

**Mechanical Engineering**  
**(Forenoon Session)**  
**Exam Date- 03-02-2024**

**SECTION - A**

**GENERAL APTITUDE**

**Q.1** The real variable  $x$ ,  $y$ ,  $z$ , and the real constants  $p$ ,  $q$ ,  $r$  satisfy

$$\frac{x}{pq-r^2} = \frac{y}{qr-p^2} = \frac{z}{rp-q^2}$$

Given the denominator are non-zero, the value of  $px + qy + rz$ .

- (a)  $pqr$  (b) 1  
(c)  $p^2 + q^2 + r^2$  (d) 0

**Ans. (d)**

Let, 
$$\frac{x}{pq-r^2} = \frac{y}{qr-p^2} = \frac{z}{rp-q^2} = K$$

$$\frac{px}{p^2q - pr^2} = K$$

$$\frac{qy}{q^2r - qp^2} = K$$

$$\frac{rz}{r^2p - rq^2} = K$$

$$px + qy + rz = \frac{kp^2q - kpr^2 + kq^2r - kqp^2 + kr^2p - krq^2}{1} = 0$$

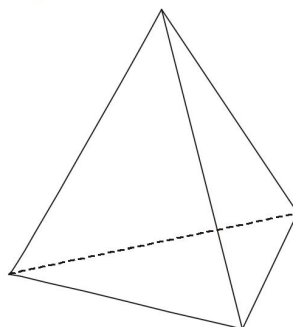
End of Solution

**Q.2** Four equilateral triangles are used to form a regular closed three-dimensional object by joining along the edges. The angle between any two faces is

- (a)  $45^\circ$  (b)  $30^\circ$   
(c)  $60^\circ$  (d)  $90^\circ$

**Ans. (\*)**

**Tetrahedron Angles :** In a regular tetrahedron, all the faces are equilateral triangles. Therefore, all the interior angles of a tetrahedron are  $60^\circ$  each.



End of Solution

- Q.3** In the given text, the blanks are numbered (i)-(iv). Select the best match for all the blanks.  
 Prof. P  (i)  merely a man who narrated funny stories  (ii)  in his blackest moments he was capable of self-depreciating humor.  
 Prof. Q  (iii)  a man who hardly narrated funny stories  (iv)  in his blackest moments was he able to find humor.
- (a) (i) wasn't (ii) Even, (iii) was, (iv) Only  
 (b) (i) wasn't (ii) Only, (iii) was, (iv) Even  
 (c) (i) was (ii) Only, (iii) wasn't, (iv) Even  
 (d) (i) was (ii) Even, (iii) wasn't, (iv) Only

**Ans. (a)**

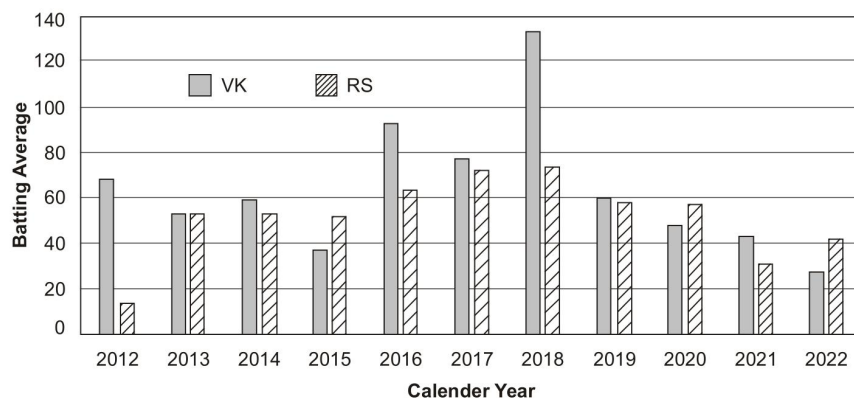
**End of Solution**

- Q.4** How many combinations of non-null sets  $A, B, C$  are possible from the subsets of  $\{2, 3, 5\}$  satisfying the condition: (i)  $A$  is a subset of  $B$ , and (ii)  $B$  is a subset of  $C$ ?
- (a) 18 (b) 27  
 (c) 28 (d) 19

**Ans. (\*)**

**End of Solution**

- Q.5** The bar chart gives the batting averages of VK and RS for 11 calendar years from 2012 to 2022. Considering that 2015 and 2019 are world cup years, which one of the following options is true?



- (a) VK has a higher yearly batting average than that of RS in every world cup year.  
 (b) VK's yearly batting average is consistently higher than that of RS between the two world cup years.  
 (c) RS's yearly batting average is consistently higher than that of VK in the last three years.  
 (d) RS has a higher yearly batting average than that of VK in every world cup year.

**Ans. (b)**

**End of Solution**

**Q.6** Take two long dice (rectangular parallelepiped), each having four rectangular faces labelled as 2, 3, 5, and 7. If thrown, the long dice cannot land on the square faces and has  $\frac{1}{4}$  probability of landing on any of the four rectangular faces. The label on the top face of the dice is the score of the throw.

If thrown together, what is the probability of getting the sum of the two long dice scores greater than 11?

(a)  $\frac{3}{16}$

(b)  $\frac{1}{16}$

(c)  $\frac{1}{8}$

(d)  $\frac{3}{8}$

**Ans. (a)**

Total outcomes : (2, 2), (2, 3), (2, 5), (2, 7)  
 (3, 2), (3, 3), (3, 5), (3, 7)  
 (5, 2), (5, 3), (5, 5), (5, 7)  
 (7, 2), (7, 3), (7, 5), (7, 7)

Favourable outcomes (sum > 11)

: (5, 7), (7, 5), (7, 7)

$$\text{Probability} = \frac{\text{Number of favourable outcomes}}{\text{Number of total outcomes}} = \frac{3}{16}$$

**End of Solution**

**Q.7** Find the odd one out in the set: {19, 37, 21, 17, 23, 29, 31, 11}

(a) 23

(b) 21

(c) 29

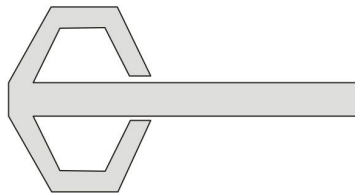
(d) 37

**Ans. (b)**

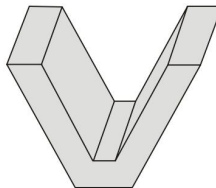
Except 21 all are prime number.

**End of Solution**

**Q.8** A planar rectangular paper has two V-shaped pieces attached as shown below:



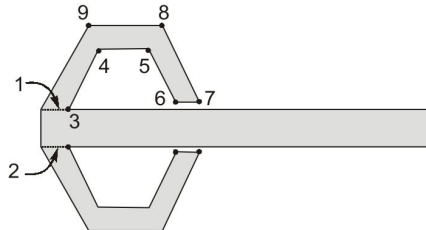
This piece of paper is folded to make the following closed three-dimensions object.



The number of folds required to form the above object is

- (a) 8 (b) 7  
(c) 9 (d) 11

Ans. (c)



Hence, the total number of folds required to form the above object is 9.

End of Solution

**Q.9** If '→' denotes increasing order of intensity, then the meaning of the words [smile → giggle → laugh] is analogous to [disapprove → \_\_\_\_\_ → chide]. Which one of the given options is appropriate to fill the blank?

- (a) reprove (b) grieve  
(c) reprise (d) praise

Ans. (a)

[smile → giggle → laugh] is similarly related as  
[disapprove → reprove → chide]

End of Solution

**Q.10** In the following series, identify the number that needs to be changed to form the Fibonacci series.

1, 1, 2, 3, 6, 8, 13, 21, .....

- (a) 13 (b) 8  
(c) 21 (d) 6

Ans. (d)

In Fibonacci series, every number is equal to the sum of previous two numbers of the series. i.e. 0, 1, 1, 2, 3, 5, 8, 13, 21, .....

$$0 + 1 = 1$$

$$1 + 1 = 2$$

$$2 + 1 = 3$$

$$3 + 2 = 5$$

$$5 + 3 = 8$$

$$8 + 5 = 13$$

$$13 + 8 = 21$$

End of Solution





## SECTION - B

## TECHNICAL

**Q.1** The most suitable electrode materials used for joining low alloy steels using Gas Metal Arc Welding (GMAW) process is

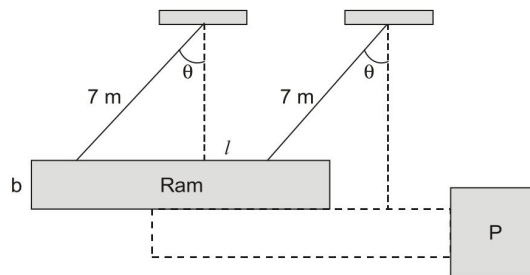
- (a) Copper (b) Cadmium  
(c) Low alloy steel (d) tungsten

**Ans. (c)**

In GMAW, consumable electrodes are used which has to be same composition as base metal. Thus, low alloy steels electrodes are the most suitable electrode material.

End of Solution

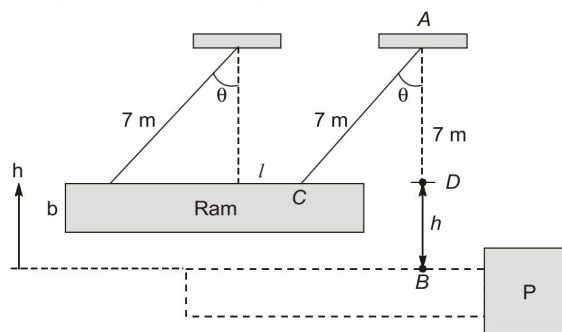
**Q.2** A ram in the form of a rectangular body of size,  $l = 9$  cm and  $b = 2$  m is suspended by two parallel ropes of lengths 7 m. Assume the center-of-mass of the body is at its geometric center and  $g = 9.81$  m/s<sup>2</sup>. For striking the object P with horizontal velocity of 5 m/s, what is the angle  $\theta$  with the vertical from which the ram should be released from rest?



- (a) 67.1° (b) 35.1°  
(c) 40.2° (d) 79.5°

**Ans. (b)**

Given,  $AB = 7$  m,  $u = 0$ ,  $v = 5$  m/s



According to question, the rope will become vertical on striking the object, hence  $AB = 7$  m.

From geometry,  $AD = 7 \cos \theta$   
 $\therefore h = AB - AD = 7 - 7 \cos \theta$   
 $= 7(1 - \cos \theta)$

From energy conservation,

Initial potential energy = Final kinetic energy

$$\Rightarrow mgh = \frac{1}{2}mv^2$$

$$9.81 \times 7(1 - \cos\theta) = \frac{1}{2} \times 5^2$$

$$\cos\theta = \frac{9.81 \times 7 - 12.5}{9.81 \times 7}$$

$$\Rightarrow \theta = 35.1^\circ$$

End of Solution

**Q.3** A furnace can supply heat steadily at 1200 K at a rate of 24000 kJ/min. The maximum amount of power (in kW) that can be produced by using the heat supplied by this furnace in an environment at 300 K is

- (a) 0 (b) 150  
(c) 300 (d) 18000

**Ans. (c)**

$$Q_{in} = 24000 \text{ kJ/min} = 400 \text{ kW}$$

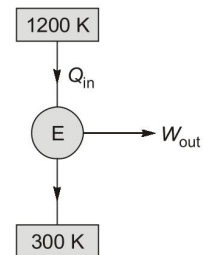
$$T_1 = 1200 \text{ K}, T_2 = 300 \text{ K}$$

Maximum power can be obtained when the heat supplied is used in Carnot engine,,

$$\therefore W_{max} = Q_{in} \times \eta_{carnot}$$

$$= 400 \left( 1 - \frac{300}{1200} \right)$$

$$W_{max} = 300 \text{ kW}$$



End of Solution

**Q.4** The value of the surface integral

$$\oiint_S z dx dy$$

where S is the external surface of the sphere  $x^2 + y^2 + z^2 = R^2$  is

- (a)  $4\pi R^3$  (b)  $\frac{4\pi R^3}{3}$   
(c) 0 (d)  $\pi R^3$

**Ans. (b)**

To evaluate,  $\oiint_S z dx dy$

where  $S(\text{sphere}) = x^2 + y^2 + z^2 = R^2$

$$= \oiint_S z dx dy$$

$$= \text{Volume of sphere} = \frac{4\pi}{3} R^3$$

End of Solution

**Q.5** Let  $f(\cdot)$  be a twice differentiable function from  $\mathbb{R}^2 \rightarrow \mathbb{R}$ . If  $p, x_o \in \mathbb{R}^2$  where  $\|p\|$  is sufficiently small (here,  $\|\cdot\|$  is the Euclidean norm or distance function), then  $f(x_o + p)$

$$= f(x_o) + \nabla f(x_o)^T p + \frac{1}{2} p^T \nabla^2 f(\psi) p \text{ where } \psi \in \mathbb{R}^2 \text{ is a point on the line segment joining } x_o \text{ and } x_o + p.$$

If  $x_o$  is a strict local minimum of  $f(x)$ , then which one of the following statements is TRUE?

- (a)  $\nabla f(x_o)^T p = 0$  and  $p^T \nabla^2 f(\psi) p > 0$       (b)  $\nabla f(x_o)^T p = 0$  and  $p^T \nabla^2 f(\psi) p = 0$
- (c)  $\nabla f(x_o)^T p > 0$  and  $p^T \nabla^2 f(\psi) p = 0$       (d)  $\nabla f(x_o)^T p = 0$  and  $p^T \nabla^2 f(\psi) p < 0$

**Ans. (a)**

It can be assumed that the function is parabolic nature i.e. of the form  $ax^2 + bx + c$ . For a strictly local minimum to exist, the coefficient of  $x^2$  i.e. 'a' should be positive. Also, for a local minimum to exist, its first derivative should be zero while the second derivative should be positive. Similar conditions can be observed in option (a) only. Hence, option (a) is correct.

**End of Solution**

**Q.6** For a ball bearing the fatigue life in millions of revolutions is given by  $L = \left(\frac{C}{P}\right)^n$ , where

P is the constant applied load and C is the basic dynamic load rating. Which one of the following statements is TRUE?

- (a)  $n = 3$ , assuming that the inner racing is fixed and outer racing is revolving.
- (b)  $n = 1/3$ , assuming that the inner racing is fixed and outer racing is revolving.
- (c)  $n = 3$ , assuming that the outer racing is fixed and inner racing is revolving.
- (d)  $n = 1/3$ , assuming that the outer racing is fixed and inner racing is revolving.

**Ans. (c)**

- For a ball bearing, the fatigue life in million revolution is given by,

$$L_{90} = \left[\frac{C}{P}\right]^3$$

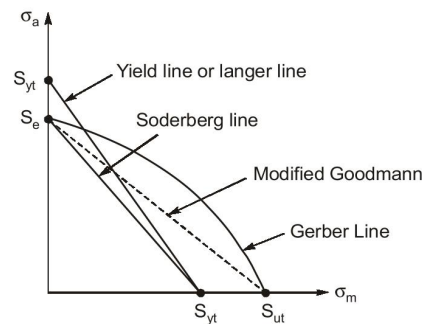
- In the following case, it is assumed that the outer race is fixed and the inner race is revolving.

**End of Solution**

**Q.7** Which one of the following failure theories is the most conservative design approach against fatigue failure?

- (a) Modified Goodman line      (b) Yield line
- (c) Gerber line      (d) Soderberge line

Ans. (d)



Soderberg line is the most conservative fatigue failure criterion.

End of Solution

Q.8 Consider the system of linear equations

$$x + 2y + z = 5$$

$$2x + ay + 4z = 12$$

$$2x + 4y + 6z = b$$

The values of  $a$  and  $b$  such that there exists a non-trivial null space and the system admits infinite solutions are

(a)  $a = 4, b = 12$

(b)  $a = 8, b = 12$

(c)  $a = 4, b = 14$

(d)  $a = 8, b = 14$

Ans. (c)

$$x + 2y + z = 5$$

$$2x + ay + 4z = 12$$

$$2x + 4y + 6z = b$$

Performing row operations on augmented matrix,

$$\left[ \begin{array}{ccc|c} 1 & 2 & 1 & 5 \\ 2 & a & 4 & 12 \\ 2 & 4 & 6 & b \end{array} \right] \xrightarrow[R_3=R_2-2R_1]{R_2=R_2-2R_1} \left[ \begin{array}{ccc|c} 1 & 2 & 1 & 5 \\ 0 & a-4 & 2 & 2 \\ 0 & 0 & 4 & b-10 \end{array} \right]$$

For infinitely many solutions,

$$\rho(A) = \rho(AB) = 2$$

$$\Rightarrow a - 4 = 0$$

$$a = 4$$

and,  $\rho(AB) = 2$

for this  $R_3 = 2R_2$

$$b - 10 = 4$$

$$b = 14$$

End of Solution

Q.9 Let  $f(z)$  be an analytic function, where  $z = x + iy$ . If the real part of  $f(z)$  is  $\cosh x \cos y$ , and the imaginary part of  $f(z)$  is zero for  $y = 0$ , then  $f(z)$  is

(a)  $\cosh x \exp(-iy)$

(b)  $\cosh z \exp z$

(c)  $\cosh z \cos y$

(d)  $\cosh z$

Ans. (d)

Let,

$$\begin{aligned} f(z) &= u + iv \\ u &= \cosh z \cos y \\ &= \left( \frac{e^x + e^{-x}}{2} \right) \cos y \\ u &= \left( \frac{e^x + e^{-x}}{2} \right) \cos y \end{aligned}$$

by Milne Thomson method

$$u(x, y) = \left( \frac{e^x + e^{-x}}{2} \right) \cos y$$

Partial differentiate w.r.t.  $x, y$

$$u_x = \left( \frac{e^x - e^{-x}}{2} \right) \cos y; \quad u_y = \left( \frac{e^x + e^{-x}}{2} \right) (-\sin y)$$

$$u_x = \left( \frac{e^x - e^{-x}}{2} \right) \cos y \quad u_y = -\frac{(e^x + e^{-x})}{2} \sin y$$

$$u_x(z, 0) = \frac{e^z - e^{-z}}{2} = \sinh z \quad u_y(z, 0) = -0$$

$$\begin{aligned} f(z) &= \int (u_x - iu_y) dz + c \\ &= \int (\sinh z - a) dz + c \\ &= \int \sinh z + c \\ &= \cosh z + c \end{aligned}$$

End of Solution

**Q.10** A set of jobs U, V, W, X, Y, Z arrive at time  $t = 0$  to a production line consisting of two workstations in series. Each job must be processed by both workstations in sequence (i.e., the first followed by the second). The process times (in minutes) for each job on each workstation in the production line are given below:

Job	U	V	W	X	Y	Z
Workstation 1	5	7	3	4	6	8
Workstation 2	4	6	6	8	5	7

The sequence in which the jobs must be processed by the production line if the total makespan of production is to be minimized is

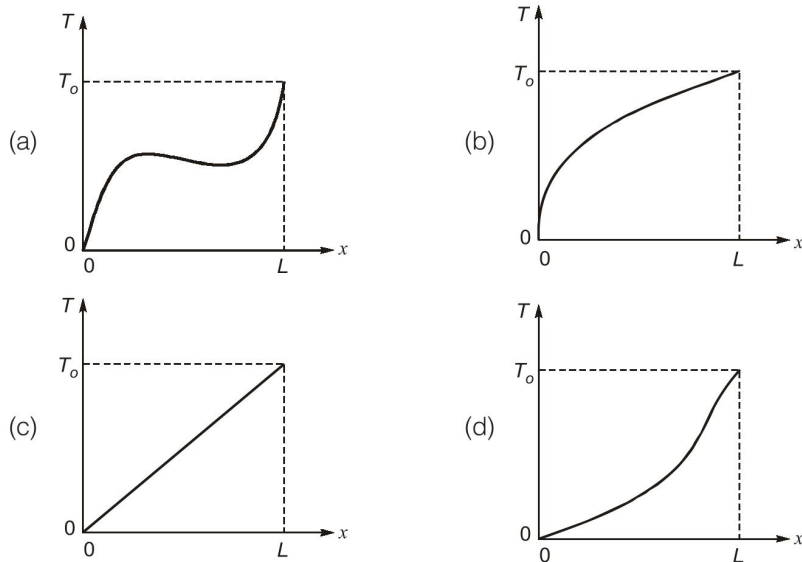
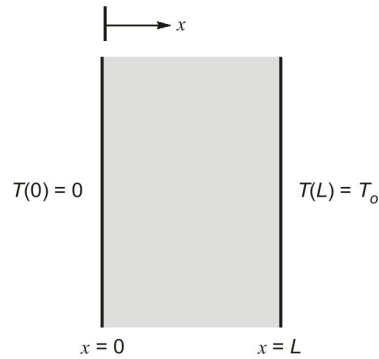
- (a) U-Y-V-Z-X-W (b) W-X-Z-V-Y-U  
(c) W-X-V-Z-Y-U (d) W-U-Z-V-Y-X

Ans. (b)

As per Johnson's algorithm the optimum sequence is W-X-Z-V-Y-U.

End of Solution

- Q.11** A plane, solid slab of thickness  $L$ , shown in the figure, has thermal conductivity  $k$  that varies with the spatial coordinate  $x$  as  $k = A + Bx$ , where  $A$  and  $B$  are positive constant ( $A > 0, B > 0$ ). The slab walls are maintained at fixed temperature of  $T(x = 0) = 0$  and  $T(x = L) = T_o > 0$ . The slab has no internal heat sources. Considering one-dimensional heat transfer, which one of the following plots qualitatively depicts the steady-state temperature distribution within the slab?



**Ans. (b)**

$$k = A + Bx$$

where  $A$  and  $B$  are positive real number

So with increase with  $x$ ,  $k$  increases

Energy equation;

$$\frac{\partial}{\partial x} \left( k \frac{\partial T}{\partial x} \right) + \dot{q} = 0$$

$$\frac{\partial}{\partial x} \left( (A + Bx) \cdot \frac{\partial T}{\partial x} \right) = 0$$

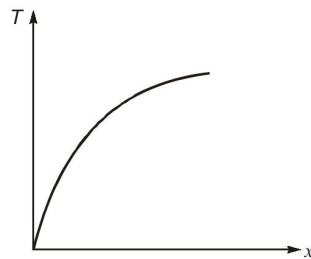
On integration,

$$(A + Bx) \cdot \frac{\partial T}{\partial x} = C_1$$
$$\frac{\partial T}{\partial x} = \frac{C_1}{A + Bx}$$

On integration,

$T = \text{logarithmic function of } x$

Hence, variation of  $T$  with  $x$  is given as;



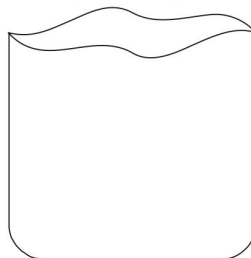
End of Solution

**Q.12** The “Earing” phenomenon in metal forming is associated with

- (a) deep drawing
- (b) extrusion
- (c) rolling
- (d) forging

**Ans. (a)**

Anisotropy plays an important role in the performance of deep drawing processes. The anisotropy is of two types. In normal anisotropy the properties differ in the thickness direction. In planar anisotropy, the properties vary with the orientation in the plane of the sheet. Whereas deep drawability of sheets increases with normal anisotropy, anisotropy leads to the formation of ears in cup drawing. Ears cause the wavy edge of a drawn cup

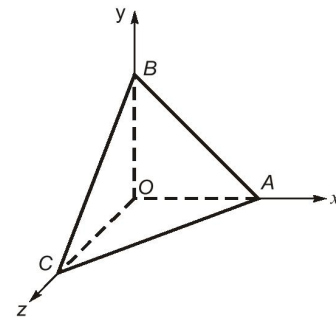


An earing defect showing the formation of ears

End of Solution



**Q.13** A rigid massless tetrahedron is placed such that vertex  $O$  is at the origin and the other three vertices  $A$ ,  $B$ ,  $C$  lie on the coordinate axes as shown in the figure. The body is acted on by three point loads, of which one is acting at  $A$  along  $x$ -axis and another at point  $B$  along  $y$ -axis. For the body to be in equilibrium, the third point load acting at point  $O$  must be



- (a) In  $y$ - $z$  plane but not along  $y$  or  $z$  axis
- (b) along  $z$ -axis
- (c) in  $z$ - $x$  plane but not along  $z$  or  $x$  axis
- (d) in  $x$ - $y$  plane but not along  $x$  or  $y$  axis

**Ans. (d)**

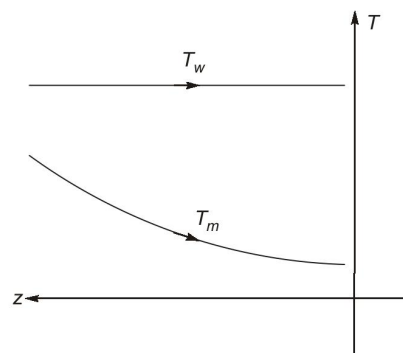
For the body to be in equilibrium, the three forces can be coplanar, parallel or concurrent.

**End of Solution**

**Q.14** Consider a hydrodynamically fully developed laminar flow through a circular pipe with the flow along the axis (i.e.  $z$  direction). In the following statements,  $T$  is the temperature of the fluid,  $T_w$  is the wall temperature and  $T_m$  is the bulk mean temperature of the fluid. Which one of the following statements is TRUE?

- (a) Nusselt number varies linearly along the  $z$ -direction for a thermally fully developed flow.
- (b) For a thermally fully developed flow,  $\frac{\partial T}{\partial z} = 0$ , always.
- (c) For constant wall temperature of the duct,  $\frac{dT_m}{dz} = \text{constant}$ .
- (d) For constant wall temperature ( $T_w > T_m$ ) of the duct,  $\frac{dT_m}{dz}$  increase exponentially with distance along  $z$ -direction.

**Ans. (d)**



The rate of increase in bulk mean temperature along the flow  $\left(\frac{dT_m}{dz}\right)$  increase exponentially along the flow direction ( $z$ ).

**End of Solution**

- Q.15** A linear spring-mass-dashpot system with a mass of 2 kg is set in motion with viscous damping. If the natural frequency is 15 Hz, and the amplitudes of two successive cycles measured are 7.75 mm and 7.20 mm, the coefficient of viscous damping (in N.s/m) is
- (a) 4.41 (b) 6.11  
(c) 2.52 (d) 7.51

**Ans. (a)**

$$m = 2 \text{ kg}$$

$$f_n = 15 \text{ Hz}$$

$$\omega_n = 2\pi(f_n) = 2\pi \times 15 = 30\pi \text{ rad/s}$$

$$x_n = 7.75 \text{ mm}, x_{n+1} = 7.20 \text{ mm}$$

Decrement ratio,  $\frac{x_n}{x_{n+1}} = \frac{7.75}{7.20} = 1.07638$

Logarithmic decrement,

$$\delta = \log_e \left( \frac{x_n}{x_{n+1}} \right) = \log_e(1.07638)$$

$$\delta = 0.0736$$

$$\frac{2\pi\xi}{\sqrt{1-\xi^2}} = 0.0736$$

$$\Rightarrow \xi = 0.0117$$

But,  $2\xi\omega_n = \frac{C}{m}$

$$\Rightarrow 2 \times 0.0117 \times 30\pi = \frac{C}{2}$$

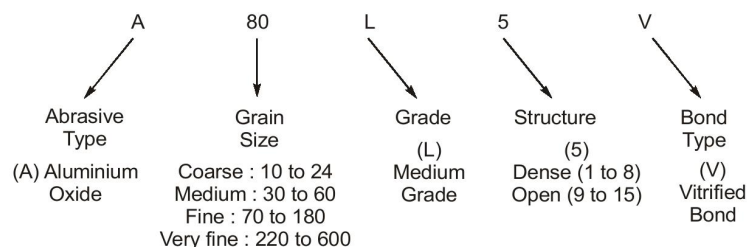
$$\Rightarrow C = 4.41 \text{ N/m/s}$$

**End of Solution**

- Q.16** The grinding wheel used to provide the best surface finish is
- (a) A60L5V (b) A80L5V  
(c) A36L5V (d) A54L5V

**Ans. (b)**

In grinding operation, surface finish is decided by the grain size. Fine and very fine grain size will result in best surface finish.



**End of Solution**

- Q.17** A queueing system has one single server workstation that admits an infinitely long queue. The rate of arrival of jobs, to the queueing system follows the Poisson distribution with a mean of 5 jobs/hour. The service time of the server is exponentially distributed with a mean of 6 minutes. In steady state operation of the queueing system, the probability that the server is not busy at any point in time is
- (a) 0.83 (b) 0.50  
(c) 0.17 (d) 0.20

**Ans. (b)**

Given:  $\lambda = 5/\text{hr}$ ,  $\mu = 10/\text{hr}$

$$\therefore \rho = \frac{\lambda}{\mu} = \frac{5}{10} = \frac{1}{2}$$

Now, probability for the system to be idle,

$$P_o = 1 - \rho = 1 - 0.5 = 0.5$$

**End of Solution**

- Q.18** In order to numerically solve the ordinary differential equation  $\frac{dy}{dt} = -y$  for  $t > 0$ , with an initial condition  $y(0) = 1$ , the following scheme is employed

$$\frac{y_{n+1} - y_n}{\Delta t} = \frac{1}{2}(y_{n+1} + y_n)$$

Here,  $\Delta t$  is the time step and  $y_n = y(n\Delta t)$  for  $n = 0, 1, 2, \dots$ . This numerical scheme will yield a solution with non-physical oscillations for  $\Delta t > h$ . The value of  $h$  is

- (a) 1 (b)  $\frac{3}{2}$   
(c)  $\frac{1}{2}$  (d) 2

**Ans. (d)**

Given, 
$$\frac{y_{n+1} - y_n}{\Delta t} = \frac{1}{2}(y_{n+1} + y_n)$$

$$y_{n+1} - y_n = \frac{\Delta t}{2}(y_{n+1} + y_n)$$

$$y_{n+1} - \frac{\Delta t}{2}y_{n+1} = y_n + \frac{\Delta t}{2}y_n$$

$$y_{n+1} = \frac{2 + \Delta t}{2 - \Delta t}y_n$$

For stability or non physical oscillations,

$$\left| \frac{2 + \Delta t}{2 - \Delta t} \right| \leq 1$$

$$-1 \leq \frac{2 + \Delta t}{2 - \Delta t} \leq 1$$

$$-1 \leq -1 + \frac{4}{2 - \Delta t} \leq 1$$

$$0 \leq \frac{4}{2 - \Delta t} \leq 2$$

$$\frac{1}{2} \leq \frac{2 - \Delta t}{4} \leq 0$$

$$2 \leq 2 - \Delta t \leq 0$$

$$0 \leq -\Delta t \leq -2$$

$$\therefore \Delta t \geq 2$$

Thus from given options, option (d) is correct.

End of Solution

**Q.19** The velocity field of a two-dimensional, incompressible flow is given by

$$\vec{V} = 2 \sinh x \hat{i} + v(x, y) \hat{j}$$

where  $\hat{i}$  and  $\hat{j}$  denote the unit vector in  $x$  and  $y$  direction, respectively. If  $v(x, 0) = \cosh x$ , then  $v(0, -1)$  is

- (a) 1 (b) 2  
(c) 4 (d) 3

**Ans. (d)**

The velocity field of a two-dimensional, incompressible flow is given by,

$$\vec{V} = 2 \sinh x \hat{i} + V(x, y) \hat{j}$$

$$\text{and } V(x, 0) = \cosh x,$$

For an incompressible flow,  $\nabla \cdot \vec{V} = 0$

$$\Rightarrow \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$2 \cosh x + \frac{\partial v}{\partial y} = 0$$

$$\Rightarrow \frac{\partial v}{\partial y} = -2 \cosh x$$

$$\Rightarrow \int \partial v = \int -2 \cosh x \cdot \partial y$$

$$V = -2y \cdot \cosh x + f(x)$$

$$\text{For, } V(x, 0) = \cosh x$$

$$\Rightarrow -2y \cdot \cosh x + f(x) = \cosh x$$

$$\Rightarrow f(x) = \cosh x \quad (\text{at } x, 0)$$

$$\Rightarrow V = -2 \cdot y \cdot \cosh x + \cosh x$$

$$V = (1 - 2y) \cosh x$$

$$V(0, -1) = [1 - 2 \times (-1)] \times \cosh(0)$$

$$= 3$$

End of Solution

**Q.20** The preparatory functions in Computer Numerical Controlled (CNC) machine programme are denoted by the alphabet

- (a) P (b) O  
(c) M (d) G

**Ans. (d)**

Preparatory function of CNC programming is denoted by G-codes. Preparatory functions are the G-codes that identify the type of activities the machine will execute. The preparatory functions describe the way in which the machine axes have to move, the method of interpolation, the dimension system, the time delay of program execution and the activation of specific operational modes in the control.

**End of Solution**

**Q.21** Consider incompressible laminar flow over a flat plate with freestream velocity of  $U_\infty$ . The Nusselt number corresponding to this flow velocity is  $Nu_1$ . If the freestream velocity is doubled, the Nusselt number changes to  $Nu_2$ . Choose the correct option for  $\frac{Nu_2}{Nu_1}$ .

- (a) 1 (b)  $\sqrt{2}$   
(c) 2 (d) 1.26

**Ans. (b)**

For laminar flow over flat plate,  
 $Nu \propto Re^{0.5} Pr^{1/3}$ , where Re is Reynolds number and Pr is the Prandtl number.

$$\begin{aligned}\therefore Re &= \frac{U_\infty x}{\nu} \\ \therefore Nu &\propto U_\infty^{0.5} \\ \text{or, } \frac{Nu_2}{Nu_1} &= \sqrt{\frac{U_{\infty,2}}{U_{\infty,1}}} = \sqrt{2}\end{aligned}$$

**End of Solution**

**Q.22** Which one of the following statements regarding a Rankine cycle is FALSE?

- (a) Cycle efficiency increases as boiler pressure decreases.  
(b) Superheating the steam in the boiler increases the cycle efficiency.  
(c) The pressure at the turbine outlet depends on the condenser temperature.  
(d) Cycle efficiency increases as condenser pressure decreases.

**Ans. (a)**

- Superheating in Rankine cycle increases the cycle efficiency because of increase in mean temperature of heat addition.
- With increase in pressure of boiler, the cycle efficiency increases. So, the given statement is wrong.
- With decrease in condenser pressure, the cycle efficiency increases because of decrease in mean temperature of heat rejection.

- The pressure of turbine outlet is governed by the condenser temperature. Decreasing the cooling water temperature, creates more vacuum in condenser which results in pressure drop and vice-versa.

End of Solution

- Q.23** The change in kinetic energy  $\Delta E$  of an engine is 300 J, and minimum and maximum shaft speeds are  $\omega_{\min} = 220$  rad/s and  $\omega_{\max} = 280$  rad/s, respectively. Assume that the torque-time function is purely-harmonic. To achieve a coefficient of fluctuation of 0.05, the moment of inertia (in kg.m<sup>2</sup>) of the flywheel to be mounted on the engine shaft is
- (a) 0.053 (b) 0.113  
(c) 0.096 (d) 0.071

**Ans. (c)**

Given:  $\Delta E = 300$  J,  $\omega_{\min} = 220$  rad/s,  $\omega_{\max} = 280$  rad/s,  $C_s = 0.05$ ,  $I = ?$

Energy fluctuation,  $\Delta E = I\omega^2 C_s$

$$\Rightarrow \Delta E = I \times \left( \frac{\omega_{\max} + \omega_{\min}}{2} \right)^2 \times C_s$$

$$\Rightarrow 300 = I \times \left( \frac{280 + 220}{2} \right)^2 \times 0.05$$

$$\Rightarrow I = 0.096 \text{ kgm}^2$$

End of Solution

- Q.24** The allowance provided to a pattern for easy withdrawal from a sand mold is
- (a) finishing allowance (b) shake allowance  
(c) shrinkage allowance (d) distortion allowance

**Ans. (b)**

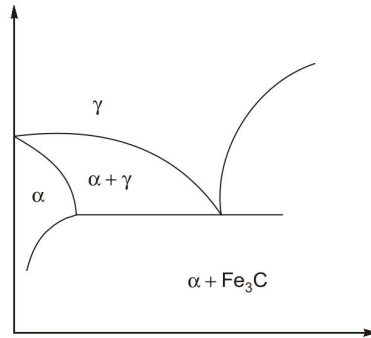
Shake allowance is also known as rapping allowance. Before the withdrawal from the sand mould, the pattern is rapped/shaken all around the vertical faces to enlarge the mould cavity slightly, which facilitates its easy removal.

End of Solution

**Q.25** The phases present in pearlite are

- (a) cementite and austenite                      (b) ferrite and cementite  
(c) austenite and ferrite                         (d) martensite and ferrite

**Ans. (b)**



**End of Solution**

**Q.26** At the current basic feasible solution (bfs)  $\mathbf{v}_o (\mathbf{v}_o \in \mathbb{R}^5)$ , the simplex method yields the following form of a linear programming problem in standard form.

$$\begin{aligned} \text{Minimize,} \quad & z = -x_1 - 2x_2 \\ \text{st.} \quad & x_3 = 2 + 2x_1 - x_2 \\ & x_4 = 7 + x_1 - 2x_2 \\ & x_5 = 3 - x_1 \end{aligned}$$

$$x_1, x_2, x_3, x_4, x_5 \geq 0$$

Here the objective function is written as a function of the non-basic variables. if the simplex method moves to the adjacent bfs  $\mathbf{v}_1 (\mathbf{v}_1 \in \mathbb{R}^5)$  that best improves the objective function, which of the following represents the objective function at  $\mathbf{v}_1$ , assuming that the objective function is written in the same manner as above?

- (a)  $z = -4 - 5x_1 + 2x_4$                       (b)  $z = -4 - 5x_1 + 2x_3$   
(c)  $z = -6 - 5x_1 + 2x_3$                       (d)  $z = -3 - x_5 - 2x_2$

**Ans. (b)**

$$z = -x_1 - 2x_2$$

$$x_3 = 2 + 2x_1 - x_2 \quad \dots(i)$$

$$x_4 = 7 + x_1 - 2x_2 \quad \dots(ii)$$

$$x_5 = 3 - x_1 \quad \dots(iii)$$

$$x_1, x_2, x_3, x_4, x_5 \geq 0 \quad \dots(iv)$$



From equation (i);

$$x_2 = 2 + 2x_1 - x_3$$

On substituting this value in the objective function,

$$z = -x_1 - 2(2 + 2x_1 - x_3)$$

$$\Rightarrow z = -4 - 5x_1 + 2x_3$$

Hence option (b) is correct.

Also from equation (iii);

$$x_1 = 3 - x_5$$

On substituting this value in the objective function,

$$z = -(3 - x_5) - 2x_2$$

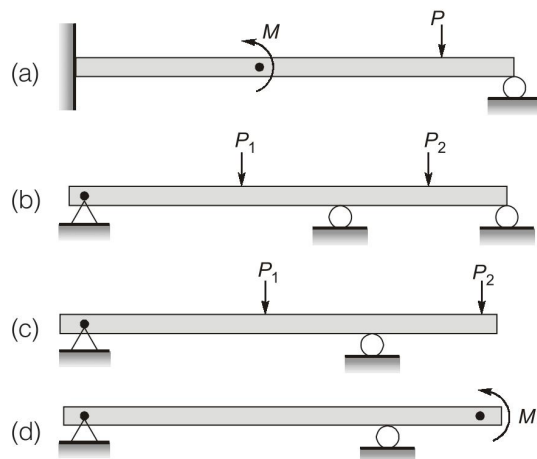
$$\Rightarrow z = -3 + x_5 - 2x_2$$

Hence option (d) is correct.

As per the given options, (b) and (d) both are correct. But since it is an MCQ problem, we will mark option (b) as correct option.

End of Solution

**Q.27** Which of the following beam(s) is/are statically indeterminate?



**Ans. (b)**

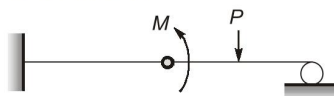
Statically indeterminate structures are those structures that cannot be analyzed using statics or equations of equilibrium. In such cases, the number unknowns exceeds the number of equilibrium equation available.

Checking each option:

**For option (a),**

Number of unknown = 3

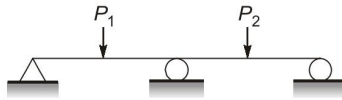
Number of equilibrium equation = 3



**For option (b),**

Number of unknown = 3

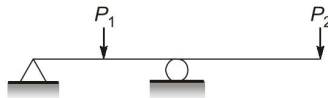
Number of equilibrium equation = 2



For option (c),

Number of unknown = 2

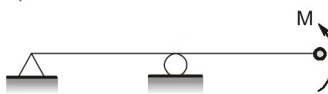
Number of equilibrium equation = 2



For option (d),

Number of unknown = 2

Number of equilibrium equation = 2

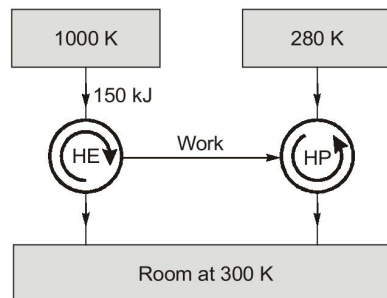


So, (b) is statically indeterminate structures.

Note : Official answer key has been given as option (a) and (b), which can be challenged by students.

End of Solution

- Q.28** A heat pump (HP) is driven by the work output of a heat engine (HE) as shown in the figure. The heat engine extracts 150 kJ of heat from the source at 1000 K. The heat pump absorbs heat from the ambient at 280 K and delivers heat to the room which is maintained at 300 K. Considering the combined system to be ideal, the total amount of heat delivered to the room together by the heat engine and heat pump is \_\_\_\_ kJ. [Answer in integer]



**Ans. (1620) (1620 to 1620)**

Given:

$$T_1 = 1000 \text{ K}, \quad T_2 = 300 \text{ K} = T_4$$

$$Q_1 = 150 \text{ kJ}$$

$$T_3 = 280 \text{ K}$$

For heat engine,

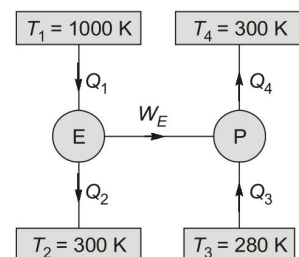
$$\eta = 1 - \frac{T_2}{T_1} = 1 - \frac{300}{1000} = 1 - \frac{Q_2}{Q_1}$$

or

$$Q_2 = 0.3Q_1 = 0.3 \times 150 = 45 \text{ kJ}$$

Now,

$$Q_1 - Q_2 = 150 - 45$$



$$\Rightarrow W_E = 105 \text{ kJ}$$

Now,

$$\text{COP}_{\text{HP}} = \frac{Q_4}{W_E} = \frac{T_4}{T_4 - T_3}$$

$$\therefore \frac{Q_4}{105} = \frac{300}{300 - 280}$$

$$\therefore Q_4 = 1575 \text{ kJ}$$

$$\therefore Q_2 + Q_4 = 1575 + 45 = 1620 \text{ kJ}$$

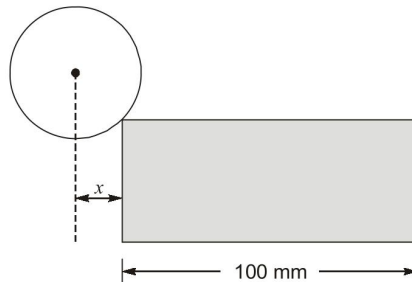
Hence, total amount of heat delivered to the room together by the heat engine and heat pump is 1620 kJ.

**End of Solution**

**Q.29** A flat surface of a C60 steel having dimensions of 100 mm (length)  $\times$  200 mm (width) is produced by a HSS slab mill cutter. The 8-toothed cutter has 100 mm diameter and 200 mm width. The feed per tooth is 0.1 mm, cutting velocity is 20 m/min and depth of cut is 2 mm. The machining time required to remove the entire stock is \_\_\_\_\_ minutes. [Rounded off to 2 decimal places]

**Ans. (2.24) (2.20 to 2.65)**

Given:  $L \times B = 100 \text{ mm} \times 200 \text{ mm}$ ,  $Z = 8$ ,  $D = 100 \text{ mm}$ ,  $W = 200 \text{ mm}$ ,  $f_t = 0.1 \text{ mm/tooth}$ ,  $V = 20 \text{ m/min}$ ,  $d = 2 \text{ mm}$



$$\text{Approach length, } x = \sqrt{d(D-d)} = \sqrt{2 \times (100-2)} = 14 \text{ mm}$$

$\therefore$  The effective length,  $L_e = L + x = 100 + 14 = 114 \text{ mm}$

$$N = \frac{V \times 1000}{\pi \times D} = \frac{20 \times 1000}{\pi \times 100} = 63.662 \text{ rpm}$$

$$F = f_t \times NZ = 0.1 \times 63.662 \times 8 = 50.93 \text{ mm/min}$$

$\therefore$  The machining time required to remove the entire stock;

$$t_{m/c} = \frac{L_e}{F} = \frac{114}{50.93} = 2.238 \text{ minutes}$$

$$\simeq 2.24 \text{ minutes}$$

**End of Solution**

**Q.30** Let  $X$  be a continuous random variable defined on  $[0, 1]$  such that its probability density function  $f(x) = 1$  for  $0 \leq x \leq 1$  and 0 otherwise. Let  $Y = \log_e(X + 1)$ . Then the expected value of  $Y$  is \_\_\_\_\_. [Rounded off to 2 decimal places]

**Ans.** (0.39) (0.38 to 0.39)

$$f(x) = \begin{cases} 1 & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Expected value of  $Y$ ,

$$\begin{aligned} E[Y] &= \int_{-\infty}^{\infty} Y \cdot f(x) \\ &= \int_0^1 1 \cdot \ln(x+1) dx = \int_0^1 \ln(x+1) dx \\ &= \int_1^2 \ln t \, dt \quad \left[ \begin{array}{l} \because x+1=t \\ \therefore dx=dt \end{array} \right] \\ &= [t \cdot \ln t - t]_1^2 = (2\ln 2 - 2) - (1\ln 1 - 1) \\ &= -0.6137 + 1 \\ &= 0.3863 \simeq 0.39 \end{aligned}$$

**End of Solution**

**Q.31** The matrix  $\begin{bmatrix} 1 & a \\ 8 & 3 \end{bmatrix}$  (where  $a > 0$ ) has a negative eigen value if  $a$  is greater than

- |                   |                   |
|-------------------|-------------------|
| (a) $\frac{1}{8}$ | (b) $\frac{1}{4}$ |
| (c) $\frac{3}{8}$ | (d) $\frac{1}{5}$ |

**Ans.** (c)

For a negative eigen value, product of eigen values  $< 0$

$$\Rightarrow \text{Determinant} < 0$$

$$\Rightarrow 3 - 8a < 0$$

$$\Rightarrow 8a > 3$$

$$\Rightarrow a > \frac{3}{8}$$

**End of Solution**

**Q.32** A company orders gears in conditions identical to those considered in the economic order quantity (EOQ) model in inventory control. The annual demand is 8000 gears, the cost per order is 300 rupees, and the holding cost is 12 rupees per month per gear. The company uses an order size that is 25% more than the optimal order quantity determined by the EOQ model. The percentage change in the total cost of ordering and holding inventory from that associated with the optimal order quantity is

- (a) 0  
(c) 12.5
- (b) 5  
(d) 2.5

**Ans. (d)**

Given:  $D = 8000$  units/yr,  $C_o = \text{Rs. } 300/\text{order}$ ,  $C_h = \text{Rs. } 12 \times 12 = \text{Rs. } 144$  per year

$$\text{Now, } \text{EOQ} = \sqrt{\frac{2C_o D}{C_h}} = \sqrt{\frac{2 \times 300 \times 8000}{144}}$$

$$\therefore Q^* = 182.5 \text{ units}$$

$$\text{Now, Actual quantity, } Q = 1.25Q^* = 1.25 \times 182.5 \\ = 228.125 \text{ units}$$

$$\text{Total cost at EOQ, } \text{TIC}_1 = \sqrt{2C_o C_h D} = \sqrt{2 \times 300 \times 144 \times 8000} \\ = \text{Rs. } 26290.68$$

Now, Total cost at  $Q = 228.125$  units,

$$\text{TIC}_2 = \frac{Q}{2} \times C_h + \frac{D}{Q} \times C_o \\ = \frac{228.125}{2} \times 144 + \frac{8000}{228.125} \times 300 \\ = \text{Rs. } 26945.54$$

$$\therefore \% \text{ increase} = \frac{26945.54 - 26290.68}{26290.68} \times 100 = 2.5\%$$

**Alternative solution**

We know, if  $Q = kQ^*$

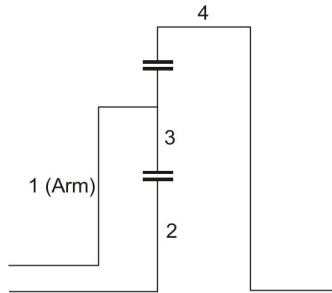
$$\text{then, } \frac{\text{TIC}(Q)}{\text{TIC}(Q^*)} = \frac{1}{2} \left[ k + \frac{1}{k} \right]$$

$\therefore$  Percentage increase,

$$\frac{\text{TIC}(Q) - \text{TIC}(Q^*)}{\text{TIC}(Q^*)} \times 100 = \left( \frac{\text{TIC}(Q)}{\text{TIC}(Q^*)} - 1 \right) \times 100 \\ = \left( \frac{1}{2} \left[ k + \frac{1}{k} \right] - 1 \right) \times 100 = \left( \frac{1}{2} \left[ 1.25 + \frac{1}{1.25} \right] - 1 \right) \times 100 \\ = 0.025 \times 100 \\ = 2.5\%$$

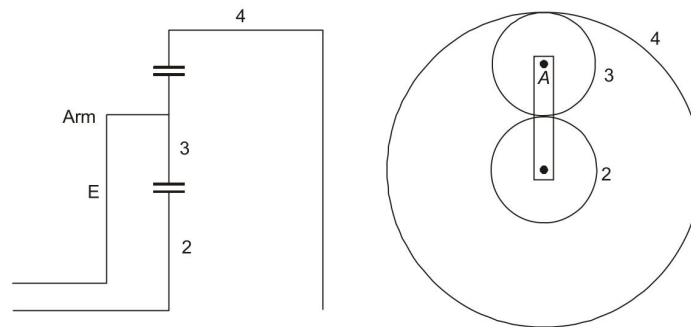
**End of Solution**

**Q.33** The Levai type-A train illustrated in the figure has gears with module  $m = 8 \text{ mm/tooth}$ . Gears 2 and 3 have 19 and 24 teeth respectively. Gear 2 is fixed and internal gear 4 rotates at 20 rev/min counter-clockwise. The magnitude of angular velocity of the arm is \_\_\_\_ rev/min. (rounded off to 2 decimal places)



**Ans.** (15.58) (15.00 to 16.00)

**Gear train:**



Given:  $T_2 = 19$ ,  $T_3 = 24$ ,  $N_4 = 0$ ,  $N_4 = 20 \text{ rpm (CCW)}$

As

$$R_2 + 2R_3 = R_4$$

$$\Rightarrow T_2 + 2T_3 = T_4$$

$$\Rightarrow T_4 = 19 + 2 \times 24 = 67$$

	A	Gear 2	Gear 3	Gear 4
Arm is fixed	0	$+x$	$-x \times \frac{T_2}{T_3} = -x \times \frac{19}{24}$	$-x \times \frac{T_2}{T_3} \times \frac{T_3}{T_4} = -x \times \frac{19}{27} \times \frac{27}{67}$ $= -x \times \frac{19}{67}$
Arm is free	$+y$	$+y + x$	$+y - x \times \frac{19}{24}$	$+y - x \times \frac{19}{67}$

$$N_2 = y + x \Rightarrow x + y = 0 \quad \dots (i)$$

$$\text{and} \quad N_4 = 20 \text{ rpm} \Rightarrow y - x \times \frac{19}{67} = 20 \quad \dots (ii)$$

On solving equation (i) and (ii);

$$x = -15.58 \text{ rpm} = 15.58 \text{ rpm (CW)}$$

$$y = 15.58 \text{ (CCW)}$$

**End of Solution**

**Q.34** If  $x(t)$  satisfies the differential equation

$$t \frac{dx}{dt} + (t - x) = 0$$

subject to the condition  $x(1) = 0$ , then the value of  $x(2)$  is \_\_\_\_\_. (rounded off to 2 decimal places)

**Ans.** (-1.39) (-1.40 to -1.38)

$$t \frac{dx}{dt} + t - x = 0$$

$$\frac{dx}{dt} - \frac{x}{t} + 1 = 0$$

$$\frac{dx}{dt} - \frac{1}{t}x = -1$$

This is a linear DE of the form,

$$\frac{dx}{dt} + Px = Q$$

where,  $P = -\frac{1}{t}$  and  $Q = -1$

Integrating factor,  $IF = e^{\int -\frac{1}{t} dx} = e^{-\ln t} = e^{\ln\left(\frac{1}{t}\right)} = \frac{1}{t}$

Thus, the solutions is

$$x \times IF = \int Q \cdot IF dt + c$$

$$x \cdot \frac{1}{t} = \int -1 \cdot \frac{1}{t} dt + c$$

$$\frac{x}{t} = -\ln t + c \Rightarrow \frac{x}{t} = \ln\left(\frac{1}{t}\right) + c$$

$$\frac{x}{t} = \ln\frac{1}{t} + c \Rightarrow x = t \cdot \ln\frac{1}{t} + ct$$

Given, at  $t = 1$ ,  $x = 0$

$$\Rightarrow 0 = 0 + c \Rightarrow c = 0$$

Thus, particular solution is,

$$x = t \cdot \ln\frac{1}{t}$$

Now, for  $t = 2$ ,

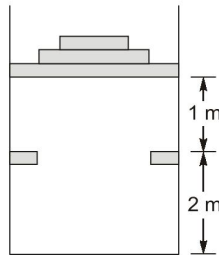
$$x = 2 \ln\frac{1}{2} ; \quad x = -2 \ln 2$$

$$x = -2 \times 0.693 = -1.386 \simeq -1.39$$

End of Solution

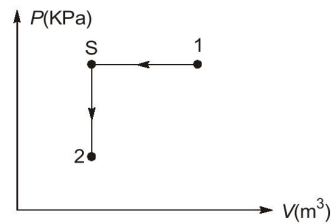
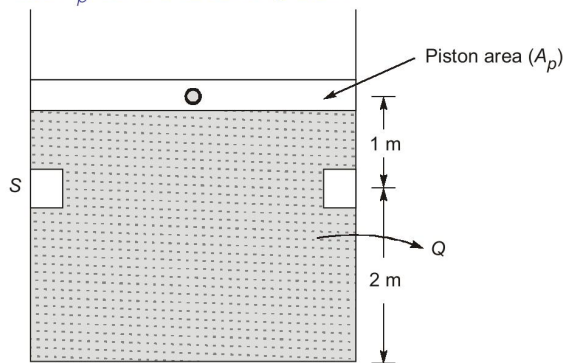


- Q.35** A piston-cylinder arrangement shown in the figure has a stop located 2 m above the base. The cylinder initially contains air at 140 kPa and 350°C and the piston is resting in equilibrium at a position which is 1 m above the stops. The system is now cooled to the ambient temperature of 25°C. Consider air to be an ideal gas with a value of gas constant  $R = 0.287 \text{ kJ/kgK}$ . The absolute value of specific work done during the process is \_\_\_\_\_ kJ/kg. (rounded off to 1 decimal place)



**Ans. (59.6) (58.0 to 61.0)**

Let  $A_p$  be the area of piston.



Given:

$$P_1 = 140 \text{ KPa}$$

$$T_1 = 350^\circ\text{C} = 350 + 273 = 623 \text{ K}$$

$$T_2 = 25^\circ\text{C} = 25 + 273 = 298 \text{ K}$$

$$\begin{aligned} W_{1-s} &= P_1 V_1 - P_s V_s \\ &= mR(T_1 - T_s) \end{aligned}$$

For constant pressure process:

$$V \propto T$$

$$\Rightarrow \frac{V_1}{V_s} = \frac{T_1}{T_s}$$

$$\Rightarrow \frac{A_p \times 3}{A_p \times 2} = \frac{623}{T_s} \Rightarrow T_s = 415.33 \text{ K}$$

$$W_{1-s} = 1 \times 0.287 \times (623 - 415.33) = 59.60 \text{ kJ/kg}$$

$$W_{s-2} = 0 \quad [\because \text{Constant volume process}]$$

$$\begin{aligned} \therefore W_{1-2} &= W_{1-s} + W_{s-2} \\ &= 59.60 + 0 = 59.60 \text{ kJ/kg} \end{aligned}$$

End of Solution

**Q.36** Aluminium is casted in a cube-shaped mold having dimensions as 20 mm × 20 mm × 20 mm. Another mold of the same mold material is used to cast a sphere of aluminium having a diameter of 20 mm. The pouring temperature for both cases is the same. The ratio of the solidification times of the cube-shaped mold to the spherical mold is \_\_\_\_\_. (answer in integer)

**Ans. (1) (1 to 1)**

Given: Cube shaped mould = 20 mm × 20 mm × 20 mm

Sphere shaped mould,  $D = 20$  mm

By Chvorinov's rule, solidification time ( $t_s$ ) is proportional to square of ratio of volume to surface area i.e.,

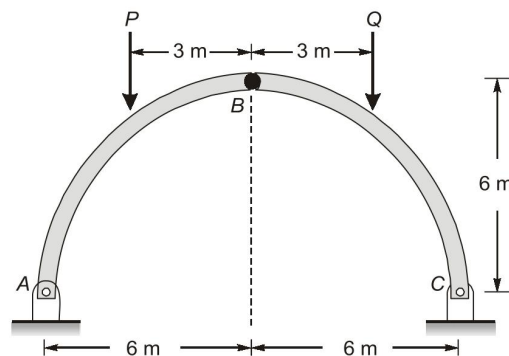
$$t_s \propto \left( \frac{V}{A} \right)^2$$

$$\frac{(t_s)_c}{(t_s)_{sp}} = \frac{(V/A)_c^2}{(V/A)_{sp}^2} = \frac{(a^3/6a^2)^2}{\left( \frac{\pi D^3}{6} \right)^2}$$

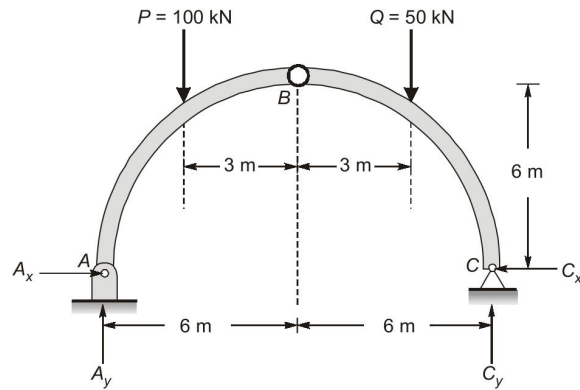
$$= \frac{(a/6)^2}{(D/6)^2} = 1 \quad [\because a = D = 20 \text{ mm}]$$

**End of Solution**

**Q.37** A three-hinge arch ABC in the form of semi-circle is shown in the figure. The arch is in static equilibrium under vertical loads of  $P = 100$  kN and  $Q = 50$  kN. Neglect friction at all the hinges. The magnitude of the horizontal reaction at B is \_\_\_\_\_. kN. (rounded off to 1 decimal place)



Ans. (37.5) (37.0 to 38.0)



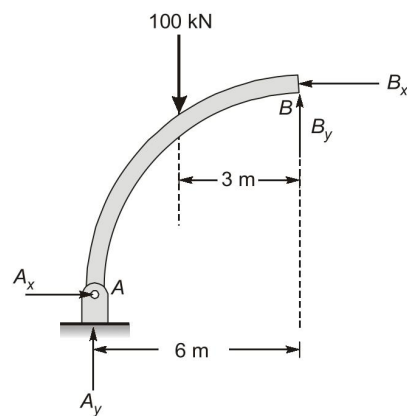
Initially, taking moment about hinged point C,

$$\Sigma M_C = 0$$

$$A_y \times 12 - 100 \times 9 - 50 \times 3 = 0$$

$$\Rightarrow A_y = 87.5 \text{ kN}$$

FBD for left half section:



Now taking moment about point B.

$$\Sigma M_B = 0$$

$$A_x \times 6 - 87.5 \times 6 + 100 \times 3 = 0$$

$$\Rightarrow A_x = 37.5 \text{ kN}$$

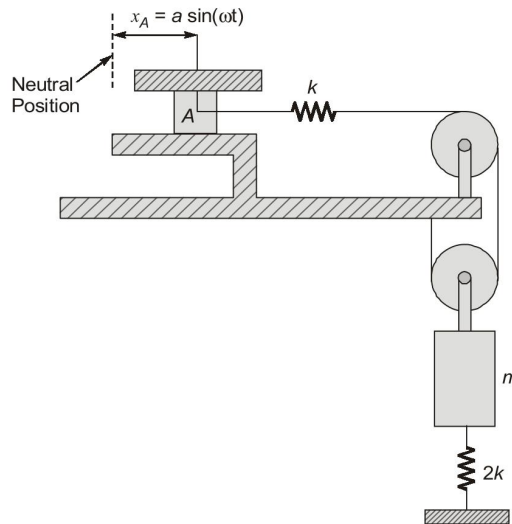
$$\Sigma F_x = 0$$

$$A_x - B_x = 0$$

$$\Rightarrow B_x = A_x = 37.5 \text{ kN}$$

End of Solution

- Q.38** A vibratory system consists of mass  $m$ , a vertical spring of stiffness  $2k$  and a horizontal spring of stiffness  $k$ . The end A of the horizontal spring is given a horizontal motion  $x_A = a \sin \omega t$ . The other end of the spring is connected to an inextensible rope that passes over two massless pulleys as shown. Assume  $m = 10 \text{ kg}$ ,  $k = 1.5 \text{ kN/m}$ , and neglect friction. The magnitude of critical driving frequency for which the oscillations of mass  $m$  tend to become excessively large is \_\_\_\_\_ rad/s. (answer in integer)



**Ans.** (30) (29 to 31)

Given :  $m = 10 \text{ kg}$ ;  $k = 1.5 \text{ kN/m} = 1500 \text{ N/m}$ ;  $\omega_n = ?$

At time  $t = t$ ;

Using D'Alembert's principle;

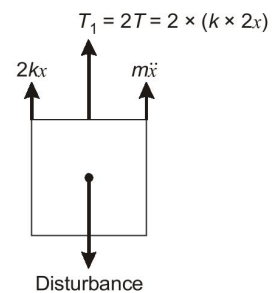
$$m\ddot{x} + 4kx + 2kx = 0$$

$$m\ddot{x} + 6kx = 0$$

$$\Rightarrow \ddot{x} + \left(\frac{6k}{m}\right)x = 0; \quad \text{equation of motion}$$

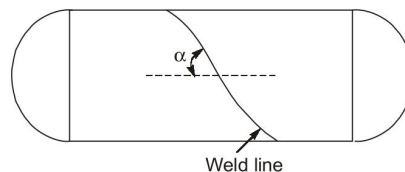
So,

$$\omega_n = \sqrt{\frac{6k}{m}} = \sqrt{\frac{6 \times 1500}{10}} = 30 \text{ rad/s}$$



**End of Solution**

- Q.39** The figure shows a thin cylindrical pressure vessel constructed by welding plates together along a line that makes an angle  $\alpha = 60^\circ$  with the horizontal. The closed vessel has a wall thickness of 10 mm and diameter of 2 m. When subjected to an internal pressure of 200 kPa, the magnitude of the normal stress acting on the weld is \_\_\_\_\_ MPa. (rounded off to 1 decimal place)

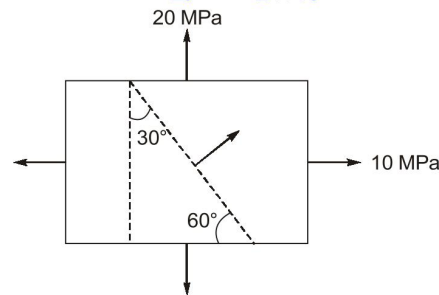


Ans. (12.5) (12.3 to 12.7)

Given:  $P = 0.2 \text{ MPa}$ ,  $D = 2 \text{ m}$ ,  $t = 10 \text{ mm}$

$$\sigma_x = \sigma_L = \frac{PD}{4t} = \frac{0.2 \times 2000}{4 \times 10} = 10 \text{ MPa}$$

$$\sigma_y = \sigma_h = \frac{PD}{2t} = \frac{0.2 \times 2000}{2 \times 10} = 20 \text{ MPa}$$



$$\text{Normal stress, } (\sigma_n)_\theta = \left( \frac{\sigma_x + \sigma_y}{2} \right) + \left( \frac{\sigma_x - \sigma_y}{2} \right) \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$(\sigma_n)_{\theta=30^\circ} = \left( \frac{10 + 20}{2} \right) + \left( \frac{10 - 20}{2} \right) \cos(2 \times 30^\circ) = 12.5 \text{ MPa}$$

End of Solution

**Q.40** A liquid fills a horizontal capillary tube whose one end is dipped in a large pool of the liquid. Experiments show that the distance  $L$  travelled by the liquid meniscus inside the capillary in time  $t$  is given by

$$L = k\gamma^a R^b \mu^c \sqrt{t},$$

where  $\gamma$  is the surface tension,  $R$  is the inner radius of the capillary, and  $\mu$  is the dynamic viscosity of the liquid. If  $k$  is a dimensionless constant, then the exponent  $a$  is \_\_\_\_\_ (rounded off to 1 decimal place)

Ans. (0.5) (0.5 to 0.5)

Dimension of surface tension =  $[MT^{-2}]$

Dimension of radius =  $[L]$

Dimension of dynamic viscosity =  $[ML^{-1}T^{-1}]$

Dimension of time =  $[T]$

Now, 
$$L = k\gamma^a R^b \mu^c \sqrt{t}$$

$$\Rightarrow [L] = [M^0 L^0 T^0] [MT^{-2}]^a [L]^b [ML^{-1}T^{-1}]^c [T]^{1/2}$$

$$\Rightarrow a + c = 0 \quad \dots(i)$$

$$\Rightarrow b - c = 1 \quad \dots(ii)$$

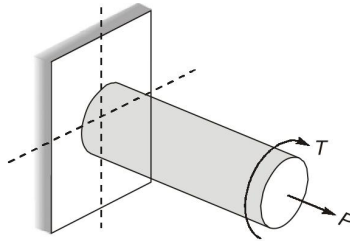
$$-2a - c + \frac{1}{2} = 0 \quad \dots(iii)$$

On solving equation (i), (ii) and (iii), we get

$$a = 0.5, b = 0.5 \text{ and } c = -0.5$$

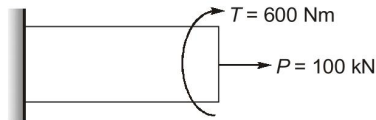
End of Solution

- Q.41** A solid massless cylindrical member of 50 mm diameter is rigidly attached at one end, and is subjected to an axial force  $P = 100$  kN and a torque  $T = 600$  Nm at the other end as shown. Assume that the axis of the cylinder is normal to the support. Considering distortion energy theory with allowable yield stress as 300 MPa, the factor of safety in the design is \_\_\_\_\_ (rounded off to 1 decimal place)



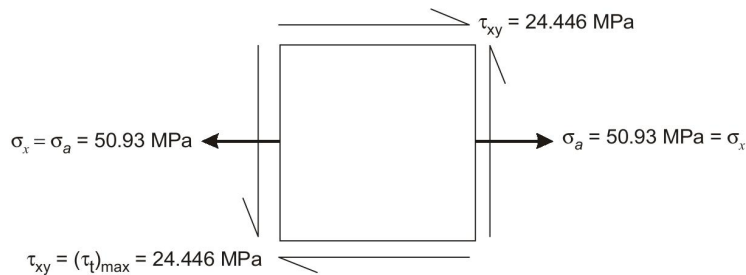
**Ans. (4.5) (4.3 to 4.7)**

Given :  $D = 50$  mm;  $S_{yt} = 300$  MPa



$$\text{Axial stress due to } P, \sigma_x = \sigma_a = \frac{P}{A} = \frac{100 \times 10^3}{\left(\frac{\pi \times 50^2}{4}\right)} = 50.93 \text{ MPa}$$

$$\text{Shear stress due to } T, \tau_{xy} = (\tau_t)_{\max} = \frac{16 \times T}{\pi D^3} = \frac{16 \times 600 \times 10^3}{\pi \times 50^3} = 24.446 \text{ MPa}$$



$$\sigma_y = 0$$

$$(\sigma_t)_{\text{per}} \geq \sqrt{\sigma_x^2 + 3\tau_{xy}^2}$$

$$\frac{S_{yt}}{N} = \frac{300}{N} \geq \sqrt{50.93^2 + 3 \times 24.446^2}$$

$$N \leq \frac{300}{66.232}$$

$$N \leq 4.529$$

$$N = 4.5$$

End of Solution

**Q.42** In an arc welding process, the voltage and current as 30 V and 200 A, respectively. The cross-sectional area of the joint is 20 mm<sup>2</sup> and the welding speed is 5 mm/s. The heat required to melt the material is 20 J/s. The percentage of heat lost to the surrounding during the welding process is \_\_\_\_\_. (rounded off to 2 decimal places)

**Ans. (\*)**

Given :  $V = 30 \text{ V}$ ;  $I = 200 \text{ Amp}$ ,  $A_c = 20 \text{ mm}^2$ ,  $v = 5 \text{ mm/sec}$

Assuming,  $H_{\text{req}} = 20 \text{ J/s}$

Heat required to melt ( $H_m$ ) = 20 J/s

$$\text{Heat lost to surrounding} = \left( \frac{H_s - H_m}{H_s} \right) \times 100$$

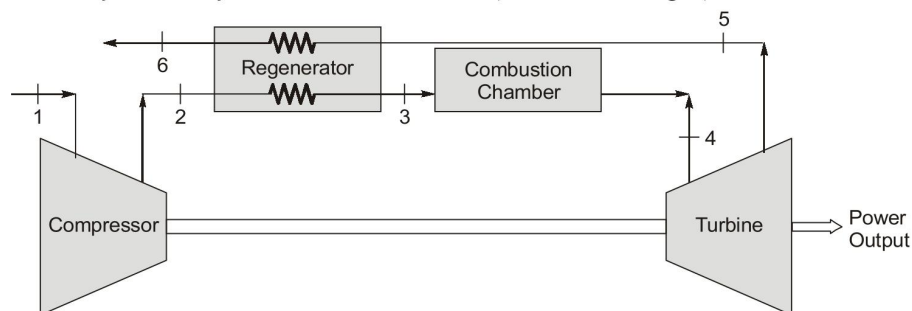
$$\text{Heat supplied } (H_s) = VI = 30 \times 200 = 6000 \text{ W} = 6000 \text{ J/s}$$

$$\text{Heat lost} = \left( \frac{6000 - 20}{6000} \right) \times 100 = 99.67\%$$

**Note :** Official answer key needs to be challenged by students as 66.67% can be the answer if heat required to melt the material were given as 20 J/mm<sup>3</sup>.

**End of Solution**

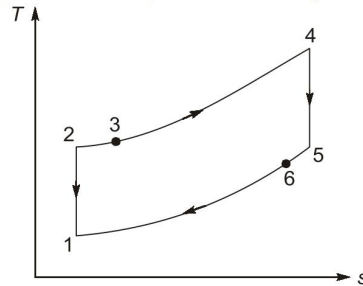
**Q.43** Consider an air-standard Brayton cycle with adiabatic compressor and turbine, and a regenerator, as shown in the figure. Air enters the compressor at 100 kPa and 300 K and exits the compressor at 600 kPa and 550 K. The air exits the combustion chamber at 1250 K and exits the adiabatic turbine at 100 kPa and 800 K. The exhaust air exits the regenerator (state 6) at 600 K. There is no pressure drop across the regenerator and the combustion chamber. Also, there is no heat loss from the regenerator to the surroundings. The ratio of specific heats at constant pressure and volume is  $c_p / c_v = 1.4$ . The thermal efficiency of the cycle is \_\_\_\_\_. (answer in integer)





Ans. (40) (40 to 40)

Given:  $T_1 = 300$  K,  $T_2 = 550$  K,  $T_4 = 1250$  K,  $T_5 = 800$  K,  $T_6 = 600$  K,



From energy balance across regenerator

$$\begin{aligned} T_3 - T_2 &= T_5 - T_6 \\ T_3 &= 550 = 800 - 600 \\ T_3 &= 750 \text{ K} \end{aligned}$$

Now,

$$\begin{aligned} \eta &= 1 - \frac{Q_R}{Q_s} = 1 - \frac{(T_6 - T_1)}{(T_4 - T_3)} = 1 - \frac{(600 - 300)}{(1250 - 750)} \\ \eta &= 1 - \frac{300}{500} = 0.4 \text{ or } 40\% \end{aligned}$$

End of Solution

**Q.44** In a supplier-retailer supply chain, the demand of each retailer, the capacity of each supplier, and the unit cost in rupees of material supply from each supplier to each retailer are tabulated below. The supply chain manager wishes to minimize the total cost of transportation across the supply chain.

	Retailer I	Retailer II	Retailer III	Retailer IV	Capacity
Supplier A	11	16	19	13	300
Supplier B	5	10	7	8	300
Supplier C	12	14	17	11	300
Supplier D	8	15	11	9	300
Demand	300	300	300	300	

The optimal cost of satisfying the total demand from all retailers is \_\_\_\_ rupees. (answer in integer)

Ans. (12300) (12300 to 12300)

As per Hungarian method, subtracting the smallest value from each column from other elements of that column, we get

6	6	12	5
0	0	0	0
7	4	10	3
3	5	4	1

Similarly, for each row, we get the following matrix.

1	1	7	0
0	0	0	0
4	1	7	0
2	4	3	0

Since, the minimum number of lines to cover each zero of every row and column is less than the order of matrix. So to get the opportunity matrix, we will subtract smallest value from every element without lines and adding the same at intersection we will get the following matrix.

0	0	6	0
0	0	0	0
3	0	6	0
1	3	2	0

Since the minimum number of lines is equal to the order of matrix. Assignment can be done in the matrix as shown below.

0	0	6	X
X	X	0	0
3	0	6	X
1	3	2	0

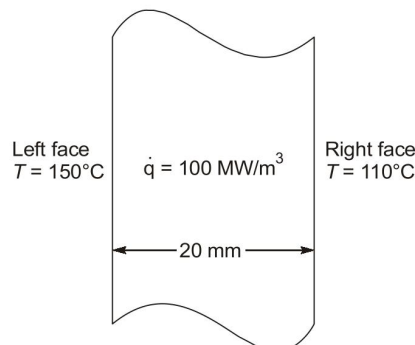
So, optimal solution is

$$= (11 + 7 + 14 + 9) \times 300$$

$$= ₹12300$$

End of Solution

- Q.45** Consider a slab of 20 mm thickness. There is a uniform heat generation of  $\dot{q} = 100 \text{ MW/m}^3$  inside the slab. The left right faces of the slab are maintained at  $150^\circ\text{C}$  and  $110^\circ\text{C}$ , respectively. The plate has a constant thermal conductivity of  $200 \text{ W/(mK)}$ . Considering a 1-D steady state heat conduction, the location of the maximum temperature from the left face will be at \_\_\_\_\_ mm. (answer in integer)



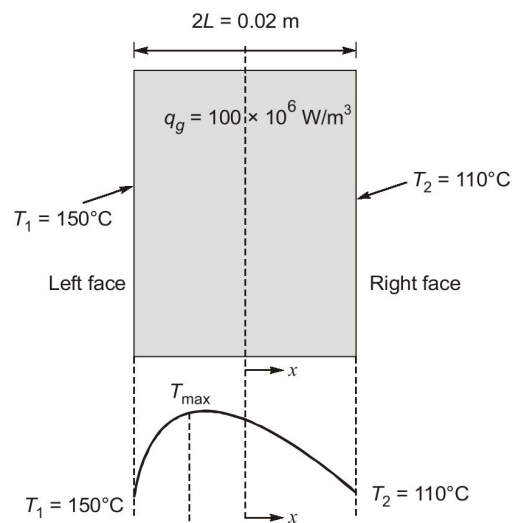
Ans. (6) (6 to 6)

For uniform internal heat generation and constant thermal conductivity in a rectangular slab,

$$\frac{\partial^2 T}{\partial x^2} + \frac{q_g}{k} = 0$$

$$\Rightarrow T = -\frac{q_g x^2}{2k} + C_1 x + C_2$$

Temperature variation is parabolic inside the slab



At  $x = -L$ ,  $T = T_1 = 150^\circ\text{C}$

$$\Rightarrow 150 = -\frac{q_g x^2}{2k} - C_1 L + C_2 \quad \dots (i)$$

and at  $x = L$ ,  $T = T_2 = 110^\circ\text{C}$

$$110 = -\frac{q_g L^2}{2k} + C_1 L + C_2 \quad \dots (ii)$$

Using equation (i) and (ii)

$$C_1 = \frac{T_2 - T_1}{2L} = \frac{110 - 150}{2 \times (0.01)} = -2000$$

At  $T_{\max}$ ,  $\frac{dT}{dx} = 0$

$$\Rightarrow \frac{dT}{dx} = -\frac{q_g x}{k} + C_1$$

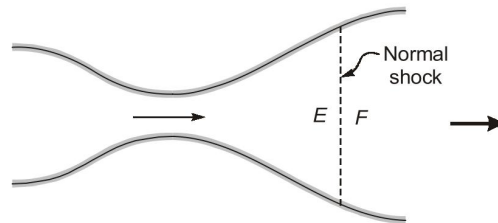
$$\Rightarrow 0 = -\frac{(100 \times 10^6)x}{200} - 2000$$

$$\Rightarrow x = -4 \times 10^{-3} \text{ m} = -4 \text{ mm}$$

Distance of  $T_{\max}$  left face  $= L - |x| = 10 - 4 = 6 \text{ mm}$

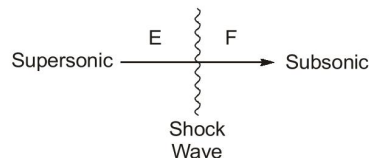
End of Solution

- Q.46** Steady, compressible flow of air takes place through an adiabatic converging-diverging nozzle, as shown in the figure. For a particular value of pressure difference across the nozzle, a stationary normal shock wave forms in the diverging section of the nozzle. If E and F denote the flow conditions just upstream and downstream of the normal shock, respectively, which of the following statement(s) is/are TRUE?



- (a) Mach number at E is lower than the Mach number at F
- (b) Density at E is lower than the density at F
- (c) Specific entropy at E is lower than the specific entropy at F
- (d) Static pressure at E is lower than the static pressure at F

**Ans. (b, c, d)**



**After normal shock:**

- Stagnation temperature remains the same.
- Density after shock will increase, i.e.,  $\rho_E < \rho_F$ .
- Mach number will decrease, i.e.,  $M_E > M_F$ .
- Entropy across a shockwave always increases as it is a highly irreversible process.

**End of Solution**

- Q.47** A blanking operation is performed on C20 steel sheet to obtain a circular disc having a diameter of 20 mm and a thickness of 2 mm. An allowance of 0.04 is provided. The punch size used for the operation is \_\_\_\_\_ mm (rounded off to 2 decimal places)

**Ans. (19.84) (19.84 to 19.84)**

Diameter of blank = 20 mm

Sheet thickness,  $t = 2$  mm

Clearance allowance,  $A_c = 0.04$

$\therefore$  Clearance,  $C = A_c t = 0.04 \times 2 = 0.08$

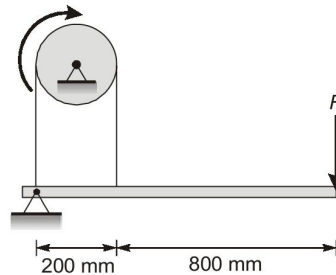
Blank size = Die size = 20 mm

Punch size = Die size  $- 2 \times C$

$= 20 - 2 \times 0.08 = 19.84$  mm

**End of Solution**

- Q.48** A band brake shown in the figure has a coefficient of friction of 0.3. The band can take a maximum force of 1.5 kN. The maximum braking force ( $F$ ) that can be safely applied is \_\_\_\_\_ N (rounded off to the nearest integer)



**Ans. (117) (115 to 119)**

Given:  $\mu = 0.3$ ,  $T_1 = 1.5$  kN

Taking moment about hinge point O.

$$\Sigma M_O = 0$$

$$\Rightarrow F \times 1000 - T_2 \times 200 = 0$$

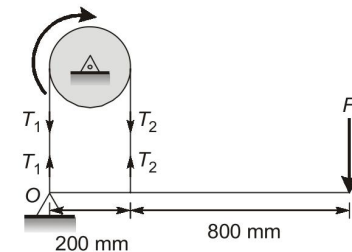
$$\Rightarrow F = \frac{T_2 \times 200}{1000} \quad \dots (i)$$

Also, 
$$\frac{T_1}{T_2} = e^{\mu\theta} \Rightarrow \frac{1.5}{T_2} = e^{0.3 \times \pi}$$

$$\Rightarrow T_2 = \frac{1.5 \times 1000}{e^{0.3 \times \pi}} = 584.49 \text{ N}$$

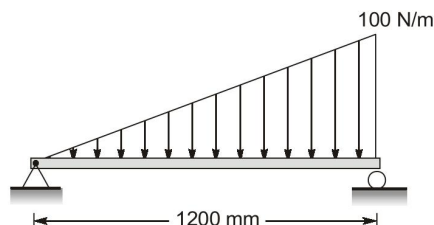
From equation (i), we have

$$F = \frac{200 \times 584.49}{1000} = 116.89 \text{ N} \simeq 117 \text{ N}$$



**End of Solution**

- Q.49** A horizontal beam of length 1200 mm is pinned at the left end and is resting on a roller at the other end as shown in the figure. A linearly varying distribution load is applied on the beam. The magnitude of maximum bending moment acting on the beam is \_\_\_\_\_ Nm. (round off to 1 decimal place)



Ans. (9.2) (9.0 to 9.5)

Reactions at the ends,

$$R_A + R_B = \frac{1}{2} \times 1.2 \times 100$$

$$\therefore R_A + R_B = 60 \text{ N} \quad \dots (i)$$

$$\Sigma M_B = 0$$

$$\therefore R_A \times 1.2 = \left( \frac{1}{2} \times 1.2 \times 100 \right) \times \frac{1.2}{3}$$

$$\therefore R_A = 20 \text{ N}$$

$$\therefore R_B = 40 \text{ N}$$

Now, SF at section 'x-x' at a distance of  $x$

$$SF_x = 0$$

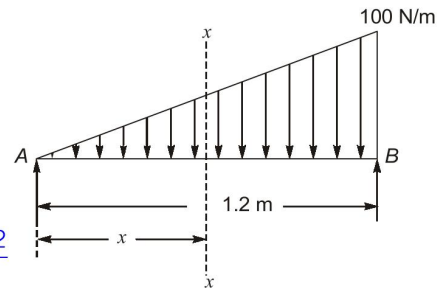
$$\therefore 20 - \frac{1}{2} \times x \times \frac{100x}{1.2} = 0$$

$$\therefore x = 0.693 \text{ m}$$

$\therefore$  At  $x = 0.693$ , BM will be maximum,

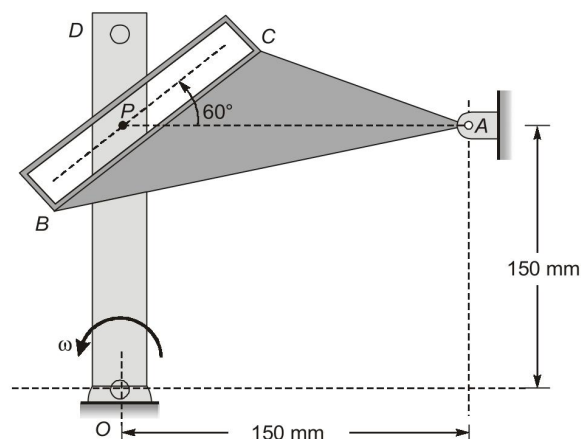
$$\therefore M_{\max} = 20 \times 0.693 - \frac{1}{2} \times \frac{0.693 \times 100 \times 0.693}{1.2} \times \frac{0.693}{3}$$

$$\therefore M_{\max} = 9.237 \text{ N-m} \simeq 9.2 \text{ N.m}$$



End of Solution

**Q.50** At the instant when OP is vertical and AP is horizontal, the link OD is rotating counter clockwise at a constant rate  $\omega = 7 \text{ rad/s}$ . Pin P on link OD slides in the slot BC of link ABC which is hinged at A, and causes a clockwise rotation of the link ABC. The magnitude of angular velocity of link ABC for this instant is \_\_\_\_\_ rad/s. (rounded off to 2 decimal places)



**Ans. (0) (0 to 0)**

At the instant when AP is horizontal and OP is vertical,  $V_P$  will come in the line of AP, then perpendicular component of  $V_P$  will be zero.

$$\Rightarrow V_P = (OP) \times \omega_{OP} \\ = (OP) \times 7 \text{ m/s; direction is along AP}$$

Hence,  $\omega_{ABC} = 0$  ; No perpendicular component of velocity

**End of Solution**

**Q.51** A condenser is used as a heat exchanger in a large steam power plant in which steam is condensed to liquid water. The condenser is a shell and tube heat exchanger which consists of 1 shell and 20000 tubes. Water flows through each of the tubes at a rate of 1 kg/s with an inlet temperature of 30°C. The steam in the condenser shell condenses at the rate of 430 kg/s at a temperature of 50°C. If the heat of vaporization is 2.326 MJ/kg and specific heat of water is 4 kJ/(kg.K), the effectiveness of the heat exchanger is \_\_\_\_\_ (rounded off to 3 decimal places)

**Ans. (0.625) (0.620 to 0.630)**

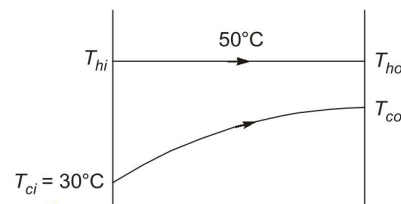
Given:  $\dot{m}_h = 430 \text{ kg/s}$ ,  $LH = 2.326 \text{ MJ/kg}$

Effectiveness,  $\epsilon = \frac{q}{q_{\max}}$

Now,  $q = \dot{m}_h \times LH = 430 \times 2.326 \times 10^6$   
 $= 1000.18 \text{ MW}$

and  $q_{\max} = C_{\min} (T_{hi} - T_{ci})$   
 $= 20000 \times 1 \times 4000 (50 - 30)$   
 $= 1600 \text{ MW}$

$\therefore \epsilon = \frac{1000.18}{1600} = 0.625$



**End of Solution**

**Q.52** A cutting tool provides a tool life of 60 minutes while machining with the cutting speed of 60 m/min. When the same tool is used for machining the same material, it provides a tool life of 10 minutes for a cutting speed of 100 m/min. If the cutting speed is changed to 80 m/min for the same tool and work material combination, the tool life computed using Taylor's tool life model is \_\_\_\_ minutes. (rounded off to 2 decimal places)

**Ans. (21.87) (21.00 to 22.50)**

Given :  $V_1 = 60 \text{ m/min}$ ;  $V_2 = 100 \text{ m/min}$ ;  $V_3 = 80 \text{ m/min}$   
 $T_1 = 60 \text{ min}$ ;  $T_2 = 10 \text{ min}$ ;  $T_3 = ?$

By using Taylor's equation,

$$V_1 T_1^n = V_2 T_2^n \\ \Rightarrow 60 \times (60)^n = 100 \times (10)^n$$



$$\Rightarrow \left(\frac{60}{10}\right)^n = \frac{100}{60} \Rightarrow n = \frac{\ln\left(\frac{100}{60}\right)}{\ln\left(\frac{60}{10}\right)} = 0.285$$

$$\begin{aligned} \text{Also, } V_1 T_1^n &= V_3 T_3^n \\ \Rightarrow 60 \times (60)^{0.285} &= 80 \times (T_3)^{0.285} \\ \Rightarrow T_3 &= 21.87 \text{ min.} \end{aligned}$$

End of Solution

**Q.53** If the value of the double integral

$$\int_{x=3}^4 \int_{y=1}^2 \frac{dy dx}{(x+y)^2}$$

is  $\log_e(a/24)$ , the  $a$  is \_\_\_\_\_ (answer in integer)

**Ans. (25) (25 to 25)**

$$\begin{aligned} \text{To evaluate, } \int_{x=3}^4 \int_{y=1}^2 \frac{dx dy}{(x+y)^2} &= \int_{x=3}^4 \left[ -\frac{1}{(x+y)} \right]_{y=1}^{y=2} = - \int_{x=3}^4 \left[ \frac{1}{2+x} - \frac{1}{x+1} \right] \\ &= - \left[ \ln(2+x) - \ln(x+1) \right]_{x=3}^{x=4} \\ &= - \left[ \ln\left(\frac{2+x}{x+1}\right) \right]_3^4 = - \left[ \ln\left(\frac{6}{5}\right) - \ln\left(\frac{5}{4}\right) \right] \\ &= - \left[ \ln\left(\frac{6 \times 4}{5 \times 5}\right) \right] = - \ln\left(\frac{24}{25}\right) = \ln\left(\frac{25}{24}\right) \end{aligned}$$

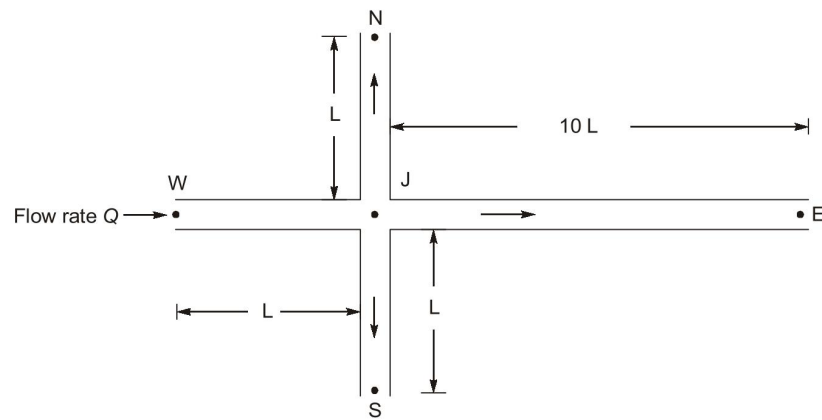
$$\begin{aligned} \text{Thus, } \log_e\left(\frac{a}{24}\right) &= \ln\left(\frac{25}{24}\right) \\ \therefore a &= 25 \end{aligned}$$

End of Solution

**Q.54** In the pipe network shown in the figure, all pipes have the same cross-section and can be assumed to have the same friction factor. The pipes, connecting points W, N, and S with point J have an equal length  $L$ . The pipe connecting points J and E has a length  $10L$ . The pressure at the ends N, E, and S are equal. The flow rate in the pipe connecting W and J is  $Q$ . Assume that the fluid flow is steady, incompressible, and the pressure losses at the pipe entrance and junction are negligible. Consider the following statements:

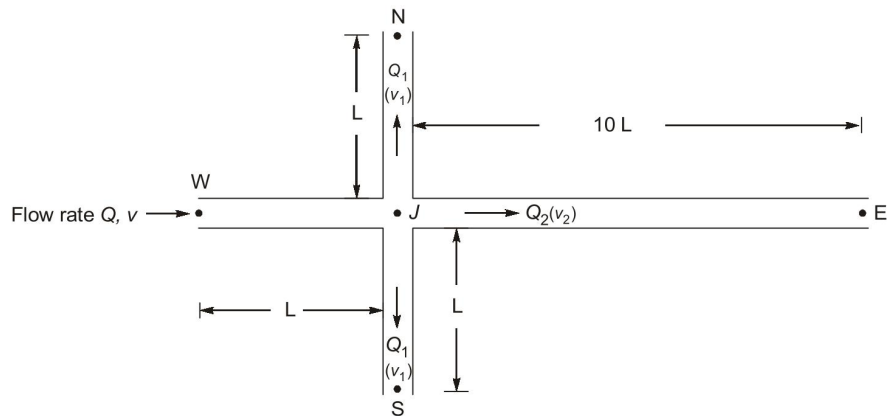
- The flow rate in pipe connecting J and E is  $Q/21$ .
- The pressure difference between J and N is equal to the pressure difference between J and E.

Which one of the following option is CORRECT?



- (a) I is False and II is True      (b) I is True and II is False  
(c) Both I and II are False      (d) Both I and II are True

Ans. (d)



$$P_N - P_J = P_E - P_J$$

$$\Rightarrow \frac{P_N - P_J}{\rho g} = \frac{P_E - P_J}{\rho g}$$

$$\Rightarrow (h_f)_{NJ} = (h_f)_{EJ}$$

$$\Rightarrow \frac{32\mu v_1 L}{\rho g d} = \frac{32\mu v_2 (10L)}{\rho g d}$$

$$\Rightarrow v_1 = 10 v_2 \quad \dots(i)$$

Using continuity equation,

$$Q = 2Q_1 + Q_2$$

$$\Rightarrow \frac{\pi}{4} d^2 v = \frac{\pi}{4} d^2 [2v_1 + v_2]$$

$$\Rightarrow v = 2v_1 + v_2 \quad \dots(ii)$$

Using equation (i) and (ii)

$$v_2 = \frac{1}{21}v \text{ and } v_1 = 10v_2 = \frac{10}{21}v$$

Discharge,  $Q_2 = \frac{\pi}{4}d^2v_2 = \frac{\pi}{4}d^2\left(\frac{1}{21}v\right) = \left(\frac{\pi}{4}d^2v\right)\frac{1}{21} = \frac{Q}{21}$

Similarly,  $Q_1 = \frac{\pi}{4}d^2v_1 = \frac{\pi}{4}d^2\left(\frac{10}{21}v\right) = \left(\frac{\pi}{4}d^2v\right)\frac{10}{21} = \frac{10Q}{21}$

Now, clearly  $Q_2 = \frac{Q}{21}$

Hence, statement I is correct.

and  $(\text{Pressure})_N = (\text{Pressure})_E$

$$\Rightarrow P_N = P_E$$

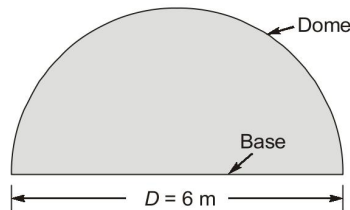
$$\Rightarrow P_J - P_N = P_J - P_E$$

Hence, the pressure difference between J and N is equal to the pressure difference between J and E.

So, statement II is true.

**End of Solution**

- Q.55** Consider a hemispherical furnace of diameter  $D = 6$  m with a flat base. The dome of the furnace has an emissivity of 0.7 and the flat base is a blackbody. The base and the dome are maintained at uniform temperature of 300 K and 1200 K, respectively. Under steady state conditions, the rate of radiation heat transfer from the dome to the base is \_\_\_\_\_ kW. (rounded off to the nearest integer).  
Use Stefan-Boltzmann constant  $= 5.67 \times 10^{-8} \text{ W(m}^2\text{K}^4)$

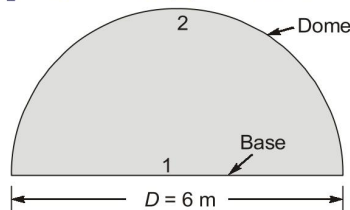


**Ans. (2727) (2720 to 2740)**

Given:  $T_1 = 300$  K,  $T_2 = 1200$  K,  $D = 6$  m,  $F_{11} = 0$ ,  $F_{12} = 1$ ,  $\epsilon_1 = 1$ ,  $\epsilon_2 = 0.7$

$$A_1 = \pi R^2 = \pi(3)^2 = 9\pi \text{ m}^2$$

$$A_2 = 2\pi R^2 = 2\pi(3)^2 = 18\pi \text{ m}^2$$



Heat transfer,

$$\begin{aligned} q &= \frac{\sigma(T_2^4 - T_1^4)}{\frac{1 - \epsilon_1}{\epsilon_1 A_1} + \frac{1}{A_1 F_{12}} + \frac{1 - \epsilon_2}{\epsilon_2 A_2}} \\ &= \frac{(5.67 \times 10^{-8}) \times (1200^4 - 300^4)}{\frac{1}{9\pi \times 1} + \frac{1 - 0.7}{0.7 \times 18\pi}} \\ &= 2726966.198 \text{ W} \quad \left[ \because \frac{1 - \epsilon_1}{\epsilon_1 A_1} = 0 \right] \\ &= 2726.966198 \text{ kW} \\ &\simeq 2726.97 \text{ kW} \end{aligned}$$

**End of Solution**

