Mathematics
Class – XII

Time allowed: **3** hours | Maximum Marks: **100**

General Instructions:

a) All questions are compulsory.
b) The question paper consists of 26 questions divided into three sections A, B and C. Section A comprises of 6 questions of one mark each, Section B comprises of 13 questions of four marks each and Section C comprises of 7 questions of six marks each.
c) All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
d) Use of calculators is not permitted.

Section A
(1 marks)

1. Let A = |R| {1} and define f! A → A by $f(x)=\frac{x}{x-1}$ for all x ∈ A, Then find f is injective.

2. what is $|\vec{a}|$, if $(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{5})$ = 3 and $2|\vec{b}|=|\vec{a}|$

3. If $[1\,0\,4]$ and $B=\begin{bmatrix}2\\5\\6\end{bmatrix}$ find Ab

4. If $P(A)=\frac{2}{5}, P(B)=\frac{1}{3}, P(A\Delta B)=\frac{1}{3}$ then find $P\left(\frac{A}{B}\right)$

Section B
(2 marks)

5. Find $(\vec{x})$ if for a unit vector $\vec{a}$

$$(\vec{x})\ (\vec{x}-\vec{a})(\vec{x}+\vec{a})=12$$

6. Five male and 3 female candidates are available for selection as an manager in a company find the probability that male is selected.

7. Solve $\int \frac{x+\cos 6x}{3x^2+\sin 6x}dx$

8. write the value of $\tan^{-1}\left[2\sin\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right]$

9. If A is a square matrix such that $A^2 = A$, then write the value of 7a – $(I + A)^3$, where I is an identity matrix.

10. If x = 0(cosθ + θ sinθ) and y = (a sinθ – θ cosθ) find $\frac{dy}{dx}$

11. Find the condition for the curve $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ and xy = c2 to intersect orthogonally.

12. Show that the relation R in the set R of real Ne defined as $\{(a,b): a \le b^2)$ is neither reflexive nor symmetric nor transitive.

Section C
(4 marks)

13. If $x=\frac{\sin^3 t}{\sqrt{\cos 2t}}, y=\frac{\cos^3 t}{\sqrt{\cos 2t}}$ solve it.

14. Prove it $\begin{vmatrix} x+a & b & c \\ a & x+b & 3 \\ a & b & x+c \end{vmatrix} = x^2$ (x + a + b+ c)

15. How many times must a man toss a fair coin, so that the probability of having at least one head is more than 80%.

16. Verify y = e^x cosx is a sol^x of this different equation

$$\frac{d^2y}{dx^2}-2\frac{dy}{dx}+2y=0$$

17. Find the magnitude of two vector $\vec{a}$ and $\vec{b}$ having same magnitude and are s·t· the angle between then 60° and their scalar product is $\frac{1}{2}$.

18. Solve $\int \frac{3x-2}{(x+1)^2(x+3)}dx$

19. Find the values of P so that the line $\frac{1-x}{3}=\frac{7y-14}{2P}=\frac{z-3}{2}$ and $\frac{7-7x}{3P}=\frac{y-5}{1}=\frac{6-z}{5}$ are right angled.

20. Show that $y=\log(1+x)-\frac{2x}{2+x}, x>-1$ and increasing function of x through out its domain.

21. If $f(x)=\begin{bmatrix}\cos-\sin x & & 0\\ \sin n & \cos n & 0\\ 0 & 0 & 1\end{bmatrix}$ show that f(x)f(y) = f(x + y)

22. Let the vectors $\vec{a},\vec{b},\vec{c}$ given as $a_1\hat{i}+a_2\hat{j}+a_3\hat{k},\ b_1\hat{i}+b_2\hat{j}+b_3\hat{k},\ c_1\hat{i}+c_2\hat{j}+c_3\hat{k},$ Then show that $\vec{a}\times(\vec{b}+\vec{c})=\vec{a}\times\vec{b}+\vec{a}\times\vec{c}$

23. Determine whether the function f(x) = 3sin(2x) satisfies condition of Rolle's Theorem.

Section D
(6 marks)

24. An Aeroplan can carry a maximum of 200 passengers. A profit of Rs.1000 is made on each executive class ticket and a profit of Rs.600 is made an each economy class ticket. The airline reserves at least 20 seat for the executive class. However, at least 4 times as many tickets of each type must be sold, in order to maximize profit.

25. Prove that $\begin{vmatrix}x & x^2 & 1+px^3\\ y & y^2 & 1+py^3\\ z & z^2 & 1+pz^3\end{vmatrix}=(1+Pxyz)(x-y(y-z)(z-x)$

26. Find the area of the region in the 1st quadrant enclosed by x – axis, $x=\sqrt{3}y$ and circle is $x^2+y^2=4$

27. Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of sphere.

28. Find shortest distance b/w line $\frac{x+1}{7}=\frac{y+1}{-6}=\frac{z+1}{1}$ and $\frac{x-3}{1}=\frac{y-5}{-2}=\frac{z-7}{1}$

29. A can hit a target 4 times in 5 shots, B can hit a target 3 times in 4 shots, e can hit a target 2 times in 3 shot. Calculate probability

1) A, B, C all may hit

2) B, C hit and A may lose

3) None of them will hit the target.

(Solution)
Class – XII Mathematics

Section A

Sol.1 take $x_1, x_2 \in A$ and $f(x_1) = f(x_2)$

Then $\frac{x_1}{x-1} = \frac{x_2}{x_2-1}$

$X_1(x_2 - 1) = x_2(x_1 - 1)$

$X_1x_2 - x_1 = x_2x_1 - 1$

$X_1x_2 - x_1 = x_2x_1 - x_2$

$-x_1 = -x_2$

$\boxed{x_1 - x_2}$

So, f is injective.

Sol2. $(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b}) = 3$

$|\vec{a}|^2 = |\vec{b}|^2 = 3$

Given that $2|\vec{b}| = |\vec{a}|$

$(2|\vec{b}|)^2 - |\vec{b}|^2$

$4|\vec{b}|^2 - |\vec{b}|^2 = 3$

$3|\vec{b}|^2 = 3$

$|\vec{b}| = \sqrt{1};\ |\vec{a}| = 2\times 1 = 2$

Sol.3 $A = [1\ 0\ 4],\ B = \begin{bmatrix} 2 \\ 5 \\ 6 \end{bmatrix}$

$Ab = [1\ 0\ 4]\begin{bmatrix} 2 \\ 5 \\ 6 \end{bmatrix}$

$= [2+0+24] = [26]$

Sol.4 $P(A) = \frac{2}{5};\ P(B) = \frac{1}{3}$

$P(A\Delta B) = \frac{3}{3}$ then $P\left(\frac{A}{B}\right) = ?$

$P\left(\frac{A}{B}\right) = \frac{P(A\Delta B)}{P(B)}$

$$= \frac{43}{43} = 1$$

Sol.5 $(\vec{x} - \vec{a}) \cdot (\vec{x} + \vec{a}) = 12$

$\Rightarrow \quad \vec{x} \cdot \vec{x} + \vec{x} \cdot \vec{a} - \vec{a} \cdot \vec{x} - \vec{a} \cdot \vec{a} = 12$

$\Rightarrow \quad |\vec{x}|^2 - |\vec{a}|^2 = 12$

$\Rightarrow \quad |\vec{x}|^2 - 1 = 12$

$\Rightarrow \quad |\vec{x}|^2 = 13$

$\therefore \quad |\vec{x}| = \sqrt{13}$

Sol.6 No. of males = 5

No. of females = 3

Total member = 8

Total outcomes = 8

No. of outcomes for a male getting selected = 5

$\therefore$ Probability of a male getting selected $= \frac{5}{8}$

Sol.7 $I = \int \frac{x + \cos 6x}{3x^2 + \sin 6x} dx$

Put $3x^2 + \sin 6x = t$

$\Rightarrow \quad 6x + 6\cos 6x = \frac{dt}{dx}$

$\Rightarrow \quad (x + \cos 6x)dx = \frac{1}{6} dt$

$\therefore \quad \int \frac{dt}{6t} = \frac{1}{6} \log |t| c$

$= \frac{1}{6}\left[\log\left|3x^2 + \sin 6x\right|\right] + c$

Sol.8 $\tan^{-1}\left[2\sin\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right]$

$= \tan^{-1}\left[2\sin\left\{\cos^{-1}\left(2 \cdot \frac{3}{4} - 1\right)\right\}\right]$

$= \tan^{-1}\left[2\sin\left\{\cos^{-1}\left(\frac{3}{2} - 1\right)\right\}\right]$

$= \tan^{-1}\left[2\sin\left\{\cos^{-1}\left(\frac{1}{2}\right)\right\}\right]$

$$= \tan^{-1}\left[2\sin\left\{\cos^{-1}\left(\cos\frac{\pi}{3}\right)\right\}\right]$$

$$= \tan^{-1}\left[2\sin\frac{\pi}{3}\right]$$

$$= \tan^{-1}\left(2\cdot\frac{\sqrt{3}}{2}\right)$$

$$= \tan^{-1}\left(\sqrt{3}\right) = \tan^{-1}\left(\tan\frac{\pi}{3}\right) = \frac{\pi}{3}$$

Sol9. We have, $A^2 = A$

Now,

$$\Rightarrow \quad 7A = (I + A)^3 = 7A - \left[I^3 + A^3 + 3IA(I + A)\right]$$

$$\Rightarrow \quad 7A - \left[I + A^2 \cdot A + 3A(I + A)\right]$$

$$\Rightarrow \quad 7A - \left[I + A \cdot A + 3AI + 3A^2\right]$$

$$\Rightarrow \quad 7A - \left[I + A + 3A + 3A\right]$$

$$\Rightarrow \quad 7A - \left[I + 7A\right] = -1$$

Sol.10

$$\frac{dx}{d\theta} = a\left[-\sin\theta + \theta\cos\theta + \sin\theta\right]$$

$$\frac{dx}{d\theta} = a\left[\cos\theta + \theta\sin\theta - \cos\theta\right]$$

$$\frac{dx}{d\theta} = a\theta\sin\theta$$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{a\theta\sin\theta}{a\theta\cos\theta}$$

$$\frac{dy}{dx} = \tan\theta$$

Sol.11 Let the curves intersect at (x_1, y_1). Therefore,

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\Rightarrow \quad \frac{2x}{a^2} - \frac{2y}{a^2} - \frac{2y}{b^2}\frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{b^2 x}{a^2 y}$$

$\Rightarrow$ Slope of tangent at the point of intersection $(m_1) = \frac{b^2 x1}{a^2 y1}$

Again xy = c²

$\Rightarrow \quad x\frac{dy}{dx} + y = 0$

$\Rightarrow \quad \frac{dy}{dx} = \frac{-y}{x} \Rightarrow m_2 = \frac{-y_1}{x_1}$

For orthogonality $m_1 \times m_2 = -1$

$\Rightarrow \quad \frac{b^2}{a^2} = 1$ or $a^2 - b^2 = 0$

Sol.12 $R = \{(a,b); a \leq b^2 y$

It can observed $\left(\frac{1}{2}, \frac{1}{2}\right) \notin R,$ since $\frac{1}{2} > \left(\frac{1}{2}\right) = \frac{1}{4}$

∴ (4, 1) ∉R, R is not symmetric

Now (3, 2) (2, 1·5) ϵR

$3 < 2^2 = 4$ and $2 < (1{\cdot}5)^2 = 2{\cdot}25$

∴ R is not transits

Sol.13 Given equation are

$x = \frac{\sin^2 t}{\sqrt{\cos 2t}}, y = \frac{\cos^3 t}{\sqrt{\cos 2t}}$

Then $\frac{dx}{dt} = \frac{\sqrt{\cos 2t} \cdot 3\sin^2 t \cos 2 - \frac{\sin^3 t(-\sin t)}{2\sqrt{\cos 2t}}}{\cos 2t}$

$= \frac{3\cos 2 + \sin^2 t + \sin^3 t \sin^2 t}{\cos 2t\sqrt{\cos 2t}}$

$\frac{dy}{dt} = \frac{\sqrt{\cos 2t}\frac{d}{dt}(\cos^3 t) - \cos^3 t\frac{d}{dt}\sqrt{\cos 2t}}{\cos 2t}$

$= \frac{3\sqrt{\cos 2t} \cdot \cos^2 t(-\sin t) - \frac{\cos^3 t \cdot \times 1(-2\sin 2t)}{2\sqrt{\cos 2t}}}{\cos 2t}$

$\frac{dy}{dt} = \frac{-3\cos 2 + \cos^2 t \sin t + \cos^3 + \sin 2t}{\cos 2t\sqrt{\cos 2t}}$

$\therefore \frac{dy}{dx} = \frac{-3\cos 2 + \cos^2 t \sin t + \cos^3 + \sin 2t}{3\cos 2t \sin^2 t \cos t + \sin^3 t \sin 2t}$

$$= \frac{\left[-3(2\cos^2 t - 1\right]\cos t + 2\cos^3 t}{\left[3(1-2\sin^2 t)\sin t + 2\sin^3 t\right.}$$

$$= \frac{-4\cos^3 t + 3\cos t}{3\sin + \cos 3t}$$

$$= \frac{-\cos 3t}{\sin 3t}$$

$$= -\cot 3t$$

Sol.14 Given that

$$\Rightarrow \begin{vmatrix} x+a & b & c \\ a & a+b & c \\ a & b & x+c \end{vmatrix}$$

Applying $C_1 \rightarrow C_1 + C_2 + C_3$

$$\Rightarrow \begin{vmatrix} x+a+b+c & b & c \\ x+a+b+c & x+b & c \\ x+a+b+c & b & x+c \end{vmatrix}$$

Common from C_1

$$\Rightarrow (x+a+b+c)\begin{vmatrix} 1 & b & c \\ 1 & x+b & c \\ 1 & b & x+c \end{vmatrix}$$

$R_1 + R_1 - R_2$, $R_2 + R_2 - R_3$

$$\Rightarrow (x+a+b+c)\begin{vmatrix} 0 & -x & 0 \\ 0 & x & -x \\ 1 & b & x+c \end{vmatrix}$$

Now expanding

We get

$$\Rightarrow \quad (x+a+b+c)\left[x^2\right)$$

$$\Rightarrow \quad x^2(x+a+b+c)$$

Sol.15 Let P denotes the probability that the man will get the head.

Then, $P = \frac{1}{2}, E = 1 - \frac{1}{2} = \frac{1}{2}$

Let x denotes the No. of times he get head in n-trails

$\therefore \quad P(x=r) = {}^n c_r p^r q^{n-r}$

Now

$$P(x \geq 1) > \frac{80}{100}$$

$\Rightarrow \quad P(x=1)+P(x=2)+.......+ P(x=n) > \frac{8}{10}$

$\Rightarrow \quad [P(x=0)+P(x+1)......] - P(x=0) > \frac{8}{10}$

$\Rightarrow \quad 1-p(x=0) > \frac{8}{10}$

$\Rightarrow \quad P(x=0) < 1-\frac{8}{10}$

$\Rightarrow \quad P(x=0) < \frac{2}{10}$

$\Rightarrow {}^{n}c_0 \left(\frac{1}{2}\right)^n < \frac{1}{5}$

$\Rightarrow \quad \left(\frac{1}{2}\right)^n < \frac{1}{5}$

Now, $\left(\frac{1}{2}\right)^n < \frac{1}{5}$ is true only when the value of n is equal to or greater than 3

$\therefore {}^{n}c_0 \left(\frac{1}{2}\right)^n < \frac{1}{5}$

$\Rightarrow \quad n = 3, 4, 5$

Hence, the man must toss atleast 3 times.

Sol.16 y = e^x cos x

$\frac{dy}{dx} = -e^x \sin x + \cos x e^x$

$\frac{dy}{dx} = -e^x (\cos x - \sin x)$

$\frac{d^2y}{dx^2} = e^x(-\sin x - \cos x) + (\cos x - \sin x)e^x$

$= -e^x \sin x - e^x \cos x + e^x \cos x - e^x \sin x$

$\frac{d^2y}{dx^2} = -2e^n \sin x$

Use in $\frac{d^2y}{dx^2} = -2\frac{dy}{dx} + 2y = 0$

L.H.S

$= -2e^x \sin x - 2(e^x(\cos x - \sin x) + 2e^x \cos x$

$= -2e^x \sin x - 2e^x \cos x + 2e^x \sin x + 2e^x \cos x = 0$

= R.H.S

Yes y = e^x cosx is solution of given equation.

Sol.17 Given that

$|\vec{a}| = |\vec{b}|$ .....(1)

Q = 60° .....(2)

$\vec{a} \cdot \vec{b} = \frac{1}{2}$ ......(3)

Angle formula

$$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}$$

$$\cos\theta = \frac{\frac{1}{2}}{|\vec{a}||\vec{b}|} \quad .....(4)$$

$$\cos 60° = \frac{\frac{1}{2}}{|\vec{a}| \cdot |\vec{b}|}$$

Because $|\vec{a}| = |\vec{b}|$

$$\frac{1}{2} = \frac{\frac{1}{2}}{|\vec{a}|^2}$$

$$|\vec{a}|^2 = \frac{\frac{1}{2}}{\frac{1}{2}}$$

$$|\vec{a}|^2 = 1$$

$$|\vec{a}| = 1$$

$$|\vec{a}| = |\vec{b}| = 1$$

Sol.18 $\int \frac{3x-2}{(1+x)^2(x+3)}dx$

Let

$$\frac{3x-2}{(1+x)^2(x+3)} = \frac{A}{(1+x)} + \frac{B}{(1+x)^2} + \frac{C}{(x+3)}$$

$$(3x-2) = A(1+x)(x+3) + B(x+3) + C(1+x)^2$$

$$= A(x+x^2+3+3x) + B(x+3) + C(1+x^2+2x)$$

Comparing terms

X^2	X	constant
0 = A + C	3 = 4A + B + 2C	-2 = 3A + 3B + C

Solving 2 and 3rd

3(4A + B + 2C = 3)

3A + 3B + C = -2

$$12A+3B+6C=9$$
$$\underline{\pm 3A \pm 3B \pm C = -2} \quad ....4$$
$$9A+5C=11$$

Solving Ist and 2nd

9(A + C = 0

9A + 5C = 11

$$9A+9C=0$$
$$\underline{\pm 9A \pm 5C = \pm 11}$$
$$4C=-11$$

$$\boxed{C=-\frac{11}{4}}$$

Use in Ist

A = -C

$$\boxed{A=\frac{11}{4}}$$

Use in 2nd

$$4\times\frac{11}{4}+B+2\times\frac{-11}{4}=3$$

$$11-\frac{11}{2}+B=3$$

$$\frac{11}{2}+B=3$$

$$B=3-\frac{11}{2}$$

$$=\frac{6-11}{2}$$

$$B=\frac{-5}{2}$$

$$\int\frac{3x-2}{(1+x)^2(x+3)}=\frac{11}{4}\int\frac{dx}{(x+1)}\frac{-5}{2}\int\frac{dx}{(1+x)^2}\frac{-11}{4}\int\frac{dx}{x+3}$$

$$=\frac{11}{4}\log|x+1|\frac{+5}{2}\left[\frac{1}{1+x}\right]-\frac{11}{4}\log|x+3|+c$$

$$=\frac{11}{4}\log\left|\frac{x+1}{x+3}\right|\frac{+5}{2}\left|\frac{1}{1+x}\right|+c$$

Sol.19 The given equation can be written in the standard form as

$$\frac{x-1}{-3}=\frac{y-2}{\frac{2P}{7}}=\frac{z-3}{2}$$

$$\frac{x-1}{\frac{-3P}{7}}=\frac{y-5}{1}=\frac{3-6}{-5}$$

The dir. ratios of lines are $-3,\frac{2P}{7},2$ and $\frac{-3P}{7},1,-5$ respectively

Two lines with dir. ratios a_1, b_1, c_1 and a_2, b_2, c_2 perpendicular to each other if

$a_1a_2+b_1b_2+c_1c_2=0$

$\therefore \quad (-3)\left(\frac{-3P}{7}\right)+\left(\frac{2P}{7}\right)\cdot 1+2(-50)=0$

$\Rightarrow \quad \frac{9P}{7}+\frac{2P}{7}=10$

$\Rightarrow \quad 11P=70$

$\Rightarrow \quad \boxed{P=\frac{70}{11}}$

Sol.20 we have,

$y=\log(1+x)-\frac{2x}{2+x}$

$\therefore \quad \frac{dy}{dx}=\frac{1}{1+x}-\frac{(2+x)\cdot 2-2(x)}{(2+x)^2}$

$=\frac{1}{1+x}-\frac{4}{-}=\frac{x^2}{(2+x)^2}$

$=\frac{x^2}{(2+x)^2}$

Now $\frac{dy}{dx}=0$

$\Rightarrow \quad \frac{x^2}{(2+x)^2}=0$

$\Rightarrow \quad x^2=0, \Rightarrow x=0$

Since x > - 1, point x = 0 divides the domain (-1, ∞) in two dis joint intervals

i.e, -1 < x < 0, we have

X < 0 ⇒ $x^2 > 0$

X > -1 ⇒ (2 + x) > 0

⇒ $(2 + x)^2 > 0$

$\therefore \quad y^1=\frac{x^2}{(2+x)^2}>0$

Also, when x > 0

X > 0 ⇒ $x^2 > 0$, $(2 + x)^2 > 0$

$\therefore \quad y^1 = \dfrac{x^2}{(2+x)^2} > 0$

Hence function f is increasing through this domain.

Sol.21 $f(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$$f(y) = \begin{bmatrix} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$f(x+y) = \begin{bmatrix} \cos(x+y) & -\sin(x+y) & 0 \\ \sin(x+y) & \cos(x+y) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$f(x)f(y) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos x \cos y - \sin x \sin y & -\cos x \sin y - \sin x \cos y & 0 \\ \sin x \cos y + \cos x \cos y & -\sin x \sin y + \cos x \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos(x+y) & -\sin(x+y) & 0 \\ \sin(x+y) & \cos(x+y) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= f(x+y)$$

$\therefore \quad f(x)f(y) = f(x+y)$

Sol.22 We have

$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

$$(\vec{b} + \vec{c}) = (\vec{b}_1 + \vec{c}_1)\hat{i} + (\vec{b}_2 + \vec{c}_2)\hat{j} + (\vec{b}_3 + \vec{c}_3)\hat{k}$$

Now, $\vec{a} \times (\vec{b} + \vec{c}) \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 + c_1 & b_2 + c_2 & b_3 + c_3 \end{vmatrix}$

$$= \hat{i}\left[a_2(b_3 + c_3) - a_3(b_2 + c_2)\right] - \hat{j}\left[a_1(b_3 + c_3) - a_3(b_1 + c_1)\right]$$

$+\hat{k}\left[a_1(b_2 + c_2) - a_2(b_1 + c_1)\right]$ ......(1)

$$= \hat{i}\left[a_2b_3 + a_2c_3 - a_3b_2 - a_3c_2\right] + \hat{j}\left[-a_1b_3 - a_1c_3 + a_3b_1 + a_3c_1\right]$$

$$+\hat{k}\left[a_1b_2 + a_1c_2 - a_2b_1 - a_2c_1\right]$$

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

$$= \hat{i}\left[a_2c_3 - a_3c_2\right] + \hat{j}\left[a_3c_1 - a_1c_3\right] + \hat{k}\left[a_1c_2 - a_2c_1\right]$$

Add $(\vec{a}\times\vec{b}) + (\vec{a}\times\vec{c})$

$$= i\left[a_2b_3 + a_2c_3 - a_3b_2 - a_3c_2\right]$$

$$+\hat{j}\left[b_1a_3 + a_3c_1 - a_1b_3 - a_1c_3\right]$$

$$+\hat{k}\left[a_1b_2 + a_1c_2 - a_2b_1 - a_2c_1\right] \quad ....(4)$$

From 1 & 4^{th}

$$\vec{a}\times(\vec{b}+\vec{c}) = \vec{a}\times\vec{b} + \vec{a}\times\vec{c}$$

Hence, the given result is proved.

Sol23. Given

$f(x) = 3\sin(2x)$

Is continuous on [0, π] and differentiable on the interval (0, π)

I know $f^1(c)$ **=** 0 in Rolle's theorem

The derivative 3cos(2c) (2)=0

And then 6cos2c = 0

Cos2c = 0

$$2c = \cos^{-1} 0 = \frac{\pi}{2}$$

$$2c = \frac{\pi}{2}$$

$$c = \frac{\pi}{4}$$

Sol.24 Let the executive class air tickets and economy class ticked sold be x and y

Now as the seating capacity of the aeroplane is 200, so

X + y ≤ 200 ……(1)

As 20 tickets for executive class are to be resolved, so

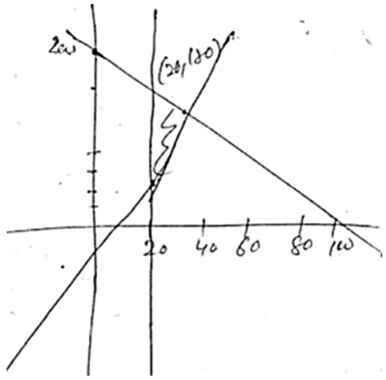
X ≥ 20 …..(2)

Economy class should be at least 4 times

Y ≥ 4x …..(3)

Profit

Z = 100x + 600y



X = y = 200

X = 20

Y = 4x

Z = 1000x + 600y

Corner points are

(20, 180) (40, 160) (20, 80)

Z = 128000

(40, 160)

Z = 136000

at (20, 80)

z = 68000

Maximum value at (40, 160)

Hence x = 40, y = 160

Sol.25 $$\Delta = \begin{vmatrix} x & x^2 & 1+px^3 \\ y & y^2 & 1+py^3 \\ z & z^2 & 1+pz^3 \end{vmatrix}$$

Applying $R_2 \to R_2 - R_1$

$R_3 \to R_3 - R_1$

$$\Delta = \begin{vmatrix} x & x^2 & 1+px^3 \\ y-x & y^2-x^2 & p(y^3-x^3) \\ z-x & z^2-x^2 & p(z^3-x^3) \end{vmatrix}$$

$$= (y-x)(z-x)\begin{vmatrix} x & x^2 & 1+px^3 \\ 1 & y+x & p(y^3-x^3) \\ 1 & z+x & p(z^3-x^3) \end{vmatrix}$$

$$= (y-x)(z-x)\begin{vmatrix} x & x^2 & 1+px^3 \\ 1 & y+x & p(y^2+x^2+xy) \\ 1 & z+x & p(z^2+x^2+xz) \end{vmatrix}$$

$R_3 \to R_3 - R_2$

$$= (y-x)(z-x)\begin{vmatrix} x & x^2 & 1+px^3 \\ 1 & y+x & P(y^2+x^2+2xy) \\ 0 & Z-y & p(z-y)(x+y+z) \end{vmatrix}$$

$$= (y-x)(z-x)(z-y)\begin{vmatrix} x & x^2 & 1+px^3 \\ 1 & y+x & P(y^2x^2+xy) \\ 0 & 1 & P(x+y+z) \end{vmatrix}$$

Expanding R_3

$\Delta = (x-y)(y-z)(z-x)$

$\left[(-1)P(xy^2+x^3+x^2y)+1+px^3 \;+P(x+y+z)(xy)\right]$

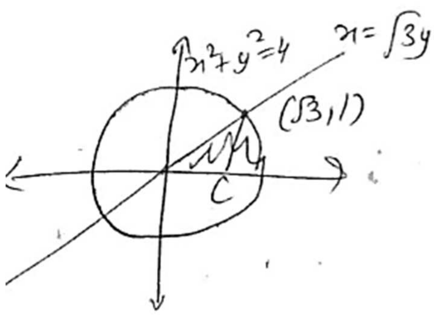
$= (x-y)(y-z)(z-x)$

$\left[-Pxy^2 - Px^3 - Px^2y + 1 + Px^3 \;+px^2y + Pxy^2 + Pxyz)\right.$

$= (x-y)(y-z)(3-x)$

Sol.26 The area of the region bounded by the circle $x^2 + y^2 = 4$

$x = \sqrt{3}y,$ and the X-axis is the area OAB



The point of intersection of the line and the circle in the Ist Quadrant is $(\sqrt{3},1)$

Area OAB = Area OCA + Area ACB

Area of $OAC = \frac{1}{2}\times\sqrt{3}\times 1 = \frac{\sqrt{3}}{2}$ .....(1)

Area $ABC = \int_{\sqrt{3}}^{2} y\,dx$

$= \int_{\sqrt{3}}^{2} \sqrt{4-x^2}\,dx$

$= \left[\frac{x}{2}\sqrt{-4-x^2} + \frac{4}{2}\sin^{-1}\frac{x}{2}\right]^2$

$$= \left[2 \times \frac{\pi}{2} - \frac{\sqrt{3}}{2}\sqrt{4-3} - 2\sin^{-1}\frac{3}{2}\right]$$

$$= \left[\pi - \frac{\sqrt{3}}{2} - \frac{2\pi}{3}\right]$$

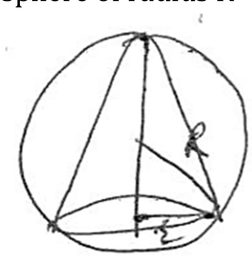
$$\left[\frac{\pi}{3} - \frac{\sqrt{3}}{2}\right] \quad .....(2)$$

Area enclosed by x-axis

The line $x = \sqrt{3}y$ and

The circle $x^2 + y^2 = 4$ in the Ist Quadrant $\frac{\pi}{3}$ unit

Sol.27 Let r and h be the radius and the height of the cone respectively inscribed in a sphere of radius R



Let v be the volume

$$v = \frac{1}{3}\pi r^2 h$$

height

$$h = R + \sqrt{R^2 - r^2}$$

$$v = \frac{1}{3}\pi r^2 \left(R + \sqrt{R^2 - r^2}\right.$$

$$\frac{dv}{dr} = \frac{2}{3}\pi r R + \frac{2}{3}\pi r\sqrt{R^2 - r^2} + \frac{1}{3}\pi r^2 \cdot \frac{(-2r)}{2\sqrt{R^2 - r^2}}$$

$$= \frac{2}{3}\pi R + \frac{2}{3}\pi r\sqrt{R^2 - r^2} - \frac{1}{3}\frac{\pi r^3}{\sqrt{R^2 - r^2}}$$

$$= \frac{2}{3}\pi R + \frac{2\pi(R^2 - r^2) - \pi r^3}{3\sqrt{R^2 - r^2}}$$

$$\frac{d^2v}{dr^2} = \frac{2}{3}\pi R \quad \frac{9(R^2 - r^2)(2\pi R^2 - 9\pi r^2) + 2\pi r^1 R^2 + 3\pi r^4}{27(R^2 r^2)\frac{1}{2}}$$

$$\frac{dv}{dr} = 0 \Rightarrow \qquad 2\pi rR$$

$$= \frac{3\pi r^3 - 2\pi R^2}{3\sqrt{R^2 r^2}}$$

$$\Rightarrow \qquad 2R = \frac{3r^2 - 2R^2}{\sqrt{R^2 - r^2}}$$

$$\Rightarrow \qquad 2R\sqrt{R^2 - r^2} = 3r^2 - 2R^2$$

St.

$$\Rightarrow \qquad 4R(R^2 - r^2) = (3r^2 - 2R^2)^2$$

$$\Rightarrow \qquad 9r^4 = 8R^2 r^2$$

$$\Rightarrow \qquad r^2 = \frac{8}{9}R^2$$

When $r^2 = \frac{8}{9}R^2$

Then $\frac{d^2v}{dr^2} < 0$

By 2nd derivative

The volume of cone is the max. when $r^2 = \frac{8}{9}R^2$

When $r^2 = \frac{8}{9}R^2$

$$h = R + \sqrt{R^2 - \frac{8}{9}R^2}$$

$$= R + \sqrt{\frac{1}{9}R^2}$$

$$= R + \frac{R}{3} = \frac{4}{3}R$$

$$v = \frac{1}{3}\pi\left(\frac{8}{9}R^2\right)\left(\frac{4}{3}R\right)$$

$$= \frac{8}{27}\left(\frac{4}{3}\pi R^3\right)$$

$$= \frac{8}{27}(\text{volume of sphere})$$

Hence proved.

Sol.28 Line are

$$\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$$

And $\frac{x-3}{1}=\frac{y-5}{-2}=\frac{z-7}{1}$

Distance b/w line are

$$d=\left|\frac{(\vec{b_1}\times\vec{b_2}\cdot(\vec{a_2}-\vec{a_1})}{|\vec{b_1}\times\vec{b_2}}\right|$$

$\vec{a_1}=-\hat{i}-\hat{j}-\hat{k}$

$\vec{a_2}=+\hat{i}+5\hat{j}+7\hat{k}$

$\vec{b_1}=9\hat{i}-6\hat{j}+\hat{k}$

$\vec{b_2}=\hat{i}-2\hat{j}+\hat{k}$

$$\vec{b_1}\times\vec{b_2}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 7 & -6 & 1\\ 1 & -2 & 1\end{vmatrix}$$

$=\hat{i}(-6+2)-\hat{j}(7-1)+\hat{k}(-14+6)$

$=-4\hat{i}-6\hat{j}-8\hat{k}$

$|\vec{b_1}\times\vec{b_2}|=\sqrt{16+36+64}$

$=\sqrt{116}=2\sqrt{29}$

$\vec{a_2}-\vec{a_1}=(\hat{i}+5\hat{j}+7\hat{k})-(-\hat{i}-\hat{j}-\hat{k})$

$=2\hat{i}+6\hat{j}+8\hat{k}$

$$d=\left|\frac{(-4\hat{i}-6\hat{j}-8\hat{k})\cdot(2\hat{i}+6\hat{j}+8\hat{k})}{2\sqrt{29}}\right|$$

$$=\left|\frac{-8-36-64}{2\sqrt{29}}\right|$$

$$=\frac{108}{2\sqrt{29}}=\frac{54}{\sqrt{29}}$$

Distance is $\frac{54}{\sqrt{29}}$

Sol.29 Probability of A of hit a target is $\frac{4}{5}$ probability of $B=\frac{3}{4}$

And probability of $C=\frac{2}{3}$

Can't hit

$\overline{A}=\frac{1}{5}$

$\overline{B} = \frac{1}{4}$

$\overline{C} = \frac{1}{2}$

1) A, B, C may hit

P(A) + P(B) + P(C)

$\frac{4}{5} + \frac{3}{4} + \frac{2}{3}$

$\frac{48 + 45 + 40}{60} = \frac{133}{60}$

2) $P(B) + P(C) + P(\overline{A})$

$\frac{3}{4} + \frac{2}{3} + \frac{1}{5}$

$\frac{45 + 40 + 12}{60} = \frac{95}{60}$

3) None of them hit

$P(\overline{A}) + P(\overline{B}) + P(\overline{C})$

$\frac{1}{5} + \frac{1}{4} + \frac{1}{3}$

$\frac{12 + 15 + 20}{60} = \frac{47}{60}$