

# Chapter 11

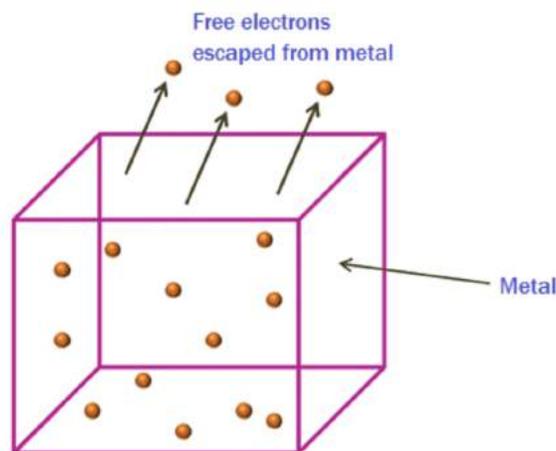
## Dual Nature of Radiation and Matter

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### Electron Emission, Photoelectric Effect & Wave Theory of Light

#### What is Electron Emission?

- When light is incident on a metal surface it was observed that electrons are ejected from a metal surface some times even when incredibly dim light such as that from stars and distant galaxies incident on it and some time electrons does not come out from the metal surface even high energetic or high-intensity light falling on the metal surface.
- This shows that the electron emission from a metal surface does not depend on the intensity of incident light but it basically depends on the energy of the incident.
- Photons no matters in the number of photons are very less in dim light, photoelectric effect can be seen.
- During the phenomenon of the photoelectric effect, one incident photon on the metal surface can eject at most only one electron.
- A photon is an energy packet that is fully absorbed not partially. Thus one photon can not be absorbed by more than one electron.
- The minimum amount of energy of photon required to eject an electron out of a metal surface is called work function It is denoted by  $\phi$ .



However, the surface barrier can be broken by providing a certain minimum amount of energy to the free electrons which increases their kinetic energy and consequently help them escape the metal surface.

This minimum amount of energy is known as the work function of the metal. And when the work function is provided to the metal, the consequent liberation of electrons from the metal surface is known as electron emission.

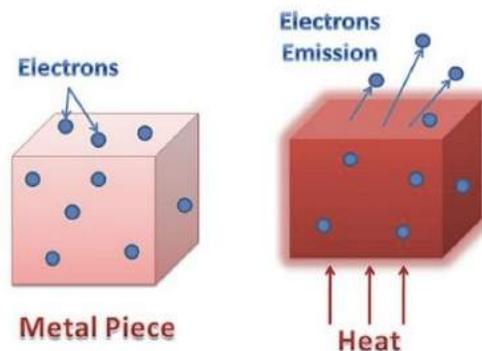
**The work function of a metal depends on:**

1. The electron emission from metal only depends on the work function or energy of one photon.
2. But how many electrons come out from the metal is depends on the intensity of the falling light on the energy of the light.
3. The energy of photon incident on metal will not necessarily cause emission of an electron even if its energy is more than work function. The electron after absorption may be involved in many other processes like collision etc in which it can lose energy hence the ratio of no. of electrons emitted to the no. of photons incident on the metal surface is less than unity.

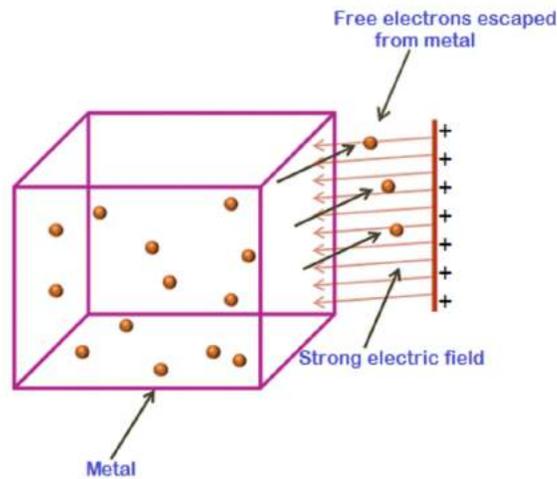
### Types of Electron Emission

The electron emission is possible only if sufficient energy (equal to the work function of the metal) is supplied to the metal in the form of heat energy, light energy, etc. Depending on the source of energy, electron emission can be of the following types:

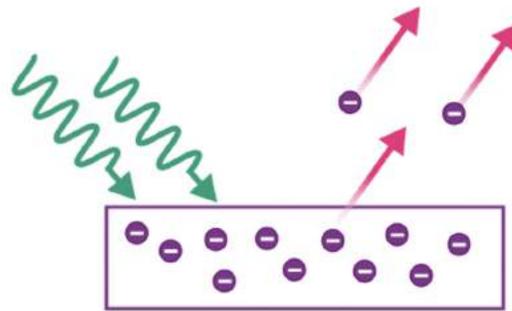
1. **Thermionic Emission:** In this type, the metal is heated to a sufficient temperature to enable the free electrons to come out of its surface.



2. **Field Emission:** In this type, a very strong electric field is applied to the metal which pulls the electrons out of the surface due to the attraction of the positive field.



3. **Photoelectric Emission:** In this type, the light of a certain frequency is made to fall on the metal surface which leads to the emission of electrons.



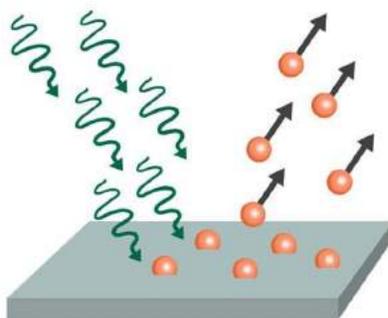
### Hertz And Lenard's Observations of Photoelectric Effect

- It was in 1887 when Heinrich Hertz was conducting experiments to prove Maxwell's electromagnetic theory of light, that he noticed a strange phenomenon. Hertz used a spark gap (two sharp electrodes placed at a small distance so that electric sparks can be generated) to detect the presence of electromagnetic waves. To get a closer look, he placed it in a dark box and found that the spark length was reduced. When he used a glass box, the spark length increased and when he replaced it with a quartz box, the spark length increased further. This was the first observation of the photoelectric effect.
- A year later, Wilhelm Hallwachs confirmed these results and showed that UV light on a Zinc plate connected to a battery generated a current (because of electron emission). In 1898, J.J. Thompson found that the amount of current varied with the intensity and frequency of the radiation used.

- In 1902, Lenard observed that the kinetic energy of electrons emitted increased with the frequency of radiation used. This could not be explained as Maxwell's electromagnetic theory (which Hertz proved correct) predicted that the kinetic energy should be only dependent on light intensity (not frequency).
- The resolution would only come a few years later by Einstein when he would provide an explanation to the photoelectric effect.

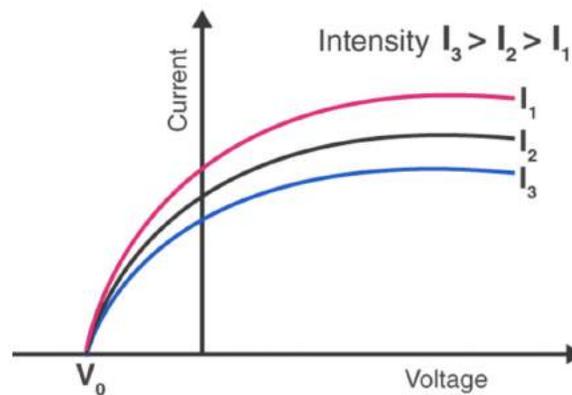
### Experimental Set-Up

J.J. Thompson's set-up (later improved by Lenard) to study this effect is of great importance. It consists of two zinc plate electrodes placed on the opposite ends of an evacuated (a vacuum is maintained) glass tube. A small quartz window illuminates one of the electrodes that is made the cathode. Quartz is used because ordinary glass blocks Ultra-Violet light. A variable voltage is exerted across the two electrodes using a battery and a potentiometer. The current in the circuit can be recorded using an ammeter as the potential and light intensity is changed. The set-up is shown below:

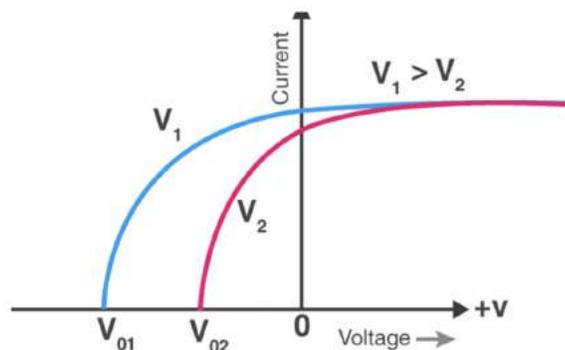


### Observations

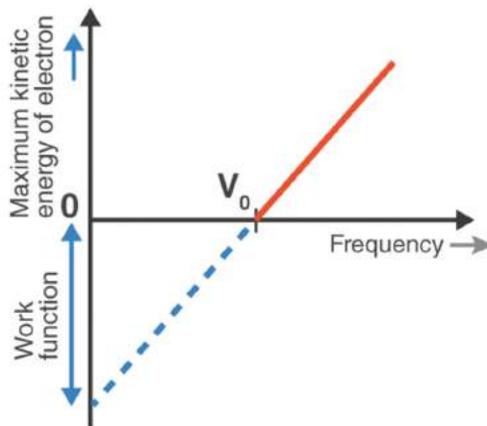
1. The photoelectric current (same as the rate of emission of electrons) is directly proportional to the intensity of light falling on the electrode. Note from the figure below that with increasing intensity the current is increasing. Also, observe that as the voltage has decreased the current also decreases. But to obtain zero current, the voltage has to be reversed to a certain  $V_0$  known as the stopping potential. The voltage must be reversed to such an extent that the electrons cannot reach the anode. This is the maximum kinetic energy an emitted electron can achieve, Maximum Kinetic energy,  $KE = eV_0$   
( $e$  is the charge of the electron)



1. Note that the stopping potential is independent of the intensity of light.
2. The Maximum kinetic energy increases with an increase in the frequency of light. With a higher frequency of light ( $\nu$ ), the stopping potential becomes more negative which implies that the kinetic energy of electrons also increases.



3. All frequencies of light, however, cannot cause a photoelectric current to develop. Only light above a certain frequency ( $\nu_0$ ) can produce a photoelectric current. This frequency is known as the threshold frequency. This varies with the electrode material. Also, the maximum kinetic energy of the electrons increases linearly with increasing light frequency. If we extend the graph below the x-axis, the intercept on the Kinetic energy axis represents the minimum energy required for emission of the electron; this is known as the work function of the material.



4. Lastly, the electron emission occurs instantly without any time lag.

### Threshold Frequency and Threshold Wavelength

- We have discussed that to start photoelectric emission, the energy of incident photon on the metal surface must be more than the work function of the metal. If  $\phi$  is the work function of the metal then there must be a minimum frequency of the incident light photon which is just able to eject the electron from the metal surface. This minimum

frequency or threshold frequency  $\nu_{th}$  can be given as  $h\nu_{th} = \phi$

- Threshold frequency  $\nu_{th}$  is a characteristic property of metal as it is the minimum frequency of the light radiation required to eject a free electron from the metal surface.
- As the threshold frequency is defined, we can also define the threshold wavelength  $\lambda_{th}$  for a metal surface. Threshold wavelength is also called cut off wavelength. For a given metal surface threshold wavelength is the longest wavelength at which photoelectric effect is possible. Thus

$$\frac{hc}{\lambda_{th}} = \phi$$

we have

- So for the wavelength of incident light  $\lambda > \lambda_{th}$ , the energy of incident photons will become less than the work function of the metal and hence photoelectric effect will not start.
- Thus for a given metal surface photoelectric emission will start at  $\nu > \nu_{th}$  or  $\lambda > \lambda_{th}$

**Example 1: The photoelectric threshold of the photoelectric effect of a certain metal is 2750 Å. Find**

(i) The work function of emission of an electron from this metal,

- (ii) Maximum kinetic energy of these electrons,  
 (iii) The maximum velocity of the electrons ejected from the metal by light with a wavelength of 1800 Å.

**Ans: (i)** Given that the threshold wavelength of a metal is  $\lambda_{th} = 2750 \text{ \AA}$ . Thus work function of metal can be given as  $\phi =$

$$\frac{hc}{\lambda_{th}} = \frac{12431}{2750} \text{ eV}$$

- (ii) The energy of an incident photon of wavelength 1800 Å on metal in eV

$$\text{is } E = \frac{12431}{1800} \text{ eV} =$$

6.9 eV

Thus maximum kinetic energy of ejected electrons is

$$KE_{max} = E - \phi = 6.9 - 4.52 \text{ eV} = 2.38 \text{ eV}$$

- (iii) If the maximum speed of ejected electrons is  $v_{max}$  then we have  $\frac{1}{2}mv_{max}^2 =$   
 2.38 eV

$$\text{or } v_{max} = \sqrt{\frac{2 \times 2.38 \times 1.6 \times 10^{-19}}{9.1 \times 10^{-31}}} = 9.15 \times 10^5 \text{ m / s}$$

**Example 2:** Light quanta with an energy of 4.9 eV eject photoelectrons from metal with work function of 4.5 eV. Find the maximum impulse transmitted to the surface of the metal when each electron flies out.

**Ans:** According to Einstein's photoelectric equation

$$E = \frac{1}{2}mv_{max}^2 = hv - \phi = 4.9 - 4.5 = 0.4 \text{ eV}$$

If E be the energy of each ejected photoelectron momentum of electrons is p

$$= \sqrt{2mE}$$

We know that change of momentum is impulse. Here the whole momentum of the electron is gained when it is ejected out thus impulse on the surface is

$$\text{Impulse} = \sqrt{2mE}$$

Substituting the values, we get

$$\text{Maximum impulse} = \sqrt{2 \times 9.1 \times 10^{-31} \times 0.4 \times 1.6 \times 10^{-19}} = 3.45 \times 10^{-25} \text{ kg m / sec}$$

**Example 3:** In an experiment tungsten cathode which has a threshold of 2300 Å is irradiated by the ultraviolet light of wavelength 1800 Å. Calculate

(i) Maximum energy of emitted photoelectron and

(ii) Work function for tungsten

(Mention both the results in electron-volts)

Given Plank's constant  $h = 6.6 \times 10^{-34}$  joule-sec,  $1 \text{ eV} = 1.6 \times 10^{-19}$  joule and velocity of light  $c = 3 \times 10^8$  m/sec

$$\phi = \frac{hc}{\lambda_{th}} = \frac{12431}{2300} \text{ eV} = 5.4 \text{ eV}$$

**Ans:** The work function of tungsten cathode is

The energy in eV of incident photons is

$$E = \frac{hc}{\lambda} = \frac{12431}{1800} \text{ eV}$$

The maximum kinetic energy of ejected electrons can be given as

$$KE_{max} = E - \phi = 6.9 - 5.4 \text{ eV} = 1.5 \text{ eV}$$

**Example 4:** Light of wavelength 1800 Å ejects photoelectrons from a plate of a metal whose work functions is 2 eV. If a uniform magnetic field of  $5 \times 10^{-5}$  tesla is applied parallel to the plate, what would be the radius of the path followed by electrons ejected normally from the plate with maximum energy.

**Ans:** Energy of incident photons in eV is given as

$$E = \frac{12431}{1800} \text{ eV}$$

As work function of metal is 2 eV, the maximum kinetic energy of ejected electrons is  $KE_{max} = E - \phi = 6.9 - 2 \text{ eV} = 4.9 \text{ eV}$

If  $v_{max}$  be the speed of fasted electrons then we have  $\frac{1}{2}mv_{max}^2 = 4.9 \times 1.6 \times 10^{-19}$  joule

$$\text{or } \sqrt{\frac{2 \times 4.9 \times 1.6 \times 10^{-19}}{9.1 \times 10^{-31}}} = 1.31 \times 10^6 \text{ m/s}$$

When an electron with this speed enters a uniform magnetic field normally it follows a circular path whose radius can be given by

$$r = \frac{mv}{qB} \quad \left[ \text{As } qvB = \frac{mv^2}{r} \right]$$

$$\text{or } r = \frac{9.1 \times 10^{-31} \times 1.31 \times 10^6}{1.6 \times 10^{-19} \times 5 \times 10^{-5}} \quad \text{or } r = 0.149 \text{ m}$$

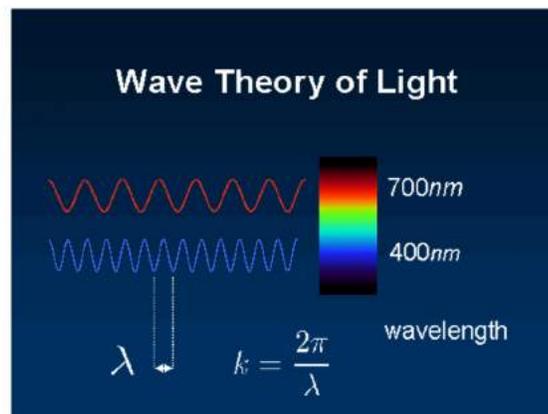
## Wave Theory of Light

Light always piqued the curiosity of thinkers and scientists. But it wasn't until the late 17th century that scientists began to comprehend the properties of light. Sir Isaac Newton proposed that light was made of tiny particles known as photons while Christian Huygens believed that light was made of waves propagating perpendicular to the direction of its movement.

In 1678, Huygens proposed that every point that a luminous disturbance meets turns into a source of the spherical wave itself. The sum of the secondary waves, which are the result of the disturbance, determines what form the new wave will take. This theory of light is known as the 'Huygens' Principle'.

Using the above-stated principle, Huygens was successful in deriving the laws of reflection and refraction of light. He was also successful in explaining the linear and spherical propagation of light using this theory. However, he wasn't able to explain the diffraction effects of light. Later, in 1803, the experiment conducted by Thomas Young on the interference of light proved the Huygens wave theory of light to be correct. Later in 1815, Fresnel provided mathematical equations for Young's experiment.

Max Planck proposed that light is made of finite packets of energy known as a light quantum and it depends on the frequency and velocity of light. Later, in 1905, Einstein proposed that light possessed the characteristics of both particle and wave. He suggested that light is made of small particles called photons. Quantum mechanics gave proof of the dual nature of light.



**Features of the Photoelectric effect cannot be explained in terms of the classical wave theory of light.**

**The intensity problem:** Wave theory requires that the oscillating electric field vector  $E$  of the light wave increases in amplitude as the intensity of the light beam is increased. Since the force applied to the electron is  $eE$ , this suggests that the kinetic energy of the photoelectrons should also be increased the light beam is made more intense. However, observation shows that maximum kinetic energy is independent of the light intensity.

**The frequency problem:** According to the wave theory, the photoelectric effect should occur for any frequency of the light, provided only that the light is intense enough to supply the energy needed to eject the photoelectron. However observations show that there exists for each surface a characteristic cutoff frequency  $\nu_0$ , for frequency less than  $\nu_0$ , the photoelectric effect does not occur, no matter how intense is light beam.

**The time delay problem:** If the energy acquired by a photoelectron is absorbed directly from the wave incident on the metal plate, the "effective target area" for an electron in the metal is limited and probably not much more than that of a circle of diameter roughly equal to that of an atom. In classical theory, the light energy is uniformly distributed over the wavefront. Thus, if the light is feeble enough, there should be a measurable time lag, between the impinging of the light on the surface and the ejection of the photoelectron. During this interval, the electron should be absorbing energy from the beam until it had accumulated enough to escape. However, no detectable time lag has ever been measured.

## Modern Physics

## Modern Physics

### 1. Nature of light

It was a matter of great interest for scientists to know that what exactly from the light is made up of or how the light behaves. This is briefly described over here

#### 1.1 Newton's Corpuscular theory:

Newton was the first scientist who said that light is made up of tiny elastic particles called "Corpuscles" which travel with the velocity of light. So according to Newton, light is a particle.

## 1.2 Huygen's wave theory

Huygen was a scientist working parallel to Newton who come with a drastically different idea for nature of light & said that light is not a particle but a wave.

## 1.3 Maxwell's electromagnetic wave theory:

During the time of Huygen, his views regarding nature of light were not accepted as newton was a popular scientist of his time. but, when maxwell asserted that light is a electromagnetic wave, scientists started believing that light is a wave.

## 1.4 Max Planck's quantum theory of light:

Once again when scientists started believing that the light is a wave max plank came with different idea & asserted that light is not a wave but a photon (i.e. a particle) which he proved through black body radiation spectrum. At this time there was a great confusion about the nature of light which was solved by de-broglie from where origin of theory of matter wave come into picture.

## 1.5 Debroglie Hypothesis

It supports dual nature of light (wave nature and particle nature). According to him the light consists of particles associated with definite amount of energy and momentum. These particles were later named as photons.

The photon posses momentum and is given by

$$P = \frac{h}{\lambda} \quad (1)$$

P = momentum of one photon

$\lambda$  = wavelength of wave.

h = Plank's constant =  $6.62 \times 10^{-34}$  Js.

A photon is a packet of energy. It posses energy given by

$$E = \frac{hc}{\lambda} \quad (2)$$

where c = speed of light

Debroglie relates particle property (momentum) with wave property (wavelength) i.e. he favours dual nature of light.

**Electron volt:** It is the energy gained by an electron when it is accelerated through a potential difference of one volt.

1 eV =  $1.6 \times 10^{-19}$  Joule.

Now from eq. (2)

$$E = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{\lambda} \quad \text{in Joule.}$$

$$E = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{\lambda \times 1.6 \times 10^{-19}} \text{ eV}$$

$$E = \frac{12400}{\lambda} \text{ eV}$$

where  $\lambda$  is in  $\text{\AA}$

**Properties of Photon:**

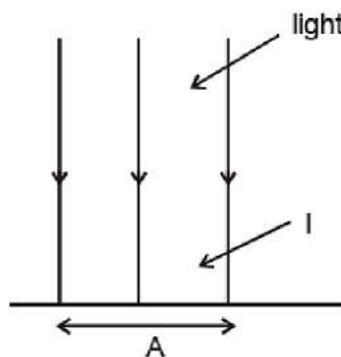
1. Photon travels with speed of light.
2. The rest mass of a photon is zero.
3. There is no concept of photon conservation.
4. All the photons of a particular frequency or wavelength possess the same energy irrespective of the intensity of the radiation.
5. The increase in the intensity of the radiation implies an increase in the number of photons crossing a given area per second.

- When light travels from one medium to another medium then

frequency = const (because it is the property of source)

but  $v, \lambda$  changes

**Ex.1** A beam of light having wavelength  $l$  and intensity  $I$  falls normally on an area  $A$  of a clean surface then find out the number of photons incident on the surface.



**Sol.** Total energy incident in time  $t = I A t$

Energy of one photon  $E = \frac{hc}{\lambda}$

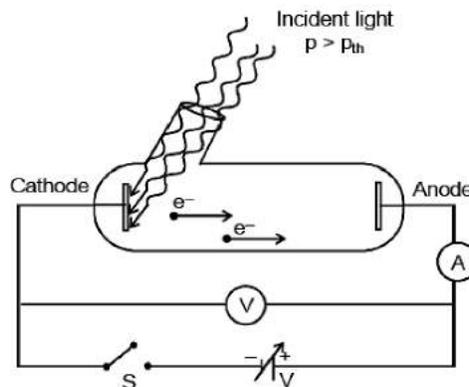
Then number of photons incident in time  $t$

$$\frac{\text{Total energy incident}}{\text{energy of one photon}} = \frac{IA t \lambda}{hc}$$

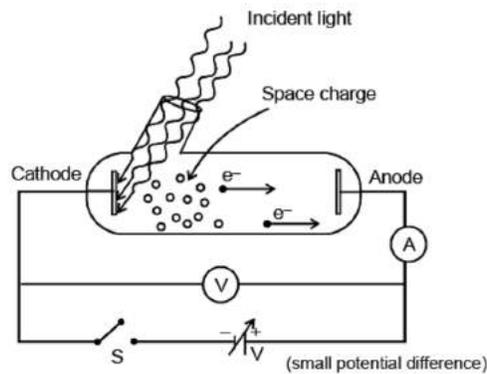
## Experimental Study of Photoelectric Effect

### 1.8 Experimental Study of Photo Electric Effect:

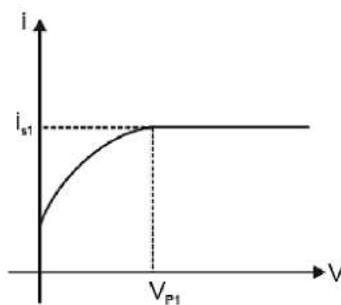
Experiments with the photoelectric effect are performed in a discharge tube apparatus as illustrated in figure shown. The cathode of discharge tube is made up of a metal which shows photoelectric effect on which experiment is being carried out.



A high potential is applied to a discharge tube through a variable voltage source and a voltmeter and an ammeter are connected to measure the potential difference across the electrodes and to measure photoelectric current. Light with frequency more than threshold frequency of cathode metal is incident on it, due to which photoelectrons are emitted from the cathode. These electrons will reach the anode and constitute the photoelectric current which the ammeter will show. Now we start the experiment by closing the switch S. Initially the variable battery source is set at zero potential. Even at zero potential variable source, ammeter will show some current because due to the initial kinetic energy some electrons will reach the anode and cause some small current will flow. But as we know majority of ejected electrons have low values of kinetic energies which are collected outside the cathode and create a cloud of negative charge, we call space charge, as shown in figure shown.



If the potential difference applied across the discharge tube is gradually increased from the variable source, positive potential of anode starts pulling electrons from the space charge. As potential difference increases, space charge decrease and simultaneously the photoelectric current in circuit also increases. This we can also see in the variation graph of current with potential difference as shown in figure shown.

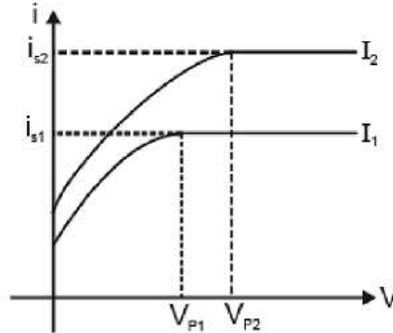


As shown in graph, we can see as potential difference increases, current in circuit increases. But at a higher voltage  $V_{P1}$  space charge vanishes and at this voltage anode is able to pull the slowest electron (zero kinetic energy) ejected by the cathode. Now as all the ejected electrons from cathode start reaching anode. If further potential difference is increased, it will not make any difference in the number of electrons reaching the anode hence, further increases in potential difference will not increase the current. This we can see in figure shown that beyond  $V_{P1}$  current in circuit becomes constant. This current  $i_{s1}$  is called saturation current. This potential difference  $V_{P1}$  at which current becomes saturated is called "pinch off voltage".

Now if the frequency of incident light is kept constant and its intensity is further increased, then the number of incident photons will increase which increases the number of ejected photo electrons so current in circuit increases and now in this case at higher intensity of incident light, current will not get saturated at potential difference  $V_{P1}$  as now due to more electron emission, space charge will be more and

it will not vanish at  $V_{P1}$ . To pull all the electrons emitted from cathode more potential difference is required. This we can see from figure shown, that at higher intensity  $I_2$  ( $I_2 > I_1$ ) current becomes saturated at higher value of potential difference  $V_{P2}$ .

**Intensity  $I_2 > I_1$**



Beyond  $V_{P2}$ , we can see that all the electrons ejected from cathode are reaching the anode and current becomes saturated at  $i_{s2}$  because of more electrons. Another point we can see from figure shown that when  $V = 0$  then also current is more at high intensity incident radiation as the number of electrons of high kinetic energy are also more in the beginning which will reach anode by penetrating the space charge.

### 1.9. Kinetic Energies of Electrons Reaching Anode

We know that when electrons are ejected from cathode then kinetic energies may

vary from 0 to  $\frac{1}{2}mv_{\max}^2$ . If  $V$  is the potential difference applied across the discharge tube then it will accelerate the electron while reaching the anode. The electron which is ejected from cathode with zero kinetic energy will be the slowest one reaching the anode if its speed is  $v_1$  at anode then we have

$$0 + eV = \frac{1}{2}mv_1^2$$

Similarly the electron ejected from cathode with maximum kinetic

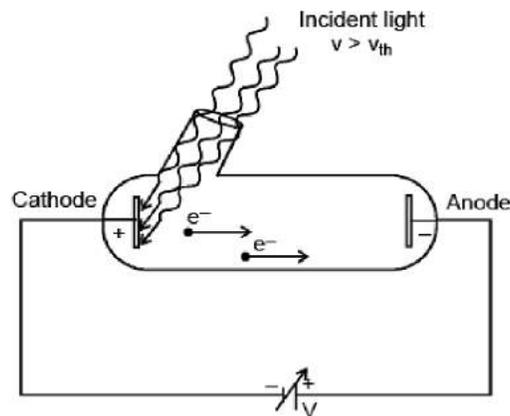
energy  $\frac{1}{2}mv_{\max}^2$  will be the fastest one when it will reach anode. If its speed is  $v_2$  at anode then we have

$$\frac{1}{2}mv_{\max}^2 + eV = \frac{1}{2}mv_2^2$$

Thus we can say that all the electrons reaching anode will have their speeds distributed from  $v_1$  to  $v_2$ .

### 1.10 Reversed Potential Across Discharge Tube:

Now the experiment is repeated with changing the polarity of source across the discharge tube. Now positive terminal of source is connected to the cathode of discharge tube. When a light beam incident on the cathode with  $(h\nu > \phi)$ , photoelectrons are ejected and move towards anode with negative polarity.



Now the electrons which are ejected with very low kinetic energy are attracted back to the cathode because of its positive polarity. Those electrons which have high kinetic energies will rush toward, anode and may constitute the current in circuit. In this case the fastest electron ejected from cathode will be retarded during its journey to anode. As the maximum kinetic energy just after emission at cathode

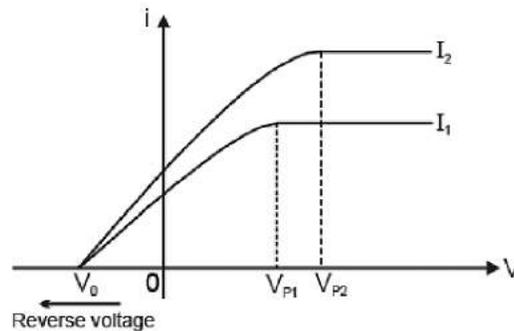
is  $\frac{1}{2}mv_{max}^2$ , if potential difference across the discharge tube is  $V$  then the speed  $v_f$  with which electrons will reach anode can be given as

$$\frac{1}{2}mv_{max}^2 - eV = \frac{1}{2}mv_f^2 \quad (1)$$

Thus all the electrons which are reaching anode will have speed less than or equal to  $v_f$ . Remaining electrons which have relatively low kinetic energy will either be attracted to cathode just after ejection or will return during their journey from cathode to anode. Only those electrons will cause current of flow in circuit which have high kinetic energies more than  $eV$  which can overcome the electric work against electric forces on electron due to opposite polarity of source.

### 1.11 Cut off Potential or Stopping Potential:

We have seen with reverse polarity electrons are retarded in the discharge tube. If the potential difference is increased with reverse polarity, the number of electrons reaching anode will decrease hence photo electric current in circuit also decreases, this we can see from figure shown which shows variation of current with increase in voltage across discharge tube in opposite direction. Here we can see that at a particular reverse voltage  $V_0$ , current in circuit becomes zero. This is the voltage at which the faster electron from cathode will be retarded and stopped just before reaching the anode.



**Intensity  $I_2 > I_1$  Frequency  $\nu$  (same for both radiation)**

This voltage  $V_0$ , we can calculate from equation (1) by substituting  $v_f = 0$  hence

$$\frac{1}{2}mv_{\max}^2 - eV_0 = 0$$

$$\text{or } eV_0 = \frac{1}{2}mv_{\max}^2$$

$$\text{or } V_0 = \frac{\frac{1}{2}mv_{\max}^2}{e} \quad \dots(2)$$

$$\text{or } V_0 = \frac{h\nu - \phi}{e} \quad \dots(3)$$

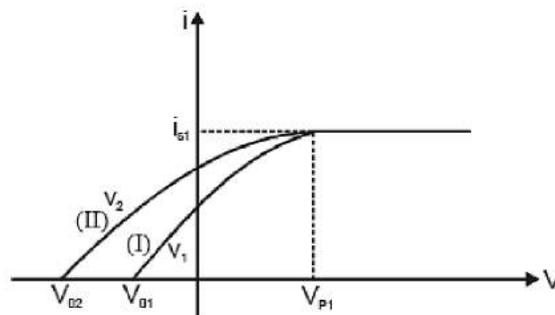
We can see one more thing in figure shown that the graphs plotted for two different intensities  $I_1$  and  $I_2$ ,  $V_0$  is same. Current in both the cases is cut off at same reverse potential  $V_0$ . The reason for this is equation-(2) and (3). It is clear that the value of  $V_0$  depends only on the maximum kinetic energy of the ejected electrons which depends only on frequency of light and not on intensity of light. Thus in above two graphs as frequency of incident light is same, the value of  $V_0$  is also same. This

reverse potential difference  $V_0$  at which the fastest photoelectron is stopped and current in the circuit becomes zero is called cut off potential or stopping potential.

### 1.12 Effect of Change in Frequency of Light on Stopping Potential:

If we repeat the experiment by increasing the frequency of incident light with number of incident photons constant, the variation graph of current with voltage will be plotted as shown in figure shown.

Frequency ( $\nu_2 > \nu_1$ )



This graph is plotted for two incident light beams of different frequency  $\nu_1$  and  $\nu_2$  and having same photon flux. As the number of ejected photoelectrons are same in the two cases of incident light here we can see that the pinch off voltage  $V_{01}$  as well as saturation current  $i_{s1}$  are same. But as in the two cases the kinetic energy of fastest electron are different as frequencies are different, the stopping potential for the two cases will be different. In graph II as frequency of incident light is more, the maximum kinetic energy of photoelectrons will also be high and to stop it high value of stopping potential is needed. These here  $V_{01}$  and  $V_{02}$  can be given as

$$V_{01} = \frac{h\nu_1 - \phi}{e} \quad \dots(4)$$

and 
$$V_{02} = \frac{h\nu_2 - \phi}{e} \quad (5)$$

In general for a given metal with work function  $\phi$ , if  $V_0$  is the stopping potential for an incident light of frequency  $\nu$  then we have

$$eV_0 = h\nu - \phi$$

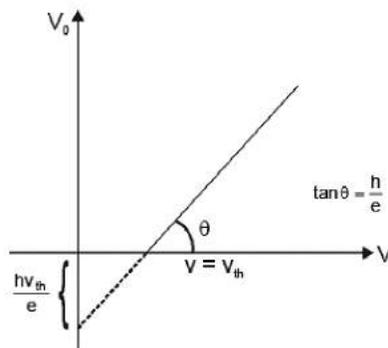
$$\text{or } eV_0 = h\nu - h\nu_{th} \dots(6)$$

$$\text{or } V_0 = \left(\frac{h}{e}\right) \nu - \frac{h\nu_{th}}{e} \quad (7)$$

Equation (7) shows that stopping potential  $V_0$  is linearly proportional to the frequency  $\nu$  of incident light. The variation of stopping potential with frequency  $\nu$  can be shown in figure shown. Here equation (6) can be written as

$$\frac{1}{2}mv_{\max}^2 = eV_0 - h(\nu - \nu_{\text{th}}) \dots (8)$$

This equation (8) is called Einstein's Photo Electric Effect equation which gives a direction relationship between the maximum kinetic energy stopping potential frequency of incident light and the threshold frequency.



**Ex. 6** Find the frequency of light which ejects electrons from a metal surface fully stopped by a retarding potential of 3 V. The photo electric effect begins in this metal at frequency of  $6 \times 10^{14} \text{ sec}^{-1}$ . Find the work function for this metal.

**Sol.** The threshold frequency for the given metal surface is

$$\nu_{\text{th}} = 6 \times 10^{14} \text{ Hz}$$

Thus the work function for metal surface is

$$\phi = h\nu_{\text{th}} = 6.63 \times 10^{-34} \times 6 \times 10^{14} = 3.978 \times 10^{-19} \text{ J}$$

As stopping potential for the ejected electrons is 3V, the maximum kinetic energy of ejected electrons will be

$$KE_{\max} = 3\text{eV} = 3 \times 1.6 \times 10^{-19} \text{ J} = 4.8 \times 10^{-19} \text{ J}$$

According to photo electric effect equation, we have

$$h\nu = h\nu_{\text{th}} + KE_{\max}$$

or frequency of incident light is

$$\nu = \frac{\phi + KE_{\max}}{h} = \frac{3.978 \times 10^{-19} + 4.8 \times 10^{-19}}{6.63 \times 10^{-34}} = 1.32 \times 10^{15} \text{ Hz}$$

**Ex.7** Electrons with maximum kinetic energy 3eV are ejected from a metal surface by ultraviolet radiation of wavelength 1500 Å. Determine the work function of the metal, the threshold wavelength of metal and the stopping potential difference required to stop the emission of electrons.

**Sol.** Energy of incident photon in eV is

$$E = \frac{12431}{1500} \text{ eV}$$

According to photo electric effect equation, we have

$$E = \phi + KE_{\max} \Rightarrow \text{or } \phi = E - KE_{\max}$$

$$\text{or } = 8.29 - 3 \text{ eV or } = 5.29 \text{ eV}$$

Threshold wavelength for the metal surface corresponding to work function 5.29 eV is given as

$$\lambda_{\text{th}} = \frac{12431}{5.29} \text{ \AA} = 2349.9 \text{ \AA}$$

Stopping potential for the ejected electrons can be given as

$$V_0 = \frac{KE_{\max}}{e} = \frac{3\text{eV}}{e} = 3 \text{ volt}$$

**Ex.8** Calculate the velocity of a photo-electron, if the work function of the target material is 1.24 eV and the wavelength of incident light is 4360 Å. What retarding potential is necessary to stop the emission of the electrons?

**Sol.** Energy of incident photons in eV on metal surface is

$$E = \frac{12431}{4360} \text{ eV} = 2.85 \text{ eV}$$

According to photo electric effect equation we have

$$E = \phi + \frac{1}{2}mv_{\max}^2 \quad \text{or} \quad \frac{1}{2}mv_{\max}^2 = E - \phi$$

$$= 2.85 - 1.24 \text{ eV} = 1.61 \text{ eV}$$

The stopping potential for these ejected electrons can be given as

$$V_0 = \frac{1/2mv_{\max}^2}{e} = \frac{1.61 \text{ eV}}{e}$$

**Ex.9** Determine the Planck's constant h if photoelectrons emitted from a surface of a certain metal by light of frequency  $2.2 \times 10^{15}$  Hz are fully retarded by a reverse

potential of 6.6 V and those ejected by light of frequency  $4.6 \times 10^{15}$  Hz by a reverse potential of 16.5 eV.

**Sol.** From photo electric effect equation, we have

$$\text{Here } hv_1 = \phi + 2 eV_{01} \quad (1)$$

$$\text{and } hv_2 = \phi + 2 eV_{02} \quad (2)$$

Subtracting equation (1) from equation (2), we get

$$h = \frac{(v_{02} - v_{01})(1.6 \times 10^{-19})}{(v_2 - v_1)}$$

or

$$h = \frac{(16.5 - 6.6)(1.6 \times 10^{-19})}{(4.6 - 2.2) \times 10^{15}}$$

or

$$\text{or } = 6.6 \times 10^{-34} \text{ J-s}$$

**Ex.10** When a surface is irradiated with light of wavelength 4950 Å, a photo current appears which vanishes if a retarding potential greater than 0.6 volt is applied across the photo tube. When a different source of light is used, it is found that the critical retarding potential is changed to 1.1 volt. Find the work function of the emitting surface and the wavelength of second source. If the photo electrons (after emission from the surface) are subjected to a magnetic field of 10 tesla, what changes will be observed in the above two retarding potentials.

**Sol.** In first case the energy of incident photon in eV is

$$E_1 = \frac{12431}{4950} \text{ eV} = 2.51 \text{ eV}$$

The maximum kinetic energy of ejected electrons is

$$KE_{\max 1} = eV_{01} = 0.6 \text{ eV}$$

Thus work function of metal surface is given as

$$\phi = E_1 - KE_{\max 1} = 2.51 - 0.6 \text{ eV} = 1.91 \text{ eV}$$

In second case the maximum kinetic energy of ejected electrons will become

$$KE_{\max 2} = eV_{02} = 1.1 \text{ eV}$$

Thus the incident energy of photons can be given as

$$E_2 = \phi + KE_{\max 2}$$

$$E_2 = 1.91 + 1.1 \text{ eV} = 3.01 \text{ eV}$$

Thus the wavelength of incident photons in second case will be

$$\lambda = \frac{12431}{3.01} \text{ \AA} = 4129.9 \text{ \AA}$$

When magnetic field is present there will be no effect on the stopping potential as magnetic force can not change the kinetic energy of ejected electrons.

**Ex.11 (a)** If the wavelength of the light incident on a photoelectric cell be reduced from  $\lambda_1$  to  $\lambda_2$  \AA, then what will be the change in the cut-off potential ?

**(b)** Light is incident on the cathode of a photocell and the stopping voltages are measured from light of two different wavelengths. From the data given below, determine the work function of the metal of the cathode in eV and the value of the universal constant  $hc/e$ .

Wavelength (\AA)	Stopping voltage (volt)
4000	1.3
4500	0.9

**Sol.** (a) Let the work function of the surface be  $\phi$ . If  $\nu$  be the frequency of the light falling on the surface, then according to Einstein's photoelectric equation, the maximum kinetic energy  $KE_{\max}$  of emitted electron is given by

$$KE_{\max} = h\nu - \phi = \frac{hc}{\lambda} - \phi$$

We know that,  $KE_{\max} = eV_0$   
Where  $V_0$  = cut-off potential.

$$eV_0 = \frac{hc}{\lambda} - \phi \quad \text{or} \quad V_0 = \frac{hc}{e\lambda} - \frac{\phi}{e}$$

Now,  $\Delta V_0 = V_{02} - V_{01}$

$$\begin{aligned} &= \left( \frac{hc}{e\lambda_2} - \frac{\phi}{e} \right) - \left( \frac{hc}{e\lambda_1} - \frac{\phi}{e} \right) \\ &= \frac{hc}{e} \left( \frac{1}{\lambda_2} - \frac{1}{\lambda_1} \right) = \frac{hc}{e} \left( \frac{\lambda_1 - \lambda_2}{\lambda_1\lambda_2} \right) \quad \dots(1) \end{aligned}$$

(b) From equation (1), we have

$$\frac{hc}{e} = \frac{\Delta V_0(\lambda_1\lambda_2)}{\lambda_1 - \lambda_2}$$

$$= \frac{(1.3 - 0.9)[(4000 \times 10^{-10}) \times (4500 \times 10^{-10})]}{500 \times 10^{-10}} = 1.44 \times 10^{-6} \text{ V/m}$$

$$\text{Now, } V_0 = \frac{hc}{e\lambda} - \frac{\phi}{e}$$

$$\text{or } \frac{\phi}{e} = \frac{hc}{e\lambda} - V_0 = \frac{1.44 \times 10^{-6}}{4000 \times 10^{-10}}$$

$$\text{or } \phi = 2.3 \text{ eV}$$

**Ex.12** A low intensity ultraviolet light of wavelength 2271 Å irradiates a photocell made of molybdenum metal. If the stopping potential is 1.3 V, find the work function of the metal. Will the photocell work if it is irradiated by a high intensity red light of wavelength 6328 Å?

**Sol.** The energy in eV of incident photons is

$$E = \frac{12431}{2271} \text{ eV} = 5.47 \text{ eV}$$

As stopping potential for ejected electrons is 1.3 V, the maximum kinetic energy of ejected electrons will be

$$KE_{\max} = eV_0 = 1.3 \text{ eV}$$

Now from photoelectric effect equation, we have

$$E = \phi + KE_{\max}$$

$$\text{or } \phi = E - KE_{\max}$$

$$\text{or } \phi = 5.47 - 1.3 \text{ eV} = 4.17 \text{ eV}$$

Energy in eV for photons for red light of wavelength 6328 Å is

$$E' = \frac{12431}{6328} \text{ eV} = 1.96 \text{ eV}$$

As  $E < \phi$ , photocell will not work if irradiated by this red light no matter however intense the light will be.

## 2. FORCE DUE TO RADIATION (PHOTON)

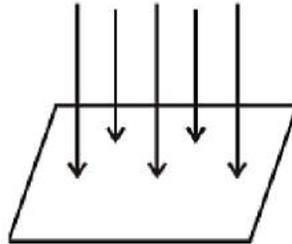
Each photon has a definite energy and a definite linear momentum. All photons of

light of a particular wavelength  $\lambda$  have the same energy  $E = \frac{hc}{\lambda}$  and the same

momentum  $p = \frac{h}{\lambda}$

When light of intensity  $I$  falls on a surface, it exerts force on that surface. Assume absorption and reflection coefficient of surface be ' $a$ ' and ' $r$ ' and assuming no transmission.

Assume light beam falls on surface of surface area ' $A$ ' perpendicularly as shown in figure.



For calculating the force exerted by beam on surface, we consider following cases.

**Case (I):**

$$a = 1, r = 0$$

$$\text{initial momentum of the photon} = \frac{h}{\lambda}$$

$$\text{final momentum of photon} = 0$$

$$\text{change in momentum of photon} = \frac{h}{\lambda} \text{ (upward)}$$

$$\Delta p = \frac{h}{\lambda}$$

$$\text{energy incident per unit time} = IA$$

$$\text{no. of photons incident per unit time} = \frac{IA}{h\nu} = \frac{IA\lambda}{hc} \text{ Therefore, total change in}$$

$$\text{momentum per unit time} = n \Delta p = \frac{IA\lambda}{hc} \times \frac{h}{\lambda} = \frac{IA}{c} \text{ (upward)}$$

$$\text{force on photons} = \text{total change in momentum per unit time} = \frac{IA}{c} \text{ (upward)}$$

$$\text{Therefore, force on plate due to photon (F)} = \frac{IA}{c} \text{ (downward)}$$

$$\text{pressure} = \frac{F}{A} = \frac{IA}{cA} = \frac{I}{c}$$

**Case: (II)**

when  $r = 1$ ,  $a = 0$

initial momentum of the photon =  $\frac{h}{\lambda}$  (downward)

final momentum of photon =  $\frac{h}{\lambda}$  (upward)

$$= \frac{h}{\lambda} + \frac{h}{\lambda} = \frac{2h}{\lambda}$$

change in momentum

Therefore, energy incident per unit time =  $I A$

no. of photons incident per unit time =  $\frac{IA\lambda}{hc}$

Therefore, total change in momentum per unit time =  $n \cdot DP = \frac{IA\lambda}{hc} \cdot \frac{2h}{\lambda} = \frac{2IA}{c}$   
force = total change in momentum per unit time

$F = \frac{2IA}{c}$  (upward on photons and downward on the plate)

$$P = \frac{F}{A} = \frac{2IA}{cA} = \frac{2I}{c}$$

pressure

**Case: (III)**

When  $0 < r < 1$      $a + r = 1$

change in momentum of photon when it is reflected =  $\frac{2h}{\lambda}$  (upward)

change in momentum of photon when it is absorbed =  $\frac{h}{\lambda}$  (upward)

no. of photons incident per unit time =  $\frac{IA\lambda}{hc}$

No. of photons reflected per unit time =  $\frac{IA\lambda}{hc} \cdot r$

No. of photon absorbed per unit time =  $\frac{IA\lambda}{hc} (1-r)$

force due to absorbed photon ( $F_a$ )  $\frac{IA\lambda}{hc} (1-r) \cdot \frac{h}{\lambda} = \frac{IA}{c} (1-r)$  (downward)

$$\frac{IA\lambda}{hc} \cdot r \frac{2h}{\lambda} = \frac{2IA\lambda}{c} \quad (\text{downward})$$

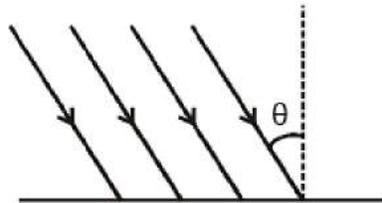
Force due to reflected photon ( $F_r$ )

$$\text{total force} = F_a + F_r$$

$$= \frac{IA}{c}(1-r) + \frac{2IAr}{c} = \frac{IA}{c}(1+r)$$

$$\text{Now pressure } P = \frac{IA}{c}(1+r) \times \frac{1}{A} = \frac{I}{c}(1+r)$$

**Ex.13** Calculate force exerted by light beam if light is incident on surface at an angle  $q$  as shown in figure. Consider all cases.



**Sol. Case - I**

When  $a = 1, r = 0$   
 initial momentum of photon (in downward direction at an angle  $q$  with vertical) is  $h/\lambda$   
 final momentum of photon = 0  
 change in momentum (in upward direction at an angle  $q$  with vertical)

$$= \frac{h}{\lambda} \left[ \begin{array}{c} \text{.....} \\ \theta \\ \downarrow \end{array} \right]$$

energy incident per unit time =  $I A \cos \theta$   
 Intensity = power per unit normal area

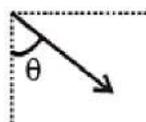
$$I = \frac{P}{A \cos \theta} \quad P = I A \cos \theta$$

No. of photons incident per unit time =  $\frac{IA \cos \theta}{hc} \cdot \lambda$   
 total change in momentum per unit time (in upward direction at an angle  $q$  with vertical)

$$= \frac{IA \cos \theta \lambda}{hc} \cdot \frac{h}{\lambda} = \frac{IA \cos \theta}{c} \left[ \begin{array}{c} \nearrow \theta \\ \text{---} \\ \searrow \theta \end{array} \right]$$

Force (F) = total change in momentum per unit time

$$F = \frac{IA \cos \theta}{c} \text{ (direction } \begin{array}{c} \nearrow \theta \\ \text{---} \\ \searrow \theta \end{array}$$

on photon and  on the plate)

Pressure = normal force per unit Area

$$\text{Pressure} = \frac{F \cos \theta}{A} \quad P = \frac{IA \cos^2 \theta}{cA} = \frac{I}{c} \cos^2 \theta$$

### Case II

When  $r = 1$ ,  $a = 0$

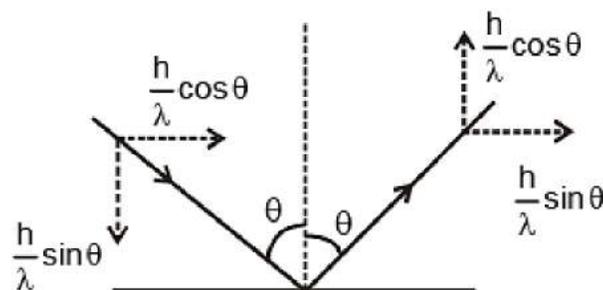
Therefore, change in momentum of one photon

$$= \frac{2h}{\lambda} \cos \theta \text{ (upward)}$$

No. of photons incident per unit time

$$= \frac{\text{energy incident per unit time}}{h\nu}$$

$$= \frac{IA \cos \theta \lambda}{hc}$$



Therefore,

$\therefore$  total change in momentum per unit time

$$= \frac{IA \cos \theta \lambda}{hc} \times \frac{2h}{\lambda} \cos \theta = \frac{2IA \cos^2 \theta}{c} \quad (\text{upward})$$

Therefore,

$$\therefore \text{force on the plate} = \frac{2IA \cos^2 \theta}{c} \quad (\text{downward})$$

$$\text{Pressure} = \frac{2IA \cos^2 \theta}{cA} \quad p = \frac{2I \cos^2 \theta}{c}$$

**Case III**

when  $0 < r < 1$ ,  $a + r = 1$

$$\text{change in momentum of photon when it is reflected} = \frac{2h}{\lambda} \cos \theta \quad (\text{downward})$$

$$\text{change in momentum of photon when it is absorbed} = \frac{h}{\lambda} \quad (\text{in the opposite direction of incident beam})$$

energy incident per unit time =  $I A \cos \theta$

$$\text{no. of photons incident per unit time} = \frac{IA \cos \theta \lambda}{hc}$$

$$\text{no. of reflected photon } (n_r) = \frac{IA \cos \theta \lambda r}{hc}$$

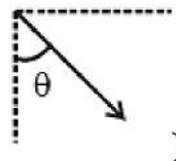
$$\text{no. of absorbed photon } (n_a) = \frac{IA \cos \theta \lambda (1 - r)}{hc}$$

force on plate due to absorbed photons  $F_a = n_a \Delta P_a$

$$= \frac{IA \cos \theta \lambda}{hc} (1 - r) \frac{h}{\lambda}$$

$$= \frac{IA \cos \theta}{hc} (1 - r)$$

(at an angle  $\theta$  with vertical



force on plate due to reflected photons  $F_r = n_r \Delta P_r$

$$= \frac{IA \cos \theta \lambda}{hc} \times \frac{2h}{\lambda} \cos \theta \quad (\text{vertically downward})$$

$$= \frac{IA \cos^2 \theta}{c} 2r$$

now resultant force is given by  $= \sqrt{F_r^2 + F_a^2 + 2F_a F_r \cos \theta}$   
 $= \frac{IA \cos \theta}{c} \sqrt{(1-r)^2 + (2r)^2 \cos^2 \theta + 4r(r-1) \cos^2 \theta}$

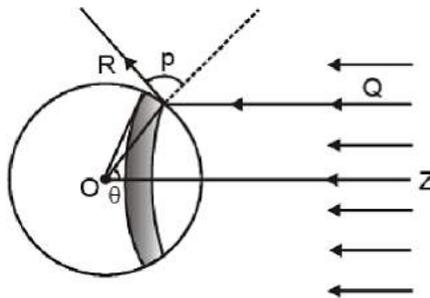
and, pressure  $P = \frac{F_a \cos \theta + F_r}{A}$

$$= \frac{IA \cos \theta (1-r) \cos \theta}{cA} + \frac{IA \cos^2 \theta \cdot 2r}{cA}$$

$$= \frac{I \cos^2 \theta}{c} (1-r) + \frac{I \cos^2 \theta}{c} 2r = \frac{I \cos^2 \theta}{c} (1+r)$$

**Ex.14** A perfectly reflecting solid sphere of radius  $r$  is kept in the path of a parallel beam of light of large aperture. If the beam carries an intensity  $I$ , find the force exerted by the beam on the sphere.

**Sol.** Let  $O$  be the centre of the sphere and  $OZ$  be the line opposite to the incident beam (figure). Consider a radius about  $OZ$  to get a making an angle  $q$  with  $OZ$ . Rotate this radius about  $OZ$  to get a circle on the sphere. Change  $q$  to  $q + dq$  and rotate the radius about  $OZ$  to get another circle on the sphere. The part of the sphere between these circles is a ring of area  $2\pi r^2 \sin q dq$ . Consider a small part  $DA$  of this ring at  $P$ . Energy of light falling on this part in time  $Dt$  is



$$\Delta U = I \delta t (\Delta A \cos \theta)$$

The momentum of this light falling on  $\Delta A$  is  $\Delta U/c$  along  $QP$ . The light is reflected by the sphere along  $PR$ . The change in momentum is

$$\Delta p = 2 \frac{\Delta u}{c} \cos \theta = \frac{2}{c} \Delta t (\Delta A \cos^2 \theta) \text{ (direction along } \vec{OP} \text{)}$$

The force on  $\Delta A$  due to the light falling on it, is

$$\frac{\Delta p}{\Delta t} = \frac{2}{c} \Delta A \cos^2 \theta \text{ (direction along } \vec{OP} \text{)}$$

The resultant force on the ring as well as on the sphere is along ZO by symmetry.  
The component of the force on DA along ZO

$$\frac{\Delta p}{\Delta t} \cos \theta = \frac{2}{c} I \Delta A \cos^2 \theta \text{ (along } \vec{ZO} \text{)}$$

The force acting on the ring is  $dF = \frac{2}{c} I (2\pi r^2 \sin \theta d\theta) \cos^3 \theta$

$$F = \int_0^{\pi/2} \frac{4\pi r^2 I}{c} \cos^3 \theta d\theta$$

The force on the entire sphere is

$$F = \int_0^{\pi/2} \frac{4\pi r^2 I}{c} \cos^3 \theta d(\cos \theta) = - \int_{\theta=0}^{\pi/2} \frac{4\pi r^2 I}{c} \left[ \frac{\cos^4 \theta}{4} \right]_0^{\pi/2} = \frac{\pi r^2 I}{c}$$

Note that integration is done only for the hemisphere that faces the incident beam.

### 3. De-broglie wavelength of matter wave

A photon of frequency  $\nu$  and wavelength  $\lambda$  has energy.

$$E = h\nu = \frac{hc}{\lambda}$$

By Einstein's energy mass relation,  $E = mc^2$  the equivalent mass  $m$  of the photon is given by.

$$m = \frac{E}{c^2} = \frac{h\nu}{c^2} = \frac{h}{\lambda c} \dots(i)$$

$$\text{or } \lambda = \frac{h}{mc} \quad \text{or } \lambda = \frac{h}{p} \dots(ii)$$

Here  $p$  is the momentum of photon. By analogy de-Broglie suggested that a particle of mass  $m$  moving with speed  $v$  behaves in some ways like waves of wavelength  $\lambda$  (called de-Broglie wavelength and the wave is called matter wave) given by,

$$\lambda = \frac{h}{mv} = \frac{h}{p} \dots(iii)$$

where  $p$  is the momentum of the particle. Momentum is related to the kinetic energy by the equation,

$$p = \sqrt{2Km}$$

and a charge  $q$  when accelerated by a potential difference  $V$  gains a kinetic energy  $K = qV$ . Combining all these relations Eq. (iii), can be written as,

$$\lambda = \frac{h}{mv} = \frac{h}{p} = \frac{h}{\sqrt{2Km}} = \frac{h}{\sqrt{2qVm}} \quad (\text{de=Broglie wavelength}) \quad (\text{iv})$$

### 3.1 de-Broglie wavelength for an electron

If an electron (charge = e) is accelerated by a potential of V volts, it acquires a kinetic energy,

$$K = eV$$

Substituting the value of h, m and q in Eq. (iv), we get a simple formula for calculating de-Broglie wavelength of an electron.

$$\lambda \text{ (in } \text{\AA}) = \sqrt{\frac{150}{V(\text{in volts})}}$$

### 3.2 de-Broglie wavelength of a gas molecule:

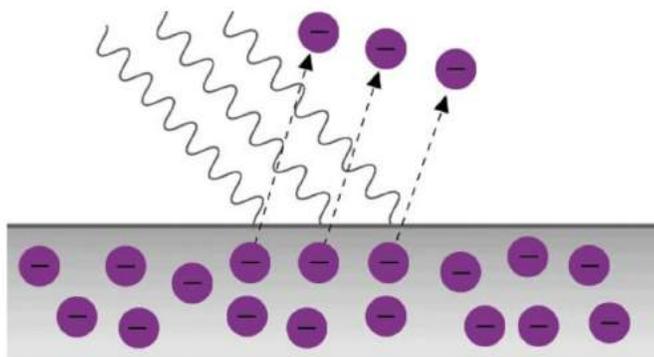
let us consider a gas molecule at absolute temperature T. Kinetic energy of gas molecule is given by

$$\text{K.E.} = \frac{3}{2} kT; \quad k = \text{Boltzman constant}$$

$$\lambda_{\text{gas molecules}} = \frac{h}{\sqrt{2mkT}}$$

## Einstein's Photoelectric Equation: Energy Quantum of Radiation

### Introduction



The photoelectric effect is a phenomenon where electrons are emitted from the metal surface when the light of sufficient frequency is incident upon. The concept of the photoelectric effect was first documented in 1887 by Heinrich Hertz and later by Lenard in 1902. But both the observations of the

photoelectric effect could not be explained by Maxwell's electromagnetic wave theory of light. Hertz (who had proved the wave theory) himself did not pursue the matter as he felt sure that it could be explained by the wave theory. However, the concept failed in the following accounts:

- According to the wave theory, energy is uniformly distributed across the wavefront and is dependent only on the intensity of the beam. This implies that the kinetic energy of electrons increases with light intensity. However, the kinetic energy was independent of light intensity.
- Wave theory says that light of any frequency should be capable of ejecting electrons. But electron emission occurred only for frequencies larger than a threshold frequency ( $\nu_0$ ).
- Since energy is dependent on intensity according to wave theory, the low-intensity light should emit electrons after some time so that the electrons can acquire sufficient energy to get emitted. However, electron emission was spontaneous no matter how small the intensity of light.

### **Einstein's Explanation of Photoelectric Effect**

Einstein resolved this problem using Planck's revolutionary idea that light was a particle. The energy carried by each particle of light (called quanta or photon) is dependent on the light's frequency ( $\nu$ ) as shown:

$$E = h\nu$$

Where  $h$  = Planck's constant =  $6.6261 \times 10^{-34}$  Js.

Since light is bundled up into photons, Einstein theorized that when a photon falls on the surface of a metal, the entire photon's energy is transferred to the electron.

A part of this energy is used to remove the electron from the metal atom's grasp and the rest is given to the ejected electron as kinetic energy. Electrons emitted from underneath the metal surface lose some kinetic energy during the collision. But the surface electrons carry all the kinetic energy imparted by the photon and have the maximum kinetic energy.

**We can write this mathematically as:**

Energy of photon = energy required to eject an electron (work function) +  
Maximum kinetic energy of the electron

$$E = W + KE$$

$$h\nu = W + KE$$

$$KE = h\nu - w$$

At the threshold frequency,  $\nu_0$  electrons are just ejected and do not have any

kinetic energy. Below this frequency, there is no electron emission. Thus, the energy of a photon with this frequency must be the work function of the metal.

$$w = hv_0$$

Thus, Maximum kinetic energy equation becomes:

$$KE = \frac{1}{2}mv^2_{\max} = hv - hv_0$$

$$\frac{1}{2}mv^2_{\max} = h(v - v_0)$$

$V_{\max}$  is the maximum kinetic energy of the electron. It is calculated experimentally using the stopping potential.

$$\text{Stopping potential} = eV_0 = \frac{1}{2}mv^2_{\max}$$

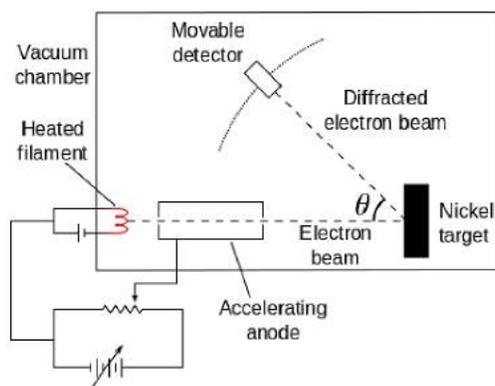
Thus, Einstein explained the Photoelectric effect by using the particle nature of light.

### Davisson Germer Experiment: Electron Diffraction

#### Introduction

Initial atomic models proposed by scientists could only explain the particle nature of electrons but failed to explain the properties related to their wave nature. C.J. Davisson and L.H. Germer in the year 1927 carried out an experiment, popularly known as Davisson Germer's experiment to explain the wave nature of electrons through electron diffraction. In this, we will learn about the observations and conclusions of the experiment.

#### Setup of Davisson Germer Experiment



#### Davisson Germer experiment

The experimental arrangement of the Davisson Germer experiment is discussed below:

- An electron gun comprising a tungsten filament F was coated with barium oxide and heated through a low voltage power supply.

- While applying suitable potential difference from a high voltage power supply, the electron gun emits electrons which were again accelerated to a particular velocity.
- In a cylinder perforated with fine holes along its axis, these emitted electrons were made to pass through it, thus producing a fine collimated beam.
- The beam produced from the cylinder is again made to fall on the surface of a nickel crystal. Due to this, the electrons scatter in various directions.
- The beam of electrons produced has a certain amount of intensity which is measured by the electron detector and after it is connected to a sensitive galvanometer (to record the current), it is then moved on a circular scale.
- By moving the detector on the circular scale at different positions that is changing the  $\theta$  (angle between the incident and the scattered electron beams), the intensity of the scattered electron beam is measured for different values of angle of scattering.

### Observations of Davisson Germer experiment

From this experiment, we can derive the below observations:

- We obtained the variation of the intensity ( $I$ ) of the scattered electrons by changing the angle of scattering,  $\theta$ .
- By changing the accelerating potential difference, the accelerated voltage was varied from 44V to 68 V.
- With the intensity ( $I$ ) of the scattered electron for an accelerating voltage of 54V at a scattering angle  $\theta = 50^\circ$ , we could see a strong peak in the intensity.
- This peak was the result of constructive interference of the electrons scattered from different layers of the regularly spaced atoms of the crystals.
- With the help of electron diffraction, the wavelength of matter waves was calculated to be 0.165 nm.

### Co-relating Davisson Germer experiment and de Broglie relation

According to de Broglie,

$$\lambda = h / p$$

$$\lambda = (1.227 / \sqrt{54}) = 0.167\text{nm}$$

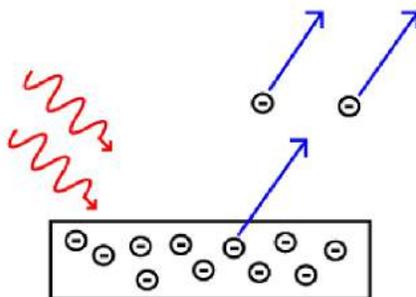
$\Lambda$  = wavelength associated with electrons

Thus, Davisson Germer experiment confirms the wave nature of electrons and the de Broglie relation.

## Particle Nature of Light, Wave Nature of Matter & de Broglie's Equation

### Particle Nature of Light

The emission of free electrons from a metal surface when the light is shone on it, it is called the photoemission or the photoelectric effect. This effect led to the conclusion that light is made up of packets or quantum of energy. Now the question was whether the light quantum theory was indicative of the particle nature of light. Einstein already associated the light quantum with momentum. This strongly supported the particle nature of light and these particles were named photons. Thus, the **wave-particle duality** of light came into the picture.



Photoelectric Emission

### Photons

Some points to be kept in mind are:

- A photon is an elementary particle. It is a quantum of light.
- Energy of a photon is given by  $E = h\nu$ . Its momentum is  $p = h\nu / c$  and speed is  $c$ , which is the speed of light.
- Irrespective of the intensity of radiation, every photon of a frequency  $\nu$  has the same momentum  $p = h / \nu c$  and energy  $E = h\nu$ .
- The increase in the intensity of light only increases the number of photons crossing an area per unit time. It does not affect the energy of the radiation.
- A photon remains unaffected by electric and **magnetic fields**. It is electrically neutral.
- A photon has a zero mass, i.e. it is massless.
- It is a stable particle.

- Photons can be created or destroyed when radiation is emitted or absorbed.
- The total energy and momentum are conserved during a photon-electron collision.
- A photon cannot decay on its own.
- The energy of a photon can be transferred during an interaction with other particles.
- A photon is a spin-1 particle, unlike electrons which are  $\frac{1}{2}$  spin. It's spin axis is parallel to the direction of travel. It is this property of photons which supports the polarization of light.

### Wave Nature of Matter

The wave nature of matter is one of the most counter-intuitive concepts in Physics. You have seen examples of both particle nature of light and wave nature of light. You know about the Photoelectric effect due to Albert Einstein's courtesy.

In the photoelectric effect, the electrons and photons exhibit the properties of a particle, just like a billiard ball. But you surely remember the Diffraction experiment and the Interference Rings. Just like how two ripples on the surface of a pond interact. We see the wave nature of light in these cases. It's an amazing mystery. It even involves our sight! The gathering and focusing mechanism of light by the eye-lens conform to the wave nature of light. But its absorption by the rods and cones of the retina conforms to the particle nature of light! While we were still struggling to understand this mystery, along came Louis de Broglie to make it even more complicated with his de Broglie Relation.

### De Broglie's Equation

De Broglie's hypothesis stated that there is symmetry in nature and that if light and radiation behave as both particles and waves, matter too will have both the particle and wave nature.

$$\lambda = h / p = h / mv$$

Through the de Broglie's relationship, we now had a wave theory of matter. The 'Lambda' here represents the wavelength of the particle and 'p' represents the momentum of the particle. The significance of the de Broglie relationship is that it proves mathematically that matter can behave as a wave. In layman terms, de Broglie equation says that every moving particle – microscopic or macroscopic –has its own wavelength.

For macroscopic objects, the wave nature of matter is observable.

For larger objects, the wavelength gets smaller with the increasing size of the object, quickly becoming so small as to become unnoticeable which is why macroscopic objects in real life don't show wave-like properties. Even the cricket ball you throw

has a wavelength that is too small for you to observe. The wavelength and the momentum in the equation are connected by the Planck's constant.

### **Heisenberg's Uncertainty**

The Davisson-Germer experiment proved beyond doubt the wave nature of matter by diffracting electrons through a crystal. In 1929, de Broglie was awarded the Nobel Prize for his matter wave theory and for opening up a whole new field of Quantum Physics. The matter-wave theory was gracefully incorporated by Heisenberg's Uncertainty Principle. The Uncertainty Principle states that for an electron or any other particle, both the momentum and position cannot be known accurately at the same time. There is always some uncertainty with either the position 'delta x' or with the momentum, 'delta p'.

### **Heisenberg's Uncertainty Equation:**

$$\sigma_x \sigma_p \leq h / 2$$

Say you measure the momentum of the particle accurately so that 'delta p' is zero. To satisfy the equation above, the uncertainty in the position of the particle, 'delta x' has to be infinite. From de Broglie's equation, we know that a particle with a definite momentum has a definite wavelength 'Lambda'. A definite wavelength extends all over space all the way to infinity. By Born's Probability Interpretation this means that the particle is not localized in space and therefore the uncertainty of position becomes infinite.

In real life though, the wavelengths have a finite boundary and are not infinite and thus both the position and momentum uncertainties have a finite value. De Broglie's equation and **Heisenberg's Uncertainty Principle** are apples of the same tree.