Long Answer Type Questions

[4 marks]

Que 1. Verify that the numbers given alongside the cubic polynomial below are their zeros, Also verify the relationship between the zeros and the coefficients.

$\boldsymbol{x^3 - 4x^2 + 5x - 2;\ \ 2, 1, 1}$

Sol. Let $p(x) = x^3 - 4x^3 + 5x - 2$

On comparing with general polynomial $p(x) = ax^3 + bx^2 + cx + d,$ we get a = 1, b = -4, c = 5 and d = -2.

Given zeros 2, 1, 1.

$\therefore \quad p(x) = (2)^3 - 4(2)^2 + 5(2) - 2 = 8 - 16 + 10 - 2 = 0$

And $p(1) = (1)^3 - 4(1)^2 + 5(1) - 2 = 1 - 4 + 5 - 2 = 0.$

Hence, 2, 1 and 1 are the zeros of the given cubic polynomial.

Again, consider a = 2, b = 1, y = 1

$\therefore \quad \alpha + \beta + \gamma = 2 + 1 + 1 = 4$

$$and \quad \alpha + \beta + \gamma = \frac{-(coefficient\ of\ x^2)}{Coefficient\ of\ x^3} = \frac{-b}{a} = \frac{-(-4)}{1} = 4$$

$\alpha\beta + \beta\gamma + \gamma\alpha = (2)(1) + (1)(1) + (1)(2) = 2 + 1 + 2 = 5$

$$and \quad \alpha\beta + \beta\gamma + \gamma\alpha = \frac{Coefficient\ x}{Coefficient\ of\ x^3} = \frac{c}{a} = \frac{5}{1} = 5$$

$\alpha\beta\gamma = (2)(1)(1) = 2$

$$and \quad \alpha\beta\gamma = \frac{-(Constant\ term)}{Coefficient\ of\ x^3} = \frac{-d}{a} = \frac{-(2)}{1} = 2.$$

Que 2. Find a cubic polynomial with the sum the zeros, sum of the products of its zeros taken two at a time, and the product of its zeros as 2, -7, -14 respectively.

Sol. Let the cubic polynomial be $p(x) = ax^3 + cx + d.$ Then

Sum of zeros = $\frac{-b}{a} = 2$

Sum of the product of zeros taken two at a time = $\frac{c}{a} = -7$

And product of the zeros = $\frac{-d}{a} = -14$

$\Rightarrow \frac{b}{a} = -2, \frac{c}{a} = -7, -\frac{d}{a} = -14 \quad or \quad \frac{d}{a} = 14$

$\therefore \quad p(x) = ax^3 + bx^2 + cx + d \;\Rightarrow\; p(x) = a\left[x^3 + \frac{b}{a}x^2 + \frac{c}{a}x + \frac{d}{a}\right]$

$p(x) = a[x^3 + (-2)x^2 + (-7)x + 14] \qquad \Rightarrow \qquad p(x) = a\left[x^3 + \frac{b}{a}x^2 + \frac{c}{a}x + \frac{d}{a}\right]$

For real value of $a = 1, p(x) = x^3 - 2x^2 - 7x + 14$

Que 3. Find the zeros of the polynomial $f(x) = x^3 - 5x^2 - 2x + 24$, if it is given that the product of its two zeros is 12.

Sol. Let $\alpha, \beta\ and\ \gamma$ be the zeros of polynomial $f(x)$ such that $ab = 12$.

We have, $\alpha + \beta + \gamma = \frac{-b}{a} = \frac{-(-5)}{1} = 5$

$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a} = \frac{-2}{1} = -2 \ and\ \alpha\beta\gamma = \frac{-d}{a} = \frac{-24}{1} = -24$

Putting $\alpha\beta = 12\ in\ \alpha\beta\gamma = -24, we\ get$

$12\gamma = -24 \qquad \Rightarrow \qquad \gamma = -\frac{24}{12} = -2$

Now, $\alpha + \beta + \gamma = 5 \qquad \Rightarrow \qquad \alpha + \beta - 2 = 5$

$\Rightarrow \alpha + \beta = 7 \qquad \Rightarrow \qquad \alpha = 7 - \beta$

$\therefore \quad \alpha\beta = 12$

$\Rightarrow (7 - \beta)\beta = 12 \qquad \Rightarrow \qquad 7\beta - \beta^2 = 12$

$\Rightarrow \beta^2 - 7\beta + 12 = 0 \qquad \Rightarrow \qquad \beta^2 - 3\beta - 4\beta + 12 = 0$

$\Rightarrow \beta = 4 \quad or \quad \beta = 3$

$\therefore \quad \alpha = 3 \quad or \quad \alpha = 4$

Que 4. If the remainder on division of $x^2 - kx^2 + 13x - 21\ by\ 2x - 1\ is - 21$, find the quotient and the value of k. Hence, find the zeros of the cubic polynomial $x^3 - kx^2 + 13x$.

Sol. Let $f(x) = x^3 - kx^2 + 13x - 21$

Then $f\left(\frac{1}{2}\right) = -21 \qquad \Rightarrow \qquad \left(\frac{1}{2}\right)^3 - k\left(\frac{1}{2}\right)^2 + 13\left(\frac{1}{2}\right) - 21 = -21$

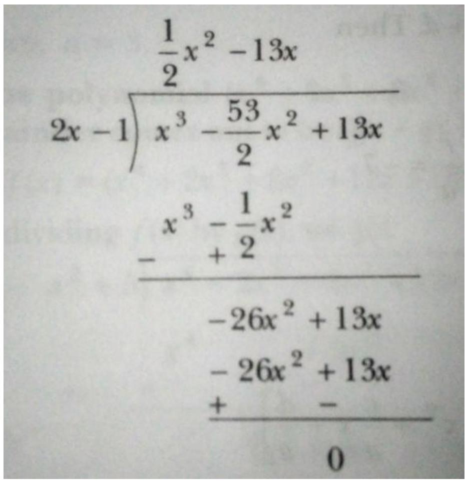
Or $\quad \frac{1}{8} - \frac{1}{4}k + \frac{13}{2} - 21 + 21 = 0 \qquad or \qquad \frac{k}{4} = \frac{53}{8} \qquad \Rightarrow \qquad k = \frac{53}{2}$

$\therefore f(x) = x^3 - \frac{53}{2}x^2 + 13x - 21$

Now, $f(x) = q(x)(2x - 1) - 21$

$$\Rightarrow \qquad x^3 - \frac{53}{2}x^2 + 13x - 21 = q(x)(2x-1) - 21$$

$$\Rightarrow \qquad \left(x^3 - \frac{53}{2}x^2 + 13x\right) \div (2x-1) = q(x)$$



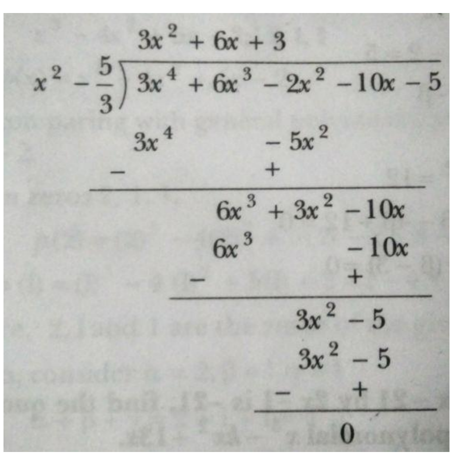
$$i.e., \quad x^3 - \frac{53}{2}x^2 + 13x = (2x-1)\left(\frac{1}{2}x^2 - 13x\right) = \frac{1}{2}x(2x-1)(x-26)$$

For zeros, $\quad x^3 - \frac{53}{2}x^2 + 13x = 0 \quad \Rightarrow \quad \frac{1}{2}x(2x-1)(x-26) = 0 \quad \Rightarrow \quad x = 0, \frac{1}{2}, 26$

Que 5. Obtain all other zeros of $3x^4 + 6x^2 - 10x - 5$, if two of its zeros are $\sqrt{\frac{5}{3}}$ $and -\sqrt{\frac{5}{3}}$.

Sol. Since two zeros are $\sqrt{\frac{5}{3}}$ $and -\sqrt{\frac{5}{3}}, so$ $\left(x - \sqrt{\frac{5}{3}}\right)\left(x + \sqrt{\frac{5}{3}}\right) = x^2 - \frac{5}{3}$ is a factor of the given polynomial.

Now, we divide the given polynomial by $\left(x^2 - \frac{5}{3}\right)$ to obtain other zeros.



So, $3x^4 + 6x^3 - 2x^2 - 10x - 5 = \left(x^2 - \frac{5}{3}\right)(3x^2 + 6x + 3)$

Now, $3x^2 + 6x + 3 = 3(x^2 + 2x + 1) = 3(x + 1)^2 = 3(x + 1)(x + 1)$

So its zeros are -1, - 1.

Thus, all the zeros of given polynomial are $\sqrt{\frac{5}{3}}, -\sqrt{\frac{5}{3}}, -1 \; and -1.$

Que 6. Given that $\sqrt{2}$ is a zero of the cubic polynomial space $6x^3 + \sqrt{2}x^2 - 10x - 4\sqrt{2}$, find its other zeros.

Sol. The given polynomial is $f(x) = (6x^3 + \sqrt{2}x^2 - 10x - 4\sqrt{2})$. Since $\sqrt{2}$ is the zero of $f(x)$, it followa that $(x - \sqrt{2})$ is a factor of $f(x)$.

On dividing $f(x)$ by $(x - \sqrt{2})$, we get

$$x - \sqrt{2} \,\big)\, 6x^3 + \sqrt{2}x^2 - 10x - 4\sqrt{2} \,\big(\, 6x^2 + 7\sqrt{2}x + 4$$

$$\begin{array}{l} 6x^3 - 6\sqrt{2}x^2 \\ - \quad + \\ \hline 7\sqrt{2}x^2 - 10x \\ 7\sqrt{2}x^2 - 14x \\ - \quad + \\ \hline 4x - 4\sqrt{2} \\ 4x - 4\sqrt{2} \\ \hline 0 \end{array}$$

$\therefore f(x) = 0 \Rightarrow (x - \sqrt{2})(6x^2 + 7\sqrt{2x} + 4) = 0 \quad \Rightarrow \quad (x - \sqrt{2})(3\sqrt{2x} + 4)(\sqrt{2x} + 1) = 0$

$x - \sqrt{2} = 0, 3\sqrt{2x} + 4 = 0, \sqrt{2x} + 1 = 0$

Hence, $x = \sqrt{2}, x = -\frac{2\sqrt{2}}{3}, x = \frac{-\sqrt{2}}{2}$ and all zeros of $f(x) are \ \sqrt{2}, \frac{-2\sqrt{2}}{3}, \frac{-\sqrt{2}}{2}$.