ICSE 2025 EXAMINATION

Sample Question Paper - 1

Mathematics

Time: 2 ½ hours **Total Marks: 80**

General Instructions:

1. *Answers to this Paper must be written on the paper provided separately.*
2. *You will not be allowed to write during the first 15 minutes. This time is to be spent in reading the question paper.*
3. *The time given at the head of this Paper is the time allowed for writing the answers.*
4. *Attempt **all** questions from **Section A** and **any four** questions from **Section B**.*
5. *The intended marks for questions or parts of questions are given in brackets [ ]*

Section A

(Attempt all questions from this section.)

Question 1

Choose the correct answers to the questions from the given options. [15]

i) What will be the rationalizing factor for the number $\frac{2\sqrt{2}}{2-\sqrt{2}}$?

(a) $2-\sqrt{2}$
(b) $\sqrt{2}+1$
(c) $1-\sqrt{2}$
(d) $\sqrt{2}-2$

ii) Raj invests Rs. 12000 for 5 years. If the rate of interest is compounded half-yearly, how many times will the interest be calculated?

(a) 1 time
(b) 5 times
(c) 10 times
(d) 15 times

iii) If $a^2 + b^2 + c^2 = 14$ and $ab + bc + ca = 5$, what is the value of $(a + b + c)^2$?

(a) 28
(b) 19
(c) 10
(d) 24

iv) The relation between (a – b) and ($a^2 - b^2$):

Statement 1: $a^2 - b^2 = (a - b)^2 + 4ab$

Statement 2: $a^2 - b^2 = (a + b)(a - b)$

Which of the following is valid?

(a) Both the statements are true.
(b) Both the statements are false.
(c) Statement 1 is true, and Statement 2 is false.
(d) Statement 1 is false, and Statement 2 is true.

v) Which of the following ordered pair (the values of x and y respectively) satisfies the two linear equations $x + y = 12$ and $8x - 19y = -12$?

(a) (4, 8)
(b) (8, 4)
(c) (7, 5)
(d) (5, 7)

vi) If $\left(\frac{1}{\sqrt{16}}\right)^{-2} = 2^m$, then the value of m is

(a) 4
(b) –4
(c) 8
(d) –8

vii) Which of the following congruency criteria is applicable when two angles and one side of one triangle are congruent to the corresponding angles and side of another triangle?

(a) SAS
(b) AAS
(c) Both (a) and (b)
(d) No such congruency criteria exist

viii) The measures of two sides of a triangle are 7 cm and 24 cm. What can be the length of the third side which makes it a right-angled triangle?

(a) 9 cm
(b) 17 cm
(c) 25 cm
(d) 31 cm

ix) If measure of an angle subtended by an arc on the circumference of a circle is x^o, then the measure of an angle subtended by the same arc at the centre of the circle is equal to

(a) ½ x^o
(b) x^o
(c) $2x^o$
(d) $3x^o$

x) Find the median of the data: 19, 17, 23, 10, 12, 6, 11, 14

(a) 11
(b) 12
(c) 13
(d) 14

xi) The marks obtained by 15 students in a test (out of hundred) are given below:
81, 72, 90, 90, 80, 55, 72, 66, 69, 80, 36, 54, 62, 56 and 58
The range of data is:

(a) 46
(b) 54
(c) 90
(d) 100

xii) A cubical box is 15 cm long. What could be its lateral surface area?

(a) 225 cm^2
(b) 900 cm^2
(c) 1800 cm^2
(d) 1350 cm^2

xiii) If tan $(90^o - x) = 0$, then the measure of x is

(a) 0^o
(b) 30^o
(c) 60^o
(d) 90^o

xiv) Find the co-ordinates of a point whose ordinate is $\frac{3}{2}$ and lies on the y-axis.

(a) (3/2, 0)
(b) (0, 3/2)
(c) (3, 2)
(d) (2, 3)

xv) **Assertion (A):** The perimeter of a triangle with vertices (−4, 0), (0, 3) and (0, 0) is 12 units.

Reason (R): The perimeter of a triangle is the sum of lengths of three sides of a triangle.

(a) A is true, R is false

(b) A is false, R is true

(c) Both A and R are true, and R is the correct reason for A.

(d) Both A and R are true, and R is the incorrect reason for A.

Question 2

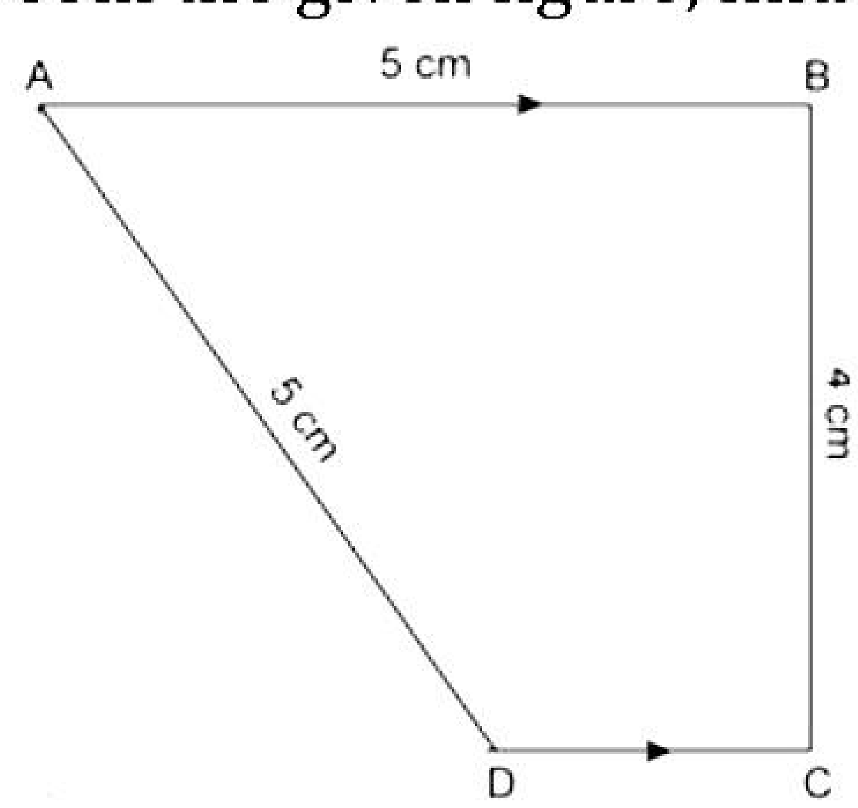
i) Ram borrows Rs. 62500 from Arjun for 2 years at 10% per annum simple interest. He immediately lends out this sum to Kunal at 10% per annum for the same period compounded annually. Calculate Ram's profit in the transaction at the end of two years (Without using formula). [4]

ii) Solve: $3x + 2y = 2xy$, $6x + 2y = 3xy$, where $x \neq 0$ and $y \neq 0$ [4]

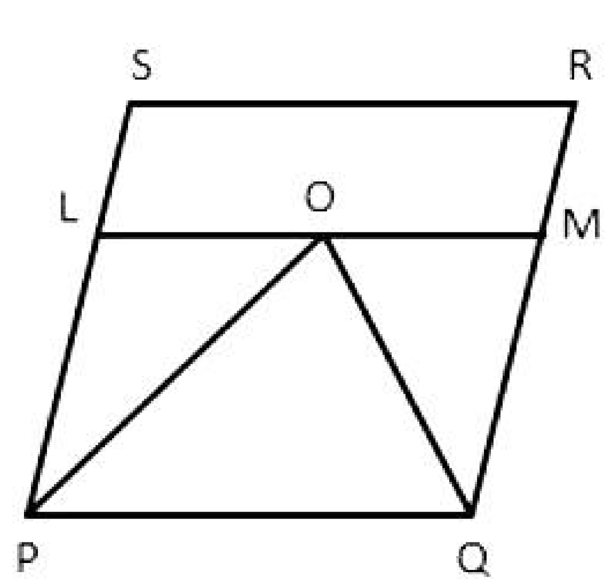
iii) In triangle ABC, D is a point on AB, such that AD = ¼ AB, and E is a point on AC such that AE = ¼ AC. Prove that DE = ¼ BC. [4]

Question 3

i) From the given figure, find the area of trapezium ABCD. [4]



ii) In the figure, PQRS is a parallelogram. OP and OQ bisects ∠P and ∠Q respectively. LOM is a straight line drawn parallel to PQ. [4]



Prove that PL = QM and LO = OM.

iii) Factorise: [5]

A. $2x^7 - 128x$

B. $x^2 + \frac{1}{4}x - \frac{1}{8}$

Section B

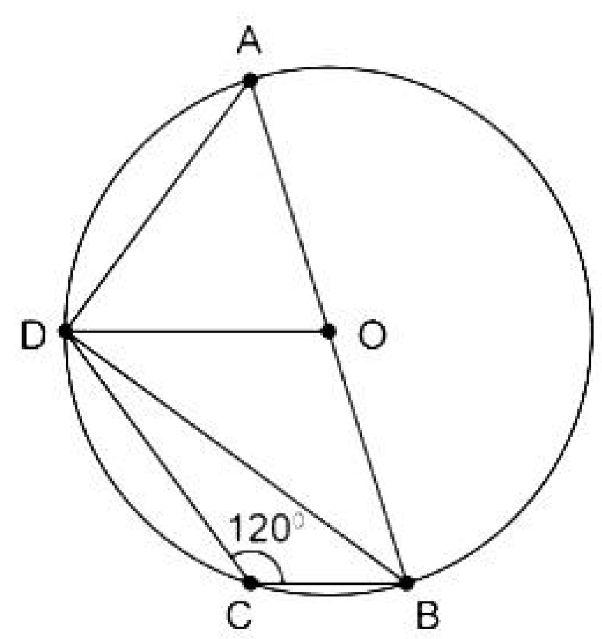
(Attempt any four questions from this Section.)

Question 4

i) Rationalise the denominator of $\frac{1}{3+\sqrt{2}}$ and $\frac{1}{3\sqrt{7}}$. [3]

ii) The population of a town is 64,000. Calculate the population after 3 years if the annual birth rate is 11.7% and the annual death rate is 4.2%. [3]

iii) In the given figure, AB is a diameter of a circle with centre O and DO || CB. [4]



If $\angle BCD = 120°$, calculate:

A. $\angle BAD$

B. $\angle ABD$

C. $\angle CBD$

D. $\angle ADC$

Also, show that ΔAOD is an equilateral triangle.

Question 5

i) If $(3a + 4b) = 16$ and $ab = 4$, find the value of $(9a^2 + 16b^2)$. [3]

ii) Show that $97^3 + 14^3$ is divisible by 111. [3]

iii) Find the median of [4]

A. 15, 6, 16, 8, 22, 21, 9, 18, 25

B. 10, 75, 3, 15, 9, 47, 12, 48, 4, 81, 17, 27

Question 6

i) Solve the following system of linear equations using elimination by substitution:

$5x - 9 = \frac{1}{y}$; $x + \frac{1}{y} = 3$ [3]

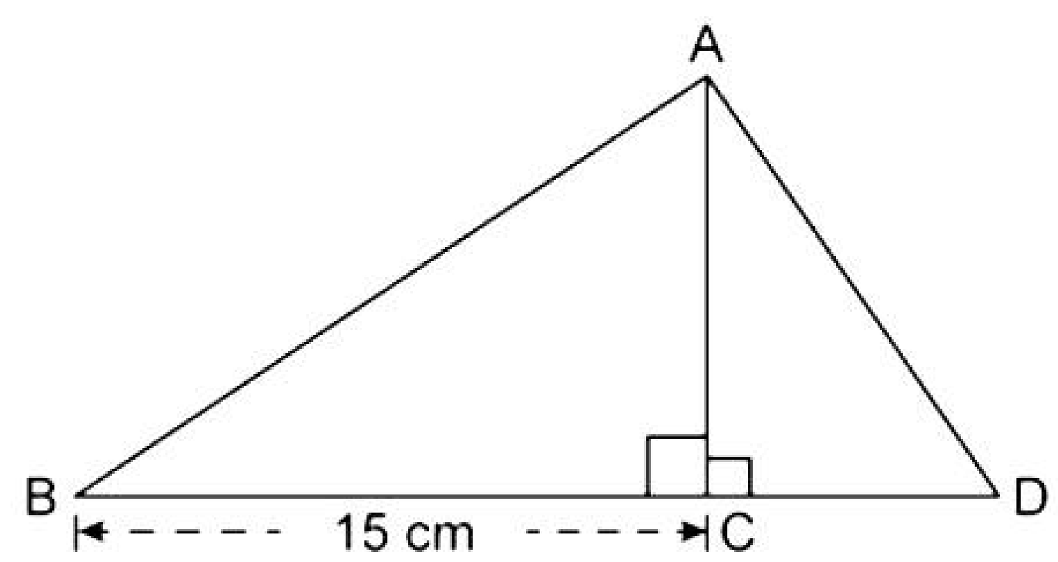
ii) Simplify: $\frac{a+b+c}{\left(a^{-1}b^{-1} + b^{-1}c^{-1} + c^{-1}a^{-1}\right)}$ [3]

iii) Construct a combined histogram and frequency polygon for the following distribution: [4]

Class interval	20–25	25–30	30–35	35–40	40–45	45–50
Frequency	30	24	52	28	46	10

Question 7

i) In the given figure, BC = 15 cm and $\sin B = \frac{4}{5}$. [5]



A. Calculate the lengths of AB and AC.

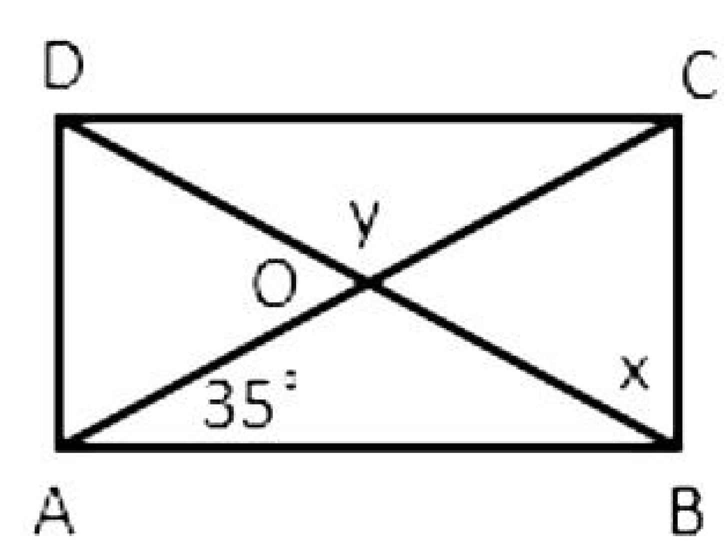
B. Now, if tan ÐADC = 1, calculate the lengths of CD and AD.

C. Also, show that $\tan^2 B - \frac{1}{\cos^2 B} = -1$.

ii) Find the area of a trapezium whose parallel sides are 11 m and 25 m long, and the non-parallel sides are 15 m and 13 m long. [5]

Question 8

i) In the figure, ABCD is a rectangle. Find the values of x and y. [3]

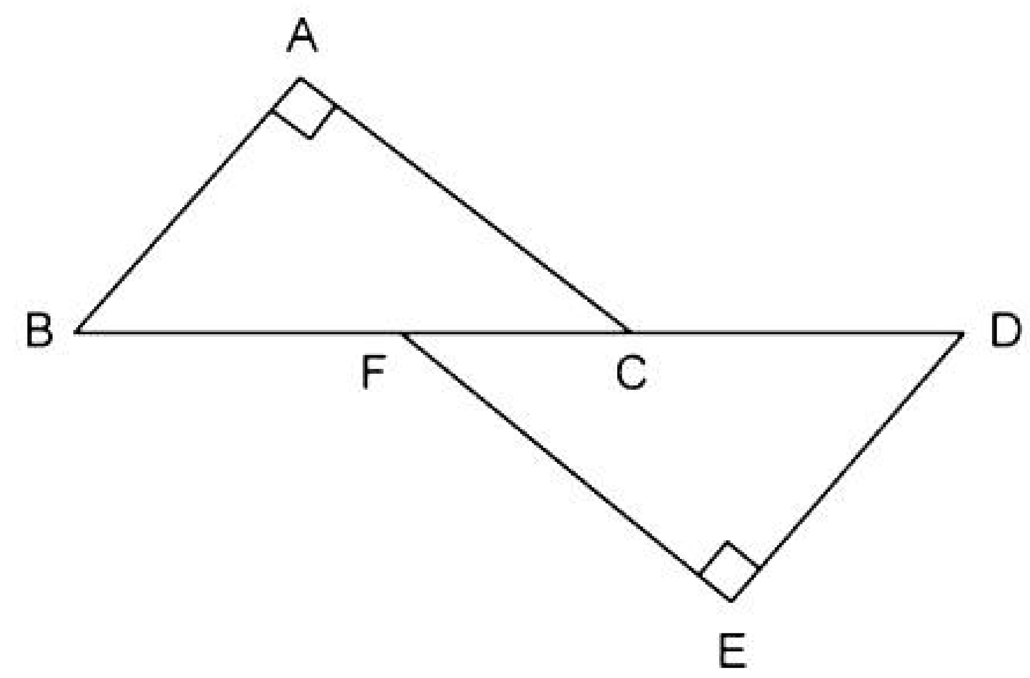


ii) AB and AC are two chords of a circle of radius r such that AB = 2AC. If p and q are the distances of AB and AC from the centre, then prove that $4q^2 = p^2 + 3r^2$. [3]

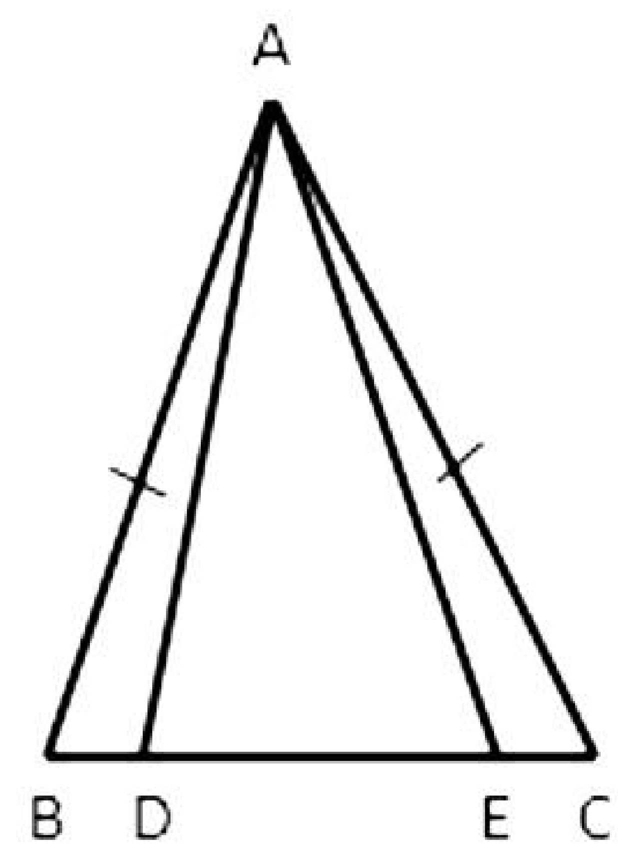
iii) A godown measures 40 m × 25 m × 10 m. Find the maximum number of wooden crates each measuring 1.5 m × 1.25 m × 0.5 m which can be stored in the godown. [4]

Question 9

i) In the given figure, BA ^ AC and DE ^ EF such that BA = DE and BF = DC. Prove that AC = EF. [3]



ii) In the given figure, AB = AC, D and E are points on BC such that BE = DC. Prove that AD = AE. [3]



iii) The perimeter of an isosceles triangle is 42 cm and its base is $1\frac{1}{2}$ times with each of the equal sides. Find [4]

A. the length of the equal sides of the triangle
B. the area of the triangle and
C. the height of the triangle.

Question 10

i) Find the capacity of a closed rectangular cistern whose length is 8 m, breadth 6 m and depth 2.5 m. Also, find the area of the iron sheet required to make the cistern. [3]

ii) Find the slope and the y-intercepts of each of the following lines: [3]

A. $5x - 3y - 6 = 0$
B. $4x + 3y - 7 = 0$
C. $5y - 4 = 0$

iii) Solve the following simultaneous equations using the graphical method: [4]
$x + y = 8; x - y = 2$

Solution

Section A

Solution 1

i) Correct option: (b)

Explanation:

$$\frac{2\sqrt{2}}{2-\sqrt{2}} = \frac{2\sqrt{2}}{\sqrt{2}\left(\sqrt{2}-1\right)} = \frac{2}{\sqrt{2}-1}$$

So, the rationalising factor is $\sqrt{2}+1$.

ii) Correct option: (c)

Explanation:

The rate of interest is compounded half-yearly.

Total time of investment = 5 years = 10 half-years.

So, the interest will be calculated 10 times.

iii) Correct option: (d)

Explanation:

Given: $a^2 + b^2 + c^2 = 14$ and $ab + bc + ca = 5$

Now, $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

$= 14 + 2 \times 5$

$= 24$

iv) Correct option: (d)

Explanation:

Statement 1:

$(a - b)^2 + 4ab = a^2 + b^2 - 2ab + 4ab$

$= a^2 + b^2 + 2ab$

$= (a + b)^2$

$\neq a^2 - b^2$

Statement 2:

$(a + b)(a - b) = a(a - b) + b(a - b)$

$= a^2 - ab + ab - b^2$

$= a^2 - b^2$

v) Correct option: (b)

Explanation:

When $x = 8$ and $y = 4$

$x + y = 8 + 4 = 12$

$8x - 19y = (8 \times 8) - (19 \times 4) = 64 - 76 = -12$

Hence, the ordered pair (8, 4) satisfies the two linear equations.

vi) Correct option: (a)

Explanation:

$$\left(\frac{1}{\sqrt{16}}\right)^{-2} = 2^m$$

$$\Rightarrow \left(\frac{1}{4}\right)^{-2} = 2^m$$

$$\Rightarrow \frac{1}{4^{-2}} = 2^m$$

$\Rightarrow 4^2 = 2^m$

$\Rightarrow (2^2)^2 = 2^m$

$\Rightarrow 2^4 = 2^m$

Therefore, $m = 4$.

vii) Correct option: (b)

Explanation:

In two triangles, when two angles and one side of one triangle are congruent to the corresponding angles and side of another triangle, AAS criteria can be applied.

viii) Correct option: (c)

Explanation:

Here, $7^2 + (24)^2 = 49 + 576 = 625 = (25)^2$

So, the length of the third side will be 25 cm.

ix) Correct option: (c)

Explanation:

We know that the measure of an angle subtended by an arc at the centre of the circle is twice the measure of an angle subtended by the same arc on the circumference of the circle.

$\Rightarrow$ Measure of an angle subtended at the centre of a circle = $2x^o$

x) Correct option: (c)

Explanation:

Arranging in ascending order: 6, 10, 11, 12, 14, 17, 19, 23

There are two middle terms i.e. 12 and 14.

So, the median = $(12 + 14)/2 = 13$

xi) Correct option: (b)

Explanation:

Range = Maximum marks – Minimum marks

= 90 – 36

= 54

xii) Correct option: (b)

Explanation:

For the given cubical box, l = 15 cm

Lateral surface area of the cubical box = $4l^2$

$= 4(15)^2$

$= 4 \times 225$

$= 900 \text{ cm}^2$

xiii) Correct option: (d)

Explanation:

We know that, tan $(90^\circ - x) = \cot x$

$\Rightarrow \cot x = 0$

$\Rightarrow \cot x = \cot 90^\circ$

$\Rightarrow x = 90^\circ$

xiv) Correct option: (b)

Explanation:

Since, the point lies on the y-axis.

So, it's x-coordinate will be 0.

Also, ordinate of a point is its y-coordinate.

$\Rightarrow$ Co-ordinates of a point are $(0, \frac{3}{2})$.

xv) Correct option: (c)

Explanation:

Side 1 $= \sqrt{(4)^2 + (3)^2} = 5$ units

Side 2 $= \sqrt{(4)^2 + (0)^2} = 4$ units

Side 3 $= \sqrt{(0)^2 + (-3)^2} = 3$ units

$\Rightarrow$ Perimeter $= 5 + 4 + 3 = 12$ units

Hence, the assertion is true.

The statement given in reason is correct.

Hence, the reason is true and is the correct reason for the assertion.

Solution 2

i) For Simple Interest:

Here, P = Rs. 62,500, R = 10% and N = 2 years

$\therefore$ Simple Interest paid by Ram $= \dfrac{P \times R \times N}{100} = \dfrac{62500 \times 10 \times 2}{100}$ = Rs. 12500

For Compound Interest:

For the 1st year:

P = Rs. 62,500, R = 10% and N = 1

$I = \dfrac{P \times R \times N}{100} = \dfrac{62500 \times 10 \times 1}{100}$ = Rs. 6250

Amount = P + I = Rs. (62500 + 6250) =Rs. 68750

For the 2nd year:

P = Rs. 68,750, R = 10% and N = 1

$I = \dfrac{P \times R \times N}{100} = \dfrac{68750 \times 10 \times 1}{100}$ = Rs. 6875

Amount = P + I = 68750 + 6875 = Rs. 75625

Total Compound Interest = Rs. (75625 – 62500) = Rs. 13,125

Therefore, Ram's profit in the transaction at the end of two years

= Rs. (13125 – 12500)

= Rs. 625

ii) $3x + 2y = 2xy$ ....(i)

$6x + 2y = 3xy$ ....(ii)

Dividing equation (i) and (ii) by xy, we get

$\dfrac{3}{y} + \dfrac{2}{x} = 2$ .....(iii)

$\dfrac{6}{y} + \dfrac{2}{x} = 3$ .....(iv)

Putting $\dfrac{1}{x} = b$ and $\dfrac{1}{y} = a$,

$3a + 2b = 2$ ....(v)

$6a + 2b = 3$ ....(vi)

Subtracting (v) from (vi), we get

$3a = 1$

$\Rightarrow a = \dfrac{1}{3}$

Putting $a = \dfrac{1}{3}$ in (v), we get

$3\left(\dfrac{1}{3}\right) + 2b = 2$

$\Rightarrow 1 + 2b = 2$

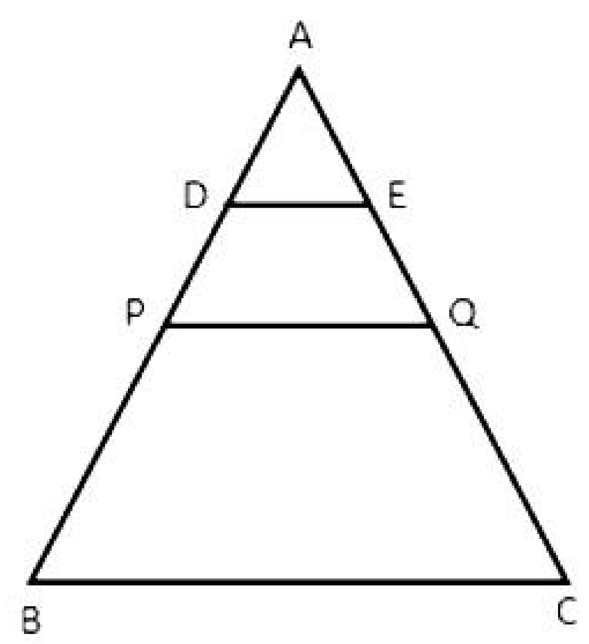
$\Rightarrow 2b = 1$

$\Rightarrow b = \frac{1}{2}$

$\therefore \frac{1}{x} = \frac{1}{2} \Rightarrow x = 2$ and $\frac{1}{y} = \frac{1}{3} \Rightarrow y = 3$

Hence, the solution is x = 2 and y = 3.

iii)



Given: In triangle ABC, AD = ¼ AB and AE = ¼ AC

To prove: DE = ¼ BC

Proof:

Let PQ be the line segment joining the mid-points of sides AB and AC.

$\Rightarrow$ AB = 2 AP

$\Rightarrow$ AC = 2 AQ

$\Rightarrow$ PQ = ½ BC ... (By mid-point theorem)

Now, AD = ¼ AB = ½ AP and AE = ¼ AC = ½ AQ

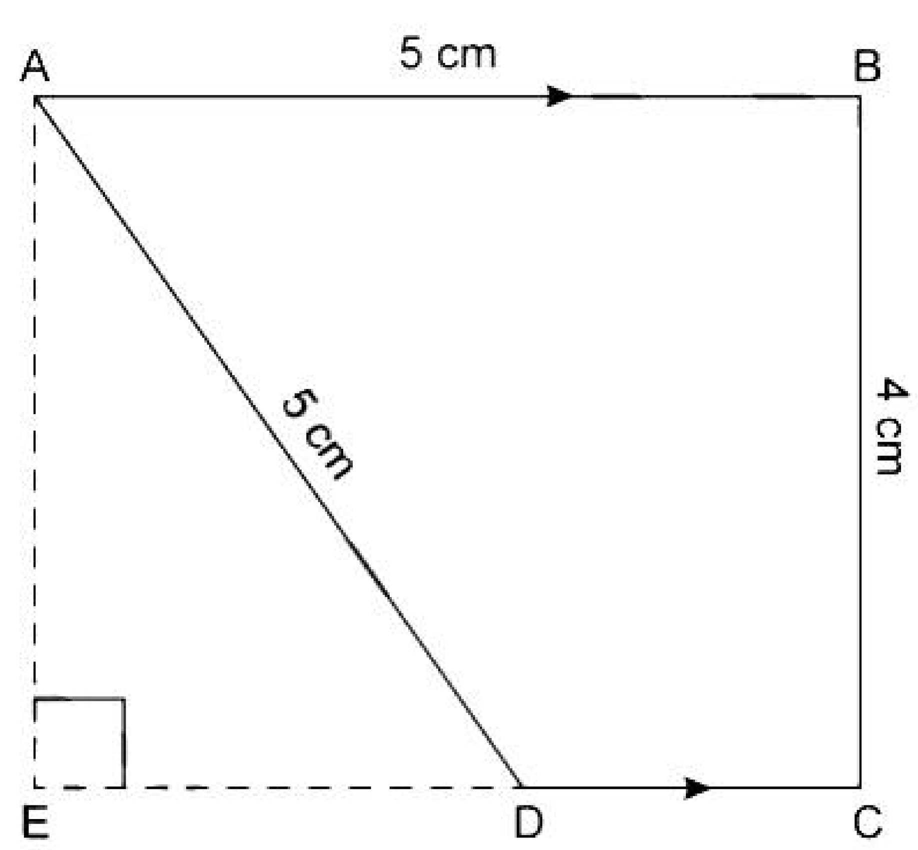
$\Rightarrow$ D and E are the mid-points of AP and AQ respectively.

$\Rightarrow$ DE = ½ PQ ... (By mid-point theorem)

$\Rightarrow$ DE = ½ (½ BC) = ¼ BC

Solution 3

i)



Construction: Extend CD and draw AE $\perp$ to extended CD such that C – D – E.

From the figure,

AE $\perp$ DE and $\angle B = \angle C = \angle A = 90°$

Also, AB = CE = 5 cm and BC = AE = 4 cm ... (Since □ABCE is a rectangle)

In ΔADE, $\angle E = 90°$.

Then, by Pythagoras' theorem,

$AD^2 = AE^2 + DE^2$

$\Rightarrow DE^2 = AD^2 - AE^2 = 5^2 - 4^2 = 25 - 16 = 9$

$\Rightarrow DE = 3$ cm

$\Rightarrow CD = CE - DE = 5 - 3 = 2$ cm

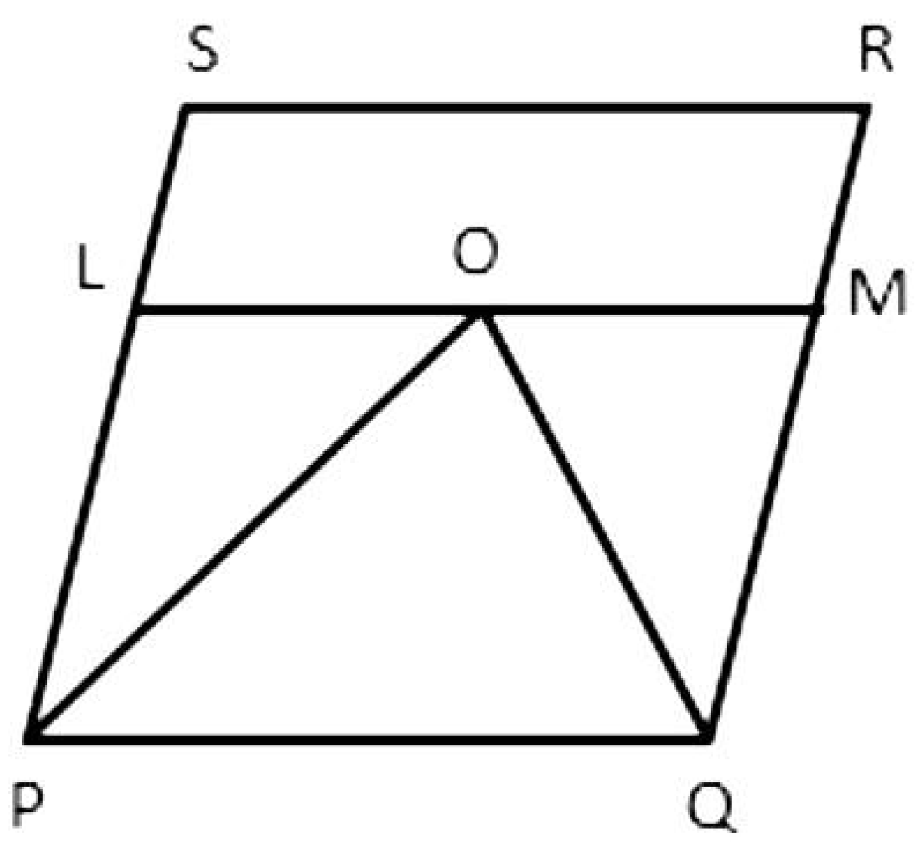
Area of trapezium ABCD = ½ × (sum of parallel sides) × height

$= \frac{1}{2} \times (5 + 2) \times 4$

$= 14 \text{ cm}^2$

Therefore, the area of a trapezium ABCD is 14 cm^2.

ii)



Given: PQRS is a parallelogram. OP bisects ∠P, OQ bisects ∠Q. LM || PQ

To prove: PL = QM and LO = OM

Proof:

We have, LM || PQ

And, LP || MQ ... (∵ PS || QR)

⇒ PQML is a parallelogram

⇒ PL = QM ....(Opposite sides of $||^{gm}$ are equal)

∠OPL = ∠OPQ ... (i) (OP bisects ∠P)

∠OPQ = ∠POL ... (ii) (Alternate angles)

⇒ ∠OPL = ∠POL [From (i) and (ii)]

⇒ PL = LO ...(iii)

∠OQM = ∠OQP ... (iv) (OQ bisects ∠Q)

∠OQP = ∠QOM ... (v) (Alternate angles)

⇒ ∠OQM = ∠QOM [From (v) and (iv)]

⇒ QM = MO

∴ PL = OM (∴ LO = OM)

iii)

A.

$2x^7 - 128x$

$= 2x(x^6 - 64)$

$= 2x\,(x^6 - 2^6)$

$= 2x\,[(x^3)^2 - (2^3)^2]$

$= 2x\,[(x^3 - 2^3)\,(x^3 + 2^3)]$ ... Since $(a^2 - b^2) = (a - b)(a + b)$

Since $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$ and $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

$\therefore 2x^7 - 128x = 2x\,[(x - 2)(x^2 + 2x + 4)(x + 2)(x^2 - 2x + 4)]$

$= 2x(x - 2)(x + 2)(x^2 + 2x + 4)(x^2 - 2x + 4)$

B.

$x^2 + \frac{1}{4}x - \frac{1}{8}$

Since $\frac{1}{2} - \frac{1}{4} = \frac{1}{4}$ and $\frac{1}{2}\left(-\frac{1}{4}\right) = -\frac{1}{8}$

$$\therefore x^2 + \frac{1}{4}x - \frac{1}{8} = x^2 + \frac{1}{2}x - \frac{1}{4}x - \frac{1}{8}$$

$$= x\left(x + \frac{1}{2}\right) - \frac{1}{4}\left(x + \frac{1}{2}\right)$$

$$= \left(x + \frac{1}{2}\right)\left(x - \frac{1}{4}\right)$$

Section B

Solution 4

i)

$$\frac{1}{3+\sqrt{2}} = \frac{1}{3+\sqrt{2}} \times \frac{3-\sqrt{2}}{3-\sqrt{2}} \quad \text{...Rationalising the denomintor}$$

$$= \frac{3-\sqrt{2}}{9-2}$$

$$= \frac{3-\sqrt{2}}{7}$$

$$\frac{1}{3\sqrt{7}} = \frac{1}{3\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} \quad \text{...Rationalising the denomintor}$$

$$= \frac{\sqrt{7}}{21}$$

ii) Birth rate = 11.7%

Death rate = 4.2%

Net growth rate = 11.7% – 4.2% = 7.5%

Here, R = 7.5%, n = 3 years, P = 64000

$$\text{Compound interest } = P\left(1+\frac{R}{100}\right)^n$$

$$= 64000\left(1+\frac{7.5}{100}\right)^3$$

$$= 64000\left(1+\frac{3}{40}\right)^3$$

$$= 64000 \times \frac{43}{40} \times \frac{43}{40} \times \frac{43}{40}$$

$$= 79507$$

Therefore, the population of a town after 3 years will be 79507.

iii)

AB is a diameter of a circle with centre O and DO||CB, $\angle BCD = 120°$

A. Since ABCD is a cyclic quadrilateral

$\therefore \angle BCD + \angle BAD = 180°$

$\Rightarrow 120° + \angle BAD = 180°$

$\Rightarrow \angle BAD = 180° - 120° = 60°$

B. $\angle BDA = 90^\circ$ [angle in a semi circle]

In ΔABD,

$\angle BDA + \angle BAD + \angle ABD = 180^\circ$

$\Rightarrow 90^\circ + 60^\circ + \angle ABD = 180^\circ$

$\Rightarrow \angle ABD = 180^\circ - 150^\circ = 30^\circ$

C. $OD = OA$

$\Rightarrow \angle ODA = \angle OAD = \angle BAD = 60^\circ$

Now, $\angle ODB = 90^\circ - \angle ODA = 90^\circ - 60^\circ = 30^\circ$

Also, $\angle CBD = \angle ODB = 30^\circ$ (alternate angles)

D. $\angle ADC = \angle ADB + \angle CDB = 90^\circ + 30^\circ = 120^\circ$

In ΔAOD,

$\angle ODA + \angle OAD + \angle AOD = 180^\circ$

$\Rightarrow 60^\circ + 60^\circ + \angle AOD = 180^\circ$

$\Rightarrow \angle AOD = 180^\circ - 120^\circ = 60^\circ$

Since all the angles of ΔAOD are of 60° each, AOD is an equilateral triangle.

Solution 5

i) We know that $(x + y)^2 = x^2 + y^2 + 2xy$

Here, $x = 3a$ and $y = 4b$

$$\therefore (3a + 4b)^2 = (3a)^2 + (4b)^2 + 2 \times 3a \times 4b$$

$$= 9a^2 + 16b^2 + 24ab$$

$$\Rightarrow 9a^2 + 16b^2 + 24ab = (3a + 4b)^2$$

$$\Rightarrow 9a^2 + 16b^2 = (3a + 4b)^2 - 24ab$$

$$= (16)^2 - 24 \times 4$$

$$= 256 - 96$$

$$= 160$$

ii) $97^3 + 14^3$

$= (97 + 14)(97^2 - 97 \times 14 + 14^2)$ ... [$\because a^3 + b^3 = (a + b)(a^2 - ab + b^2)$]

$= 111 \times (9409 - 1358 + 196)$

$= 111 \times 8247$, which is divisible by 111.

$\Rightarrow 97^3 + 14^3$ is divisible by 111.

iii)

A. 15, 6, 16, 8, 22, 21, 9, 18, 25

Ascending order: 6, 8, 9, 15, 16, 18, 21, 22, 25

Here, n = 9 (odd)

$$\therefore \text{Median} = \left(\frac{n+1}{2}\right)^{th} \text{ observation}$$

$$= \left(\frac{9+1}{2}\right)^{th} \text{ observation}$$

$$= 5^{th} \text{ observation}$$

$$= 16$$

B. 10, 75, 3, 15, 9, 47, 12, 48, 4, 81, 17, 27

Ascending order: 3, 4, 9, 10, 12, 15, 17, 27, 47, 48, 75, 81

Here, n = 12 (even)

$$\therefore \text{Median} = \frac{\left(\frac{n}{2}\right)^{th} \text{ observation} + \left(\frac{n}{2}+1\right)^{th} \text{ observation}}{2}$$

$$= \frac{\left(\frac{12}{2}\right)^{th} \text{ observation} + \left(\frac{12}{2}+1\right)^{th} \text{ observation}}{2}$$

$$= \frac{6^{th} \text{ observation} + 7^{th} \text{ observation}}{2}$$

$$= \frac{15+17}{2}$$

$$= \frac{32}{2}$$

$$= 16$$

Solution 6

i)

$$5x - 9 = \frac{1}{y} \quad ....(i)$$

$$x + \frac{1}{y} = 3$$

$$\Rightarrow x = 3 - \frac{1}{y} \quad ...(ii)$$

Putting the value of x from (ii) in (i), we get

$$5x - 9 = \frac{1}{y}$$

$$\Rightarrow 5\left(3 - \frac{1}{y}\right) - 9 = \frac{1}{y}$$

$$\Rightarrow 15 - \frac{5}{y} - 9 = \frac{1}{y}$$

$$\Rightarrow 15 - 9 = \frac{1}{y} + \frac{5}{y}$$

$$\Rightarrow 6 = \frac{6}{y}$$

$$\Rightarrow y = 1$$

Putting y = 1 in equation (i), we get

Since, $5x - 9 = \frac{1}{y}$

$\Rightarrow$ 5x – 9 = 1

$\Rightarrow$ 5x = 10

$\Rightarrow$ x = 2

Hence, the solution is x = 2 and y = 1.

ii)

$$\frac{a + b + c}{\left(a^{-1}b^{-1} + b^{-1}c^{-1} + c^{-1}a^{-1}\right)}$$

$$= \frac{a + b + c}{\left(\frac{1}{a} \times \frac{1}{b} + \frac{1}{b} \times \frac{1}{c} + \frac{1}{c} \times \frac{1}{a}\right)}$$

$$= \frac{a + b + c}{\left(\frac{1}{ab} + \frac{1}{bc} + \frac{1}{ac}\right)}$$

$$= \frac{a + b + c}{\left(\frac{c}{abc} + \frac{a}{abc} + \frac{b}{abc}\right)}$$

$$= \frac{abc(a + b + c)}{(a + b + c)}$$

$$= abc$$

iii) STEPS:

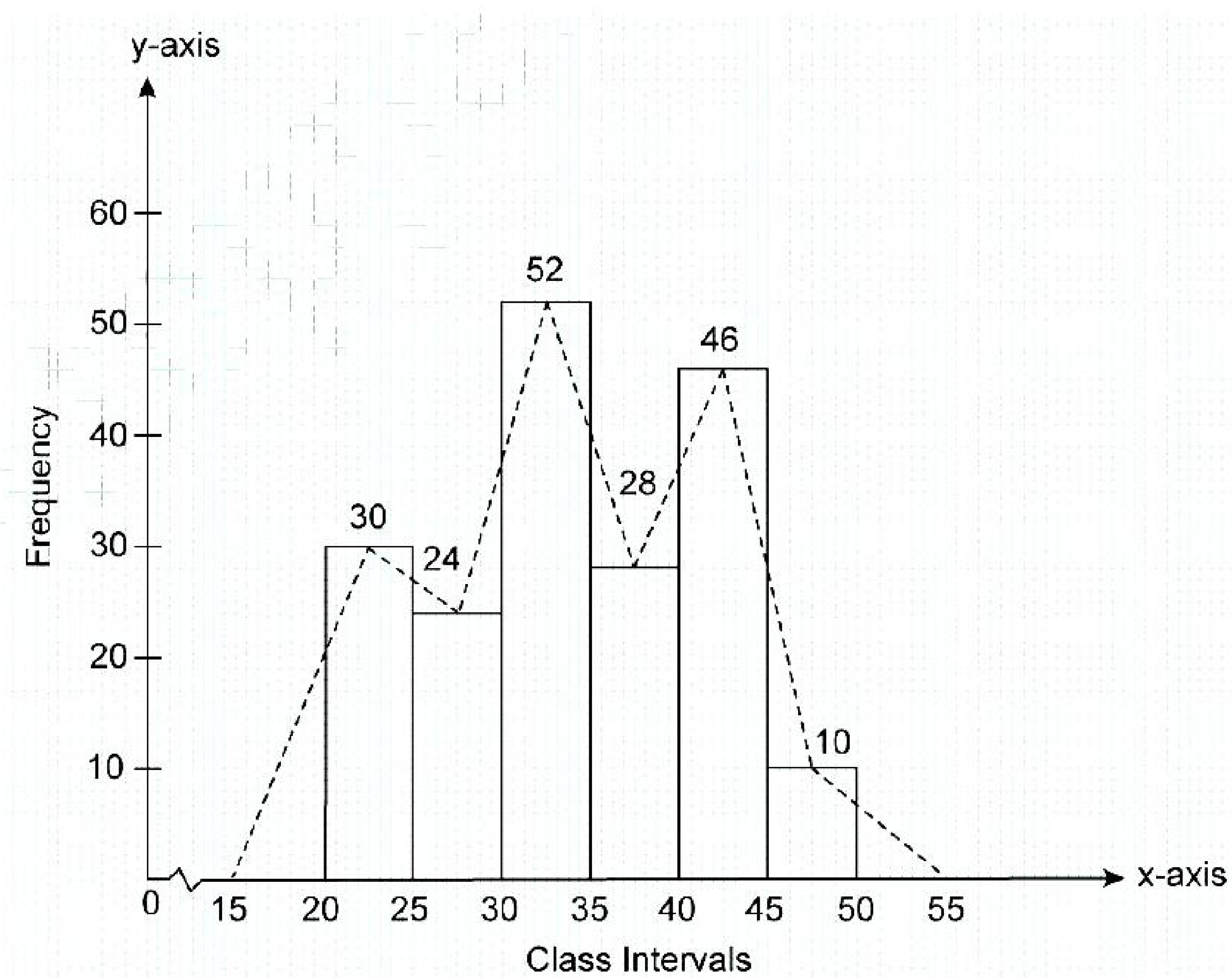
Histogram:

1. The given data is in the exclusive form.
2. Taking suitable scales, mark the class intervals on the x-axis and the frequencies on the y-axis.
3. Construct rectangles with class intervals as bases and the corresponding frequencies as heights. Thus, we get the required histogram.

Draw a Frequency Polygon:

1. Now take imaginary class intervals 15–20 at the beginning and 50–55 at the end, each with frequency zero.
2. Since the scale on the x-axis starts at 15, a kink (break) or a zigzag curve is shown near the origin to indicate that the graph is drawn to scale beginning at 15 and not at the origin itself.
3. Mark the mid-point at the top of each rectangle of the histogram drawn.
4. Draw line segments joining the consecutive points marked in step 3.

Note: Join the class mark of the class interval just before the first class and the class mark of the class interval just after the last class. This completes the required frequency polygon.



Solution 7

i) Given: $\sin B = \frac{4}{5}$

i.e. $\frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{AC}{AB} = \frac{4}{5}$

Therefore, if the length of the perpendicular = 4x, the length of the hypotenuse = 5x.

Since,

$BC^2 + AC^2 = AB^2$ [Using Pythagoras Theorem]

$AB^2 - AC^2 = BC^2$

$(5x)^2 - (4x)^2 = BC^2$

$BC^2 = 9x^2$

$\therefore BC = 3x$

Now, BC = 15

$\Rightarrow 3x = 15$

$\Rightarrow x = 5$

A. AB = 5x = 5 × 5 = 25 cm

AC = 4x = 4 × 5 = 20 cm

B. Given:

$\tan \angle ADC = \frac{1}{1}$

i.e. $\frac{\text{perpendicular}}{\text{base}} = \frac{AC}{CD} = \frac{1}{1}$

Therefore, if the length of the perpendicular = x, the length of the base = x

Since

$AC^2 + CD^2 = AD^2$ [Using Pythagoras Theorem]

$(x)^2 + (x)^2 = AD^2$

$AD^2 = 2x^2$

$\therefore AD = \sqrt{2}x$

Now,

AC = 20

$\Rightarrow x = 20$

So, $AD = \sqrt{2}x = \sqrt{2} \times 20 = 20\sqrt{2}\,\text{cm}$

And, CD = 20 cm

C. Now,

$\tan B = \frac{AC}{BC} = \frac{20}{15} = \frac{4}{3}$

$$\cos B = \frac{BC}{AB} = \frac{15}{25} = \frac{3}{5}$$

So,

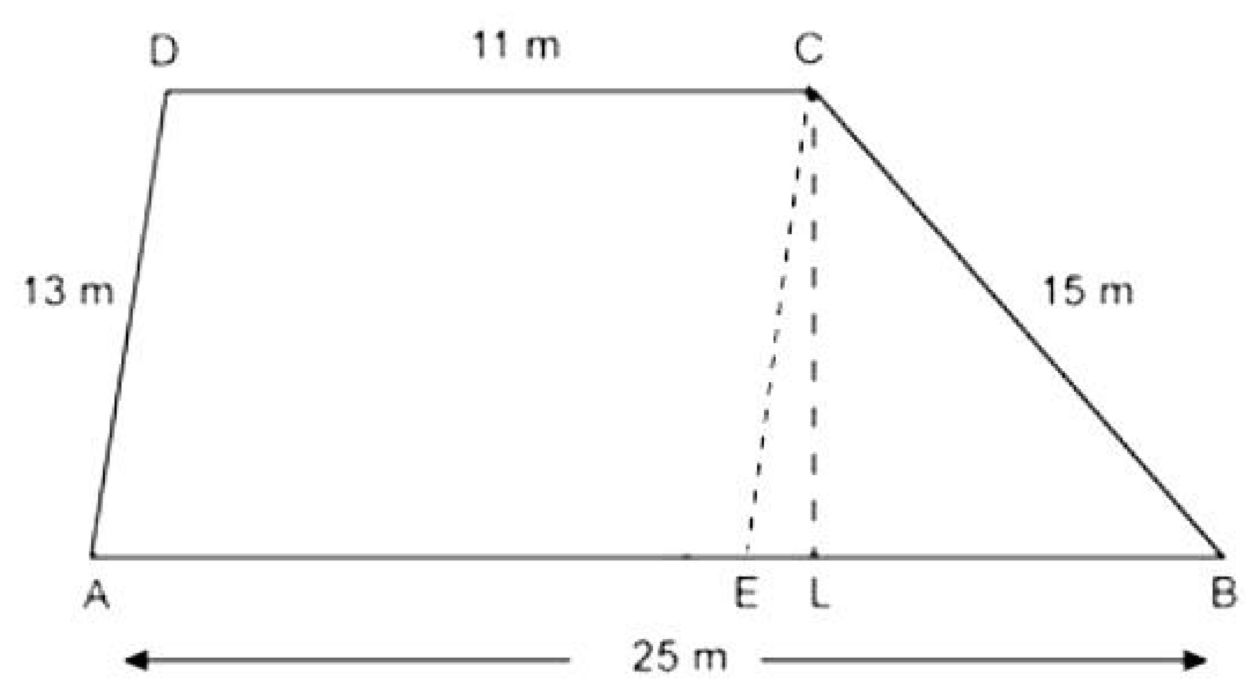
$$\tan^2 B - \frac{1}{\cos^2 B} = \left(\frac{4}{3}\right)^2 - \frac{1}{\left(\frac{3}{5}\right)^2}$$

$$= \frac{16}{9} - \frac{25}{9}$$

$$= -\frac{9}{9}$$

$$= -1$$

ii)



From C, draw CE || DA.

Clearly, ADCE is a parallelogram having AD || EC and AE || DC such that AD = 13 m and DC = 11 m.

AE = DC = 11 m and EC = AD = 13 m

$\Rightarrow$ BE = AB – AE = 25 – 11 = 14 m

Thus, in ΔBCE, we have

BC = 15 m, CE = 13 m and BE = 14 m

Let a = 15 m, b = 13 m and c = 14 m

$$\text{Semi-perimeter, } s = \frac{a+b+c}{2}$$

$$= \frac{15+13+14}{2}$$

$$= \frac{42}{2}$$

$$= 21 \text{ m}$$

$\therefore$ Area of $\Delta BCE = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{21(21-15)(21-13)(21-14)}$

$= \sqrt{21 \times 6 \times 8 \times 7}$

$= \sqrt{7 \times 3 \times 2 \times 3 \times 4 \times 2 \times 7}$

$= 7 \times 3 \times 2 \times 2$

$= 84 \text{ m}^2$

Also, area of $\Delta BCE = \frac{1}{2} \times BE \times CL$

$\Rightarrow 84 \text{ cm}^2 = \frac{1}{2} \times 14 \text{ cm} \times CL$

$\Rightarrow 84 \text{ cm}^2 = 7 \text{ cm} \times CL$

$\Rightarrow CL = 12 \text{ m}$

Now, area of trapezium ABCD

$= \frac{1}{2} \times \text{Sum of parallel sides} \times \text{Height}$

$= \frac{1}{2} \times (AB + CD) \times CL$

$= \frac{1}{2} \times (11 + 25) \times 12$

$= 36 \times 6$

$= 216 \text{ m}^2$

Solution 8

i) Diagonals of a rectangle are equal and bisect each other.

$\Rightarrow AC = BD$

$\therefore 2AO = 2BO$ (O is the mid-point of the diagonals)

$\angle ABO = \angle OAB = 35°$ (Angles opposite to equal sides are equal)

$\angle ABO + \angle OBC = 90°$ (Each angle of a rectangle is 90°)

$\therefore 35° + x = 90°$

$\therefore x = 55°$

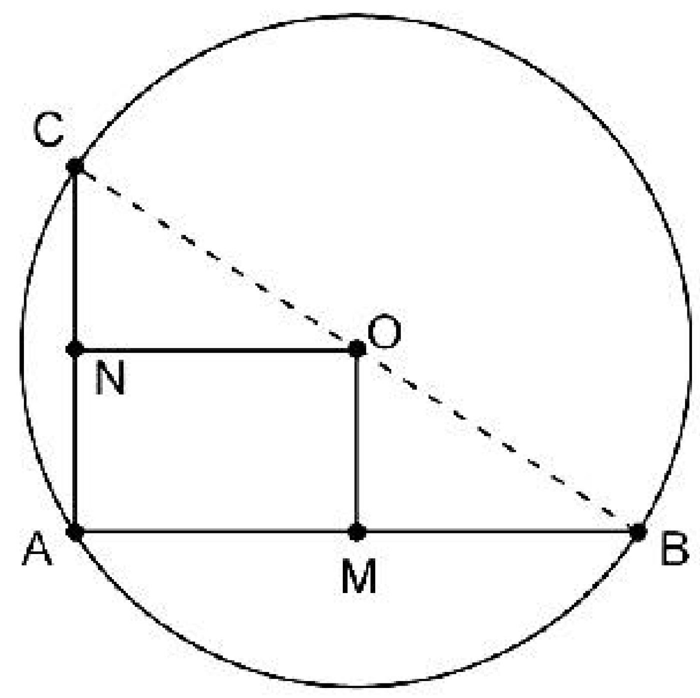
In ΔOAB,

$\angle OAB + \angle ABO + \angle AOB = 180°$

$\therefore 35° + 35° + \angle AOB = 180°$

$\therefore \angle AOB = 110°$

$\therefore \angle DOC = \angle AOB = y = 110°$ (Vertically opposite angles) ii)



Let O be the centre of a circle with radius r.

OB = OC = r

Let AC = x

Then, AB = 2x

Let OM $\perp$ AB

$\Rightarrow$ OM = p

Let ON $\perp$ AC

$\Rightarrow$ ON = q

In ΔOMB, by Pythagoras' theorem,

$OB^2 = OM^2 + BM^2$

$\Rightarrow r^2 = p^2 + \left(\frac{1}{2}AB\right)^2$ $\left(\begin{array}{l}\text{perpendicular from the centre}\\ \text{of a circle bisects the chord}\end{array}\right)$

$\Rightarrow r^2 = p^2 + \frac{1}{4} \times 4x^2$

$\Rightarrow r^2 = p^2 + x^2$

$\Rightarrow x^2 = r^2 - p^2$ ....(i)

In ΔONC, by Pythagoras' theorem,

$OC^2 = ON^2 + CN^2$

$\Rightarrow r^2 = q^2 + \left(\frac{1}{2}AC\right)^2$ $\left(\begin{array}{l}\text{perpendicular from the centre}\\ \text{of a circle bisects the chord}\end{array}\right)$

$\Rightarrow r^2 = q^2 + \frac{x^2}{4}$

$\Rightarrow q^2 = r^2 - \frac{x^2}{4}$

$\Rightarrow 4q^2 = 4r^2 - x^2$

$\Rightarrow 4q^2 = 4r^2 - (r^2 - p^2)$ [Using (i)]

$\Rightarrow 4q^2 = 3r^2 + p^2$

iii) Length (l_1) of the godown = 40 m

Breadth (b_1) of the godown = 25 m

Height (h_1) of the godown = 10 m

Volume of godown = $l_1 \times b_1 \times h_1 = (40 \times 25 \times 10)\ m^3 = 10000\ m^3$

Length (l_2) of a wooden crate = 1.5 m

Breadth (b_2) of a wooden crate = 1.25 m

Height (h_2) of a wooden crate = 0.5 m

Volume of a wooden crate = $l_2 \times b_2 \times h_2$ = $(1.5 \times 1.25 \times 0.5)$ m^3 = 0.9375 m^3

Let n wooden crates be stored in the godown.

Volume of n wooden crates = volume of the godown

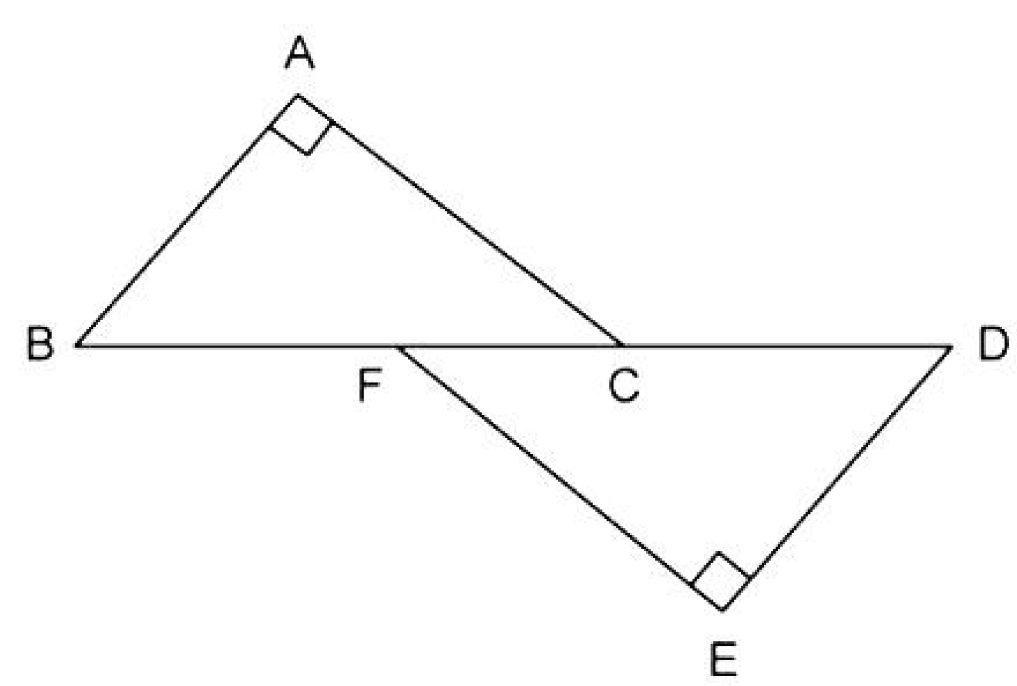
$0.9375 \times n = 10000$

$$n = \frac{10000}{0.9375} = 10666.66$$

Thus, maximum 10666 wooden crates can be stored in the godown.

Solution 9

i)



In ΔABC,

$BC = BF + FC$ ....(1)

And, in ΔDEF,

$FD = FC + CD$ ....(2)

Given, $BF = CD$ ....(3)

From (1),(2) and (3)

$\Rightarrow BC = FD$

So, in ΔABC and ΔEDF,

$AB = DE$ [Given]

$\angle BAC = \angle DEF = 90^\circ$ [Given]

$BC = FD$ [Proved above]

$\therefore \Delta ABC \cong \Delta EDF$ [By RHS]

$\Rightarrow AC = EF$ [C.P.C.T]

ii) Given: AB = AC, BE = DC

To prove: AD = AE

Proof:

BE = DC

$\Rightarrow$ BE – DE = DC – DE

$\Rightarrow BD = EC$ ....(i)

In ΔABD and ΔACE,

$AB = AC$ (given)

$\Rightarrow \angle B = \angle C$ (angles opposite to equal sides are equal)

$BD = EC$ [from (i)]

$\Rightarrow \Delta ABD \cong \Delta ACE$ (by SAS congruence)

$\Rightarrow AD = AE$ (c.p.c.t.)

iii)

A. In an isosceles triangle, the length of the lateral sides is equal.

Let the length of each lateral side be x cm

Then, base $= \frac{3}{2} \times x \text{ cm}$

A. Perimeter of an isosceles triangle = 42 cm

$\Rightarrow x + x + \frac{3}{2}x = 42$

$\Rightarrow 2x + 2x + 3x = 84$

$\Rightarrow 7x = 84$

$\Rightarrow x = \frac{84}{7} = 12$

$\therefore$ The length of each equal side of a triangle is 12 cm.

B. Area of the triangle:

Base $= \frac{3}{2}x = \frac{3}{2} \times 12 = 18$ cm

Therefore, length of the sides of the triangle are 12 cm, 12 cm and 18 cm.

Let $a = 12$ cm, $b = 12$ cm and $c = 18$ cm.

Now, $s = \frac{1}{2}(a+b+c) = \left(\frac{12+12+18}{2}\right) \text{cm} = \left(\frac{42}{2}\right) \text{cm} = 21 \text{ cm}$

$$\begin{aligned}\text{Area of the triangle} &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{21(21-12)(21-12)(21-18)} \\ &= \sqrt{21 \times 9 \times 9 \times 3} \\ &= \sqrt{3 \times 7 \times 9 \times 9 \times 3} \\ &= 27\sqrt{7} \\ &= 71.43 \text{cm}^2 \quad \ldots\left(\sqrt{7} = 2.6457\right)\end{aligned}$$

C. Area of a triangle $= \frac{1}{2} \times \text{base} \times \text{height}$

$\Rightarrow 71.43 = \frac{1}{2} \times 18 \times h$

$\Rightarrow h = \frac{71.43}{9} = 7.94\,\text{cm}$

Solution 10

i) Length of the cistern, l = 8 m

Breadth of the cistern, b = 6 m

Height (depth) of the cistern, h = 2.5 m

Capacity of the cistern = Volume of the cistern

$= (l \times b \times h)$

$= (8 \times 6 \times 2.5)\ m^3$

$= 120\ m^3$

Area of the iron sheet required = Total surface area of the cistern

$= 2(lb + bh + lh)$

$= 2(8 \times 6 + 6 \times 2.5 + 2.5 \times 8)\ m^2$

$= 2(48 + 15 + 20)\ m^2$

$= (2 \times 83)\ m^2$

$= 166\ m^2$

ii)

A. $5x - 3y - 6 = 0$

$3y = 5x - 6$

$\Rightarrow y = \frac{5}{3}x - \frac{6}{3}$

Slope = coefficient of x = $\frac{5}{3}$ and y-intercept = constant term $= -\frac{6}{3} = -2$

B. $4x + 3y - 7 = 0$

$3y = -4x + 7$

$\Rightarrow y = -\frac{4}{3}x + \frac{7}{3}$

Slope = coefficient of x = $-\frac{4}{3}$ and y-intercept = constant term = $\frac{7}{3}$

C. $5y - 4 = 0$

$\Rightarrow y = \frac{4}{5}$

Slope = coefficient of x = 0 and y-intercept = constant term = $\frac{4}{5}$

iii) The first given equation is $x + y = 8$.

$\therefore y = 8 - x$

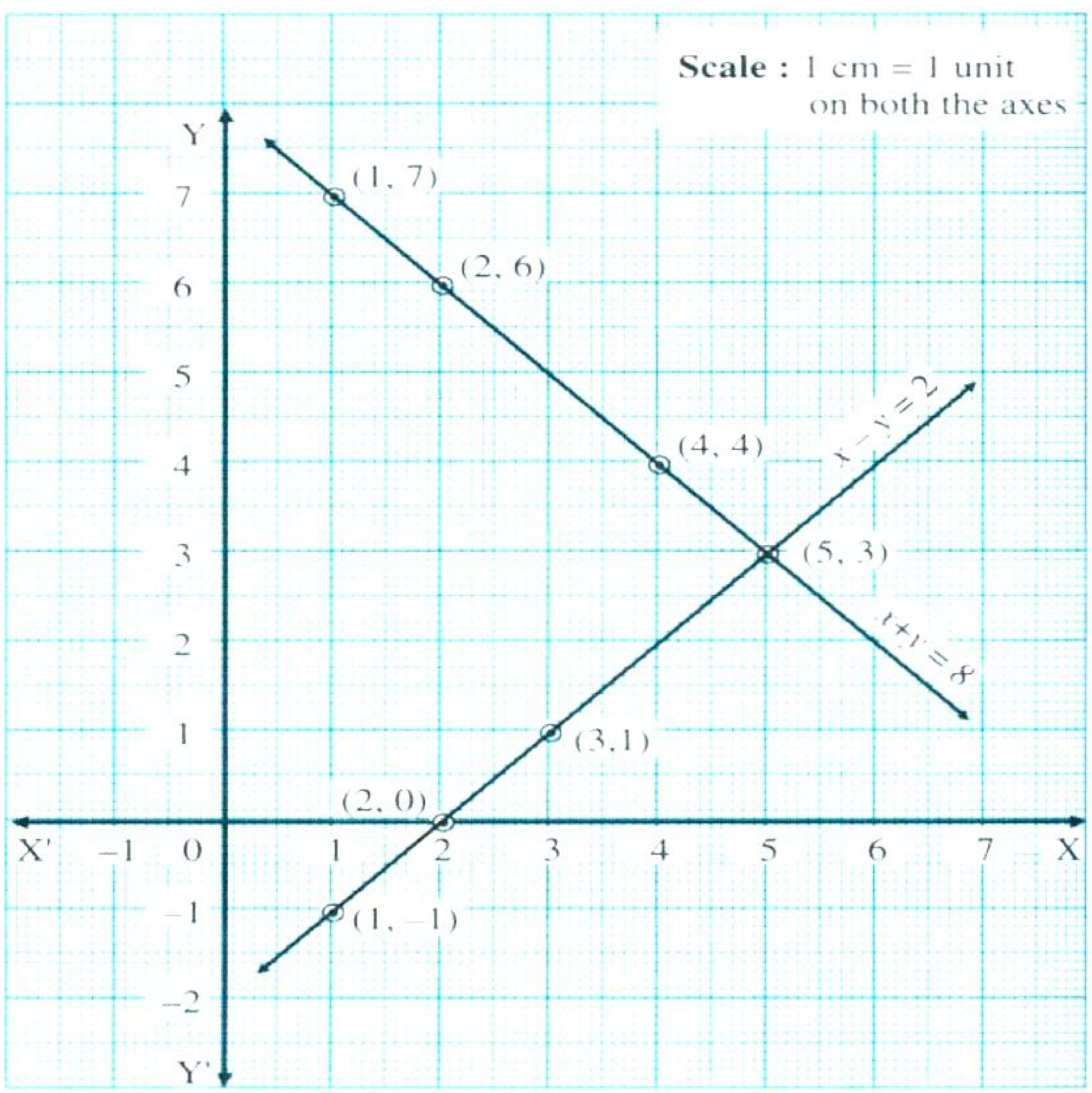
x	1	2	4
y	7	6	4
(x, y)	(1, 7)	(2, 6)	(4, 4)

The second equation is $x - y = 2$.

$\therefore y = x - 2$

x	1	2	3
y	−1	0	1
(x, y)	(1, −1)	(2, 0)	(3, 1)

Let us draw two lines corresponding to the two equations, and the co-ordinates of the point of intersection of these lines is the solution of the given simultaneous equations.



The co-ordinates of the point of intersection are (5, 3).

Therefore, the solution of the given simultaneous equations is $x = 5$ and $y = 3$.