



(a) increases with r
(b) decreases with r
(c) decreases as $\frac{1}{r^2}$
(d) zero

- 3** The magnetic field of a beam emerging from a filter facing a flood light is given by $B = 12 \times 10^{-8} \sin(1.20 \times 10^7 z - 3.60 \times 10^{15} t) \text{ T}$. What is the average intensity of the beam?

(a) 1.7 W/m^2 (b) 2.3 W/m^2
(c) 2.7 W/m^2 (d) 3.2 W/m^2

- 4** The ratio of contributions made by the electric field and magnetic field components to the intensity of an electromagnetic wave is

(a) $c:1$ (b) $c^2:1$ (c) $1:1$ (d) $\sqrt{c}:1$

- 5** An FM radio station, antenna radiates a power of 10 kW at a wavelength of 3 m. Assume the radiated power is confined to and is uniform over a hemisphere with antenna at its centre. E_{max} at a distance of 10 km from antenna is

(a) 0.62 NC^{-1} (b) 0.41 NC^{-1}
(c) 0.31 NC^{-1} (d) 0.10 NC^{-1}

- 6** Assume that all the energy from a 1000 W lamp is radiated uniformly, then the amplitude of electric field of radiation at a distance of 2 m from the lamp is

(a) 245.01 V/m (b) 17 V/m
(c) 0 (d) 2.96 V/m

- 7** A red LED emits light at 0.1W uniformly around it. The amplitude of the electric field of the light at a distance of 1m from the diode is

→ JEE Main 2015

(a) 1.73 V/m
(b) 2.45 V/m
(c) 5.48 V/m
(d) 7.75 V/m

- 8** In a transverse wave, the distance between a crest and neighbouring trough at the same instant is 4.0 cm and the distance between a crest and trough at the same place is 1.0 cm. The next crest appears at the same place after a time interval of 0.4 s. The maximum speed of the vibrating particles in the medium is

→ JEE Main (Online) 2013

- (a) $\frac{3\pi}{2}$ cm/s (b) 5π cm/s
(c) $\frac{\pi}{2}$ cm/s (d) 2π cm/s

- 9** An EM wave from air enters a medium. The electric fields are $\mathbf{E}_1 = E_{01}\hat{\mathbf{x}} \cos\left[2\pi\nu\left(\frac{z}{c} - t\right)\right]$ in air and

$\mathbf{E}_2 = E_{02}\hat{\mathbf{x}} \cos[k(2z - ct)]$ in medium, where the wave number k and frequency ν refer to their values in air. The medium is non-magnetic.

If ϵ_{r1} and ϵ_{r2} refer to relative permittivities of air and medium respectively, which of the following options is correct?

→ JEE Main 2018

- (a) $\frac{\epsilon_{r1}}{\epsilon_{r2}} = 4$ (b) $\frac{\epsilon_{r1}}{\epsilon_{r2}} = 2$ (c) $\frac{\epsilon_{r1}}{\epsilon_{r2}} = \frac{1}{4}$ (d) $\frac{\epsilon_{r1}}{\epsilon_{r2}} = \frac{1}{2}$

- 10** An electromagnetic wave of frequency $\nu = 3.0$ MHz passes from vacuum into a dielectric medium with permittivity $\epsilon = 4.0$. Then,

- (a) wavelength is doubled and the frequency remains unchanged
(b) wavelength is doubled and frequency becomes half
(c) wavelength is halved and frequency remains unchanged
(d) wavelength and frequency both remain unchanged

- 11** The magnetic field at a point between the plates of a capacitor at a perpendicular distance R from the axis of the capacitor plate radius R , having the displacement current I_D is given by

- (a) $\frac{\mu I_D r}{2\pi R^2}$ (b) $\frac{\mu_0 I_D}{2\pi R}$
(c) $\frac{\mu_0 I_D}{\pi r^2}$ (d) zero

ANSWERS

SESSION 1

- | | | | | | | | | | |
|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 1 (c) | 2 (a) | 3 (a) | 4 (b) | 5 (c) | 6 (b) | 7 (d) | 8 (b) | 9 (b) | 10 (a) |
| 11 (b) | 12 (b) | 13 (d) | 14 (d) | 15 (a) | 16 (b) | 17 (a) | 18 (b) | 19 (d) | 20 (c) |

SESSION 2

- | | | | | | | | | | |
|--------|-------|-------|-------|-------|-------|-------|-------|-------|--------|
| 1 (a) | 2 (b) | 3 (a) | 4 (c) | 5 (d) | 6 (a) | 7 (b) | 8 (b) | 9 (c) | 10 (c) |
| 11 (b) | | | | | | | | | |

Hints and Explanations

SESSION 1

- 1** Both the energy densities are equal, i.e. energy is equally divided between electric and magnetic field.

- 2** Let E = energy falling on the surface per second = 10 J

Momentum of photons,

$$p = \frac{h}{\lambda} = \frac{h}{c/f}$$

$$= \frac{hf}{c} = \frac{E}{c}$$

On reflection, change in momentum per second = force

$$= 2p = \frac{2E}{c} = \frac{2 \times 10}{3 \times 10^8}$$

$$= 6.7 \times 10^{-8} \text{ N}$$

- 3** As $B \propto r$, since the point is on the axis, where $r = 0$, so $B = 0$.

- 4** As, $I_d = C \left(\frac{V}{t} \right)$ $\left(\because I = \frac{dQ}{dt} \right)$

$$\text{or } \frac{V}{t} = \frac{I_d}{C}$$

$$= \frac{1.0}{10^{-6}} = 10^6 \text{ Vs}^{-1}$$

- 5** As, $C = \frac{\epsilon_0 K A}{d}$
- $$= \frac{(8.85 \times 10^{-12}) \times 10 \times 1}{10^{-3}}$$

$$= 8.85 \times 10^{-8} \text{ F}$$

$$\therefore I = \frac{d}{dt}(CV) = C \frac{dV}{dt}$$

$$= 8.85 \times 10^{-8} \times 25$$

$$= 2.2 \times 10^{-6} = 2.2 \mu\text{A}$$

- 6** Charge on the capacitor plates, at time t is, $q = It$

Electric field between the plates at this instant,

$$E = \frac{q}{A\epsilon_0} = \frac{It}{A\epsilon_0}$$

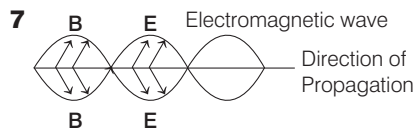
Electric flux through the given area,

$$\phi_E = \left(\frac{A}{2} \right) E = \frac{It}{2\epsilon_0}$$

Therefore, displacement current,

$$I_d = \epsilon_0 \frac{d\phi_E}{dt}$$

$$= \epsilon_0 \frac{d}{dt} \left(\frac{It}{2\epsilon_0} \right) = \frac{I}{2}$$



As electromagnetic waves contain both electric field and magnetic field. It carries both energy and momentum according to de-Broglie wave-particle duality of radiations.

8 Here, $E_0 = 18 \text{ Vm}^{-1}$

$$\therefore B_0 = \frac{E_0}{c} = \frac{18}{3 \times 10^8} = 6 \times 10^{-8} \text{ T}$$

9 $\mathbf{E} = \mathbf{B} \times \mathbf{c}$ keep value of electric field

$$|\mathbf{E}| = |\mathbf{B}|c = 20 \times 10^{-9} \times 3 \times 10^8 = 6 \text{ V/m}$$

$$\begin{aligned} \text{10 Total average energy} &= \epsilon_0 E_{\text{rms}}^2 \\ &= 8.85 \times 10^{-12} \times (720)^2 \\ &= 4.58 \times 10^{-6} \text{ Jm}^{-3} \end{aligned}$$

11 Initial momentum of surface, $p_i = E/c$ where, c = velocity of light (constant). Since, the surface is perfectly reflecting, so the same momentum will be reflected completely. Final momentum, $p_f = E/c$ [negative value]
 \therefore Change in momentum,

$$\Delta p = p_f - p_i = -\frac{E}{c} - \frac{E}{c} = -\frac{2E}{c}$$

Thus, momentum transferred to the surface is

$$\Delta p' = |\Delta p| = \frac{2E}{c}$$

12 In electromagnetic wave, the direction of propagation of wave, electric field and magnetic field are mutually perpendicular, i.e. wave propagates perpendicular to \mathbf{E} and \mathbf{B} or along $\mathbf{E} \times \mathbf{B}$. While polarisation of wave takes place parallel to electric field vector.

13 Here, in electromagnetic wave, the electric field vector is given as

$$\mathbf{E} = (E_1 \hat{i} + E_2 \hat{j}) \cos(kz - \omega t)$$

In electromagnetic wave, the associated magnetic field vector,

$$\mathbf{B} = \frac{E}{c} = \frac{E_1 \hat{i} + E_2 \hat{j}}{c} \cos(kz - \omega t)$$

Also, \mathbf{E} and \mathbf{B} are perpendicular to each other and the propagation of electromagnetic wave is perpendicular to \mathbf{E} as well as \mathbf{B} , so the given electromagnetic wave is plane polarised.

- 14** (a) Infrared waves are used to treat muscular strain.
 (b) Radio waves are used for broadcasting purposes.
 (c) X-rays are used to detect fracture of bones.
 (d) Ultraviolet waves are absorbed by ozone.

15 As, we know energy liberated,

$$E = \frac{hc}{\lambda}$$

$$\text{i.e. } E \propto \frac{1}{\lambda}$$

So, lesser the wavelength, then greater will be energy liberated by electromagnetic radiations per quantum.

As, order of wavelength is given by

X-ray, VIBGYOR, Radio waves

(C) (A) (B) (D)

\therefore Order of increasing energy of electromagnetic radiations per quantum.

$$\Rightarrow D < B < A < C$$

16 Ultraviolet radiations are electromagnetic waves. The wavelength of these waves ranges between 4000 \AA to 100 \AA , i.e. of smaller wavelength and higher frequency. They are absorbed by ozone layer of stratosphere in atmosphere. They cause skin diseases and they are harmful to eye and may cause permanent blindness.

17 Earth is heated by sun's infrared radiation. The earth also emits radiation most in infrared region. These radiations are reflected back by heavy gases like CO_2 in atmosphere. These back radiation keeps the earth's surface warm at night. This phenomenon is called greenhouse effect. When the atmosphere were absent, then temperature of the earth falls.

18 Electromagnetic waves have linear momentum as well as energy. From this we conclude that, we can exert radiation pressure by making a beam of electromagnetic radiation fall on an object. Let us assume that object is free to move and that the radiation is entirely absorbed in the object during time interval Δt . The object gains an energy ΔU from the radiation. Maxwell showed that the object also gains linear momentum, the magnitude Δp of the change in momentum of the object is related to the energy change ΔU as

$$\Delta p = \frac{\Delta U}{c} \quad (\text{total absorption})$$

19 In free space or vacuum, the speed of electromagnetic waves is

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \quad \dots(i)$$

Here, $\mu_0 = 4\pi \times 10^{-7} \text{ N s}^2 \text{ C}^{-2}$ is

permeability (constant) of free space.

$\epsilon_0 = 8.85418 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$ is the

permittivity of free space. On substituting the values in Eq. (i), we have

$$\begin{aligned} c &= \frac{1}{\sqrt{4\pi \times 10^{-7} \times 8.85418 \times 10^{-12}}} \\ &= 2.99792 \times 10^8 \text{ ms}^{-1} \end{aligned}$$

This is same as the speed of light in vacuum. From this we conclude that light is an electromagnetic wave.

20 The frequency of radio waves less than the frequency of microwaves.

\therefore Frequency of radio waves = $3 \times 10^8 \text{ Hz}$ and frequency of microwaves = 10^{10} Hz

$$\therefore \nu_{\text{radio waves}} < \nu_{\text{microwaves}}$$

SESSION 2

1 Given, capacitance of capacitor, $C = 2 \mu\text{F}$

Displacement current, $I_d = 1 \text{ mA}$

Charge, $q = CV$

$$I_d dt = C dV \quad [\because q = It]$$

$$\text{or } I_d = C \frac{dV}{dt}$$

$$1 \times 10^{-3} = 2 \times 10^{-6} \times \frac{dV}{dt}$$

$$\text{or } \frac{dV}{dt} = \frac{1}{2} \times 10^{+3} = 500 \text{ V/s}$$

Clearly, by applying a varying potential difference of 500 V/s , we would produce a displacement current of desired value.

2 A time varying magnetic field produces an electric field. The magnitude of the electric field at a distance r from the centre of a circular region of radius a , where a time varying field B exists, is given by

$$E = \frac{a^2}{2r} \frac{dB}{dt}$$

At $r = a$,

$$E = \left(\frac{a}{2}\right) \frac{dB}{dt}$$

This is the value of E at the edge of the circular region. For $r > a$, E decreases with r .

3 Magnetic field, $\mathbf{B} = B_0 \sin \omega t$.

Given equation,

$$\begin{aligned} B &= 12 \times 10^{-8} \\ &\sin(1.20 \times 10^7 z - 3.60 \times 10^{15} t) \text{ T} \end{aligned}$$

On comparing this equation with standard equation, we get

$$B_0 = 12 \times 10^{-8}$$

The average intensity of the beam,

$$\begin{aligned} I_{av} &= \frac{1}{2} \frac{B_0^2}{\mu_0} \cdot c \\ &= \frac{1}{2} \times \frac{(12 \times 10^{-8})^2 \times 3 \times 10^8}{4\pi \times 10^{-7}} \\ &= 1.7 \text{ W/m}^2 \end{aligned}$$

4 Intensity in terms of electric field,

$$U_{av} = \frac{1}{2} \epsilon_0 E_0^2$$

Intensity in terms of magnetic field,

$$U_{av} = \frac{1}{2} \frac{B_0^2}{\mu_0}$$

Now, taking the intensity in terms of electric field,

$$\begin{aligned} (U_{av})_{\text{electric field}} &= \frac{1}{2} \epsilon_0 E_0^2 \\ &= \frac{1}{2} \epsilon_0 (cB_0)^2 \quad (\because E_0 = cB_0) \end{aligned}$$

$$\text{But, } c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

$$\begin{aligned} \therefore (U_{av})_{\text{electric field}} &= \frac{1}{2} \epsilon_0 \times \frac{1}{\mu_0 \epsilon_0} B_0^2 \\ &= \frac{1}{2} \cdot \frac{B_0^2}{\mu_0} \end{aligned}$$

Thus, the energy in electromagnetic wave is divided equally between electric field vector and magnetic field vector. Therefore, the ratio of contributions by the electric field and magnetic field components to the intensity of an electromagnetic wave is 1:1.

5 As, $I = \frac{\rho}{2\pi r^2} = \frac{10^4}{2\pi (10 \times 10^3)^2} = \frac{10^{-4}}{2\pi}$

$$\text{and } I = \frac{1}{2} \epsilon_0 E_0^2 c$$

$$\begin{aligned} \Rightarrow E_0 &= \sqrt{\frac{2I}{\epsilon_0 c}} \\ &= \sqrt{\frac{10^{-4}}{\pi \times 8.85 \times 10^{-12} \times 3 \times 10^8}} \end{aligned}$$

$$\text{or } E_0 = 0.10 \text{ NC}^{-1}$$

6 Poynting vector, $S = E \times H$

$$\begin{aligned} &= EH \sin 90^\circ = EH \\ \text{Energy of lamp} &= \frac{1000 \text{ W}}{\pi r^2} \\ &= \frac{1000}{\pi \times 2^2} \text{ Jm}^{-2} \text{ s}^{-1} \end{aligned}$$

S represents energy flow per unit area per second, we have

$$EH = \frac{1000}{\pi \times 2^2} = 79.61,$$

$$\frac{E}{H} = 377$$

$$EH \times \frac{E}{H} = 79.61 \times 377 = 30015.9$$

$$\begin{aligned} \Rightarrow E &= \sqrt{30015.9} \\ &= 173.25 \text{ V/m} \end{aligned}$$

Amplitude of electric field of radiation is

$$E_0 = E\sqrt{2} = 245.01 \text{ V/m}$$

7 Consider the LED as a point source of light. Let power of the LED is P . Intensity at r from the source,

$$I = \frac{P}{4\pi r^2} \quad \dots(i)$$

As we know that,

$$I = \frac{1}{2} \epsilon_0 E_0^2 c \quad \dots(ii)$$

From Eqs.(i) and (ii), we can write

$$\frac{P}{4\pi r^2} = \frac{1}{2} \epsilon_0 E_0^2 c$$

$$\begin{aligned} \text{or } E_0^2 &= \frac{2P}{4\pi \epsilon_0 r^2 c} \\ &= \frac{2 \times 0.1 \times 9 \times 10^9}{1 \times 3 \times 10^8} \end{aligned}$$

$$\text{or } E_0^2 = 6$$

$$\Rightarrow E_0 = \sqrt{6} = 2.45 \text{ V/m}$$

8 Given, $\frac{\lambda}{4} = 4 \text{ cm}$

$$\therefore \lambda = 16 \text{ cm and } T = 0.4 \text{ s}$$

$$\text{As, } f\lambda \times T = 2\pi$$

$$\Rightarrow f = \frac{2\pi}{16 \times 0.4} = \frac{5\pi}{16} \text{ s}^{-1}$$

$$\text{Now, } v = f\lambda = \frac{5\pi}{16} \times 16 = 5\pi \text{ cm/s}$$

9 Speed of progressive wave is given by,

$$v = \frac{\omega}{k}$$

As electric field in air is,

$$E_1 = E_{01} \hat{x} \cos \left(\frac{2\pi v z}{c} - 2\pi v t \right)$$

$$\therefore \text{Speed in air} = \frac{2\pi v}{\left(\frac{2\pi v}{c} \right)} = c$$

$$\text{Also, } c = \frac{1}{\sqrt{\mu_0 \epsilon_{r1} \epsilon_0}} \quad \dots(i)$$

In medium,

$$E_2 = E_{02} \hat{x} \cos (2kz - kct)$$

$$\therefore \text{Speed in medium} = \frac{kc}{2k} = \frac{c}{2}$$

$$\text{Also, } \frac{c}{2} = \frac{1}{\sqrt{\mu_0 \epsilon_{r2} \epsilon_0}} \quad \dots(ii)$$

As medium is non-magnetic medium,

$$\mu_{\text{medium}} = \mu_{\text{air}}$$

On dividing Eq. (i) by Eq. (ii), we have

$$2 = \sqrt{\frac{\epsilon_{r2}}{\epsilon_{r1}}} \Rightarrow \frac{\epsilon_{r1}}{\epsilon_{r2}} = \frac{1}{4}$$

10 In vacuum, $\epsilon_0 = 1$

In medium, $\epsilon = 4$

\therefore Refractive index,

$$\mu = \sqrt{\frac{\epsilon}{\epsilon_0}} = \sqrt{\frac{4}{1}} = 2$$

Wavelength,

$$\lambda' = \frac{\lambda}{\mu} = \frac{\lambda}{2}$$

and wave velocity,

$$v = \frac{c}{\mu} = \frac{c}{2} \quad \left[\because \mu = \frac{c}{v} \right]$$

Hence, it is clear that wavelength and velocity will become half, but frequency remains unchanged when the wave is passing through any medium.

11 According to the Ampere-Maxwell's law, for a closed surface,

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_D$$

$$\text{As, } B(2\pi R) = \mu_0 I_D$$

$$\Rightarrow B = \frac{\mu_0 I_D}{2\pi R}$$