

Short Answer Type Questions – II

[3 marks]

Que 1. In Fig. 10.34, ABCD is a cyclic quadrilateral in which $AB \parallel CD$. If $\angle B = 65^\circ$, then find other angles.

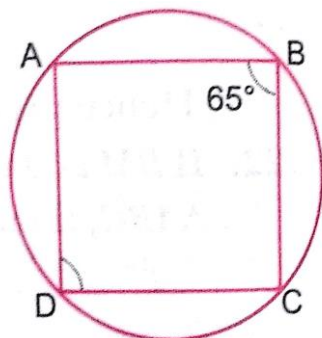


Fig. 10.34

Sol. $\angle B + \angle D = 180^\circ$ (Opp. angles of cyclic quadrilateral)

$$\Rightarrow 65^\circ + \angle D = 180^\circ$$

$$\Rightarrow \angle D = 180^\circ - 65^\circ = 115^\circ$$

Since $AB \parallel CD$ and BC is the transversal

$$\therefore \angle B + \angle C = 180^\circ \Rightarrow 65^\circ + \angle C = 180^\circ$$

$$\Rightarrow \angle C = 180^\circ - 65^\circ \Rightarrow \angle C = 115^\circ$$

Now, $\angle A + 115^\circ = 180^\circ$ (Opposite angles of cyclic quadrilateral)

$$\Rightarrow \angle A = 180^\circ - 115^\circ \Rightarrow \angle A = 65^\circ$$

Que 2. In Fig. 10.35, $\angle OAB = 30^\circ$ and $\angle OCB = 57^\circ$. Find $\angle BOC$ and $\angle AOC$.

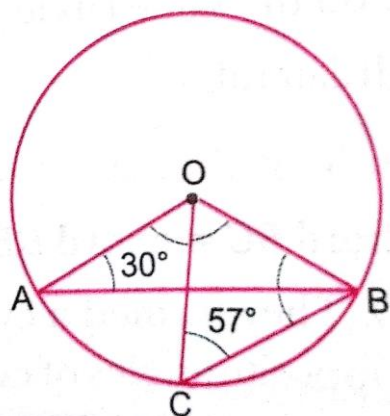


Fig. 10.35

Sol. Since, $OC = OB$ (Radii of the same circle)

$$\therefore \angle OBC = \angle OCB$$

$$\Rightarrow \angle OBC = 57^\circ$$

In $\triangle OBC$, we have

$$\angle OBC + \angle BOC + \angle OCB = 180^\circ$$

$$57^\circ + \angle BOC + 57^\circ = 180^\circ$$

$$\Rightarrow \angle BOC = 66^\circ$$

In $\triangle OAB$, we have

$$\angle OAB + \angle OBA + \angle AOB = 180^\circ \quad (\because AO = OB \therefore \angle OAB = \angle OBA)$$

$$30^\circ + 30^\circ + \angle AOB = 180^\circ$$

$$\Rightarrow \angle AOB = 180^\circ - 60^\circ$$

$$\Rightarrow \angle AOC = 120^\circ$$

$$\begin{aligned} \angle AOC &= \angle AOB - \angle BOC \\ &= 120^\circ - 66^\circ = 54^\circ \end{aligned}$$

Que 3. In Fig. 10.36, AD is a diameter of the circle. If $\angle BCD = 150^\circ$, calculate
(i) $\angle BAD$ (ii) $\angle ADB$

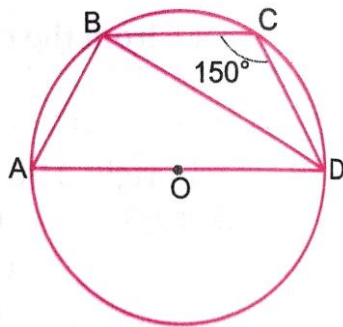


Fig. 10.36

Sol. Join BD

Now, ABCD is a cyclic quadrilateral

$\therefore \angle BAD + \angle BCD = 180^\circ$ (Opposite angles of a cyclic quadrilateral)

$$\Rightarrow \angle BAD + 150^\circ = 180^\circ$$

$$\Rightarrow \angle BAD = 180^\circ - 150^\circ = 30^\circ$$

(ii) $\angle ABD = 90^\circ$ (Angle in a semi-circle)

Now, in $\triangle ABD$, we have

$$\angle ABD + \angle BAD + \angle ADB = 180^\circ$$

$$90^\circ + 30^\circ + \angle ADB = 180^\circ$$

$$\Rightarrow \angle ADB = 180^\circ - 120^\circ = 60^\circ$$

Que 4. In Fig. 10.37, RS is diameter of the circle, PM is parallel to RS and $\angle MRS = 29^\circ$, find $\angle RPM$.

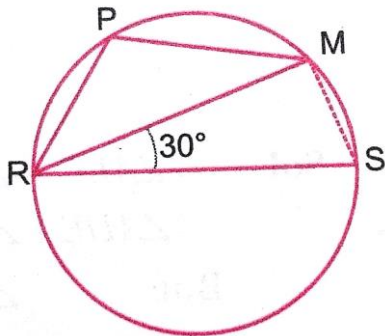


Fig. 10.37

Sol. In the given figure $\angle RMS = 90^\circ$
 (Angles in the semi-circle as RS is diameter)
 $\therefore \angle RSM = 180^\circ - (30^\circ + 90^\circ) = 60^\circ$
 $\angle RPM + \angle RSM = 180^\circ$
 (Opposite angles of cyclin quadrilateral are supplementary)
 $\angle RPM + 60^\circ = 180^\circ$
 $\Rightarrow \angle RPM = 120^\circ$

Que 5. If circle are drawn taking two sides of a triangle as diameter, prove that the point of intersection of these circles lie on the third side.

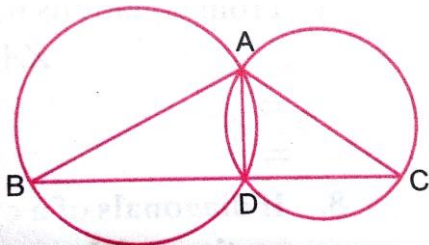


Fig. 10.38

Sol. Given: Two circles are drawn on sides AB and AC of a $\triangle ABC$ as diameters.
 The circles intersect at D.

To prove: D lies on BC

Construction: Join A and D

Proof: $\angle ADB = 90^\circ$ (Angles in the semi-circle)(i)

and $\angle ADC = 90^\circ$ (Angles in the semi-circle)(ii)

Adding (i) and (ii), we get

$$\angle ADB + \angle ADC = 90^\circ + 90^\circ$$

$$\Rightarrow \angle ADB + \angle ADC = 180^\circ$$

$\Rightarrow$ BDC is a straight line.

Hence, D lies on third side BC.

Que 6. In Fig. 10.39, O is the circumcenter of the triangle ABC and D is the mid-point of the base BC. Prove that $\angle BOD = \angle A$.

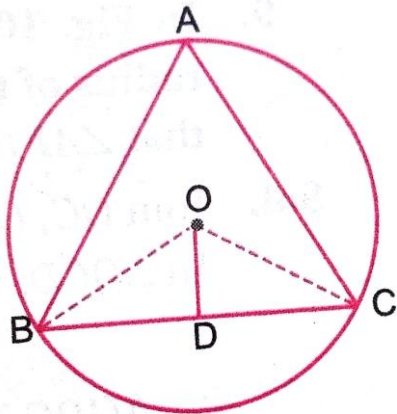


Fig. 10.39

Sol. As line drawn through the centre of a circle bisecting a chord is perpendicular to the chord.

$$\therefore OD \perp BC$$

In the right triangles OBD and OCD, We have

$$OB = OC \quad (\text{Radii of the same circle})$$

$$OD = OD \quad (\text{Common})$$

$$\angle ODB = \angle ODC \quad (\text{Each } 90^\circ)$$

$$\therefore \triangle OBD \cong \triangle OCD \quad (\text{By RHS congruence criterion})$$

$$\Rightarrow \angle BOD = \angle COD \quad (\text{CPCT})$$

$$\Rightarrow \angle BOD = \frac{1}{2} \angle BOC \quad \dots\dots(i)$$

$$\text{Also,} \quad \angle A = \frac{1}{2} \angle BOC \quad \dots\dots(ii)$$

From (i) and (ii), we have

$$\therefore \angle BOD = \angle A$$

Que 7. ABCD is a parallelogram. The circle through A, B and C intersect CD (Produce if necessary) at E. Prove that AE = AD.

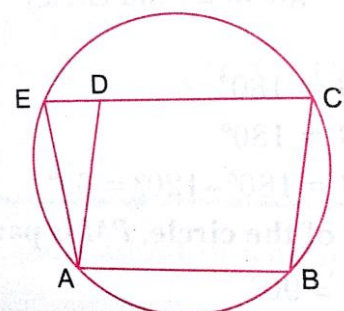


Fig. 10.40

Sol. $\angle ABC + \angle AEC = 180^\circ$ (Opposite angles of cyclin quadrilateral)(i)

$\angle ADE + \angle ADC = 180^\circ$ (Linear Pair)

But $\angle ADC = \angle ABC$ (Opposite angles of $\parallel^{\text{gm}}$)

$\therefore \angle ADE + \angle ABC = 180^\circ$

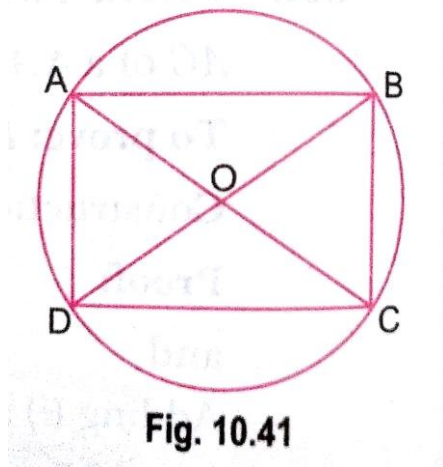
From equations (i) and (ii), we have

$$\angle ABC + \angle AEC = \angle ADE + \angle ABC$$

$$\Rightarrow \angle AEC = \angle ADE$$

$$\Rightarrow AD = AE \text{ (Sides opposite to equal angles)}$$

Que 8. If diagonals of a cyclin quadrilateral are diameter of the circle through the vertices of the quadrilateral, prove that it is a rectangle.



Sol. Let, ABCD be a cyclin quadrilateral such that its diagonal AC and BD are the diameters of the circle through the vertices A, B, C and D.

As angle in a semi-circle is 90°

$\therefore \angle ABC = 90^\circ$ and $\angle ADC = 90^\circ$

$\angle DAB = 90^\circ$ and $\angle BCD = 90^\circ$

So, $\angle ABC = \angle BCD = \angle CDA = \angle DAB = 90^\circ$

Hence, ABCD is a rectangle.

Que 9. In Fig. 10.42, AB is a diameter of the circle, CD is a chord equal to the radius of the circle. AC and BD when extended intersect at point E. Prove that

In ΔOCD , we have

1998, 1999, 2000, 2001, 2002, 2003, 2004, 2005, 2006, 2007, 2008, 2009, 2010, 2011, 2012, 2013, 2014, 2015, 2016, 2017, 2018, 2019, 2020, 2021, 2022, 2023, 2024, 2025, 2026, 2027, 2028, 2029, 2030, 2031, 2032, 2033, 2034, 2035, 2036, 2037, 2038, 2039, 2040, 2041, 2042, 2043, 2044, 2045, 2046, 2047, 2048, 2049, 2050, 2051, 2052, 2053, 2054, 2055, 2056, 2057, 2058, 2059, 2060, 2061, 2062, 2063, 2064, 2065, 2066, 2067, 2068, 2069, 2070, 2071, 2072, 2073, 2074, 2075, 2076, 2077, 2078, 2079, 2080, 2081, 2082, 2083, 2084, 2085, 2086, 2087, 2088, 2089, 2090, 2091, 2092, 2093, 2094, 2095, 2096, 2097, 2098, 2099, 2100, 2101, 2102, 2103, 2104, 2105, 2106, 2107, 2108, 2109, 2110, 2111, 2112, 2113, 2114, 2115, 2116, 2117, 2118, 2119, 2120, 2121, 2122, 2123, 2124, 2125, 2126, 2127, 2128, 2129, 2130, 2131, 2132, 2133, 2134, 2135, 2136, 2137, 2138, 2139, 2140, 2141, 2142, 2143, 2144, 2145, 2146, 2147, 2148, 2149, 2150, 2151, 2152, 2153, 2154, 2155, 2156, 2157, 2158, 2159, 2160, 2161, 2162, 2163, 2164, 2165, 2166, 2167, 2168, 2169, 2170, 2171, 2172, 2173, 2174, 2175, 2176, 2177, 2178, 2179, 2180, 2181, 2182, 2183, 2184, 2185, 2186, 2187, 2188, 2189, 2190, 2191, 2192, 2193, 2194, 2195, 2196, 2197, 2198, 2199, 2200, 2201, 2202, 2203, 2204, 2205, 2206, 2207, 2208, 2209, 2210, 2211, 2212, 2213, 2214, 2215, 2216, 2217, 2218, 2219, 2220, 2221, 2222, 2223, 2224, 2225, 2226, 2227, 2228, 2229, 2230, 2231, 2232, 2233, 2234, 2235, 2236, 2237, 2238, 2239, 2240, 2241, 2242, 2243, 2244, 2245, 2246, 2247, 2248, 2249, 2250, 2251, 2252, 2253, 2254, 2255, 2256, 2257, 2258, 2259, 2260, 2261, 2262, 2263, 2264, 2265, 2266, 2267, 2268, 2269, 2270, 2271, 2272, 2273, 2274, 2275, 2276, 2277, 2278, 2279, 2280, 2281, 2282, 2283, 2284, 2285, 2286, 2287, 2288, 2289, 2290, 2291, 2292, 2293, 2294, 2295, 2296, 2297, 2298, 2299, 2300, 2301, 2302, 2303, 2304, 2305, 2306, 2307, 2308, 2309, 2310, 2311, 2312, 2313, 2314, 2315, 2316, 2317, 2318, 2319, 2320, 2321, 2322, 2323, 2324, 2325, 2326, 2327, 2328, 2329, 2330, 2331, 2332, 2333, 2334, 2335, 2336, 2337, 2338, 2339, 2340, 2341, 2342, 2343, 2344, 2345, 2346, 2347, 2348, 2349, 2350, 2351, 2352, 2353, 2354, 2355, 2356, 2357, 2358, 2359, 2360, 2361, 2362, 2363, 2364, 2365, 2366, 2367, 2368, 2369, 2370, 2371, 2372, 2373, 2374, 2375, 2376, 2377, 2378, 2379, 2380, 2381, 2382, 2383, 2384, 2385, 2386, 2387, 2388, 2389, 2390, 2391, 2392, 2393, 2394, 2395, 2396, 2397, 2398, 2399, 2400, 2401, 2402, 2403, 2404, 2405, 2406, 2407, 2408, 2409, 2410, 2411, 2412, 2413, 2414, 2415, 2416, 2417, 2418, 2419, 2420, 2421, 2422, 2423, 2424, 2425, 2426, 2427, 2428, 2429, 2430, 2431, 2432, 2433, 2434, 2435, 2436, 2437, 2438, 2439, 2440, 2441, 2442, 2443, 2444, 2445, 2446, 2447, 2448, 2449, 2450, 2451, 2452, 2453, 2454, 2455, 2456, 2457, 2458, 2459, 2460, 2461, 2462, 2463, 2464, 2465, 2466, 2467, 2468, 2469, 2470, 2471, 2472, 2473, 2474, 2475, 2476, 2477, 2478, 2479, 2480, 2481, 2482, 2483, 2484, 2485, 2486, 2487, 2488, 2489, 2490, 2491, 2492, 2493, 2494, 2495, 2496, 2497, 2498, 2499, 2500, 2501, 2502, 2503, 2504, 2505, 2506, 2507, 2508, 2509, 2510, 2511, 2512, 2513, 2514, 2515, 2516, 2517, 2518, 2519, 2520, 2521, 2522, 2523, 2524, 2525, 2526, 2527, 2528, 2529, 2530, 2531, 2532, 2533, 2534, 2535, 2536, 2537, 2538, 2539, 2540, 2541, 2542, 2543, 2544, 2545, 2546, 2547, 2548, 2549, 2550, 2551, 2552, 2553, 2554, 2555, 2556, 2557, 2558, 2559, 2560, 2561, 2562, 2563, 2564, 2565, 2566, 2567, 2568, 2569, 2570, 2571, 2572, 2573, 2574, 2575, 2576, 2577, 2578, 2579, 2580, 2581, 2582, 2583, 2584, 2585, 2586, 2587, 2588, 2589, 2590, 2591, 2592, 2593, 2594, 2595, 2596, 2597, 2598, 2599, 2600, 2601, 2602, 2603, 2604, 2605, 2606, 2607, 2608, 2609, 2610, 2611, 2612, 2613, 2614, 2615, 2616, 2617, 2618, 2619, 2620, 2621, 2622, 2623, 2624, 2625, 2626, 2627, 2628, 2629, 2630, 2631, 2632, 2633, 2634, 2635, 2636, 2637, 2638, 2639, 2640, 2641, 2642, 2643, 2644, 2645, 2646, 2647, 2648, 2649, 2650, 2651, 2652, 2653, 2654, 2655, 2656, 2657, 2658, 2659, 2660, 2661, 2662, 2663, 2664, 2665, 2666, 2667, 2668, 2669, 2670, 2671, 2672, 2673, 2674, 2675, 2676, 2677, 2678, 2679, 26

$\therefore \triangle ODC$ is an equilateral triangle.

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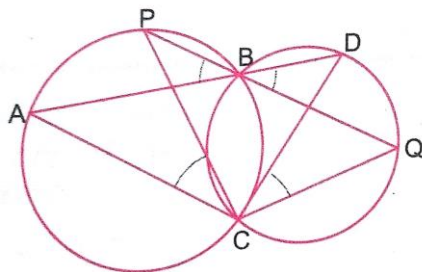
$$\Rightarrow \angle BCE = 180^\circ - \angle ACB = 180^\circ - 90^\circ = 90^\circ$$

$$\therefore \angle BCE + \angle CEB + \angle CBE = 180^\circ$$

$$\Rightarrow 90^\circ + \angle CEB + 30^\circ = 180^\circ \Rightarrow \angle CEB = 60^\circ$$

Hence, $\angle AEB = \angle CEB = 60^\circ$

1. *Journal of Management Studies*, 1996, 33, 1, 1-14.



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$$\angle ABP = \angle QBD \quad (\text{Vertically opposite angles})$$

Also, $\angle QCD = \angle QBD$ (Angles in the same segment)

(c) The following information shall be provided:

$$\therefore \angle ABP = \angle QCD \quad \dots\dots(ii)$$

From (i) and (ii), we have

$$\angle ACP = \angle QCD$$

Que 11. A circle has radius $\sqrt{2}$ cm. It is divided into two segment by a chord of length 2 cm. Prove that the angle subtended by chord a point in major segment is 45° .

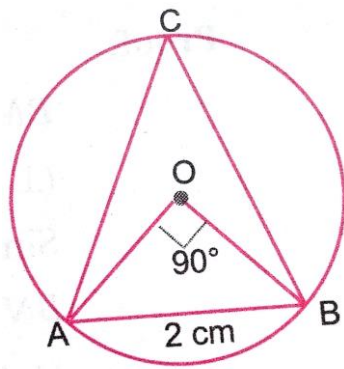


Fig. 10.44

Sol. Given: A chord AB of length 2 cm and radius of the circle is $\sqrt{2}$ cm

Proof: In $\triangle AOB$,

$$OA^2 + OB^2 = (\sqrt{2})^2 + (\sqrt{2})^2 = 2 + 2 = 4 = AB^2$$

$\Rightarrow \triangle AOB$ is a right triangle right angled at O.

i.e. $\angle AOB = 90^\circ$

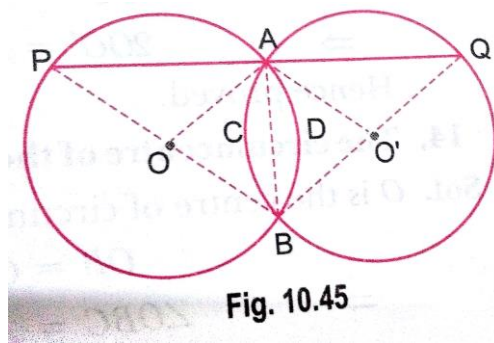
As the angle subtended by an arc at the centre is double the angle subtended by it at remaining part of the circle.

$$\therefore \angle AOB = 2\angle ACB$$

$$\Rightarrow \angle ACB = \frac{1}{2} \times 90^\circ = 45^\circ$$

Que 12. Two congruent circles intersect each other at point A and B. Through A any line segment PAQ is drawn so that P, Q lie on the two circles. Prove that

BP = BQ.



Sol. Let, O and O' be the centres of two congruent circles. As, AB is the common chord of these circles.

$$\therefore \angle ACB = \angle ADB$$

As congruent arcs subtend equal angles at the centre.

$$\angle AOB = \angle AO'B$$

$$\Rightarrow \frac{1}{2} \angle AOB = \frac{1}{2} \angle AO'B$$

$$\Rightarrow \angle BPA = \angle BQA$$

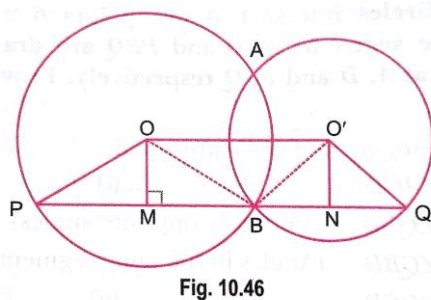
$$\Rightarrow BP = BQ \quad (\text{Sides opposite to equal angles})$$

Que 13. Two circles with centre O and O' intersect at two points A and B. A line PQ is drawn parallel to OO' through B intersecting the circles at P and Q. Prove that $PQ = 2OO'$.

Sol. Construction: Draw two circles having centres O and O' intersecting at point A and B.

Draw a parallel line PQ to OO'

Join OO', OP, O'Q, OM and O'N



To Prove: $PQ = 2OO'$

Proof: In $\triangle OPB$

$$BM = MP \quad \dots\dots(i)$$

($\perp$ from the centre to the circle bisects the chord)

Similarly in $\triangle O'BQ$

$$BN = NQ \quad \dots\dots(ii)$$

($\perp$ from the centre to the circle bisects the chord)

Adding (i) and (ii),

$$BM + BN = PM + NQ$$

Adding $BM + BN$ to both the sides

$$BM + BN + BM + BN = BM + PM + NQ + BN$$

$$2BM + 2BN = PQ$$

$$2(BM + BN) = PQ \quad \text{.....(iii)}$$

Again,

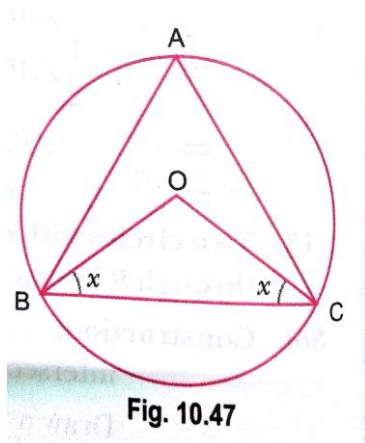
$$OO' = MN \quad [\text{As } OO'NM \text{ is a rectangle}] \quad \text{.....(iv)}$$

$$\Rightarrow 2OO' = PQ$$

Hence Proved.

Que 14. The circumcenter of the $\triangle ABC$ is O. Prove that $\angle OBC + \angle BAC = 90^\circ$

Sol.



O is the centre of circumscribed circle.

$$OB = OC = \text{radii}$$

$$\Rightarrow \angle OBC = \angle OCB = x$$

$$\therefore x + x + \angle BOC = 180^\circ \quad (\text{Angle sum property of } \triangle OBC)$$

$$2x + \angle BOC = 180^\circ$$

$$\angle BOC = 180^\circ - 2x$$

$$\text{Also, } \angle BOC = 2\angle BAC$$

$$180 - 2x = 2\angle BAC$$

$$\Rightarrow 90 - x = \angle BAC$$

$$\therefore \angle BAC + \angle OBC = (90 - x) + x$$

$$\angle BAC + \angle OBC = 90^\circ$$