Short Answer Type Questions – II

[3 marks]

Que 1. In Fig. 10.34, ABCD is a cyclic quadrilateral in which AB || CD. If $\angle$ = 65°, then find other angles.

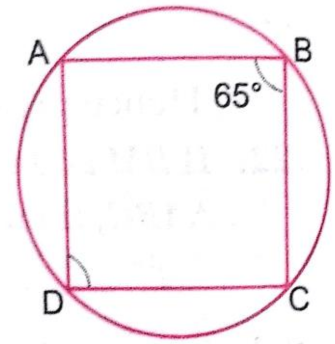

Fig. 10.34

Sol. $\angle B + \angle D = 180°$ (OPP. angles of cyclin quadrilateral)

$\Rightarrow$ $65° + \angle D = 180°$

$\Rightarrow$ $\angle D = 180° - 65° = 115°$

Since AB || CD and BC is the transversel

$\therefore$ $\angle B + \angle C = 180°$ $\Rightarrow$ $65° + \angle C = 180°$

$\Rightarrow$ $\angle C = 180° - 65°$ $\Rightarrow$ $\angle C = 115°$

Now, $\angle A + 115° = 180°$ (Opposite angles of cyclin quadrilateral)

$\Rightarrow$ $\angle A = 180° - 115°$ $\Rightarrow$ $\angle A = 65°$

Que 2. In Fig. 10.35, $\angle OAB = 30°$ and $\angle OCB = 57°$. Find $\angle BOC$ and $\angle AOC$.

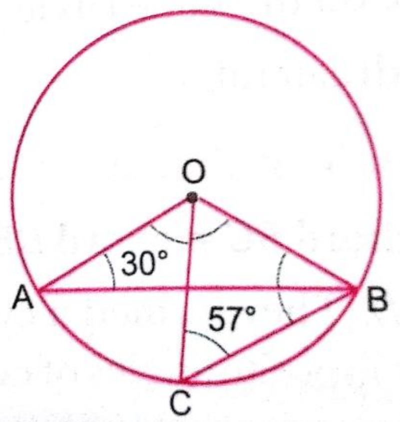

Fig. 10.35

Sol. Since, OC = OB (Radii of the same circle)

$\therefore$ $\angle OBC = \angle OCB$

$\Rightarrow \quad \angle OBC = 57°$

In ΔOBC, we have

$\angle OBC + \angle BOC + \angle OCB = 180°$

$57° + \angle BOC + 57° = 180°$

$\Rightarrow \quad \angle BOC = 66°$

In ΔOAB, we have

$\angle OAB + \angle OBA + \angle AOB = 180°$ $\quad (\because AO = OB \;\; \therefore \angle OAB = \angle OBA)$

$30° + 30° + \angle AOB = 180°$

$\Rightarrow \quad \angle AOB = 180° - 60°$

$\Rightarrow \quad \angle AOC = 120°$

$\angle AOC = \angle AOB - \angle BOC$

$= 120° - 66° = 54°$

Que 3. In Fig. 10.36, AD is a diameter of the circle. If $\angle BCD = 150°$, calculate
(i) $\angle BAD$ (ii) $\angle ADB$

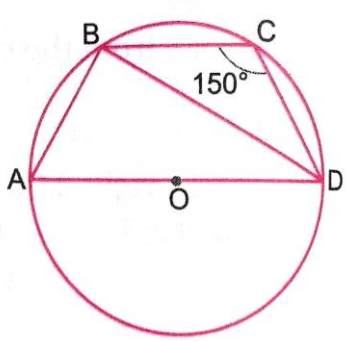

Fig. 10.36

Sol. Join BD

Now, ABCD is a cyclic quadrilateral

$\therefore \angle BAD + \angle BCD = 180°$ (Opposite angles of a cyclin quadrilateral)

$\Rightarrow \angle BAD + 150° = 180°$

$\Rightarrow \quad \angle BAD = 180° - 150° = 30°$

(ii) $\angle ABD = 90°$ (Angle in a semi-circle)

Now, in Δ ABD, we have

$\angle ABD + \angle BAD + \angle ADB = 180°$

$90° + 30° + \angle ADB = 180°$

$\Rightarrow \quad \angle ADB = 180° - 120° = 60°$

Que 4. In Fig. 10.37, RS is diameter of the circle, PM is parallel to RS and ∠MRS = 29°, find ∠RPM.

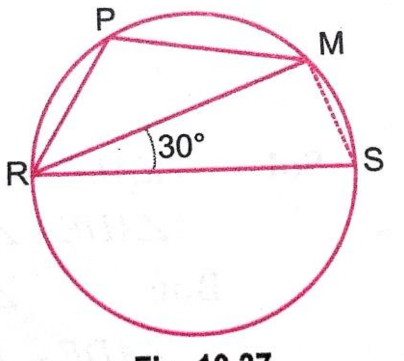

Fig. 10.37

Sol. In the given figure ∠RMS = 90°

(Angles in the semi-circle as RS is diameter)

∴ ∠RSM = 180° - (30° + 90°) = 60°

∠RPM + ∠RSM = 180°

(Opposite angles of cyclin quadrilateral are supplementary)

∠RPM + 60° = 180°

⇒ ∠RPM = 120°

Que 5. If circle are drawn talking two sides of a triangle as diameter, prove that the point of intersection of these circles lie on the third side.

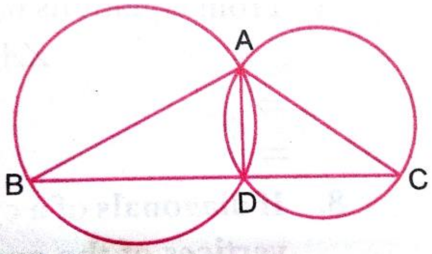

Fig. 10.38

Sol. Given: Two circles are drawn on sides AB and AC of a ΔABC as diameters. The circles intersects at D.

To prove: D lies on BC

Construction: Join A and D

Proof: ∠ADB = 90° (Angles in the semi-circle) …..(i)

and ∠ADC = 90° (Angles in the semi-circle) …..(ii)

Adding (i) and (ii), we get

∠ADB + ∠ADC = 90° + 90°

⇒ ∠ADB + ∠ADC = 180°

⇒ BDC is a straight line.

Hence, D lies on third side BC.

Que 6. In Fig. 10.39, O is the circumcenter of the triangle ABC and D is the mid-point of the base BC. Prove that ∠BOD = ∠A.

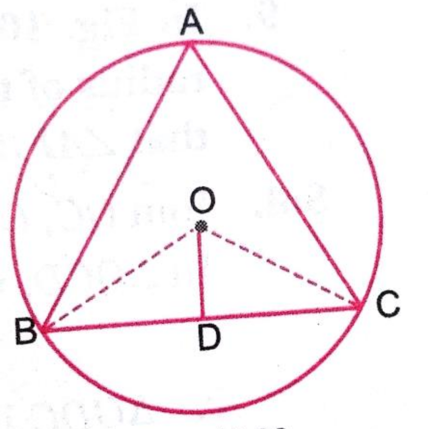

Fig. 10.39

Sol. As line drawn through the centre of a circle bisecting a chord is perpendicular to the chord.

∴ OD ⊥ BC

In the right triangles OBD and OCD, We have

OB = OC (Radii of the same circle)

OD = OD (Common)

∠ODB = ∠ODC (Each 90°)

∴ ΔOBD ≅ ΔOCD (By RHS congruence criterion)

⇒ ∠BOD = ∠COD (CPCT)

⇒ $\angle BOD = \frac{1}{2}\angle BOC$ …….(i)

Also, $\angle A = \frac{1}{2}\angle BOC$ ……..(ii)

From (i) and (ii), we have

∴ ∠BOD = ∠A

Que 7. ABCD is a parallelogram. The circle through A, B and C intersect CD (Produce if necessary) at E. Prove that AE = AD.

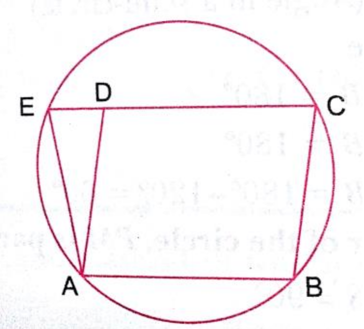

Fig. 10.40

Sol. $\angle ABC + \angle AEC = 180°$ (Opposite angles of cyclin quadrilateral) ……(i)

$\angle ADE + \angle ADC = 180°$ (Linear Pair)

But $\angle ADC = \angle ABC$ (Opposite angles of ||^{gm})

∴ $\angle ADE + \angle ABC = 180°$

From equations (i) and (ii), we have

$$\angle ABC + \angle AEC = \angle ADE + \angle ABC$$

$\Rightarrow$ $\angle AEC = \angle ADE$

$\Rightarrow$ $AD = AE$ (Sides opposite to equal angles)

Que 8. If diagonals of a cyclin quadrilateral are diameter of the circle through the vertices of the quadrilateral, prove that it is a rectangle.

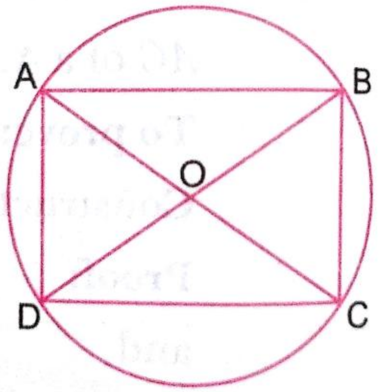

Fig. 10.41

Sol. Let, ABCD be a cyclin quadrilateral such that its diagonal AC and BD are the diameters of the circle through the vertices A, B, C and D.

As angle in a semi-circle is 90°

∴ $\angle ABC = 90°$ and $\angle ADC = 90°$

$\angle DAB = 90°$ and $\angle BCD = 90°$

So, $\angle ABC = \angle BCD = \angle CDA = \angle DAB = 90°$

Hence, ABCD is a rectangle.

Que 9. In Fig. 10.42, AB is a diameter of the circle, CD is a chord equal to the radius of the circle. AC and BD when extended intersect at point E. Prove that

∠AEB = 60°.

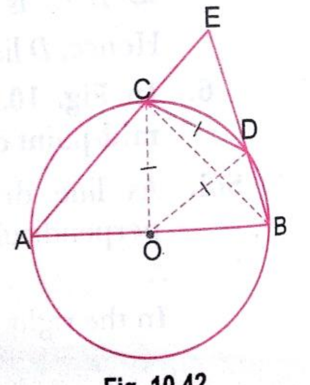

Fig. 10.42

Sol. Join OC, OD and BC

In ΔOCD, we have

OC = OD = CD (Each equal to radius)

∴ ΔODC is an equilateral triangle.

⇒ ∠COD = 60°

Also, ∠COD = 2 ∠CBD

⇒ 60° = 2 ∠CBD ⇒ ∠CBD = 30°

Since ∠ACB is angle in a semi-circle.

∠ACB = 90°

⇒ ∠BCE = 180° - ∠ACB = 180° - 90° = 90°

Thus, in ΔBCE, we have

∠BCE = 90° and ∠CBE = 30°

∴ ∠BCE + ∠CEB + ∠CBE = 180°

⇒ 90° + ∠CEB + 30° = 180° ⇒ ∠CEB = 60°

Hence, ∠AEB = ∠CEB = 60°

Que 10. In Fig. 10.43 two circles intersect at two points B and C. Through B, two line segments ABD and PBQ are drawn to intersect the circles at A, D and P, Q respectively. Prove that ∠ACP = ∠QCD.

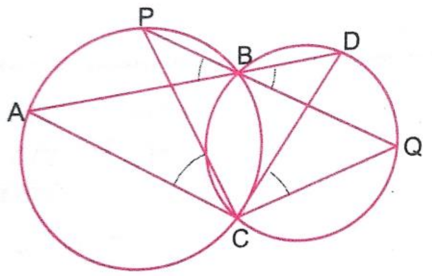

Fig. 10.43

Sol. As angles in the same segment of circle are equal

∠ABP = ∠ACP …….(i)

∠ABP = ∠QBD (Vertically opposite angles)

Also, ∠QCD = ∠QBD (Angles in the same segment)

∴ $\angle ABP = \angle QCD$ ……(ii)

From (i) and (ii), we have

$\angle ACP = \angle QCD$

Que 11. A circle has radius $\sqrt{2}$ cm. It is divided into two segment by a chord of length 2 cm. Prove that the angle subtended by chord a point in major segment is 45°.

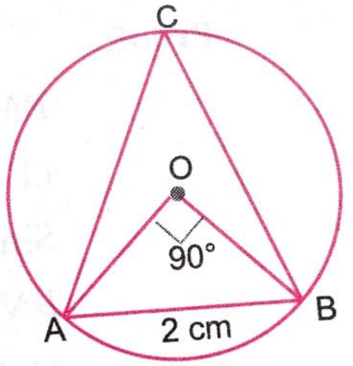

Fig. 10.44

Sol. Given: A chord AB of length 2 cm and radius of the circle is $\sqrt{2}$ cm

Proof: In ΔAOB,

$$OA^2 + OB^2 = (\sqrt{2})^2 + (\sqrt{2})^2 = 2 + 2 = 4 = AB^2$$

⇒ ΔAOB is a right triangle right angled at O.

i.e. $\angle AOB = 90°$

As the angle subtended by an arc at the centre is double the angle subtended by it at remaining part of the circle.

∴ $\angle AOB = 2\angle ACB$

⇒ $\angle ACB = \frac{1}{2} \times 90° = 45°$

Que 12. Two congruent circles intersect each other at point A and B. Through A any line segment PAQ is drawn so that P, Q lie on the two circles. Prove that

BP = BQ.

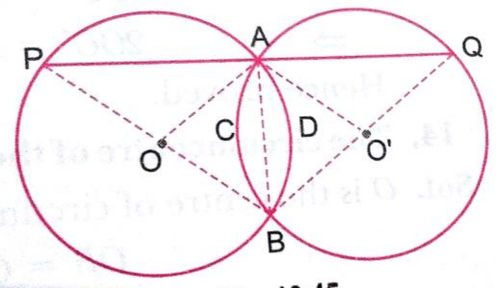

Fig. 10.45

Sol. Let, O and O’ be the centres of two congruent circles. As, AB is the common chord of these circles.

∴ ACB = ADB

As congruent arcs subtend equal angles at the centre.

∠AOB = ∠AO’B

⇒ $\frac{1}{2}\angle AOB = \frac{1}{2}\angle AO'B$

⇒ ∠BPA = ∠BQA

⇒ BP = BQ (Sides opposite to equal angles)

Que 13. Two circles with centre O and O’ intersect at two points A and B. A line PQ is drawn parallel to OO’ through B intersecting the circles at P and Q. Prove that PQ = 2OO’.

Sol. Construction: Draw two circles having centres O and O’
intersecting at point A and B.
Draw a parallel line PQ to OO’
Join OO’, OP, O’Q, OM and O,N

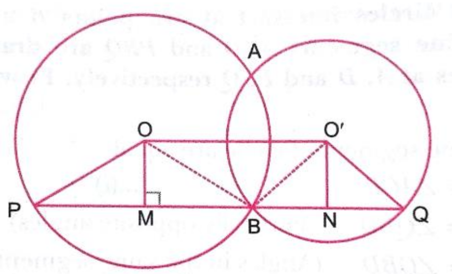

Fig. 10.46

To Prove: PQ = 2OO’

Proof: In ΔOPB

BM = MP ……(i)

(⊥ from the centre to the circle bisects the chord)

Similarly in ΔO, BQ

BN = NQ ……(ii)

(⊥ from the centre to the circle bisects the chord)

Adding (i) and (ii),

$$BM + BN = PM + NQ$$

Adding BM + BN to both the sides

$$BM + BN + BM + BN = BM + PM + NQ + BN$$

$$2BM + 2BN = PQ$$

$$2(BM + BN) = PQ \quad \ldots\ldots(iii)$$

Again,

$OO' = MN$ [As OO' NM is a rectangle] ……..(iv)

$\Rightarrow$ $2OO' = PQ$

Hence Proved.

Que 14. The circumcenter of the ΔABC is O. Prove that ∠OBC + ∠BAC = 90°

Sol.

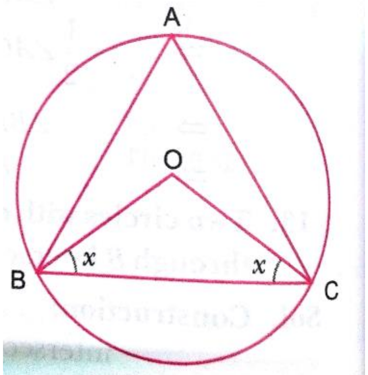

Fig. 10.47

O is the centre of circumscribed circle.

$OB = OC =$ radii

$\Rightarrow$ $\angle OBC = \angle OCB = x$

$\therefore$ $x + x + \angle BOC = 180°$ (Angle sum property of ΔOBC)

$2x + \angle BOC = 180°$

$\angle BOC = 180° - 2x$

Also, $\angle BOC = 2\angle BAC$

$180 - 2x = 2\angle BAC$

$\Rightarrow$ $90 - x = \angle BAC$

$\therefore$ $\angle BAC + \angle OBC = (90 - x) + x$

$\angle BAC + \angle OBC = 90°$