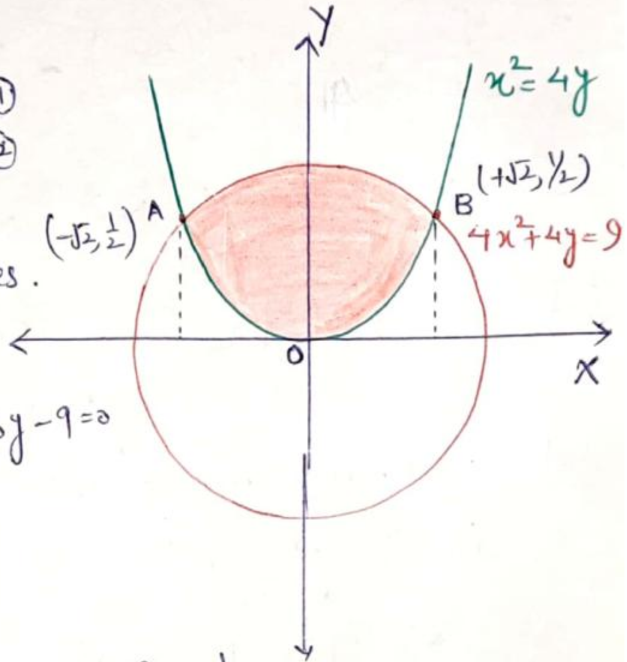
EXERCISE 8.2.

Q.No 1: Find Area of Circle $4x^2+4y^2=9$ which is interior to parabola $x^2=4y$.



Sol: Given Circle is $4x^2+4y^2=9$ — (1)

and Upward parabola $x^2=4y$ — (2)

$\therefore$ Solving (1) and (2) for points of Intersection A and B of both curves.

$$4(4y)+4y^2=9$$

or $16y+4y^2-9=0$ or $4y^2+16y-9=0$

or $$y=\frac{-16\pm\sqrt{256+144}}{8}$$

$$=\frac{-16\pm 20}{8}=-\frac{36}{8},\frac{4}{8}=-\frac{9}{2},\frac{1}{2}$$

But since $x^2=4y$ $\therefore$ $y=-9/2$ is not possible.

$\therefore y=\frac{1}{2}$ and $x^2=4y \Rightarrow x^2=4\times\frac{1}{2} \Rightarrow x^2=2 \Rightarrow x=\pm 2$

$\therefore$ A is $(-\sqrt{2},\frac{1}{2})$ and B is $(\sqrt{2},\frac{1}{2})$

Now Since required area, shown shaded is symmetric about y-axis

$\therefore$ Required Area $=2\left[\begin{array}{l}\text{Area under circle from } x=0 \text{ to } x=\sqrt{2}\\ -\text{Area Under Parabola from } x=0 \text{ to } x=\sqrt{2}\end{array}\right]$

$$=2\left[\int_0^{\sqrt{2}}\sqrt{\frac{9-4x^2}{4}}\,dx-\int_0^{\sqrt{2}}\frac{x^2}{4}\,dx\right]$$

$$=2\left[\int_0^{\sqrt{2}}\sqrt{\left(\frac{3}{2}\right)^2-(x)^2}\,dx-\frac{1}{4}\int_0^{\sqrt{2}}x^2\,dx\right]$$

$$=2\left[\frac{x}{2}\sqrt{\left(\frac{3}{2}\right)^2-x^2}+\frac{(3/2)^2}{2}\sin^{-1}\frac{x}{3/2}\right]_0^{\sqrt{2}}-\frac{2}{4}\left[\frac{x^3}{3}\right]_0^{\sqrt{2}}$$

$$= \frac{2}{2}\left[x\sqrt{\frac{9}{4}-x^2} + \frac{9}{4}\sin^{-1}\frac{2x}{3}\right]_0^{\sqrt{2}} - \frac{1}{6}\left[(\sqrt{2})^3 - 0\right]$$

$$= \frac{\sqrt{2}}{2} + \frac{9}{4}\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right) - \frac{2\sqrt{2}}{6} = \left[\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{3}\right] + \frac{9}{4}\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)$$

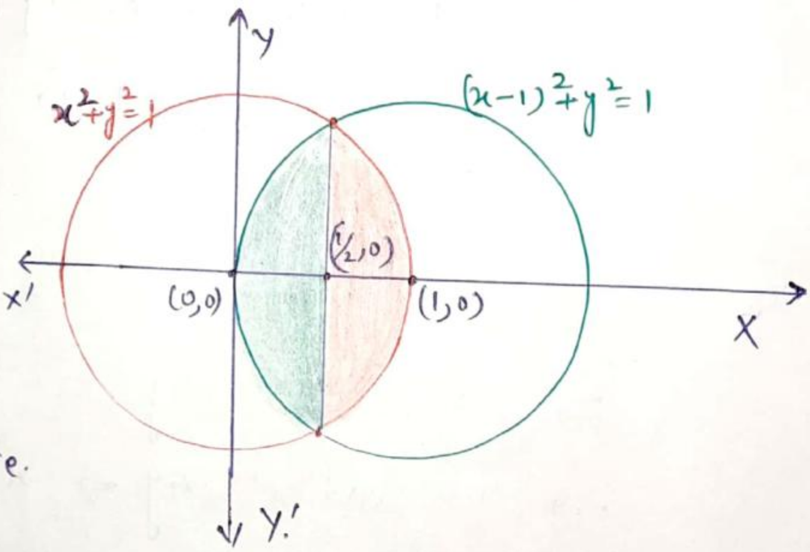
$$= \frac{\sqrt{2}}{6} + \frac{9}{4}\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right) \text{ square units.}$$

Q No. 2: Find the area bounded by the curves $(x-1)^2+y^2=1$ and $x^2+y^2=1$

Sol: Given Curves are

$(x-1)^2+y^2 = 1$ — (1)

$x^2+y^2 = 1$ — (2)



Which are both circles of radius = 1 unit.

(1) and (2) meet where.

$(x-1)^2 - x^2 = 0$

ie $x^2+1-2x-x^2=0$ or $2x=1$ or $x=1/2$

$\therefore$ Required Area (shown shaded) $= 2\times$ [Area under circle $(x-1)^2+y^2=1$ from $x=0$ to $1/2$ − Area under circle $x^2+y^2=1$ from $x=\frac{1}{2}$ to 1]

(By Symmetry)

$$= 2\left[\int_0^{1/2}\sqrt{1-(x-1)^2}\,dx - \int_{1/2}^{1}\sqrt{1-x^2}\,dx\right]$$

$$= 2\left[\frac{x-1}{2}\sqrt{1-(x-1)^2} + \frac{1}{2}\sin^{-1}\frac{x-1}{1}\right]_0^{1/2} - 2\left[\frac{x}{2}\sqrt{1-x^2} + \frac{1}{2}\sin^{-1}\frac{x}{1}\right]_{1/2}^{1}$$

$$= 2\left[\frac{\frac{1}{2}-1}{2}\sqrt{1-\frac{1}{4}} + \frac{1}{2}\sin^{-1}\left(-\frac{1}{2}\right) - \left(-\frac{1}{2}\right)(0) - \frac{1}{2}\sin^{-1}(-1)\right] + 2\left[0 + \frac{1}{2}\sin^{-1}(1) - \frac{1}{4}\sqrt{1-\frac{1}{4}} - \frac{1}{2}\sin^{-1}\frac{1}{2}\right]$$

$$= 2\left[-\frac{1}{4}\times\frac{\sqrt{3}}{2} - \frac{1}{2}\times\frac{\pi}{6} + 0 + \frac{1}{2}\times\frac{\pi}{2}\right] + \frac{\pi}{2} - \frac{1}{2}\times\frac{\sqrt{3}}{2} - \frac{\pi}{6}$$

$$= \frac{\sqrt{3}}{4} - \frac{\pi}{6} + \frac{\pi}{2} + \frac{\pi}{2} - \frac{\sqrt{3}}{4} - \frac{\pi}{6} = \left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right) \text{ square units.}$$

Q No 3: Find the area of region bounded by the curves $y = x^2+2$, $y = x$ and $x = 0$ and $x = 3$.

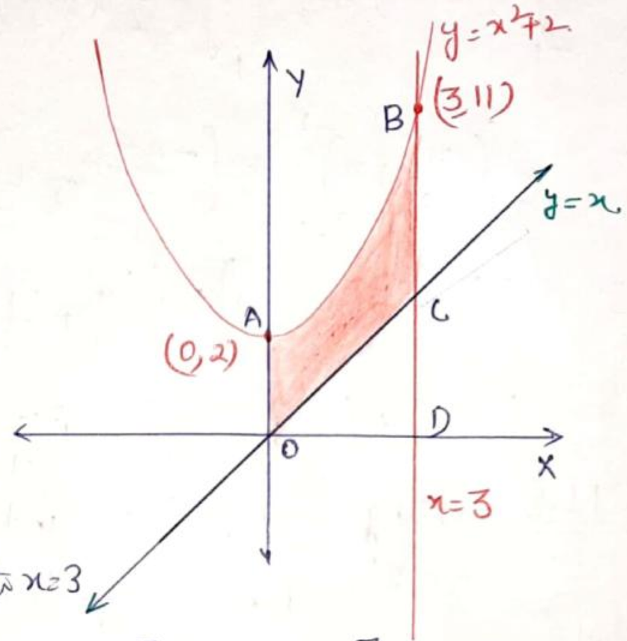
Soln: The given curves are

$y = x^2+2$

$y = x$

$x = 0$

and $x = 3$.



Required Area (shown shaded)

= Area under parabola $y = x^2+2$ from $x=0$ to $x=3$ − Area under line $y=x$ from $x=0$ to $x=3$

$$= \int_0^3 (x^2+2)\,dx - \int_0^3 x\,dx = \left[\frac{x^3}{3}+2x\right]_0^3 - \left[\frac{x^2}{2}\right]_0^3$$

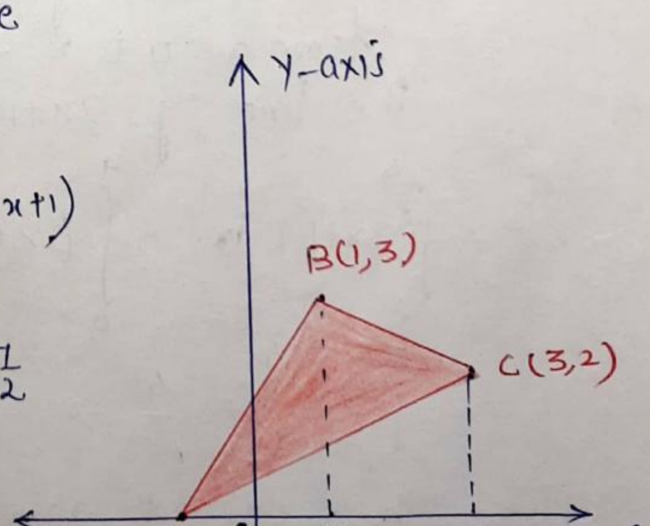
$$= \left[\frac{3^3}{3}+2\times3 - \frac{0^3}{3}+2\times0\right] - \left[\frac{3^2}{2}-\frac{0^2}{2}\right]$$

$= 9+6-9/2 = \frac{21}{2}$ square units.

Q No 4: Using Integration find the area of the region bounded by the triangle whose vertices are $(-1,0)(1,3)$ and $(3,2)$

Sol: Let the given vertices of Δ be

$A(-1,0)$ $B(1,3)$ $C(3,2)$



Then eqn of line AB is

$y-0 = \frac{3-0}{1+1}(x+1)$ or $y = \frac{3}{2}(x+1)$

Eqn of line BC is

$y-3 = \frac{2-3}{3-1}(x-1)$ or $y = -\frac{1}{2}x+\frac{7}{2}$

Eq. of Line AC is

$y-0 = \frac{2-0}{3+1}(x+1)$ or $y = \frac{1}{2}x+\frac{1}{2}$

Now Area of Δ ABC = area AMB + Area BMNC − Area AC

= Area Under line AB from $x=-1$ to $x=1$ + Area under line

BC from $x=1$ to $x=3$ − Area Under AC from $x=-1$ to $x=3$"

$$= \int_{-1}^{1} \frac{3}{2}(x+1)\,dx + \int_{1}^{3}\left(-\frac{1}{2}x+\frac{7}{2}\right)dx - \int_{-1}^{3}\left(\frac{1}{2}x+\frac{1}{2}\right)dx$$

$$= \frac{3}{2}\left(\frac{x^2}{2}+x\right)_{-1}^{1} + \left[-\frac{x^2}{4}+\frac{7}{2}x\right]_{1}^{3} - \left[\frac{x^2}{4}+\frac{1}{2}x\right]_{-1}^{3}$$

$$= \frac{3}{2}\left[\frac{1}{2}+1-\frac{1}{2}-(-1)\right] + \left[-\frac{9}{4}+\frac{21}{2}+\frac{1}{4}-\frac{7}{2}\right] - \left[\frac{9}{4}+\frac{3}{2}-\frac{1}{4}+\frac{1}{2}\right]$$

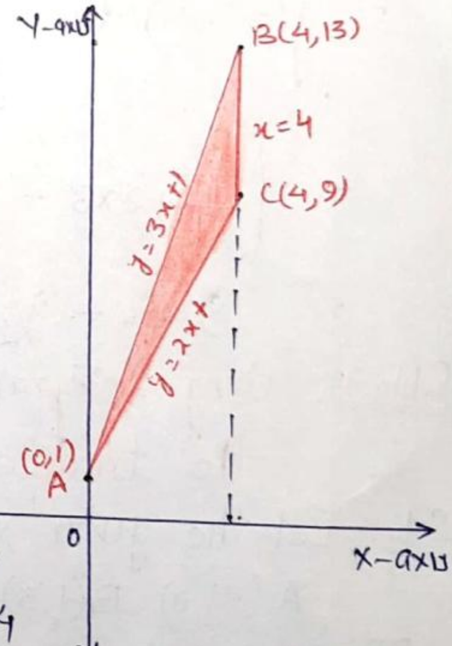
$$= \frac{3}{2}(2) + \left(\frac{43}{4}-\frac{23}{4}\right) - \left(\frac{17}{4}-\frac{1}{4}\right) = 3+5-4 = 4 \text{ square Unit.}$$

QNo5: Using Integration find the area of triangular region whose sides have equations $y=2x+1$, $y=3x+1$ and $x=4$.

Sol: Given lines are $y=2x+1$ — ①

$y=3x+1$ — ②

$x=4$ — ③



Now Solving ① and ②

$2x+1 = 3x+1 \Rightarrow x=0$

∴ ① and ② meet at A(0,1)

∴ Required Area (Shaded)

= Area Under line $y=3x+1$ from $x=0$ to $x=4$ − Area under the line $y=2x+1$ from $x=0$ to 4

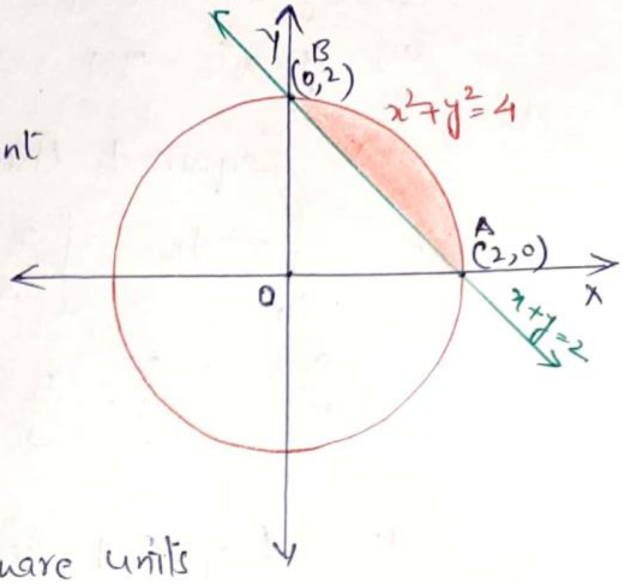
$$= \int_0^4 (3x+1)\,dx - \int_0^4 (2x+1)\,dx = \int_0^4 (3x+1-2x-1)\,dx = \int_0^4 x\,dx.$$

$$= \left[\frac{x^2}{2}\right]_0^4 = \frac{4^2}{2} - 0 = 8 \text{ square units.}$$

Choose the correct answer in the following questions. 6 and 7.

Q No. 6: Smaller area enclosed by the circle $x^2+y^2=4$ and the line $x+y=2$ is (A) $2(\pi-2)$ (B) $\pi-2$ (C) $2\pi-1$ (D) $2(\pi+2)$

Soln: As shown in fig.



Required Area = Area bounded by the circle in first quadrant − Area of Δ AOB

$= \frac{1}{4} \times$ Area of circle with radius 2 − Area of rt ∠d Δ AOB

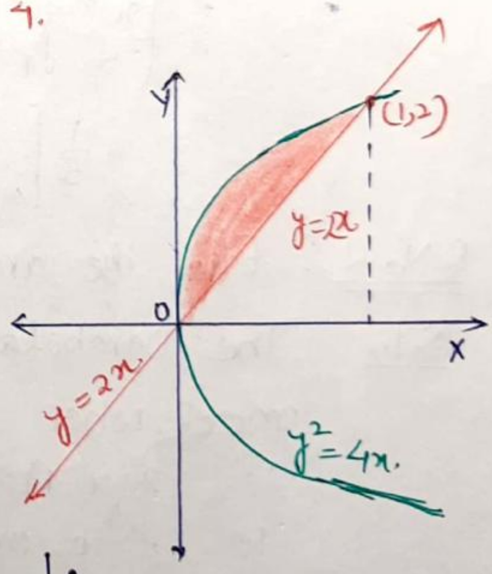
$= \frac{1}{4} \times \pi \times (2)^2 - \frac{1}{2} \times |OA| \times |OB|$.

$= \pi - \frac{1}{2} \times 2 \times 2 = (\pi - 2)$ square units

$\therefore$ (B) is the correct option.

Q No. 7: Area lying between the curves $y^2=4x$ and $y=2x$ is
(A) $\frac{2}{3}$ (B) $\frac{1}{3}$ (C) $\frac{1}{4}$ (D) $\frac{3}{4}$.

Sol: The given parabola $y^2=4x$ meets the line $y=2x$ at



$(2x)^2 = 4x \Rightarrow 4x(x-1)=0$

$\Rightarrow x = 0, 1$

When $x=0$, $y=0$

$x=1$ $y=2$.

$\therefore$ Curves meet at $(0,0)$ and $(1,2)$

$\therefore$ Required Area (Shaded) $= \int_0^1 \sqrt{4x}\, dx - \int_0^1 2x\, dx$.

$= \int_0^1 (2\sqrt{x} - 2x)\, dx = 2\int_0^1 (\sqrt{x} - x)\, dx = 2\left[\frac{x^{3/2}}{3/2} - \frac{x^2}{2}\right]_0^1$

$= 2\left[\frac{1 \times 2}{3} - \frac{1}{2} - 0 + 0\right] = 2\left[\frac{2}{3} - \frac{1}{2}\right] = 2\left[\frac{4-3}{6}\right] = 2 \times \frac{1}{6} = \frac{1}{3}$

$\therefore$ (B) is the correct option.

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