Comparing Quantities

- In many situations, comparison between quantities is made by using division i.e., by observing how many times one quantity is in relation to the other quantity. This comparison is known as **ratio.** We denote it by using the symbol ':'.

- A ratio may be treated as a fraction. For example, 3:11 can be treated as $\frac{3}{11}$.

- We can compare two quantities in terms of ratio, if these quantities are in the same unit. If they are not, then they should be expressed in the same unit before the ratio is taken.

  For example, if we want to compare 70 paise and Rs 3 in terms of ratio then we have to convert Rs 3 into paise.

  Rs 3 = 300 paise

  Hence, required ratio $\frac{70}{300} = 7:30$

- We can find equivalent ratios by multiplying or dividing the numerator and denominator by the same number.

  For example, to find the equivalent ratios of 12:20, we proceed as follows:

  $$12:20 = \frac{12}{20} = \frac{12 \div 2}{20 \div 2} = \frac{6}{10} = 6:10$$
  $$12:20 = \frac{12}{20} = \frac{12 \div 4}{20 \div 4} = \frac{3}{5} = 3:5$$
  $$12:20 = \frac{12 \times 2}{20 \times 2} = \frac{24}{40} = 24:40$$
  $$12:20 = \frac{12 \times 3}{20 \times 3} = \frac{36}{60} = 36:60$$

  Therefore, 6:10, 3:5, 24:40, 36:60 are the equivalent ratios of 12:20. In this way, we can find many equivalent ratios of 12:20.

- Two ratios are equivalent, if the product of the numerator of the first ratio and the denominator of the other ratio is equal to the product of the denominator of first ratio and the numerator of the other ratio.
- For example, 14:49 and 6:21 are equivalent as:

- $14 \times 21 = 294 = 6 \times 49$
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- A ratio is always expressed in its lowest terms.
- For example, the lowest form of 45:72 is given by,

$$45:72 = \frac{45}{72}$$
$$= \frac{45 \div 9}{72 \div 9} \quad \text{(HCF of 45 and 72 is 9)}$$
$$= \frac{5}{8}$$
$$= 5:8$$

- We can also compare and arrange the ratios using the concept of equivalent ratios. For this, we make the denominators of the all the ratios equal and then compare the ratios by comparing their numerators.

- The same ratio may occur in different situations.

  To understand this concept, let us consider the following situations.

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  - Distances of Lata's home and Ravi's home from their school are 12 km and 21 km respectively. Therefore, the ratio of the distance of Lata's home to the distance of Ravi's home from their school is $\frac{12}{21} = \frac{12 \div 3}{21 \div 3} = \frac{4}{7} = 4:7$
  - Neha has Rs 20 and Saroj has Rs 35. Therefore, the ratio of the amount of money that Neha has to that of Saroj is $\frac{20}{35} = \frac{20 \div 5}{35 \div 5} = \frac{4}{7} = 4:7$

  In this way, we can come across many situations where the ratio would be 4:7.

- The order of ratio is important.

  For example, let us consider that the length and breadth of a rectangle are 80 m and 30 m respectively. The ratio of length to the breadth of rectangle is $\frac{80}{30}$. This ratio can be written as 8: 3. However, it cannot be written as 3:8. Therefore, the order in which quantities are taken to express their ratio is important.

- Four quantities are said to be in proportion, if the ratio of first and second quantities is equal to the ratio of third and fourth quantities.

  For example, to check whether 8, 22, 12, and 33 are in proportion or not, we have to find the ratio of 8 to 22 and the ratio of 12 to 33.

  $8:22 = \frac{8}{22} = \frac{4}{11} = 4:11$

  $12:33 = \frac{12}{33} = \frac{4}{11} = 4:11$

Therefore, 8, 22, 12, and 33 are in proportion as 8:22 and 12:33 are equal.

- When four terms are in proportion, the first and fourth terms are known as extreme terms and the second and third terms are known as middle terms.

  In the above example, 8, 22, 12, and 33 were in proportion. Therefore, 8 and 33 are known as extreme terms while 22 and 12 are known as middle terms.

- If two ratios are equal then we say that they are in proportion and use the symbol '::' or '=' to equate the two ratios.

  For example, 8:36 and 14:63 are equal as 836 = 29 and 14:63 = $\frac{14}{63} = \frac{2}{9}$
  Since 8:36 and 14:63 are in proportion, we write it as 8:36 :: 14:63 or 8:36 = 14:63.

- **Percentages** are numerators of fractions with denominator 100. It is represented by the symbol % and means hundredths too, i.e., 25% $= \frac{25}{100}$ = 0.25
- Fractional numbers, whole numbers and decimals can be converted into percentages by multiplying them by 100%.

  Note: Percentages related to proper fractions are less than 100 whereas percentages related to improper fractions are more than 100.
  For example, $1\frac{1}{4} = \frac{5}{4} \times 100\% = 125\%$

- To convert ratio to percentage, we proceed as follows:

  Consider the ratio *a*:*b*.
  Sum of parts = *a* + *b*
  Percantage form = $\frac{a}{a+b} \times 100\%$

- Percentages can be converted into fractions or decimals by dividing them by 100.

  For example, 35% can be converted to decimals and fraction as follows:
  35% $= \frac{35}{100} = 0.35$
  $35\% = \frac{35}{100} = \frac{7}{20}$

- To convert percentage to ratio, we have to find the ratio of the percentages of the two quantities.
- When added, all parts of a whole give whole or 100%.

- To express *x* as a percentage of *y,* percentage = $\left(\frac{x}{y} \times 100\right)\%$

- *x* % of a given quantity = $\frac{x}{100} \times \textbf{given quanity}$
- If *x* % of a given quantity is *y*, then quantity = $\frac{y}{x} \times 100$

Example: In a bag there are 6 blue marbles, 4 red marbles and 5 green marbles. What percent of total marbles are blue?
Solution: Total number of marbles = 6 + 4 + 5 = 15
Number of blue marbles = 6
∴ Percentage of blue marbles = $\frac{6}{15} \times 100 = 40\%$

- Formula for percentage increase and decrease are:

$$\text{Percentage increase} = \frac{\text{Increase in the value}}{\text{Original value}} \times 100$$

$$\text{Percentage decrease} = \frac{\text{Decrease in the value}}{\text{Original value}} \times 100$$

Example: In the year 2007, the number of children in a locality was 1500. In the year 2008, the number of children in the locality rose to 2100. Find the percent increase in the number of children of the locality.

Solution:

Increase in the number of children = 2100 – 1500 = 600

Percent increase $= \frac{600}{1500} \times 100 = 40\%$

Thus, the required percent increase in the number of children of the locality is 40%.

- The price at which an article is bought is called its **cost price** (CP).
- The price at which an article is sold is its **selling price** (SP).
  - Conditions of profit or loss:

1. If CP < SP then profit is made and Profit = SP – CP
2. If CP = SP then there is a no profit, no loss.
3. If CP > SP then loss is incurred and Loss = CP – SP

For example, Suman bought a bottle for Rs 130 and sold it for Rs 142.
Here, SP = Rs 142, CP = Rs 130
As SP > CP, so profit is incurred.

Profit = SP – CP = Rs 142 – Rs 130 = Rs 12

- The formulae to calculate profit and loss are:
  - Profit % $= \frac{\text{Profit}}{\text{C.P}} \times 100$
  - Loss % $= \frac{\text{Loss}}{\text{C.P.}} \times 100$

Example:

A shopkeeper purchased 15 dozen cups for Rs 900. However, 9 cups cracked during transportation. The remaining cups were sold for Rs 9 each. Find the gain or loss percent.

Solution:

Cost price of 15 dozen i.e., 180 cups = Rs 900

9 cups were cracked. Therefore, number of cups left = 180 – 9 = 171

These 171 cups were sold at Rs 9 each.

∴ S.P. of 171 cups = Rs 9 × 171 = Rs 1539

⇒ Profit = SP – CP = Rs (1539 – 900) = Rs 639

$\text{Profit\%} = \frac{\text{Profit}}{\text{C.P.}} \times 100 = \frac{639}{900} \times 100 = 71\%$

- **Terminology related to simple interest:**

1. The amount of money that is borrowed is known as principal and is denoted by P.
2. The extra amount of money that one has to pay is known as interest and is denoted by I.
3. The total amount of money, A that one pays back is equal to the sum of principal and interest.
4. The simple interest (SI) on the principal (P) when borrowed for T years at R% rate of interest per year is given by the formula $SI = \frac{P \times T \times R}{100}$

   Example:
   Rashmi takes a loan of Rs 4000 from a bank at 8% rate of interest per year. Find the amount of money that Rashmi has to repay after 3 years.

   Solution:
   P = Rs 4000, R = 8% p.a., T = 3 years

$$SI = \frac{P \times R \times T}{100} \frac{4000 \times 8 \times 3}{100} = Rs.960$$

∴ Amount = P + I = Rs 4000 + Rs 960 = Rs 4960

Thus, Rashmi has to repay Rs 4960 after 3 years.

Note: Principal remains unchanged throughout the given time period while calculating the simple interest.