Permutations and Combinations

Question1

Let $\alpha = \frac{(4!)!}{(4!)^{3!}}$ and $\beta = \frac{(5!)!}{(5!)^{4!}}$. Then :

[27-Jan-2024 Shift 2]

Options:

A.

$\alpha \in N$ and $\beta \notin N$

B.

$\alpha \notin N$ and $\beta \in N$

C.

$\alpha \in N$ and $\beta \in N$

D.

$\alpha \notin N$ and $\beta \notin N$

Answer: C

Solution:

$$\alpha = \frac{(4!)!}{(4!)^{3!}}, \beta = \frac{(5!)!}{(5!)^{4!}}$$

$$\alpha = \frac{(24)!}{(4!)^{6}}, \beta = \frac{(120)!}{(5!)^{24}}$$

Let 24 distinct objects are divided into 6 groups of 4 objects in each group.

No. of ways of formation of group $= \frac{24!}{(4!)^{6}.6!} \in N$

Similarly,

Let 120 distinct objects are divided into 24 groups of 5 objects in each group.

No. of ways of formation of groups

$$= \frac{(120)!}{(5!)^{24} \cdot 24!} \in N$$

Question2

All the letters of the word "GTWENTY" are written in all possible ways with or without meaning and these words are written as in a dictionary. The serial number of the word "GTWENTY" IS

[29-Jan-2024 Shift 1]

Options:

Answer: 553

Solution:

Words starting with E = 360
Words starting with GE = 60
Words starting with GN = 60
Words starting with GTE = 24
Words starting with GTN = 24
Words starting with GTT = 24
GTWENTY = 1
Total = 553

Question3

Number of ways of arranging 8 identical books into 4 identical shelves where any number of shelves may remain empty is equal to

[29-Jan-2024 Shift 2]

Options:

A.

18

B.

16

C.

12

D.

15

Answer: D

Solution:

3 Shelf empty : $(8, 0, 0, 0) \longrightarrow 1$ way

2 shelf empty: $\left.\begin{matrix}(7, 1, 0, 0)\\(6, 2, 0, 0)\\(5, 3, 0, 0)\\(4, 4, 0, 0)\end{matrix}\right] \rightarrow 4$ ways

1 shelf empty : $\left.\begin{matrix}(6, 1, 1, 0) & (3, 3, 2, 0)\\(5, 2, 1, 0) & (4, 2, 2, 0)\\(4, 3, 1, 0) & \end{matrix}\right] \rightarrow 5$ ways

0 Shelf empty : $\left.\begin{matrix}(1, 2, 3, 2) & (5, 1, 1, 1)\\(2, 2, 2, 2) & \\(3, 3, 1, 1) & \\(4, 2, 1, 1) & \end{matrix}\right] \rightarrow 5$ ways

Total $= 15$ ways

Question4

In an examination of Mathematics paper, there are 20 questions of equal marks and the question paper is divided into three sections: A,B and C. A student is required to attempt total 15 questions taking at least 4 questions from each section. If section A has 8 questions, section B has 6 questions and section C has 6 questions, then the total number of ways a student can select 15 questions is_____

[30-Jan-2024 Shift 2]

Answer: 11376

Solution:

If 4 questions from each section are selected

Remaining 3 questions can be selected either in (1, 1, 1) or (3, 0, 0) or (2, 1, 0)

$$\therefore \text{ Total ways } = {}^8c_5 \cdot {}^6c_5 \cdot {}^6c_5 + {}^8c_6 {}^6c_5 \cdot {}^6c_4 \times 2 + {}^8c_5 \cdot {}^6c_6 \cdot {}^6c_4 \times 2 + {}^8c_4 \cdot {}^6c_6 \cdot {}^6c_5 \times 2 + {}^8c_7 \cdot {}^6c_4 \cdot {}^6c_4$$

$$= 56.6.6 + 28.6.15.2 + 56.15.2 + 70.6.2 + 8.15.15$$

$$= 2016 + 5040 + 1680 + 840 + 1800 = 11376$$

Question5

The total number of words (with or without meaning) that can be formed out of the letters of the word 'DISTRIBUTION' taken four at a time, is equal to_____

[31-Jan-2024 Shift 1]

Options:

Answer: 3734

Solution:

We have III, TT, D, S, R, B, U, O, N

Number of words with selection (a, a, a, b)

$$= {}^8C_1 \times \frac{4!}{3!} = 32$$

Number of words with selection (a, a, b, b)

$$= \frac{4!}{2!2!} = 6$$

Number of words with selection (a, a, b, c)

$$= {}^2C_1 \times {}^8C_2 \times \frac{4!}{2!} = 672$$

Number of words with selection (a, b, c, d)

$$= {}^9C_4 \times 4! = 3024$$

$$\therefore \text{ total } = 3024 + 672 + 6 + 32$$

$$= 3734$$

Question6

The number of ways in which 21 identical apples can be distributed among three children such that each child gets at least 2 apples, is

[31-Jan-2024 Shift 2]

Options:

A.

406

B.

130

C.

142

D.

136

Answer: D

Solution:

After giving 2 apples to each child 15 apples left now 15 apples can be distributed in ${}^{15+3-1}C_2 = {}^{17}C_2$ ways

$= \frac{17 \times 16}{2} = 136$

Question7

If n is the number of ways five different employees can sit into four indistinguishable offices where any office may have any number of persons including zero, then n is equal to:

[1-Feb-2024 Shift 1]

Options:

A.

47

B.

53

C.

51

D.

43

Answer: C

Solution:

Total ways to partition 5 into 4 parts are :

$5, 0, 0, 0 \Rightarrow 1$ way

$4, 1, 0, 0 \Rightarrow \frac{5!}{4!} = 5$ ways

$3, 2, 0, 0, \Rightarrow \frac{5!}{3!2!} = 10$ ways

$2, 2, 0, 1 \Rightarrow \frac{5!}{2!2!2!} = 15$ ways

$2, 1, 1, 1 \Rightarrow \frac{5!}{2!(1!)^3 3!} = 10$ ways

$3, 1, 1, 0 \Rightarrow \frac{5!}{3!2!} = 10$ ways

Total $\Rightarrow 1 + 5 + 10 + 15 + 10 + 10 = 51$ ways

Question8

The lines L_1, L_2,...,I_{20} are distinct. For n = 1, 2, 3,...,10 all the lines L_{2n-1} are parallel to each other and all the lines L_{2n} pass through a given point P. The maximum number of points of intersection of pairs of lines from the set {L_1, L_2,...,L_{20}} is equal to :

[1-Feb-2024 Shift 2]

Options:

Answer: 101

Solution:

$L_1, L_3, L_5, \text{- -} L_{19}$ are Parallel

$L_2, L_4, L_6, -- L_{20}$ are Concurrent

Total points of intersection $= {}^{20}C_2 - {}^{10}C_2 - {}^{10}C_2 + 1 = 101$

Question9

The number of 9 digit numbers, that can be formed using all the digits

of the number 123412341 so that the even digits occupy only even places, is__
[24-Jan-2023 Shift 1]

Options:

A.

Answer: 60

Solution:

Solution:
Even digits occupy at even places

$$\frac{4!}{2!2!} \times \frac{5!}{2!3!} = \frac{24 \times 120}{4 \times 12} = 60$$

Question10

A boy needs to select five courses from 12 available courses, out of which 5 courses are language courses. If he can choose at most two language courses, then the number of ways he can choose five courses is
[24-Jan-2023 Shift 1]

Options:

A.

Answer: 546

Solution:

Solution:
For at most two language courses

$$= {}^5C_2 \times {}^7C_3 + {}^5C_1 \times {}^7C_4 + {}^7C_5 = 546$$

Question11

The number of integers, greater than 7000 that can be formed, using the digits 3, 5, 6, 7, 8 without repetition, is
[24-Jan-2023 Shift 2]

Options:

A. 120

B. 168

C. 220

D. 48

Answer: B

Solution:

Solution:
Four digit numbers greater than 7000 $= 2 \times 4 \times 3 \times 2 = 48$
Five digit number $= 5! = 120$
Total number greater than 7000 $= 120 + 48 = 168$

Question12

Let S = {1, 2, 3, 5, 7, 10, 11}. The number of nonempty subsets of S that have the sum of all elements a multiple of 3 , is _______. [25-Jan-2023 Shift 1]

Options:

A.

Answer: 43

Solution:

Solution:
Elements of the type $3k = 3$
Elements of the type $3k + 1 = 1, 7, 9$
Elements of the type $3k + 2 = 2, 5, 11$
Subsets containing one element $S_1 = 1$
Subsets containing two elements
$S_2 = {}^3C_1 \times {}^3C_1 = 9$
Subsets containing three elements
$S_3 = {}^3C_1 \times {}^3C_1 + 1 + 1 = 11$
Subsets containing four elements
$S_4 = {}^3C_3 + {}^3C_3 + {}^3C_2 \times {}^3C_2 = 11$
Subsets containing five elements
$S_5 = {}^3C_2 \times {}^3C_2 \times 1 = 9$
Subsets containing six elements $S_6 = 1$
Subsets containing seven elements $S_7 = 1$
$\Rightarrow$ sum $= 43$

Question13

The number of numbers, strictly between 5000 and 10000 can be formed using the digits 1, 3, 5, 7, 9 without repetition, is [25-Jan-2023 Shift 2]

Options:

A. 6

B. 12

C. 120

D. 72

Answer: D

Solution:

Solution:
Numbers between 5000&10000
Using digits 1, 3, 5, 7, 9
Total Numbers $= 3 \times 4 \times 3 \times 2 = 72$

Question14

Suppose Anil's mother wants to give 5 whole fruits to Anil from a basket of 7 red apples, 5 white apples and 8 oranges. If in the selected 5 fruits, at least 2 orange, at least one red apple and at least one white apple must be given, then the number of ways, Anil's mother can offer 5 fruits to Anil is _______
[25-Jan-2023 Shift 2]

Options:

A.

Answer: 6860

Solution:

Solution:
7 Red apple(RA), 5 white apple(WA), 8 oranges (O)
5 fruits to be selected (Note:- fruits taken different)
Possible selections :- (2O, 1 RA, 2 WA) or (2O ,
2RA, 1WA) or (3O, 1RA, 1WA)
$\Rightarrow {}^8C_2\,{}^7C_1\,{}^5C_2 + {}^8C_2\,{}^7C_2\,{}^5C_1 + {}^8C_3\,{}^7C_1\,{}^5C_1$
$\Rightarrow 1960 + 2940 + 1960$
$\Rightarrow 6860$

Question15

If all the six digit numbers $x_1x_2x_3x_4x_5x_6$ with $0 < x_1 < x_2 < x_3 < x_4 < x_5 < x_6$ are arranged in the increasing order, then the sum of the digits in the 72^{th} number is _______.
[29-Jan-2023 Shift 1]

Options:

A.

Answer: 32

Solution:

Solution:

1	2				

$$= {}^{7}C_{4} = 35$$

1	3				

$$= {}^{6}C_{4} = 15$$

1	4				

$= {}^{5}C_4 = 5$

1	5				

$= {}^{4}C_4 = 1$

2	3				

$= {}^{6}C_4 = 15$

71 words

$245678 \to 72^{th}$ word

$$245678 \to 72 \quad \text{word}$$

$$2+4+5+6+7+8=32$$

$= {}^6C_4 = 15$
71 words
$245678 \to 72^{\text{th}}$ word
$2+4+5+6+7+8=32$

Question16

Five digit numbers are formed using the digits 1,2 , 3, 5, 7 with repetitions and are written in descending order with serial numbers. For example, the number 77777 has serial number 1. Then the serial number of 35337 is ______.
[29-Jan-2023 Shift 1]

Answer: 1436

Solution:

Solution:
No of 5 digit numbers starting with digit 1
$= 5 \times 5 \times 5 \times 5 = 625$
No of 5 digit numbers starting with digit 2
$= 5 \times 5 \times 5 \times 5 = 625$
No of 5 digit numbers starting with 31
$= 5 \times 5 \times 5 = 125$
No of 5 digit numbers starting with 32
$= 5 \times 5 \times 5 = 125$
No of 5 digit numbers starting with 33
$= 5 \times 5 \times 5 = 125$
No of 5 digit numbers starting with 351
$= 5 \times 5 = 25$
No of 5 digit numbers starting with 352
$= 5 \times 5 = 25$
No of 5 digit numbers starting with 3531 $= 5$
No of 5 digit numbers starting with 3532 $= 5$
Before 35337 will be 4 numbers,
So rank of 35337 will be 1690
So, in descending order serial number will be
$3125 - 1690 + 1 = 1436$

Question17

The number of 3 digit numbers, that are divisible by either 3 or 4 but

not divisible by 48 , is
[29-Jan-2023 Shift 2]

Options:

A. 472

B. 432

C. 507

D. 400

Answer: B

Solution:

Solution:
Total 3 digit number $= 900$
Divisible by $3 = 300$ (Using $\frac{900}{3} = 300$)
Divisible by $4 = 225$ (Using $\frac{900}{4} = 225$)
Divisible by 3 & $4 = 108, \ldots$
(Using $\frac{900}{12} = 75$)
Number divisible by either 3 or 4
$= 300 + 2250 - 75 = 450$
We have to remove divisible by 48 ,
$144, 192, \ldots., 18$ terms
Required number of numbers $= 450 - 18 = 432$

Question18

The letters of the word OUGHT are written in all possible ways and these words are arranged as in a dictionary, in a series. Then the serial number of the word TOUGH is: [29-Jan-2023 Shift 2]

Options:

A. 89

B. 84

C. 86

D. 79

Answer: A

Solution:

Solution:
Lets arrange the letters of OUGHT in alphabetical order.
G, H, O, T, U
Words starting with
$G---- \rightarrow 4!$
$H---- \rightarrow 4!$

$O----\rightarrow 4!$
$TG---\rightarrow 3!$
$TH---\rightarrow 3!$
$TOG--\rightarrow 2!$
$TOH\ --\rightarrow 2!$
$TOUGH\ \rightarrow 1!$
$-$ Total $= 89$

Question19

The total number of 4-digit numbers whose greatest common divisor with 54 is 2 , is ______.
[29-Jan-2023 Shift 2]

Answer: 3000

Solution:

N should be divisible by 2 but not by 3
N = (Numbers divisible by 2) − (Numbers divisible by 6)

$$N = \frac{9000}{2} - \frac{9000}{6} = 4500 - 1500 = 3000$$

Question20

Number of 4-digit numbers (the repetition of digits is allowed) which are made using the digits 1, 2, 3 and 5 , and are divisible by 15 , is equal to _______
[30-Jan-2023 Shift 1]

Answer: 21

Solution:

Solution:
For number to be divisible by 15 , last digit should be 5 and sum of digits must be divisible by 3 .
Possible combinations are

1	2	1	5

Numbers $= 3$

2	2	3	5

Numbers $= 3$

3	3	1	5

Numbers $= 3$

1	1	5	5

Numbers $= 3$

2	3	5	5

Numbers $= 3$

3	3	5	5

Total Numbers $= 21$

Question21

The number of ways of selecting two numbers a and b, $a \in \{2, 4, 6, \ldots, 100\}$ and $b \in \{1, 3, 5, \ldots, 99\}$ such that 2 is the remainder when $a + b$ is divided by 23 is
[30-Jan-2023 Shift 2]

Options:

A. 186

B. 54

C. 108

D. 268

Answer: C

Solution:

Solution:
$a \in \{2, 4, 6, 8, 10, \ldots, 100\}$
$b \in \{1, 3, 5, 7, 9, \ldots, 99\}$
Now, $a + b \in \{25, 71, 117, 163\}$
(i) $a + b = 25$, no. of ordered pairs (a, b) is 12
(ii) $a + b = 71$, no. of ordered pairs (a, b) is 35
(iii) $a + b = 117$, no. of ordered pairs (a, b) is 42
(iv) $a + b = 163$, no. of ordered pairs (a, b) is 19 $\therefore$ total $= 108$ pairs

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Question22

The number of seven digits odd numbers, that can be formed using all the seven digits 1, 2, 2, 2,3,3,5 is _______.
[30-Jan-2023 Shift 2]

Answer: 240

Solution:

Solution:
Digits are 1, 2, 2, 2, 3, 3, 5
If unit digit 5 , then total numbers $= \frac{6!}{3!2!}$
If unit digit 3 , then total numbers $= \frac{6!}{3!}$
If unit digit 1 , then total numbers $= \frac{6!}{3!2!}$
$\therefore$ total numbers $= 60 + 60 + 120 = 240$

Question23

Number of 4-digit numbers that are less than or equal to 2800 and either divisible by 3 or by 11 , is equal to ________.
[31-Jan-2023 Shift 1]

Answer: 710

Solution:

$1000 - 2799$
Divisible by 3
$1002 + (n - 1)3 = 2799$
$n = 600$
Divisible by 11

$$1 - 2799 \rightarrow \left[\frac{2799}{11} \right] = [254] = 254$$

$$1 - 999 = \left[\frac{999}{11} \right] = 90$$

$1000 - 2799 = 254 - 90 = 164$
Divisible by 33

$$1 - 2799 \rightarrow \left[\frac{2799}{33} \right] = 84$$

$$1 - 999 \rightarrow \left[\frac{999}{33} \right] = 30$$

$1000 - 2799 \rightarrow 54$
$\therefore n(3) + n(11) - n(33)$
$600 + 164 - 54 = 710$

Question24

Let 5 digit numbers be constructed using the digits 0, 2, 3, 4, 7, 9 with repetition allowed, and are arranged in ascending order with serial numbers. Then the serial number of the number 42923 is _______.
[31-Jan-2023 Shift 1]

Answer: 2997

Solution:

$42920 = 1$
$42922 = 1$
$42923 = 1$
$= 2997$

Question25

If $^{2n+1}P_{n-1} : {}^{2n-1}P_n = 11 : 21$, then $n^2 + n + 15$ is equal to:
[31-Jan-2023 Shift 2]

Answer: 45

Solution:

Solution:

$$\frac{(2n+1)!(n-1)!}{(n+2)!(2n-1)!} = \frac{11}{21}$$

$$\Rightarrow \frac{(2n+1)(2n)}{(n+2)(n+1)n} = \frac{11}{21}$$

$$\Rightarrow \frac{2n+1}{(n+1)(n+2)} = \frac{11}{42}$$

$\Rightarrow n = 5$

$\Rightarrow n^2 + n + 15 = 25 + 5 + 15 = 45$

Question26

The number of words, with or without meaning, that can be formed using all the letters of the word ASSASSINATION so that the vowels occur together, is ________.
[1-Feb-2023 Shift 1]

Answer: 50400

Solution:

Solution:
Vowels : A,A,A,I,I,O
Consonants : S,S,S,S,N,N,T
∵ Total number of ways in which vowels come together

$$= \frac{x8}{\lfloor 4 \mid 2} \times \frac{x6}{\lfloor x3L2} = 50400$$

Question27

Number of integral solutions to the equation x + y + z = 21, where x ≥ 1, y ≥ 3, z ≥ 4, is equal to _______.
[1-Feb-2023 Shift 2]

Answer: 105

Solution:

Solution:
$^{15}C_2 = \frac{15 \times 14}{2} = 105$

Question28

The total number of six digit numbers, formed using the digits 4, 5, 9 only and divisible by 6 , is ______.
[1-Feb-2023 Shift 2]

Answer: 81

Solution:

Solution:
Taking single digit →444444 $\frac{6!}{6!} = 1$

Taking two digit →
(4, 5) 444555 (4, 9) 444999
$\frac{5!}{3!2!} = 10$ $\frac{5!}{3!2!} = 10$

Taking three digit
$4, 5, 9, 4, 4, 4 \Rightarrow \frac{5!}{3!} = 20$

$4, 5, 9, 5, 5, 5 \Rightarrow \frac{5!}{4!} = 5$

$4, 5, 9, 9, 9, 9 \Rightarrow \frac{5!}{4!} = 5$

$4, 5, 9, 4, 5, 9 \Rightarrow \frac{5!}{2!2!} = 30$

Total = 81

Question29

The number of ways of giving 20 distinct oranges to 3 children such that each child gets at least one orange is ______.
[6-Apr-2023 shift 1]

Answer: 171

Solution:

Solution:

20 distinct oranges distributed among 3 children so

that each child gets at least one orange

$= 3^{20} - {}^{3}C_{1}\, 2^{20} + {}^{3}C_{2}\, 1^{20}$

Question30

All the letters of the word PUBLIC are written in all possible orders and these words are written as in a dictionary with serial numbers. Then the serial number of the word PUBLIC is :
[6-Apr-2023 shift 2]

Options:

A. 580

B. 578

C. 576

D. 582

Answer: D

Solution:

Solution:

B——— $= 5! = 120$
C——— $5! = 120$
I——— $= 5! = 120$
L——— $= 5! = 120$
PB−−−−−− $= 4! = 24$
PC−−−−−− $= 4! = 24$
PI−−−−−− $= 4! = 24$
PL−−−−−− $= 4! = 24$
PUBC−−−−−− $= 2! = 2$
PUBI−−−−−− $2! = 2$
PUBLC ——— $= 1$
PUBLIC——— $- \frac{= 1}{582}$
Rank $= 582$
Ans. Option 4

Question31

The number of 4-letter words, with or without meaning, each consisting of 2 vowels and 2 consonants, which can be formed from the letters of the word UNIVERSE without repetition is _______ :
[6-Apr-2023 shift 2]

Answer: 432

Solution:

Solution:
Case I 2 vowels different, 2 consonant different
$({}^{3}C_{2})({}^{4}C_{2})(4!)$
$= (3)(6)(24)$
$= 432$

Question32

The number of arrangements of the letters of the word "INDEPENDENCE" in which all the vowels always occur together is.
[8-Apr-2023 shift 1]

Options:

A. 16800

B. 14800

C. 18000

D. 33600

Answer: A

Solution:

Solution:
IEEEE,
NNN, DD, P, C
$\frac{8!}{3!2!} \times \frac{6!}{41} = 16800$

Question33

The number of ways, in which 5 girls and 7 boys can be seated at a round table so that no two girls sit together, is
[8-Apr-2023 shift 1]

Options:

A. $7(720)^2$

B. 720

C. $7(360)^2$

D. $126(5!)^2$

Answer: D

Solution:

Solution:
$6! \times {}^7C_5 \times 5!$
$\Rightarrow 720 \times 21 \times 120$
$\Rightarrow 2 \times 360 \times 7 \times 3 \times 120$
$\Rightarrow 126 \times (5!)^2$

Question34

If the number of words, with or without meaning, which can be made using all the letters of the word MATHEMATICS in which C and S do not come together, is (6!)k, is equal to
[8-Apr-2023 shift 2]

Options:

A. 1890

B. 945

C. 2835

D. 5670

Answer: D

Solution:

Solution:
$M_2A_2T_2\,HEICS$
= total words - when C&S are together
$\frac{\lfloor 11}{\lfloor 2\lfloor 2\lfloor 2} - \frac{\lfloor 10}{\lfloor 2\lfloor 2 \ll \text{corner}2} \times \lfloor 2$
$\frac{\lfloor x10}{\lfloor 2\lfloor 2 \ll \text{corner}2} \times 9$
$= \frac{9 \times 10 \times 9 \times 8 \times 7}{8}\lfloor 6$
$= 5670\lfloor 6$
$k = 5670($ Option 4)

Question35

The number of permutations of the digits 1, 2, 3, ..., 7 without repetition, which neither contain the string 153 nor the string 2467 , is ________.
[10-Apr-2023 shift 1]

Answer: 4898

Solution:

Solution:
Numbers are 1, 2, 3, 4, 5, 6, 7
Numbers having string (154) = (154), 2, 3, 6, 7 = 5 !
Numbers having string (2467) = (2467), 1, 3, 5 = 4 !
Number having string (154) and (2467)
= (154), (2467) = 2!
Now $n(154 \cup 2467) = 5! + 4! - 2!$
$= 120 + 24 - 2 = 142$
Again total numbers $= 7! = 5040$
Now required numbers $= n$ (neither 154 nor 2467)
$= 5040 - 142$
$= 4898$

Question36

Some couples participated in a mixed doubles badminton tournament. If the number of matches played, so that no couple in a match, is 840 , then the total numbers of persons, who participated in the tournament, is ________.
[10-Apr-2023 shift 1]

Answer: 16

Solution:

Solution:
Let number of couples $= n$
$\therefore {}^{n}C_2 \times {}^{n-2}C_2 \times 2 = 840$
$\Rightarrow n(n-1)(n-2)(n-3) = 840 \times 2$
$= 21 \times 40 \times 2$
$= 7 \times 3 \times 8 \times 5 \times 2$
$n(n-1)(n-2)(n-3) = 8 \times 7 \times 6 \times 5$
$\therefore n = 8$
Hence, number of persons $= 16$.

Question37

Eight persons are tobe transported from city A to city B in three cars different makes. If each car can accomodate at most three persons, then the number of ways, in which they can be transported, is
[10-Apr-2023 shift 2]

Options:

A. 1120

B. 560

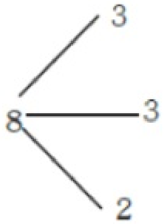
C. 3360

D. 1680

Answer: D

Solution:

Solution:



$$\text{Ways} = \frac{8!}{3!3!2!2!} \times 3!$$
$$= \frac{81 \times 7 \times 6 \times 5 \times 4}{4}$$
$$= 56 \times 30$$
$$= 1680$$

Question38

The sum of all the four-digit numbers that can be formed using all the digits 2, 1, 2, 3 is equal to ________.
[10-Apr-2023 shift 2]

Answer: 26664

Solution:

Solution:

2, 1, 2, 3

$$- - \frac{1}{2!} = 3$$

$- -$ x2 $3! = 6$

$--x3 \quad \frac{3!}{2!} = 3$

Sum of digits of unit place $= 3 \times 1 + 6 \times 2 + 3 \times 3 = 24$

Required sum

$= 24 \times 1000 + 24 \times 100 + 24 \times 10 + 24 \times 1$

$= 24 \times 1111$

$= 26664$

Question39

The number of triplets (x, y, z), where x, y, z are distinct non negative integers satisfying x + y + z = 15, is :
[11-Apr-2023 shift 1]

Options:

A. 136

B. 114

C. 80

D. 92

Answer: B

Solution:

Solution:

$x + y + z = 15$

Total no. solution $= {}^{15+3-1}C_3 = 136 \ldots$ (1)

Let $x = y \neq z$

$2x + z = 15 \Rightarrow z = 15 - 2t$

$\Rightarrow r \in \{0, 1, 2, \ldots 7\} - \{5\}$

$\therefore$ 7 solutions

$\therefore$ there are 21 solutions in which exactly

Two of x, y, z are equal ... (2)

There is one solution in which $x = y = z \ldots$ (3)

Required answer $= 136 - 21 - 1 = 144$

Question40

In an examination, 5 students have been allotted their seats as per their roll numbers. The number of ways, in which none of the students sits on the allotted seat, is _______.
[11-Apr-2023 shift 1]

Answer: 44

Solution:

Solution:
Derangement of 5 students

$$D_5 = 5!\left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!}\right)$$
$$= 120\left(\frac{1}{2} - \frac{1}{6} + \frac{1}{24} - \frac{1}{120}\right)$$
$$= 60 - 20 + 5 - 1$$
$$= 40 + 4$$
$$= 44$$

Question41

If the letters of the word MATHS are permuted and all possible words so formed are arranged as in a dictionary with serial number, then the serial number of the word THAMS is
[11-Apr-2023 shift 2]

Options:

A. 102

B. 103

C. 101

D. 104

Answer: A

Solution:

Solution:

5	2	1	3	4
T	H	A	M	S
4	1	0	0	0
4!	3!	2!	1!	0!

$$\Rightarrow 4 \times 4! + 3! \times 1 + 0 + 0 + 0$$
$$\Rightarrow 96 + 6 = 102$$
$$\text{Ran k THAMS} \ = 102 + 1 = 103$$

Question42

The number of five digit numbers, greater then 40000 and divisible by 5 , which can be formed using the digits 0, 1, 3, 5, 7 and 9 without repetition, is equal to
[12-Apr-2023 shift 1]

Options:

A. 132

B. 120

C. 72

D. 96

Answer: B

Solution:

Solution:

5	x	x	x	0
7	x	x	x	0
5	x	x	x	5
9	x	x	x	0
9	x	x	x	5

So Required numbers $= 5 \times {}^4P_3 = 120$

Question43

Let the digits a, b, c be in A.P. Nine-digit numbers are to be formed using each of these three digits thrice such that three consecutive digits are in A.P. at least once. How many such numbers can be formed?
[12-Apr-2023 shift 1]

Answer: 1260

Solution:

Solution:
abc or cba
abc
- - - - -
cba

$$\frac{{}^7C_1 \times 2 \times 6!}{2!2!2!} = 1260$$

Question44

The number of seven digit positive integers formed using the digits 1, 2, 3 and 4 only and sum of the digits equal to 12 is _______.
[13-Apr-2023 shift 1]

Answer: 413

Solution:

Solution:

$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 = 12, x_i \in \{1, 2, 3, 4\}$

No. of solutions $= {}^{5+7-1}C_{7-1} - \frac{7!}{6!} - \frac{7!}{5!} = 413$

Question45

All words, with or without meaning, are made using all the letters of the word MONDAY. These words are written as in a dictionary with serial numbers. The serial number of the word MONDAY is [13-Apr-2023 shift 2]

Options:

A. 328

B. 327

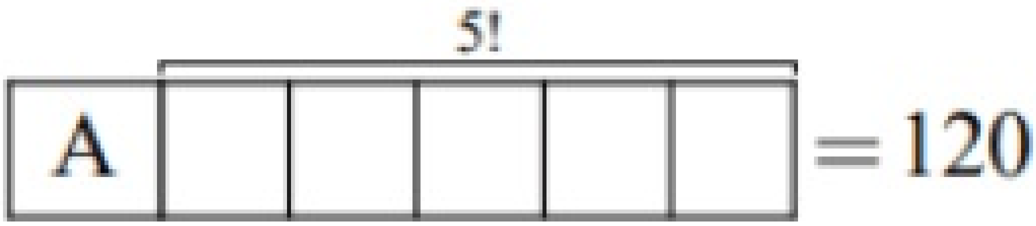
C. 324

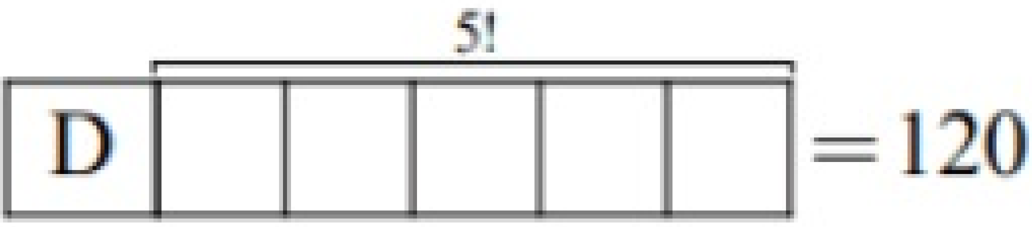
D. 326

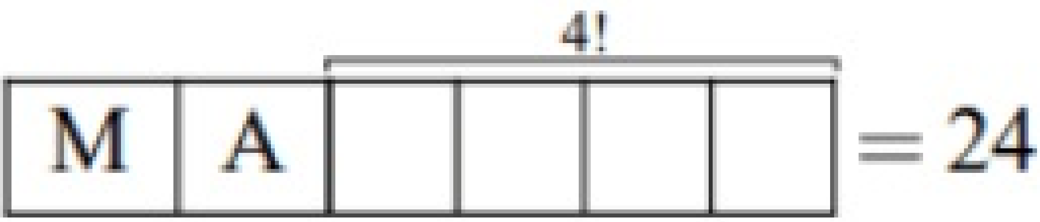
Answer: B

Solution:

Solution:







M	D					= 24

4!

M	N					= 24

3!

M	O	A				= 6

3!

M	O	D				= 6

2!

M	O	N	A			= 2

M	O	N	D	A	Y	= 1

$$\text{Rank} = 120 + 120 + 24 + 24 + 24 + 6 + 6 + 2 + 1$$

$$= 327$$

Question46

Total numbers of 3-digit numbers that are divisible by 6 and can be formed by using the digits 1, 2, 3, 4, 5 with repetition, is ______.
[13-Apr-2023 shift 2]

Answer: 16

Solution:

a	b	2

(a, b) = (1, 3), (3, 1), (2, 2), (2, 5), (5, 2), (3, 4), (4, 3), (5, 5)
= 8 numbers

a	b	4

(a, b) = (1, 1), (1, 4), (4, 1), (2, 3), (3, 2)
(4, 4), (3, 5), (5, 3) = 8 numbers
total $8 + 8 = 16$

Question47

The total number of three-digit numbers, divisible by 3 , which can be formed using the digits 1, 3, 5, 8, if repetition of digits is allowed, is [15-Apr-2023 shift 1]

Options:

A. 21

B. 18

C. 20

D. 22

Answer: D

Solution:

Solution:
(1, 1, 1)(3, 3, 3)(5, 5, 5)(8, 8,, 8)
(5, 5, 8)(8, 8, 5)(1, 3, 5)(1, 3, 8)

Total number $= 1 + 1 + 1 + 1 + \frac{3!}{2!} + \frac{3!}{2!} + 3! + 3! = 22$

Question48

A person forgets his 4-digit ATM pin code. But he remembers that in the code all the digits are different, the greatest digit is 7 and the sum of the first two digits is equal to the sum of the last two digits. Then the maximum number of trials necessary to obtain the correct code is

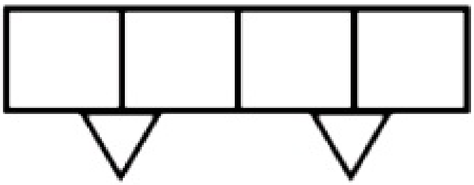
______

[15-Apr-2023 shift 1]

Answer: 72

Solution:

Solution:

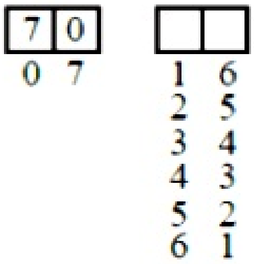


Sum of first two digits

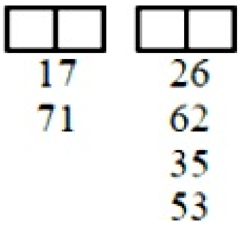
Sum of last two digits $= \alpha$

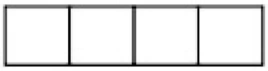
Case-I : $\alpha = 7$

$2 \times 12 = 24$ ways.



Case – II : $\alpha = 8$





2×8 ways

$= 16$ ways

Case-III : $\alpha = 9$

27 36
72 63
45
54

2×8 ways

$= 16$ ways

Case IV : $\alpha = 10$

37 46
73 64

2×4 ways

8 ways

Case V : $\alpha = 11$

47 56
74 65

2×4 ways

8 ways

Ans. $24 + 16 + 16 + 8 + 8 = 72$

Question49

The letters of the word 'MANKIND' are written in all possible orders and arranged in serial order as in an English dictionary. Then the serial number of the word 'MANKIND' is
[25-Jul-2022-Shift-1]

Answer: 1492

Solution:

Solution:

M	A	N	K	I	N	D

$$\left(\frac{4 \times 6!}{2!}\right) + (5! \times 0) + \left(\frac{4! \times 3}{2!}\right) + (3! \times 2) + (2! \times 1) + (1! \times 1) + (0! \times 0) + 1 = 1492$$

Question50

The number of 5-digit natural numbers, such that the product of their digits is 36 , is _______.
[26-Jul-2022-Shift-1]

Answer: 180

Solution:

Solution:

Factors of 36 $= 2^2 \cdot 3^2 \cdot 1$

Five-digit combinations can be

$(1, 2, 2, 3, 3)(1, 4, 3, 3, 1), (1, 9, 2, 2, 1)$

$(1, 4, 9, 11)(1, 2, 3, 6, 1)(1, 6, 6, 1, 1)$

i.e., total numbers

$\frac{5!}{2!2!} + \frac{5!}{2!2!} + \frac{5!}{2!2!} + \frac{5!}{3!} + \frac{5!}{2!} + \frac{5!}{3!2!}$

$= (30 \times 3) + 20 + 60 + 10 = 180$

Question51

Numbers are to be formed between 1000 and 3000 , which are divisible by 4 , using the digits 1, 2, 3, 4, 5 and 6 without repetition of digits. Then the total number of such numbers is _______.
[26-Jul-2022-Shift-2]

Answer: 30

Solution:

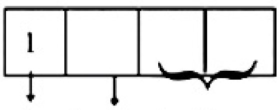
Solution:

Here 1^{st} digit is 1 or 2 only

Case-I

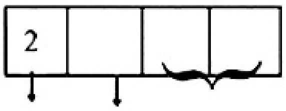
If first digit is 1

Then last two digits can be $24, 32, 36, 52, 56, 64$



$1 \times 3 \times 6 = 18$ ways

Case - IIIf first digit is 2 then last two digit can be $16, 36, 56, 64$



$1 \times 3 \times 4 = 12$ ways

Total ways $= 12 + 18 = 30$ ways

Question52

Let S be the sample space of all five digit numbers. It p is the

probability that a randomly selected number from S, is a multiple of 7 but not divisible by 5 , then 9p is equal to
[27-Jul-2022-Shift-1]

Options:

A. 1.0146

B. 1.2085

C. 1.0285

D. 1.1521

Answer: C

Solution:

Among the 5 digit numbers,
First number divisible by 7 is 10003 and last is 99995 .
⇒ Number of numbers divisible by 7 .

$$= \frac{99995 - 10003}{7} + 1$$

$= 12857$
First number divisible by 35 is 10010 and last is 99995.
⇒ Number of numbers divisible by 35

$$= \frac{99995 - 10010}{35} + 1$$

$= 2572$
Hence number of number divisible by 7 but not by 5
$= 12857 - 2572$
$= 10285$

$$9P. = \frac{10285}{90000} \times 9$$

$= 1.0285$

Question53

Let S be the set of all passwords which are six to eight characters long, where each character is either an alphabet from {A, B, C, D, E } or a number from {1, 2, 3, 4, 5} with the repetition of characters allowed. If the number of passwords in S whose at least one character is a number from {1, 2, 3, 4, 5} is $\alpha \times 5^6$, then α is equal to _______.
[28-Jul-2022-Shift-1]

Answer: 7073

Solution:

If password is 6 character long, then
Total number of ways having atleast one number $= 10^6 - 5^6$
Similarly, if 7 character long $= 10^7 - 5^7$

and if 8-character long $= 10^8 - 5^8$
Number of password $= (10^6 + 10^7 + 10^8) - (5^6 + 5^7 + 5^8)$
$= 5^6(2^6 + 5.2^7 + 25.2^8 - 1 - 5 - 25)$
$= 5^6(64 + 640 + 6400 - 31)$
$= 7073 \times 5^6$
$\therefore \alpha = 7073$

Question54

A class contains b boys and g girls. If the number of ways of selecting 3 boys and 2 girls from the class is 168 , then b + 3g is equal to ________.
[28-Jul-2022-Shift-2]

Answer: 17

Solution:

${}^bC_3 \cdot {}^gC_2 = 168$
$\Rightarrow \frac{b(b-1)(b-2)}{6} \cdot \frac{g(g-1)}{2} = 168$
$\Rightarrow b(b-1)(b-2) \ g(g-1) = 2^5 \cdot 3^2 \cdot 7$
$\Rightarrow b(b-1)(b-2) \ g(g-1) = 6.7.8.3.2$
$\therefore b = 8$ and $g = 3$
$\therefore b + 3g = 17$

Question55

Let $S = \{4, 6, 9\}$ and $T = \{9, 10, 11, \ldots, 1000\}$. If $A = \{a_1 + a_2 + \ldots + a_k : k \in N, a_1, a_2, a_3, \ldots, a_k E S\}$, then the sum of all the elements in the set $T - A$ is equal to _______.
[29-Jul-2022-Shift-1]

Answer: 11

Solution:

Here $S = \{4, 6, 9\}$
And $T = \{9, 10, 11, \ldots\ldots, 1000\}$
We have to find all numbers in the form of $4x + 6y + 9z$, where $x, y, z \in \{0, 1, 2, \ldots\ldots\}$.
If a and b are coprime number then the least number from which all the number more than or equal to it can be express as $ax + by$ where $x, y \in \{0, 1, 2, \ldots\ldots\}$ is $(a-1) . (b-1)$.
Then for $6y + 9z = 3(2y + 3z)$
All the number from $(2-1) \cdot (3-1) = 2$ and above can be express as $2x + 3z$ (say t).
Now $4x + 6y + 9z = 4x + 3(t + 2)$
$= 4x + 3t + 6$

again by same rule $4x + 3t$, all the number from $(4 - 1)(3 - 1) = 6$ and above can be express from $4x + 3t$
Then $4x + 6y + 9z$ express all the numbers from 12 and above.
again 9 and 10 can be express in form $4x + 6y + 9z$.
Then set $A = \{9, 10, 12, 13, \ldots, 1000\}$.
Then $T - A = \{11\}$
Only one element 11 is there.
Sum of elements of $T - A = 11$

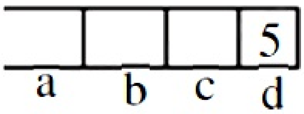
Question56

The number of natural numbers lying between 1012 and 23421 that can be formed using the digits 2, 3, 4, 5, 6 (repetition of digits is not allowed) and divisible by 55 is _______.
[29-Jul-2022-Shift-2]

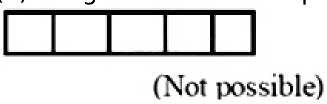
Answer: 6

Solution:

Solution:
4 digit numbers
For divisibility by 55 , no. should be div. by 5 and 11 both
Also, for divisibility by 11



$a + c = b + 5$
for $b = 1$ $a = 2$, $c = 4$
$a = 4$, $c = 2$
for $b = 2$ $a = 3$, $c = 4$
$a = 4$, $c = 3$
for $b = 3$ $a = 6$, $c = 2$
$a = 2$, $c = 6$
$\therefore$6 possible four digit no.s are div. by 55
(II) 5 digit number is not possible



Question57

In an examination, there are 5 multiple choice questions with 3 choices, out of which exactly one is correct. There are 3 marks for each correct answer, −2 marks for each wrong answer and 0 mark if the question is not attempted. Then, the number of ways a student appearing in the examination gets 5 marks is___
[24-Jun-2022-Shift-1]

Answer: 40

Solution:

Solution:

Let student marks x correct answers and y incorrect. So

$3x - 2y = 5$ and $x + y \le 5$ where $x, y \in W$

Only possible solution is $(x, y) = (3, 2)$

Students can mark correct answers by only one choice but for an incorrect answer, there are two choices. So total number of ways of scoring 5 marks = ${}^{5}C_{3}(1)^{3} \cdot (2)^{2} = 40$

Question58

The number of 7-digit numbers which are multiples of 11 and are formed using all the digits 1, 2, 3, 4, 5,7 and 9 is__ [24-Jun-2022-Shift-2]

Answer: 576

Solution:

Digits are 1, 2, 3, 4, 5, 7, 9

Multiple of 11 $\rightarrow$ Difference of sum at even backslash& odd place is divisible by 11 .

Let number of the form abcdefg

$\therefore (a+c+e+g)-(b+d+f)=11x$

$a+b+c+d+e+f=31$

$\therefore$ either $a+c+e+g=21$ or 10

$\therefore b+d+f=10$ or 21

Case-1

$a+c+e+g=21$

$b+d+f=10$

$(b, d, f) \in \{(1,2,7)(2,3,5)(1,4,5)\}$

$(a, c, e, g) \in \{(1,4,7,9),(3,4,5,9),(2,3,7,9)\}$

Case-2

$a+c+e+g=10$

$b+d+f=21$

$(a, b, e, g) \in \{1,2,3,4)\}$

(b, d,f)\& $\{(5,7,9)\}$

$\therefore$ Total number in case $2=3! \times 4!=144$

$\therefore$ Total numbers $=144+432=576$

Question59

The sum of all the elements of the set $\{\alpha \in \{1, 2, ..., 100\} : HCF(\alpha, 24) = 1\}$ is___
[24-Jun-2022-Shift-2]

Answer: 1633

Solution:

Solution:

The numbers upto 24 which gives g.c.d. with 24 equals to 1 are 1, 5, 7, 11, 13, 17, 19 and 23.

Sum of these numbers = 96

There are four such blocks and a number 97 is there upto 100 .

$\therefore$ Complete sum

$= 96 + (24 \times 8 + 96) + (48 \times 8 + 96) + (72 \times 8 + 96) + 97$

$= 1633$

Question60

The number of 3-digit odd numbers, whose sum of digits is a multiple of 7 , is____
[25-Jun-2022-Shift-1]

Answer: 63

Solution:

For odd numbers, unit place shall be 1, 3, 5, 7 or 9 .

$xxyxy$, $xxyxy$, $xxyxy$, $xxyxz$, $xxyx9$ are the type of numbers.

If $xy1$ then

$x + y = 6, 13, 20 \cdots$

Total number $= 6 + 6 + 0 + \cdots = 12$

If x y 3 then

$x + y = 4, 11, 18, \cdots$

Total number $= 4 + 8 + 1 + 0 \cdots = 13$

Similarly for x y 5 , we have

$x+y=2, 9, 16, \cdots$

Total number $=2+9+3=14$

for x y 7 we have

$x+y=0, 7, 14, \cdots$

Total number $=0+7+5=12$ ways

And for x y 9 we have

$x+y=5, 12, 19\ldots$

Total number $=5+7+0 \cdots =12$ ways

$\therefore$ Total odd numbers whose sum of digits is a multiple of 7 is 63 .

Question61

The total number of three-digit numbers, with one digit repeated exactly two times, is___
[25-Jun-2022-Shift-2]

Answer: 243

Solution:

C − 1 : All digits are non-zero
$^{9}C_{2}\cdot 2\cdot \frac{3!}{2}=216$
C − 2: One digit is 0
$0, 0, x \Rightarrow {}^{9}C_{1}.1=9$
$0, x, x \Rightarrow {}^{9}C_{1}.2=18$
Total $=216+27=243$

Question62

There are ten boys B_1, B_2, ..., B_{10} and five girls G_1, G_2, ..., G_5 in a class. Then the number of ways of forming a group consisting of three boys and three girls, if both B_1 and B_2 together should not be the members of a group, is___
[26-Jun-2022-Shift-1]

Answer: 1120

Solution:

Number of ways when B_1 and B_2 are not together

= Total number of ways of selecting 3 boys $- B_1$ and B_2 are together

$= {}^{10}C_3 - {}^{8}C_1$

$= \frac{10.9.8}{1.2.3} - 8$

$= 112$

Number of ways to select 3 girls $= {}^{5}C_3 = 10$

$\therefore$ Total number of ways $= 112 \times 10 = 1120$

Question63

The total number of 3-digit numbers, whose greatest common divisor with 36 is 2 , is____
[26-Jun-2022-Shift-2]

Answer: 150

Solution:

$\because x \in [100, 999], x \in N$

Then $\frac{x}{2} \in [50, 499], \frac{x}{2} \in N$

Number whose G.C.D. with 18 is 1 in this range have the required condition. There are 6 such number from 18×3 to 18×4. Similarly from 18×4 to 18×5......,26×18 to 27×18

$\therefore$ Total numbers $= 24 \times 6 + 6 = 150$

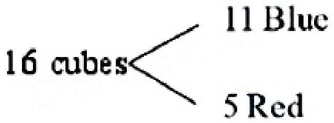
The extra numbers are 53, 487, 491, 493, 497 and 499.

Question64

The number of ways, 16 identical cubes, of which 11 are blue and rest are red, can be placed in a row so that between any two red cubes there should be at least 2 blue cubes, is
[27-Jun-2022-Shift-1]

Answer: 56

Solution:



$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 11$

$x_1, x_6 \ge 0, \quad x_2, x_3, x_4, x_5 \ge 2$

$x_2 = t_1 + 2$

$x_3 = t_3 + 2$

$x_4 = t_4 + 2$

$x_5 = t_5 + 2$

$x_1, t_2, t_3, t_4, t_5, x_6 \ge 0$

No. of solutions $= {}^{6+3-1}C_3 = {}^{8}C_3 = 56$

Question65

The total number of 5-digit numbers, formed by using the digits 1, 2, 3, 5, 6, 7 without repetition, which are multiple of 6 , is [28-Jun-2022-Shift-1]

Options:

A. 36

B. 48

C. 60

D. 72

Answer: D

Solution:

Solution:
To make a no. divisible by 3 we can use the digits 1, 2, 5, 6, 7 or 1, 2, 3, 5, 7.
Using 1, 2, 5, 6, 7, number of even numbers is
$= 4 \times 3 \times 2 \times 1 \times 2 = 48$
Using 1, 2, 3, 5, 7, number of even numbers is
$= 4 \times 3 \times 2 \times 1 \times 1 = 24$
Required answer is 72.

Question66

Let $A = \{1, a_1, a_2 \ldots\ldots a_{18}, 77\}$ be a set of integers with $1 < a_1 < a_2 < \ldots\ldots < a_{18} < 77$. Let the set $A + A = \{x + y : x, y \in A\}$ contain exactly 39 elements. Then, the value of $a_1 + a_2 + \ldots\ldots + a_{18}$ is equal to___ [28-Jun-2022-Shift-1]

Answer: 702

Solution:

$a_1, a_2, a_3, \ldots, a_{18}, 77$
are in AP i.e. 1, 5, 9, 13, …, 77.
Hence $a_1 + a_2 + a_3 + \ldots + a_{18} = 5 + 9 + 13 + \ldots$ 18 terms $= 702$

Question67

The number of ways to distribute 30 identical candies among four children C_1, C_2, C_3 and C_4 so that C_2 receives at least 4 and at most 7 candies, C_3 receives at least 2 and at most 6 candies, is equal to:
[28-Jun-2022-Shift-2]

Options:

A. 205

B. 615

C. 510

D. 430

Answer: D

Solution:

Solution:
By multinomial theorem, no. of ways to distribute 30 identical candies among four children C_1, C_2 and C_3, C_4

$= \text{ Coefficient of } x^{30} \text{ in } (x^4 + x^5 + \ldots. + x^7)(x^2 + x^3 + \ldots. + x^6)(1 + x + x^2 \ldots..)^2$

$= \text{ Coefficient of } x^{24} \text{ in } \frac{(1 - x^4)}{1 - x} \frac{(1 - x^5)}{1 - x} \frac{(1 - x^{31})^2}{(1 - x)^2}$

$= \text{ Coefficient of } x^{24} \text{ in } (1 - x^4 - x^5 + x^9)(1 - x)^{-4}$

$= {}^{27}C_{24} - {}^{23}C_{20} - {}^{22}C_{19} + {}^{18}C_{15} = 430$

Question68

Let $b_1b_2b_3b_4$ be a 4-element permutation with $b_i \in \{1, 2, 3, \ldots\ldots., 100\}$ for $1 \le i \le 4$ and $b_i \ne b_j$ for $i \ne j$, such that either b_1, b_2, b_3 are consecutive integers or b_2, b_3, b_4 are consecutive integers. Then the number of such permutations $b_1b_2b_3b_4$ is equal to___
[29-Jun-2022-Shift-1]

Answer: 18915

Solution:

$b_i \in \{1, 2, 3 \ldots\ldots..100\}$
Let A = set when $b_1b_2b_3$ are consecutive

$$n(A) = \frac{97 + 97 + \ldots\ldots + 97}{98 \text{ times}} = 97 \times 98$$

Similarly when $b_2b_3b_4$ are consecutive
$N(A) = 97 \times 98$

$$n(A \cap B) = \frac{97 + 97 + ------ - 97}{98 \text{ times}} = 97 \times 98$$

Similarly when $b_2b_3b_4$ are consecutive
$n(B) = 97 \times 98$

$n(A \cap B) = 97$
$n(AUB) = n(A) + n(B) - n(A \cap B)$
Number of permutation $= 18915$

Question69

The total number of four digit numbers such that each of first three digits is divisible by the last digit, is equal to____ [29-Jun-2022-Shift-2]

Answer: 1086

Solution:

Let the number is abcd, where a,b,c are divisible by d.
No. of such numbers
$d = 1, \; 9 \times 10 \times 10 = 900$
$d = 2, \; 4 \times 5 \times 5 = 100$
$d = 3, \; 3 \times 4 \times 4 = 48$
$d = 4, \; 2 \times 3 \times 3 = 18$
$d = 5, \; 1 \times 2 \times 2 = 4$
$d = 6, 7, 8, 9 \quad 4 \times 4 = 16$
Total =1086

Question70

The total number of 4-digit numbers whose greatest common divisor with 18 is 3, is [2021, 26 Feb. Shift-II]

Answer: 1000

Solution:

Solution:
Let x be four digit number, then $\gcd(x, 18) = 3$
This implies x is divisible by 3 but not divisible by 9 .
The 4-digit numbers which is an odd multiple of 3 are 1005, 1011, 1017, ... 9999
These are 1499 in counting i.e. total number of 4-digit numbers which is odd multiple of 3 are 1499.
Now, The 4-digit numbers which is an odd multiple of 9 are, 1017, 1035, ...999 These, are total 499.
Then, required 4-digit numbers
$= 1499 - 499 = 1000$

Question71

The total number of two digit numbers n', such that $3^n + 7^n$ is a multiple of 10 , is [2021, 25 Feb. Shift-II]

Answer: 45

Solution:

We may write, $7^n = (10-3)^n$ or $7^n = 10K + (-3)^n$ (using expansion)

$\therefore 7^n + 3^n = 10K + (-3)^n + 3^n$

$$= \begin{cases} 10k & n = \text{odd} \\ 10k + 2 \cdot 3^n & n = \text{even} \end{cases}$$

Let $n = \text{even} = 2t, t \in N$

Then, $3^n = 3^{2t} = 9^t = (10-1)^t$

$= 10p + (-1)^t$

$= 10p \pm 1$

If $n =$ even, then $7^n + 3^n$ will never be multiple of 10.

This implies $n = 0$ dd

$n = 11, 13, 15, \ldots 99$ (since, n is two digit)

$\Rightarrow 10 < n < 100$

Total possible 'n' are 45.

Question72

The total number of numbers, lying between 100 and 1000 that can be formed with the digits 1, 2, 3, 4, 5, if the repetition of digits is not allowed and numbers are divisible by either 3 or 5 , is ....
[2021, 25 Feb. Shift-I]

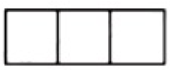
Answer: 32

Solution:

Solution:

Given, digits $= \{1, 2, 3, 4, 5\}$

Numbers divisible by 3 (sum of digits divisible by 3).



Case I When sum is $12 \rightarrow 3, 4, 5 \rightarrow 3! = 6$

Case II When sum is $9 \rightarrow 2, 3, 4 \rightarrow 3! = 6$

Case III When sum is $9 \rightarrow 1, 3, 5 \rightarrow 3! = 6$

Case IV When sum is $6 \rightarrow 1, 2, 3 \rightarrow 3! = 6$

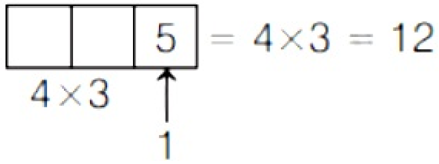
So, total numbers divisible by $3 = 6 \times 4 = 24$

Numbers divisible by 5 (ending with 5)

So, total numbers divisible by $5 = 12$

Numbers divisible by 15 , are $145, 415, 345, 435$

i.e. total 4 numbers are divisible by both 3 and 5.



i.e. divisible by 15 .
Hence, the required numbers which are divisible by 3 or 5 $= 24 + 12 - 4 = 32$

Question73

A natural number has prime factorisation given by $n = 2^x 3^y 5^z$, where y and z are such that $y + z = 5$ and $y^{-1} + z^{-1} = \frac{5}{6}$, $y > z$.
Then, the number of odd divisors of n, including 1 , is
[2021, 26 Feb. Shift-II]

Options:

A. 11

B. 6

C. 6x

D. 12

Answer: D

Solution:

Solution:
Given, $n = 2^x 3^y 5^z$
and $y + z = 5$
$\frac{1}{y} + \frac{1}{z} = \frac{5}{6}$ or $\frac{y+z}{yz} = \frac{5}{6}$
This implies,
$y + z = 5$ and $yz = 6$
Put $y = \frac{6}{z}$ in $y + z = 5$
$\Rightarrow \frac{6}{z} + z = 5$ or $z^2 - 5z + 6 = 0$
$\Rightarrow z^2 - 3z - 2z + 6 = 0$
$(z-3)(z-2) = 0$
$\Rightarrow z = 3$ or 2
Using $y = \frac{6}{z}$, we get $y = 2$ or 3
For calculating the odd divisor x must be 0 i.e. $x = 10$
$\therefore n = 2^0 3^3 5^2$ or $n = 2^0 3^2 5^3$
Total number of odd divisor of nis equal to
$= (3+1)(2+1)$
$= (4)(3) = 12$

Question74

The number of seven digit integers with sum of the digits equal to 10 and formed by using the digits 1, 2 and 3 only is

[2021, 26 Feb. Shift-I]

Options:

A. 42

B. 82

C. 77

D. 35

Answer: C

Solution:

Solution:
To form a seven digit number with sum of digits 10 , all the digits can't be 1, 2 or 3 . Hence, seven digit number must have the following cases,
Case 1. Using 1, 1, 1, 1, 1, 2, 3
Possible seven digit numbers will be
$= \frac{7!}{5!} = 7 \times 6 = 42$
Case 2. Using 2, 2, 2, 1, 1, 1, 1
Possible numbers will be
$= \frac{7!}{3!4!} = \frac{7 \times 6 \times 5}{3 \times 2} = 35$

No more cases will be forme(d)
Hence, total number of seven digit numbers possible
$= 42 + 35 = 77$

Question75

The total number of positive integral solutions (x, y, z), such that xyz = 24 is [2021, 25 Feb. Shift-1]

Options:

A. 36

B. 24

C. 45

D. 30

Answer: D

Solution:

Solution:
Given, $xyz = 24$
$\Rightarrow xyz = 2^3 \cdot 3^1$
Let $x = 2^{a_1} \cdot 3^{b_1}$,

$y = 2^{a_2} \cdot 3^{b_2}$,
$z = 2^{a_3} \cdot 3^{b_3}$
where, $a_1, a_2, a_3 \in \{0, 1, 2, 3\}$
$b_1, b_2, b_3 \in \{0, 1\}$
Case I $a_1 + a_2 + a_3 = 3$
∴ Non-negative solution
$= {}^{3+3-1}C_{3-1} = {}^5C_2 = 10$
Case II $b_1 + b_2 + b_3 = 1$
∴ Non-negative solution
$= {}^{1+3-1}C_{3-1} = {}^3C_2 = 3$
∴ Total solutions $= 10 \times 3 = 30$

Question76

The students $S_1, S_2, \ldots, S_{10}$ are to be divided into 3 groups A, B and C such that each group has at least one student and the group C has at most 3 students. Then, the total number of possibilities of forming such groups is ....
[2021, 24 Feb. Shift-II]

Answer: 31650

Solution:

Solution:
Given, total students $= 10$
number of groups $= 3$ (i.e. A, B and C)
Each group has atleast one student but group C has atmost 3 students.
∴ There are 3 cases depending on number of students in group C.

Case I C has 1 student, then $\left.\begin{matrix} A \\ B \end{matrix}\right] \leftarrow$ 9 students

∴ Number of ways $= {}^{10}C_1 \times [2^9 - 2]$

Case II C has 2 students, then $\left.\begin{matrix} A \\ B \end{matrix}\right] \leftarrow$ 8 Students.

∴ Number of ways $= {}^{10}C_2 \times [2^8 - 2]$

Case III C has 3 students, then $\left.\begin{matrix} A \\ B \end{matrix}\right] \leftarrow$ 7 Students.

∴ Number of ways $= {}^{10}C_3 \times [2^7 - 2]$
∴ Required number of possibilities
$= {}^{10}C_1(2^9 - 2) + {}^{10}C_2(2^8 - 2) + {}^{10}C_3(2^7 - 2)$
$= 2^7[{}^{10}C_1 \times 4 + {}^{10}C_2 \times 2 + {}^{10}C_3]$
$= 20 - 90 - 240$
$= 128[40 + 90 + 120] - 350$
$= (128 \times 250) - 350 = 31650$

Question77

A scientific committee is to be formed from 6 Indians and 8 foreigners,

which includes at least 2 Indians and double the number of foreigners as Indians. Then the number of ways, the committee can be formed, is : [24-Feb-2021 Shift 1]

Options:

A. 1625

B. 575

C. 560

D. 1050

Answer: A

Solution:

Solution:

Indians	Foreigners	Number of Ways
2	4	${}^{6}C_{2} \times {}^{8}C_{4} = 1050$
3	6	${}^{6}C_{3} \times {}^{8}C_{6} = 560$
4	8	${}^{6}C_{4} \times {}^{8}C_{8} = 15$

Total number of ways $= 1625$

Question78

The sum of all the 4-digit distinct numbers that can be formed with the digits 1, 2,2 and 3 is
[2021, 18 March Shift-I]

Options:

A. 26664

B. 122664

C. 122234

D. 22264

Answer: A

Solution:

Solution:
Given, digits are $= 1, 2, 2, 3$
$\therefore$ Total distinct numbers $= 4! / 2! = 12$
1 at unit place $\Rightarrow$ Number of such

numbers $= \dfrac{3!}{2!} = 3$

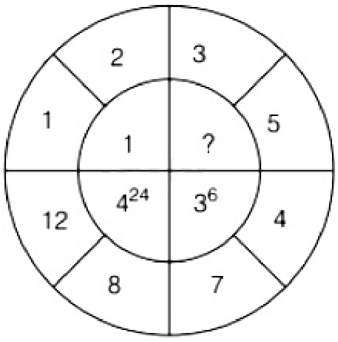
2 at unit place ⇒ Number of such numbers $= 3! = 6$

3 at unit place ⇒ Number of such numbers $= \dfrac{3!}{2!} = 3$

∴ Sum of digits at unit place is
$3 \times 1 + 6 \times 2 + 3 \times 3 = 24$
Hence, sum of all 4 digit such numbers
$= (3 + 12 + 9)(10^3 + 10^2 + 10 + 1)$
$= 1111 \times 24$
$= 26664$

Question79

The missing value in the following figure is



[2021, 18 March Shift-I]

Answer: 4

Solution:

Solution:
As, we observe the pattern Inside number
Inside number = (difference)$^{\text{(difference)!}}$
=(Greater number - Smaller number) $^{\text{(Greater number - Smaller number)!}}$

i.e. $1 = (2 - 1)^{(2-1)!}$, $4^{24} = (12 - 8)^{(12-8)!}$,
$3^6 = (7 - 4)^{(7-4)!}$
$\therefore ? = (5 - 3)^{(5-3)!}$
∴ Required number $= 2^{2!} = 2^{2 \times 1} = 4$

Question80

If $\sum\limits_{r=1}^{10} r!(r^3 + 6r^2 + 2r + 5) = \alpha(11!)$, then the value of α is equal to [2021, 18 March Shift-II]

Answer: 160

Solution:

$$\sum_{r=1}^{10} r![(r+1)(r+2)(r+3)-9(r+1)+8]$$
$$=\sum_{r=1}^{10}[\{(r+3)!-(r+1)!\}-8\{(r+1)!-r!\}]$$
$$=(13!+12!-2!-3!)-8(11!-1)$$
$$=(12\cdot 13+12-8).11!-8+8$$
$$=(160)\ (11!)$$
$$\therefore \alpha=160$$

Question81

The number of times the digit 3 will be written when listing the integers from 1 to 1000 is
[2021, 18 March Shift-1]

Answer: 300

Solution:

Let the number be xyz, $0 \leq x, y, z \leq 9$

Case I ' 3 ' appears only one time $\Rightarrow {}^{3}C_{1} \times 9 \times 9 = 243$

Case II '3' appears two times $\Rightarrow {}^{3}C_{2} \times 2 \times 9 = 54$

Case III ' 3 ' appears three times

$\Rightarrow {}^{3}C_{3} \times 3 = 3$

$\therefore$ Total $= 243 + 54 + 3 = 300$

Question82

Team A consists of 7 boys and n girls and Team B has 4 boys and 6 girls. If a total of 52 single matches can be arranged between these two teams, when a boy plays against a boy and a girl plays against a girl, then n is equal to
[2021, 17 March Shift-I]

Options:

A. 5

B. 2

C. 4

D. 6

Answer: C

Solution:

Solution:

	Boys	Girls
Team A	7	n
Team B	4	6

Number of matches between Team A and Team B when a boy play against a boy $({}^{7}C_{1} \times {}^{4}C_{1}) = 28$
Similarly, number of matches between Team A and Team B when a girl play against a girl $({}^{n}C_{1} \times {}^{6}C_{1}) = 6n$
According to question,
$28 + 6n = 52$
$6n = 24$
$n = 4$

Question83

If the sides AB, BC and CA of a triangle $\triangle ABC$ have 3,5 and 6 interior points respectively, then the total number of triangles that can be constructed using these points as vertices, is equal to [2021, 17 March Shift-II]

Options:

A. 364

B. 240

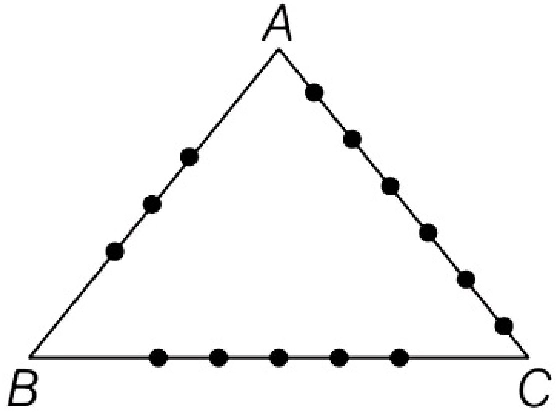
C. 333

D. 360

Answer: C

Solution:

Solution:
Method (I) (Proper Method)
Whenever we construct a triangle, we must require three non-collinear points.



∴ Total number of triangles using the points 3,5 and 6 which are on the sides AB, BC and CA
= Either taking (one point from AB, BC and CA)
or (one point from AB and two points from BC
or (one point from BC and two points from AB)
or (one point from AB and two points from AC)
or (one point from AC and two pointsfrom AB)
or (one point from BC and two points from AC)
or(one point from BC and two points from AC)
or(one point from AC and two points from BC)
⇒ Total number of triangles
$r = ({}^3C_1 \times {}^5C_1 \times {}^6C_1) + ({}^3C_1 \times {}^5C_2)$
$+({}^3C_1 \times {}^6C_2) + ({}^6C_1 \times {}^3C_2) + ({}^5C_2)$
$+ ({}^6C_2)$
$= 90 + 30 + 15 + 45 + 18 + 75 + 60$
$= 333 \left[\text{using } {}^nC_r = \frac{n!}{r!(n-r)!} \right.$
$\left. \text{and } n! = 1 \times 2 \times 3 \times \ldots \times n \right]$

Method (II) (Direct Method)
Total number of points $= 3 + 5 + 6 = 14$
Then, when we construct a triangle, we must select 3 points out of 14 but these points never be collinear.
∴ Total number of triangles formed $= {}^{14}C_3 - {}^3C_3 - {}^5C_3 - {}^6C_3 = 333$

Question84

Consider a rectangle ABCD having 5, 7, 6, 9 points in the interior of the line segments AB, CD, BC, DA, respectively. Let α be the number of triangles having these points from different sides as vertices and β be the number of quadrilaterals having these points from different sides as vertices. Then, (β − α) is equal to
[2021, 16 March Shift-II]

Options:

A. 795

B. 1173

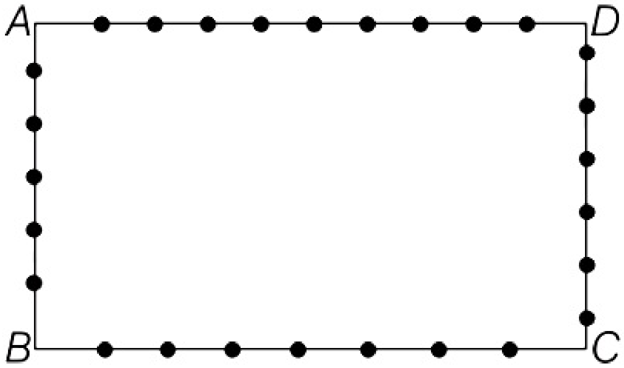
C. 1890

D. 717

Answer: D

Solution:

Solution:



Number of triangles that can be formed from the points on 3 of the sides.
${}^{5}C_1{}^{7}C_1{}^{6}C_1 + {}^{5}C_1{}^{7}C_1{}^{9}C_1 + {}^{5}C_1{}^{6}C_1{}^{9}C_1$
$+{}^{6}C_1{}^{7}C_1{}^{9}C_1$
$= 210 + 315 + 270 + 378$
$\Rightarrow \alpha = 1173$
Number of quadrilaterals that can be formed by taking one point from each of the four vertex
${}^{5}C_1{}^{7}C_1{}^{6}C_1{}^{9}C_1 = 5 \times 6 \times 7 \times 9 = 1890$
$\Rightarrow \beta = 1890$
$\therefore \beta - \alpha = 1890 - 1173$
$= 717$

Question85

Words with or without meaning are to be formed using all the letters of the word EXAMINATION. The probability that the letter M appears at the fourth position in any such word is
[2021, 20 July Shift-1]

Options:

A. 1 / 66

B. 1 / 11

C. 1 / 9

D. 2/11

Answer: B

Solution:

$\underbrace{EXA}_{3}\ M\ \underbrace{INATION}_{7}$

Let $x =$ When Mis at fourth place $= \dfrac{10!}{2!2!2!}$

Let $y =$ Total number of words $= \dfrac{11!}{2!2!2!}$

Probability $= \dfrac{x}{y} = \dfrac{1}{11}$

Question86

There are 15 players in a cricket team, out of which 6 are bowlers, 7 are batsmen and 2 are wicketkeepers. The number of ways, a team of 11 players be selected from them so as to include atleast 4 bowlers, 5 batsman and 1 wicketkeeper, is
[2021, 20 July Shift-I]

Answer: 777

Solution:

Total number of players $= 15$
Bowlers $= 6$, Batsman $= 7$, Wicket keepers $= 2$

Bowlers	Batsman	Wicket Keepers	Total
$4+1$	5	1	${}^6C_5 \times {}^7C_5 \times {}^2C_1 = 252$
4	$5+1$	1	${}^6C_4 \times {}^7C_6 \times {}^2C_1 = 210$
4	5	$1+1$	${}^6C_4 \times {}^7C_5 \times {}^2C_2 = 315$

Total $= 252 + 210 + 315 = 777$

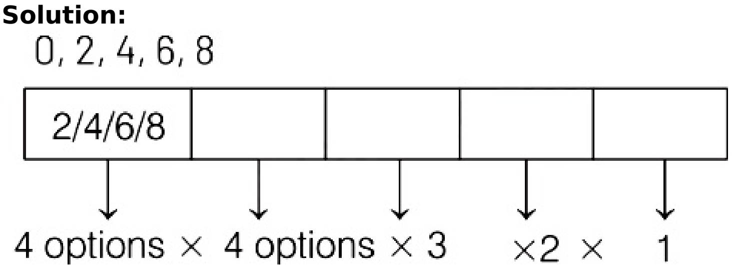
Question87

If the digits are not allowed to repeat in any number formed by using the digits 0, 2, 4, 6, 8, then the number of all numbers greater than 10000 is equal to ....
[2021, 22 July Shift-II]

Answer: 96

Solution:

Solution:



$\therefore$ Total $= 4 \times 4 \times 3 \times 2 \times 1 = 96$

Question88

If ${}^nP_r = {}^nP_{r+1}$ and ${}^nC_r = {}^nC_{r-1}$, then the value of r is equal to
[2021, 25 July Shift-II]

Options:

A. 1

B. 4

C. 2

D. 3

Answer: C

Solution:

Solution:

Given, ${}^nP_r = {}^nP_{r+1}$

$\Rightarrow \dfrac{n!}{(n-r)!} = \dfrac{n!}{(n-r-1)!}$

$\Rightarrow \dfrac{n!}{(n-r)(n-r-1)!} = \dfrac{n!}{(n-r-1)!}$

$\Rightarrow n - r = 1$ ...(i)

and

$\Rightarrow \dfrac{n!}{r!(n-r)!} = \dfrac{{}^nC_r = {}^nC_{r-1}!}{(r-1)!(n-r+1)!}$

$\Rightarrow \dfrac{1}{r(n-r)!} = \dfrac{1}{(n-r+1)(n-r)!}$

$\Rightarrow n - r + 1 = r$

From Eq. (i),

$1 + 1 = r \Rightarrow r = 2$

Question89

There are 5 students in class 10,6 students in class 11 and 8 students in class 12. If the number of ways in which 10 students can be selected from them so as to include at least 2 students from each class and at most 5 students from the total 11 students of class 10 and 11 is 100k, then k is equal to ....
[2021, 25 July Shift-I]

Answer: 238

Solution:

10(5)	11(6)	12(8)
2+1	2	2+3
2	2+1	2+3
2	2	2+4
	5	5

$\Rightarrow {}^5C_3 \times {}^6C_2 \times {}^8C_5 = 8400$
$\Rightarrow {}^5C_2 \times {}^6C_3 \times {}^8C_5 = 11200$
$\Rightarrow {}^5C_2 \times {}^6C_2 \times {}^8C_6 = 4200$
$\text{Total} = 8400 + 11200 + 4200 = 23800$
According to the question, $100K = 23800$
$K = 238$

Question90

Let $n \in N$ and $[x]$ denote the greatest integer less than or equal to x. If the sum of (n + 1) terms ${}^nC_0, 3 . {}^nC_1, 5 . {}^nC_2, 7 . {}^nC_3$....... is equal to $2^{100}.101$, then $2\left[\frac{n-1}{2}\right]$ is equal to

[2021, 25 July Shift-II]

Answer: 98

Solution:

We have,
$1{}^nC_0 + 3{}^nC_1 + 5{}^nC_2 + \ldots + (2n+1){}^nC_n$
$T_r = (2r+1){}^nC_r$
Now, $\text{sum}(S) = \sum T_r$
$S = \sum(2r+1){}^nC_r$
$= 2\sum r{}^nC_r + \sum {}^nC_r$
$= 2(n2^{n-1}) + 2^n = n \cdot 2^n + 2^n$
$\therefore S = 2^n(n+1)$
Given that, $S = 2^{100} \cdot 101$
$\Rightarrow 2^n(n+1) = 2^{100} \cdot 101$
$\Rightarrow n = 100$
Now, $2\left[\frac{n-1}{2}\right] = 2\left[\frac{100-1}{2}\right] = 2\left[\frac{99}{2}\right]$
$= 2[49.5] = 2 \times 49 = 98$
($\because [x]$ is greatest integer function)

Question91

Let n be a non-negative integer. Then the number of divisors of the form" $4n+1$" of the number $(10)^{10} \cdot (11)^{11} \cdot (13)^{13}$ is equal to [2021, 27 July shift-II]

Answer: 924

Solution:

Solution:
Let $N = (10)^{10} \cdot (11)^{11} \cdot (13)^{13}$
$N = 2^{10} \cdot 5^{10} \cdot 11^{11} \cdot 13^{13}$
Now, power of 2 must be zero. Power of 5 can be anything. Power of 13 can be anything. But power of 11 should be even.
So, required number of divisor is $= 1 \times 11 \times 14 \times 6 = 924$

Question92

The sum of all three-digit numbers less than or equal to 500, that are formed without using the digit 1 and they all are multiple of 11 , is [2021, 26 Aug. Shift-II]

Answer: 7744

Solution:

Solution:
Multiples of 11 such that they are of 3 -digit and less than 500.
$121, 132, \ldots, 495$
$cn = \frac{495-121}{11} + 1 = 35$
$S = \frac{35}{2}(121 + 495) = 10780$
Again, multiplies of 11 which are 3-digits, less than 500 and having 1 at hundred's place are $121, 132, \ldots, 198$
$n_1 = \left(\frac{198-121}{11}\right) + 1 = 8$
$S_1 = \frac{8}{2}(121 + 198) = 1276$
The multiply of 11 which are of 3 -digits, less than 500 and having 1 at ten's place are 319,418
$\therefore\ S_2 = 319 + 418 = 737$
The multiple of 11 which are 3-digits, less than 500 and having 1 at unit place are 231, 341, 451
$\therefore\ S_3 = 231 + 341 + 451 = 1023$
$\therefore$ Required sum $= S - S_1 - S_2 - S_3$
$= 7744$

Question93

The number of three-digit even numbers, formed by the digits 0,1 , 3, 4, 6, 7, if the repetition of digits is not allowed, is
[2021, 26 Aug. Shift-I]

Answer: 52

Solution:

Solution:
Case I When 0 is at unit place
$\frac{}{(5)} \times \frac{-0}{(4)} \times \frac{0}{(1)} = 20$
Case II When 4 or 6 are at unit place
$\frac{1}{(4)} \times \frac{\frac{4}{6}}{(4)} \times \frac{6}{(2)} = 32$
[0 cannot be come at hundredth place]
∴ Total number of required
$= 20 + 32 = 52$

Question94

A number is called a palindrome if it reads the same backward as well as forward For example 285582 is a six digit palindrome. The number of six digit palindromes, which are divisible by 55 , is
[2021, 27 Aug. Shift-I]

Answer: 100

Solution:

Solution:
Form of six digit palindrome number xyzzyx
This will be divisible by 55 Hence, $x = 5$ and $5yzzy5$ will be divisible by 11 .

$\Rightarrow (5 + z + y) - (y + z + 5)$ is divisible by 11 which is true for all values of y and z $\Rightarrow y$ and z can be chosen in 10×10 ways
Number of such number $= 100$

Question95

Let S = {1, 2, 3, 4, 5, 6, 9}. Then, the number of elements in the set T = { A ⊂ eqS : A ≠ φ and the sum of all the elements of A is not a multiple of 3 \} is
[2021, 27 Aug. Shift-II]

Answer: 80

Solution:

Solution:
$S = \{1, 2, 3, 4, 5, 6, 9\}$
3 n Type numbers 3, 6, 9
$3n - 1$ Type numbers 2,5
$3n - 2$ Type numbers 1,4
Let $N_P =$ Number of Subset of S
containing p element which are not divisible by 3 .
For $P = 1$
${}^2C_1 + {}^2C_1 = 4$

For $P = 2$
${}^3C_1{}^2C_1 + {}^3C_1{}^2C_1 + {}^2C_2 + {}^2C_2 = 14$
For $P = 3$
${}^3C_1({}^2C_2 + {}^2C_2) + {}^3C_2({}^2C_1 + {}^2C_1) + {}^2C_2{}^2C_1$
$+{}^2C_1{}^2C_2 = 22$
For $P = 4$
${}^3C_1[{}^2C_2{}^2C_1 + {}^2C_1{}^2C_2] + {}^3C_2({}^2C_2 + {}^2C_2)$
$+{}^3C_3({}^2C_1 + {}^2C_1) = 22$
For $P = 5$
${}^3C_2({}^2C_2{}^2C_1 + {}^2C_1{}^2C_2) + {}^3C_3({}^2C_2 + {}^2C_2) = 14$
For $P = 6$
${}^3C_3({}^2C_2{}^2C_1 + {}^2C_1{}^2C_2) = 4$

Total Subsets
$= 4 + 14 + 22 + 22 + 14 + 4 = 80$

Question96

The number of six letter words (with or without meaning), formed using all the letters of the word 'VOWELS', so that all the consonants never come together, is [2021, 31 Aug. Shift-1]

Answer: 576

Solution:

VOWELS (2 Vowel +4 consonant)
All consonants must not be together
Total possibility of formation of 6 letter word $= 6\,!$
The number of arrangement when all the consonoment comes together $= 3! \times 4\,!$
Number of arrangement when all the consonants never come together
$=$ Total - All consonant together $= 6! - 3!4! = 576$

Question97

The number of ordered pairs (r, k) for which $6 \cdot {}^{35}C_r = (k^2 - 3) \cdot {}^{36}C_{r+1}$, where k is an integer, is:
[Jan. 7, 2020 (II)]

Options:

A. 3

B. 2

C. 6

D. 4

Answer: D

Solution:

Solution:
$\frac{36}{r+1} \times {}^{35}C_r(k^2 - 3) = {}^{35}C_r \cdot 6$
$\Rightarrow k^2 - 3 = \frac{r+1}{6}$
$\Rightarrow k^2 = 3 + \frac{r+1}{6}$
r can be 5,35 for $k \in$
$r = 5, k = \pm 2$
$r = 35, k = \pm 3$
Hence, number of ordered pairs $= 4$

Question98

An urn contains 5 red marbles, 4 black marbles and 3 white marbles. Then the number of ways in which 4 marbles can be drawn so that at the most three of them are red is ________.
[NA Jan. 8, 2020 (I)]

Answer: 490

Solution:

Solution:
0 Red, 1 Red, 2 Red, 3 Red
Number of ways of selecting atmost three red balls
$= {}^7C_4 + {}^5C_1 \cdot {}^7C_3 + {}^5C_2 \cdot {}^7C_2 + {}^5C_3 \cdot {}^7C_1$
$= 35 + 175 + 210 + 70 = 490$

Question99

If a, b and c are the greatest values of ${}^{19}C_p$, ${}^{20}C_q$ and ${}^{21}C_r$ respectively, then:
[Jan. 8, 2020 (I)]

Options:

A. $\frac{a}{11} = \frac{b}{22} = \frac{c}{21}$

B. $\frac{a}{10} = \frac{b}{11} = \frac{c}{21}$

C. $\frac{a}{11} = \frac{b}{22} = \frac{c}{42}$

D. $\frac{a}{10} = \frac{b}{11} = \frac{c}{42}$

Answer: C

Solution:

Solution:
We know ${}^{n}C_r$ is greatest at middle term.
So, $a = ({}^{19}C_p)_{max} = {}^{19}C_{10} = {}^{19}C_9$
$b = ({}^{20}C_q)_{max} = {}^{20}C_{10}$
$c = ({}^{21}C_6)_{max} = {}^{21}C_{10} = {}^{21}C_{11}$

$$\text{Now, } \frac{a}{19}C_9 = \frac{b}{\frac{20}{10} \cdot {}^{19}C_9} = \frac{c}{\frac{21}{11} \cdot \frac{20}{10}{}^{19}C_9}$$

$$\Rightarrow \frac{a}{1} = \frac{b}{2} = \frac{c}{42/11} \quad \therefore \quad \frac{a}{11} = \frac{b}{22} = \frac{c}{42}$$

Question100

The number of 4 letter words (with or without meaning) that can be formed from the eleven letters of the word 'EXAMINATION' is _______.
[NA Jan. 8, 2020 (II)]

Answer: 2454

Solution:

Solution:
EXAMINATION
2N , 2A, 2I , E , X , M , T , O
Case I : If all are different, then
${}^{8}p_4 = \frac{8!}{4!} = 8.7.6.5 = 1680$

Case II : If two are same and two are different, then ${}^{3}C_1 \cdot {}^{7}C_2 \cdot \frac{4!}{2!} = 3.21.12 = 756$

Case III : If two are same and other two are same, then

$^{3}C_{2}\cdot\frac{4!}{2!2!}=3.6=18$

$\therefore$ Total cases $=1680+756+18=2454$

Question101

$\mathop{Lt}_{r}\equiv{}^{25}C_r$ and $C_0+5\cdot C_1+9\cdot C_2+...+(101)\cdot C_{25}=2^{25}\cdot k$, then k is equal to

[NA Jan. 9, 2020 (II)]

Answer: 51

Solution:

$$\sum_{r=0}^{25}(4r+1)^{25}C_r=4\sum_{r=0}^{25}r\cdot{}^{25}C_r+\sum_{r=0}^{25}{}^{25}C_r$$

$$=4\sum_{r=1}^{25}r\times\frac{25}{r}{}^{24}C_{r-1}+2^{25}=100\sum_{r=1}^{25}{}^{24}C_{r-1}+2^{25}$$

$$=100.2^{24}+2^{25}=2^{25}(50+1)=51.2^{25}$$

Hence, by comparison $k=51$

Question102

If the number of five digit numbers with distinct digits and 2 at the 10^{th} place is 336k, then k is equal to:

[Jan. 9, 2020 (I)]

Options:

A. 4

B. 6

C. 7

D. 8

Answer: D

Solution:

Solution:

Number of five digit numbers with 2 at 10^{th} place

$=8\times8\times7\times6=2688$

$\because$ It is given that, number of five digit number with 2 at 10^{th} place $=336k$

$\therefore$ $336k=2688\Rightarrow k=8$

Question103

Total number of 6 -digit numbers in which only and all the five digits 1,3,5,7 and 9 appear, is:
[Jan. 7, 2020 (I)]

Options:

A. $\frac{1}{2}(6!)$

B. 6!

C. 5^6

D. $\frac{5}{2}(6!)$

Answer: D

Solution:

Solution:
Five digits numbers be 1,3,5,7,9
For selection of one digit, we have ${}^5C_1$
choice. And six digits can be arrange in $\frac{6!}{2!}$ ways.
Hence, total such numbers $= \frac{5.6!}{2!} = \frac{5}{2}.6!$

Question104

Two families with three members each and one family with four members are to be seated in a row. In how many ways can they be seated so that the same family members are not separated?
[Sep. 06, 2020 (I)]

Options:

A. 2!3!4!

B. $(3!)^3 \cdot (4!)$

C. $(3!)^2 \cdot (4!)$

D. $3!(4!)^3$

Answer: B

Solution:

Solution:
Number of arrangement
$= (3! \times 3! \times 4!) \times 3! = (3!)^3 4!$

Question105

The value of $(2 \cdot {}^1P_0 - 3 \cdot {}^2P_1 + 4 \cdot {}^3P_2 - \ldots$ up to 51^{th} term $)$ $+(1! - 2! + 3! - \ldots$ up to 51^{th} term $)$ is equal to :
[Sep. 03, 2020 (I)]

Options:

A. $1 - 51(51)!$

B. $1 + (51)!$

C. $1 + (52)!$

D. 1

Answer: C

Solution:

Solution:

We know, $(r+1) \cdot {}^rP_{r-1} = (r+1) \cdot \frac{r!}{1!} = (r+1)!$

So, $(2 \cdot {}^1P_0 - 3 \cdot {}^2P_1 + \ldots .51$ terms $)+$

$(1! - 2! + 3! - \ldots$ upto 51 terms $)$

$= [2! - 3! + 4! - \ldots + 52!] + [1! - 2! + 3! - \ldots + 51!]$

$= 52! + 1! = 52! + 1$

Question106

If the letters of the word 'MOTHER' be permuted and all the words so formed (with or without meaning) be listed as in a dictionary, then the position of the word 'MOTHER' is _______.
[NA Sep. 02, 2020 (I)]

Answer: 309

Solution:

M - 3
O - 4
T - 6
H - 2
E - 1
R - 5

$\Rightarrow 2 \times 5! + 2 \times 4! + 3 \times 3! + 2! + 1$

$= 240 + 48 + 18 + 2 + 1 = 309$

Question107

The number of words (with or without meaning) that can be formed from all the letters of the word "LETTER" in which vowels never come together is ________.
[NA Sep. 06, 2020 (II)]

Answer: 120

Solution:

Solution:
For vowels not together

Number of ways to arrange L, T , T , R = $\frac{4!}{2!}$

Then put both E in 5 gaps formed in $^5C_2$ ways.

∴ No. of ways $= \frac{4!}{2!} \cdot {}^5C_2 = 120$

Question108

The number of words, with or without meaning, that can be formed by taking 4 letters at a time from the letters of the word 'SYLLABUS' such that two letters are distinct and two letters are alike, is ________.
[NA Sep. 05, 2020 (I)]

Answer: 240

Solution:

Solution:
$S \to 2, L \to 2, A, B, Y, U$

∴ Required number of ways $= {}^2C_1 \times {}^5C_2 \times \frac{4!}{2!} = 240$

Question109

There are 3 sections in a question paper and each section contains 5 questions. A candidate has to answer a total of 5 questions, choosing at least one question from each section. Then the number of ways, in which the candidate can choose the questions, is:
[Sep. 05, 2020 (II)]

Options:

A. 3000

B. 1500

C. 2255

D. 2250

Answer: D

Solution:

Solution:
Since, each section has 5 questions.
∴ Total number of selection of 5 questions
$= 3 \times {}^5C_1 \times {}^5C_1 \times {}^5C_3 + 3 \times {}^5C_1 \times {}^5C_2 \times {}^5C_2$
$= 3 \times 5 \times 5 \times 10 + 3 \times 5 \times 10 \times 10$
$= 750 + 1500 = 2250$

Question110

A test consists of 6 multiple choice questions, each having 4 alternative answers of which only one is correct. The number of ways, in which a candidate answers all six questions such that exactly four of the answers are correct, is
[NA Sep. 04, 2020 (II)]

Answer: 135

Solution:

Solution:
Select any 4 correct questions in ${}^6C_4$ ways.
Number of ways of answering wrong question $= 3$
∴ Required number of ways $= {}^6C_4(1)^4 \times 3^2 = 135$

Question111

The total number of 3 -digit numbers, whose sum of digits is 10, is
[NA Sep. 03, 2020 (II)]

Answer: 54

Solution:

Solution:
Let xyz be the three digit number
$x+y+z=10, x \leq 1, y \geq 0, z \geq 0$
$x-1=t \Rightarrow x=1+t \qquad x-1 \geq 0, t \geq 0$
$t+y+z=10-1=9 \qquad 0 \leq t, z, z \leq 9$
$\therefore$ Total number of non-negative integral solution $= {}^{9+3-1}C_{3-1} = {}^{11}C_2 = \dfrac{11 \cdot 10}{2} = 55$
But for $t=9$, $x=10$, so required number of integers
$= 55-1=54$

Question112

Let n > 2 be an integer. Suppose that there are n Metro stations in a city located along a circular path. Each pair of stations is connected by a straight track only. Further, each pair of nearest stations is connected by blue line, whereas all remaining pairs of stations are connected by red line. If the number of red lines is 99 times the number of blue lines, then the value of n is: [Sep. 02, 2020 (II)]

Options:

A. 201

B. 200

C. 101

D. 199

Answer: A

Solution:

Solution:
Number of two consecutive stations (Blue lines) $= n$
Number of two non-consecutive stations (Red lines) $= {}^nC_2 - n$
Now, according to the question, ${}^nC_2 - n = 99n$
$\Rightarrow \dfrac{n(n-1)}{2} - 100n = 0 \Rightarrow n(n-1-200) = 0$
$\Rightarrow n-1-200=0 \Rightarrow n=201$

Question113

Consider a class of 5 girls and 7 boys. The number of different teams consisting of 2 girls and 3 boys that can be formed from this class, if there are two specific boys A and B, who refuse to be the members of the same team, is: [Jan. 9, 2019 (I)]

Options:

A. 500

B. 200

C. 300

D. 350

Answer: C

Solution:

Solution:
Since, the number of ways to select 2 girls is ${}^5C_2$.
Now, 3 boys can be selected in 3 ways.
(a) Selection of A and selection of any 2 other boys (except B) in ${}^5C_2$ ways
(b) Selection of B and selection of any 2 twoother boys (except A) in ${}^5C_2$ ways
(c) Selection of 3 boys (except A and B) in ${}^5C_3$ ways
Hence, required number of different teams
$= {}^5C_2({}^5C_2 + {}^5C_2 + {}^5C_3) = 300$

Question114

The number of natural numbers less than 7,000 which can be formed by using the digits 0,1,3,7,9 (repetition of digits allowed) is equal to: [Jan. 09, 2019 (II)]

Options:

A. 374

B. 372

C. 375

D. 250

Answer: A

Solution:

Solution:
Number of numbers with one digit $= 4 = 4$
Number of numbers with two digits $= 4 \times 5 = 20$
Number of numbers with three digits $= 4 \times 5 \times 5$
$= 100$ Number of numbers with four digits $= 2 \times 5 \times 5 \times 5$
$= 250$
$\therefore$ Total number of numbers $= 4 + 20 + 100 + 250$
$= 374$

Question115

Let S be the set of all triangles in the xy-plane, each having one vertex at the origin and the other two vertices lie on coordinate axes with integral coordinates. If each triangle in S has area 50 sq. units, then the number of elements in the set S is:
[Jan. 09, 2019 (II)]

Options:

A. 9

B. 18

C. 36

D. 32

Answer: C

Solution:

Solution:
One of the possible ΔOAB is $A(a, 0)$ and $B(0, b)$.
Area of $\Delta OAB = \frac{1}{2}\left|ab\right|$
$\therefore |ab| = 100$
$|a||b| = 100$
But $100 = 1 \times 100, 2 \times 50, 4 \times 25, 5 \times 20$ or 10×10
$\therefore$ For 1×100, $a = 1$ or -1 and $b = 100$ or -100
$\therefore$ Total possible pairs are 8
Total possible pairs for $1 \times 100, 2 \times 50, 4 \times 25$ or 5×20 are 4×8
And for 10×10 total possible pairs are 4
$\therefore$ Total number of possible triangles with integral coordinates are $4 \times 8 + 4 = 36$

Question116

If $\sum_{i=1}^{20}\left(\frac{{}^{20}C_{i-1}}{{}^{20}C_i + {}^{20}C_{i-1}}\right)^3 = \frac{k}{21}$, then k equals:
[Jan. 10, 2019 (I)]

Options:

A. 400

B. 50

C. 200

D. 100

Answer: D

Solution:

Solution:
Consider the expression,

$$\frac{{}^{20}C_{i-1}}{{}^{20}C_i + {}^{20}C_{i-1}} = \frac{{}^{20}C_{i-1}}{{}^{21}C_1}$$

$$= \frac{20!}{(i-1)!(21-i)!} \times \frac{i!(21-i)!}{21!} = \frac{i}{21}$$

$$\therefore \sum_{i=1}^{20} \left(\frac{{}^{20}C_{i-1}}{{}^{20}C_i + {}^{20}C_{i-1}} \right)^3 = \sum_{i=1}^{20} \left(\frac{i}{21} \right)^3 = \frac{(1)}{(21)^3} \sum_{i=1}^{20} i^3$$

$$= \frac{1}{(21)^3} \times \left(\frac{20 \times 21}{2} \right)^2 = \frac{100}{21}$$

$$\therefore \sum_{i=1}^{20} \left(\frac{{}^{20}C_{i-1}}{{}^{20}C_i + {}^{20}C_{i-1}} \right)^3 = \frac{k}{21}$$

$\therefore\ k = 100$

Question117

Consider three boxes, each containing 10 balls labelled 1, 2, ..., 10. Suppose one ball is randomly drawn from each of the boxes. Denote by n_i, the label of the ball drawn from the i^{th} box, (i = 1, 2, 3). Then, the number of ways in which the balls can be chosen such that $n_1 < n_2 < n_3$ is :
[Jan. 12, 2019 (I)]

Options:

A. 120

B. 82

C. 240

D. 164

Answer: A

Solution:

Solution:
Collecting different labels of balls drawn $= 10 \times 9 \times 8$
∵ arrangement is not required.
∴ the number of ways in which the balls can be chosen is,
$\frac{10 \times 9 \times 8}{3!} = 120$

Question118

There are m men and two women participating in a chess tournament. Each participant plays two games with every other participant. If the number of games played by the men between themselves exceeds the number of games played between the men and the women by 84, then the value of m is
[Jan. 12, 2019 (II)]

Options:

A. 12

B. 11

C. 9

D. 7

Answer: A

Solution:

Solution:
$^mC_2 \times 2 = {}^mC_1 \cdot {}^2C_1 \times 2 + 84$
$m(m-1) = 4m + 84$
$m^2 - 5m - 84 = 0$
$m^2 - 12m - 7m - 84 = 0$
$m(m-12) + 7(m-12) = 0$
$m = 12, m = -7$
$\because\ m > 0$
$m = 12$

Question119

The number of four-digit numbers strictly greater than 4321 that can be formed using the digits 0,1,2,3,4,5 (repetition of digits is allowed) is: [April 08, 2019 (II)]

Options:

A. 288

B. 360

C. 306

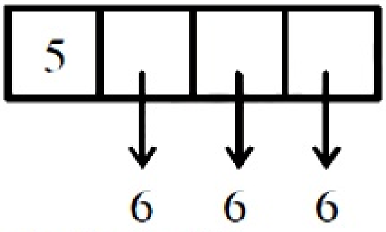
D. 310

Answer: D

Solution:

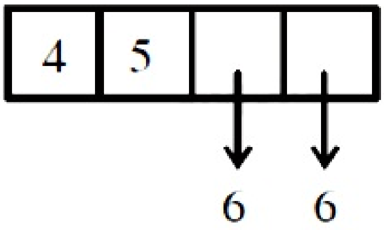
0, 1, 2, 3, 4, 5

Number of four-digit number starting with 5 is,



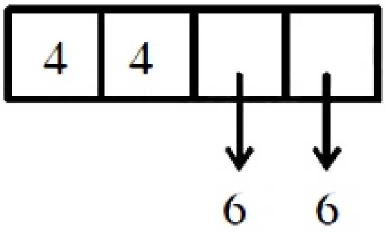
= 6 × 6 × 6 = 216

Number of four-digit numbers starting with 45 is,



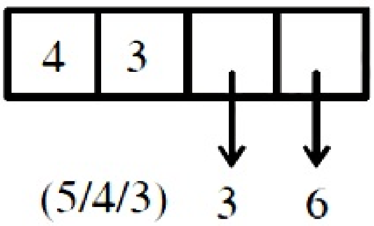
= 6 × 6 = 36

Number of four-digit numbers starting with 44 is,



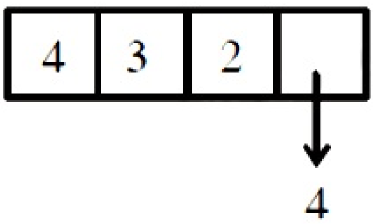
= 6 × 6 = 36

Number of four-digit numbers starting with 43 and greaterthan 4321 is,



= 3 × 6 = 18

Number of four-digit numbers starting with 432 and greaterthan 4321 is,



= 4

Hence, required numbers = 216 + 36 + 36 + 18 + 4 = 310.

Question120

A committee of 11 members is to be formed from 8 males and 5 females. If m is the number of ways the committee is formed with at least 6 males and n is the number of ways the committee is formed with at least 3 females, then:
[April 9, 2019 (I)]

Options:

A. $m + n = 68$

B. $m = n = 78$

C. $n = m - 8$

D. $m = n = 68$

Answer: B

Solution:

Solution:
Since, $m =$ number of ways the committee is formed with at least 6 males
$= {}^8C_6 \cdot {}^5C_5 + {}^8C_7 \cdot {}^5C_4 + {}^8C_8 \cdot {}^5C_3 = 78$
and $n =$ number of ways the committee is formed with at least 3 females

$= {}^5C_3 \cdot {}^8C_8 + {}^5C_4 \cdot {}^8C_7 + {}^5C_5 \cdot {}^8C_6 = 78$
Hence, $m = n = 78$

Question121

All possible numbers are formed using the digits 1,1, 2, 2, 2, 2, 3, 4, 4 taken all at a time. The number of such numbers in which the odd digits occupy even places is:
[April 8, 2019 (I)]

Options:

A. 180

B. 175

C. 160

D. 162

Answer: A

Solution:

∵ There are total 9 digits and out of which only 3 digits are odd.

∴ Number of ways to arrange odd digits first $= {}^4C_3 \cdot \frac{3!}{2!}$

Hence, total number of 9 digit numbers

$= \left({}^4C_3 \cdot \frac{3!}{2!}\right) \cdot \frac{6!}{2!4!} = 180$

Question122

The number of 6 digit numbers that can be formed using the digits 0,1,2,5,7 and 9 which are divisible by 11 and no digit is repeated, is: [April 10, 2019 (I)]

Options:

A. 72

B. 60

C. 48

D. 36

Answer: B

Solution:

Given digit 0, 1, 2, 5, 7, 9

a_1	a_2	a_3	a_4	a_5	a_6

$(a_1 + a_3 + a_5) - (a_2 + a_4 + a_6) = 11K$

Therefore, (1,2,9) (0,5,7)

Number of ways to arranging them

$= 3! \times 3! + 3! \times 2 \times 2 = 6 \times 6 + 6 \times 4 = 6 \times 10 = 60$

Question123

Suppose that 20 pillars of the same height have been erected along the boundary of a circular stadium. If the top of each pillar has been connected by beams with the top of all its non-adjacent pillars, then the total number of beams is:
[April 10, 2019 (II)]

Options:

A. 170

B. 180

C. 210

D. 190

Answer: A

Solution:

Solution:
Total number of beams $= {}^{20}C_2 - 20 = 190 - 20 = 170$

Question124

The number of ways of choosing 10 objects out of 31 objects of which 10 are identical and the remaining 21 are distinct is:
[April 12, 2019 (I)]

Options:

A. $2^{20} - 1$

B. 2^{21}

C. 2^{20}

D. $2^{20} + 1$

Answer: C

Solution:

Number of ways of selecting 10 objects
$= (10I, 0D)$ or $(9I, 1D)$ or $(8I, 1D)$ or $\ldots (0I, 10D)$
Here, D signifies distinct object and I indicates identical object
$= 1 + {}^{21}C_1 + {}^{21}C_2 + \ldots + {}^{21}C_{10} = \dfrac{2^{21}}{2} = 2^{20}$

Question125

A group of students comprises of 5 boys and n girls. If the number of ways, in which a team of 3 students can randomly be selected from this group such that there is at least one boy and at least one girl in each team, is 1750 , then n is equal to :
[April 12, 2019 (II)]

Options:

A. 28

B. 27

C. 25

D. 24

Answer: C

Solution:

Solution:
Number of ways of selecting three persons such that there is atleast one boy and atleast one girl in the selected persons
$= {}^{n+5}C_3 - {}^{n}C_3 - {}^{5}C_3 = 1750$
$\Rightarrow \dfrac{(n+5)!}{3!(n+2)!} - \dfrac{n!}{3!(n-3)!} - \dfrac{5!}{3!2!} = 1750$
$\Rightarrow \dfrac{(n+5)(n+4)(n+3)}{6} - \dfrac{n(n-1)(n-2)}{6} = 1760$
$\Rightarrow n^2 + 3n - 700 = 0 \Rightarrow n = 25$ [$n = -28$ rejected]

Question126

n− digit numbers are formed using only three digits 2,5 and 7 . The smallest value of n for which 900 such distinct numbers can be formed, is
[Online April 15, 2018]

Options:

A. 6

B. 8

C. 9

D. 7

Answer: D

Solution:

Solution:
Required n digit numbers is 3^n as each place can be filled by 2,5,7
So smallest value of n such that $3^n > 900$. Therefore $n = 7$.

Question127

The number of four letter words that can be formed using the letters of the word BARRACK is [Online April 15, 2018]

Options:

A. 144

B. 120

C. 264

D. 270

Answer: D

Solution:

Solution:
If all four letters are different then the number of words
$^5C_4 \times 4! = 120$
If two letters are R and other two different letters are chosen from B, A, C, K then the number of words
$= {}^4C_2 \times \frac{4!}{2!} = 72$
If two letters are A and other two different letters are chosen from B, R, C, K then the number of words
$= {}^4C_2 \times \frac{4!}{2!} = 72$
If word is formed using two R 's and two A 's then the number of words $= \frac{4!}{2!2!} = 6$
Therefore, the number of four-letter words that can be formed $= 120 + 72 + 72 + 6 = 270$

Question128

The number of numbers between 2,000 and 5,000 that can be formed with the digits 0, 1, 2, 3, 4, (repetition of digits is not allowed) and are multiple of 3 is? [Online April 16, 2018]

Options:

A. 30

B. 48

C. 24

D. 36

Answer: A

Solution:

Solution:
The thousands place can only be filled with 2,3 or 4, since the number is greater than 2000 .
For the remaining 3 places, we have pick out digits such that the resultant number is divisible by 3 .
It the sum of digits of the number is divisible by 3, then the number itself is divisible by 3
Case 1: If we take 2 at thousands place.
The remaining digits can be filled as:
0,1 and 3 as $2+1+0+3=6$ is divisible by 3 0,3 and 4 as $2+3+0+4=9$ is divisible by 3
In both the above combinations the remaining three digits can be arranged in $3!$ ways.
$\therefore$ Total number of numbers in this case $=2 \times 3!=12$.
Case 2: If we take 3 at thousands place. The remaining digits can be filled as:
0,1 and 2 as $3+1+0+2=6$ is divisible by 3 .
0,2 and 4 as $3+2+0+4=9$ is divisible by 3 .
In both the above combinations, the remaining three digits can be arranged in $3!$ ways. Total number of numbers in this case $=2 \times 3!=12$
Case 3 : If we take 4 at thousands place.
The remaining digits can be filled as:
0,2 and 3 as $4+2+0+3=9$ is divisible by 3 .
In the above combination, the remaining three digits can be arranged in $3!$ ways.
$\therefore$ Total number of numbers in this case $=3!=6$
$\therefore$ Total number of numbers between 2000 and 5000 divisible by 3 are $12+12+6=30$

Question129

From 6 different novels and 3 different dictionaries, 4 novels and 1 dictionary are to be selected and arranged in a row on a shelf so that the dictionary is always in the middle. The number of such arrangements is: [2018]

Options:

A. less than 500

B. at least 500 but less than 750

C. at least 750 but less than 1000

D. at least 1000

Answer: D

Solution:

Solution:
$\therefore$ Required number of ways $={}^{6}C_{4} \times {}^{3}C_{1} \times 4!$
$=15 \times 3 \times 24=1080$

Question130

If all the words, with or without meaning, are written using the letters of the word QUEEN and are arranged as in English dictionary, then the position of the word QUEEN is:
[Online April 8, 2017]

Options:

A. 44^{th}

B. 45^{th}

C. 46^{th}

D. 47^{th}

Answer: C

Solution:

Solution:
E , E , N , Q, U
(i) E .... = 4! = 24
(ii) N............... = $\frac{4!}{2}$ = 12
(iii) QE............... = 3! = 6
(iv) QN............. = $\frac{3!}{2!}$ = 3
(v) QUEEN = 1
∴ Required rank
= (24) + (12) + (6) + (c) + (a) = 46th

Question131

The number of ways in which 5 boys and 3 girls can be seated on a round table if a particular boy B_1 and a particular girl G_1 never sit adjacent to each other, is:
[Online April 9, 2017]

Options:

A. $5 \times 6!$

B. $6 \times 6!$

C. $7!$

D. $5 \times 7!$

Answer: A

Solution:

Solution:
4 boys and 2 girls in circle
$\Rightarrow 5! \times \frac{6!}{4!2!} \times 2!$

$\Rightarrow 5 \times 6!$

Question132

A man X has 7 friends, 4 of them are ladies and 3 are men. His wife Y also has 7 friends, 3 of them are ladies and 4 are men. Assume X and Y have no common friends. Then the total number of ways in which X and Y together can throw a party inviting 3 ladies and 3 men, so that 3 friends of each of X and Y are in this party, is : [2017]

Options:

A. 484

B. 485

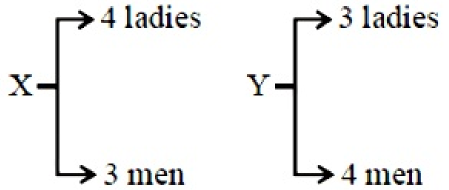
C. 468

D. 469

Answer: B

Solution:

Solution:



Possible cases for X are
(1) 3 ladies, 0 man
(2) 2 ladies, 1 man
(3) 1 lady, 2 men
(4) 0 ladies, 3 men
Possible cases for Y are
(1) 0 ladies, 3 men
(2) 1 lady, 2 men
(3) 2 ladies, 1 man
(4) 3 ladies, 0 man
No. of ways $= {}^4C_3 \cdot {}^4C_3 + ({}^4C_2 \cdot {}^3C_1)^2 + ({}^4C_1 \cdot {}^3C_2)^2 + ({}^3C_3)^2$
$= 16 + 324 + 144 + 1 = 485$

Question133

If all the words (with or without meaning) having five letters, formed using the letters of the word SMALL and arranged as in a dictionary; then the position of the word SMALL is: [2016]

Options:

A. 52^{nd}

B. 58^{th}

C. 46^{th}

D. 59^{th}

Answer: B

Solution:

Solution:
ALLMS
No. of words starting with
$A: \ A\_\_\_\_ \frac{4!}{2!} = 12$
$L: \ L\_\_\_\_ 4! = 24$
$M: \ M\_\_\_\_ \frac{4!}{2!} = 12$
$S: \ SA\_\_\_\_ \frac{3!}{2!} = 3$
$: SL\_\_\_ 3! = 6$
SMALL $\rightarrow 58^{th}$ word

Question134

If the four letter words (need not be meaningful) are to be formed using the letters from the word "MEDITERRANEAN" such that the first letter is R and the fourth letter is E , then the total number of all such words is :
[Online April 9, 2016]

Options:

A. 110

B. 59

C. $\frac{11!}{(2!)^3}$

D. 56

Answer: B

Solution:

Solution:
$M, EEE, D.I, T, RR, AA, NN$
$R - - E$
Two empty places can be filled with identical letters $[EE, AA, NN] \Rightarrow 3$ ways
Two empty places, can be filled with distinct letters $[M, E, D, I, T, R, A, N] \Rightarrow {}^8P_2$
$\therefore$ Number of words $3 + {}^8P_2 = 59$

Question135

The value of $\sum_{r=1}^{15} r^2 \left(\frac{^{15}C_r}{15C_{r-1}} \right)$ is equal to

[Online April 9, 2016]

Options:

A. 1240

B. 560

C. 1085

D. 680

Answer: D

Solution:

Solution:

$$\sum_{r=1}^{15} r^2 \left(\frac{15_{C_r}}{15_{C_{r-1}}} \right)$$

$$= \frac{16-r}{r}$$

$$= \sum_{r=1}^{15} r^2 \left(\frac{16-r}{r} \right) = \sum_{r=1}^{15} r(16-r)$$

$$= 16 \sum_{r=1}^{15} r - \sum_{r=1}^{15} r^2$$

$$= \frac{16 \times 15 \times 16}{2} - \frac{15 \times 31 \times 16}{6}$$

$$= 8 \times 15 \times 16 - 5 \times 8 \times 31 = 1920 - 1240 = 680$$

Question136

If $\frac{^{n+2}C_6}{^{n-2}P_2} = 11$, then n satisfies the equation :

[Online April 10, 2016]

Options:

A. $n^2 + n - 110 = 0$

B. $n^2 + 2n - 80 = 0$

C. $n^2 + 3n - 108 = 0$

D. $n^2 + 5n - 84 = 0$

Answer: C

Solution:

Solution:

$$\frac{n+2}{n-2p_2} = 11$$

$$\Rightarrow \frac{\frac{(n+2)(n+1)n(n-1)(n-2)(n-3)}{6.5.4.3.2.1}}{\frac{(n-2)(n-3)}{2.1}} = 11$$

$\Rightarrow (n+2)(n+1)n(n-1) = 11.10.9.4$

$\Rightarrow\ n = 9$

$n^2 + 3n - 108 = (9)^2 + 3(9) - 108$

$= 81 + 27 - 108$

$= 108 - 108 = 0$

Question137

The sum $\sum_{r=1}^{10} (r^2 + 1) \times (r!)$ is equal to [Online April 10, 2016]

Options:

A. $11 \times (11!)$

B. $10 \times (11!)$

C. $(11!)$

D. $101 \times (10!)$

Answer: B

Solution:

Solution:

$\sum_{R-1}^{10} (r^2 + 1) \mid r$

$\left|\underline{r + r}\right.$

$T_1 = (r^2 + 1 + r - r)\left|\underline{r}\right. = (r^2 + r)\left|\underline{r}\right. - (r - r)\left|\underline{r}\right.$

$T_1 = r\left|\underline{r + r}\right. - (r - 1)\left|\underline{r}\right.$

$T_1 = 1\left|\underline{2}\right. - 0$

$T_2 = 2\left|\underline{3}\right. - 1\left|\underline{2}\right.$

$T_3 = 3\left|\underline{4}\right. - 2\left|\underline{3}\right.$

$T_{10} = 10\left|\underline{11}\right. - 9\left|\underline{10}\right.$

$\sum_{R-1}^{10} (r^2 + 1)\left|\underline{r}\right. = 10\left|\underline{11}\right.$

Question138

The number of points, having both co-ordinates as integers, that lie in the interior of the triangle with vertices (0,0),(0,41) and (41,0) is : [2015]

Options:

A. 820

B. 780

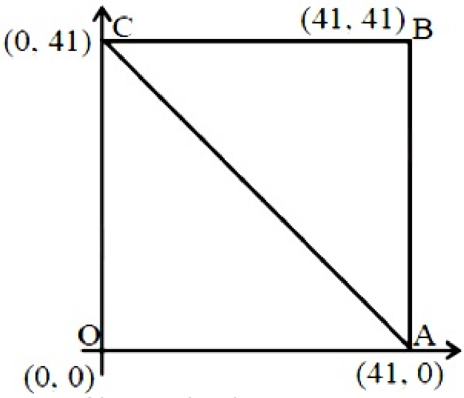
C. 901

D. 861

Answer: B

Solution:

Solution:
Total number of integral points inside the square OABC = $40 \times 40 = 1600$
No. of integral points on AC



= No. of integral points on OB
= 40[namely $(1, 1), (2, 2) \ldots (40, 40)$]
$\therefore$ No. of integral points inside the ΔOAC
$= \dfrac{1600 - 40}{2} = 780$

Question139

The number of integers greater than 6,000 that can be formed, using the digits 3,5,6,7 and 8 , without repetition, is: [2015]

Options:

A. 120

B. 72

C. 216

D. 192

Answer: D

Solution:

Solution:
Four digits number can be arranged in $3 \times 4!$ ways.
Five digits number can be arranged in $5!$ ways.
Number of integers $= 3 \times 4! + 5! = 192$

Question140

The number of ways of selecting 15 teams from 15 men and 15 women, such that each team consists of a man and a woman, is:
[Online April 10, 2015]

Options:

A. 1120

B. 1880

C. 1960

D. 1240

Answer: D

Solution:

Solution:
Number of ways of selecting a man and a woman for a team from 15 men and 15 women
$= 15 \times 15 = (15)^2$
Number of ways of selecting a man and a woman for next team out of the remaining 14 men and 14 women.
$= 14 \times 14 = (14)^2$
Similarly for other teams
Hence required number of ways
$= (15)^2 + (14)^2 + \ldots . + (1)^2 = \dfrac{15 \times 16 \times 31}{6} = 1240$

Question141

Let A and B be two sets containing four and two elements respectively. Then the number of subsets of the set A × B each having at least three elements is :
[2015]

Options:

A. 275

B. 510

C. 219

D. 256

Answer: C

Solution:

Solution:
Given
$n(A) = 4, n(B) = 2, n(A \times B) = 8$
Required number of subsets
$= {}^8C_3 + {}^8C_4 + \ldots + {}^8C_8 = 2^8 - {}^8C_0 - {}^8C_1 - {}^8C_2$
$= 256 - 1 - 8 - 28 = 219$

Question142

If in a regular polygon the number of diagonals is 54, then the number of sides of this polygon is
[Online April 11, 2015]

Options:

A. 12

B. 6

C. 10

D. 9

Answer: A

Solution:

Solution:
Number of diagonal $= 54$
$\Rightarrow \frac{n(n-3)}{2} = 54$
$\Rightarrow n^2 - 3n - 108 = 0 \Rightarrow n^2 - 12n + 9n - 108 = 0$
$\Rightarrow n(n-12) + 9(n-12) = 0$
$\Rightarrow n = 12, -9 \Rightarrow n = 12 (\because n \neq -9)$

Question143

The sum of the digits in the unit's place of all the 4 -digit numbers formed by using the numbers 3,4,5 and 6, without repetition, is:
[Online April 9, 2014]

Options:

A. 432

B. 108

C. 36

D. 18

Answer: B

Solution:

Solution:
With 3 at unit place,
total possible four digit number (without repetition) will be $3! = 6$
With 4 at unit place,
total possible four digit numbers will be $3! = 6$
With 5 at unit place, total possible four digit numbers will be $3! = 6$ With 6 at unit place, total possible four digit numbers

will be 3! = 6
Sum of unit digits of all possible numbers
= 6 x 3 + 6 x 4 + 6 x 5 + 6 x 6
= 6[3 + 4 + 5 + 6]
= 6[18] = 108

Question144

An eight digit number divisible by 9 is to be formed using digits from 0 to 9 without repeating the digits. The number of ways in which this can be done is:
[Online April 11, 2014]

Options:

A. 72(7!)

B. 18(7!)

C. 40(7!)

D. 36(7!)

Answer: D

Solution:

Solution:
We know that any number is divisible by 9 if sum of the digits of the number is divisible by 9.
Now sum of the digits from 0 to 9
= 0 + 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 = 45
Hence to form 8 digits numbers which are divisible by 9 , a pair of digits either 0 and 9,1 and 8,2 and 7,3 and 6 or 4 and 5 are not used.

Digits which are not used to form 8 digits number divisible by 9	Number of 8 digits numbers which are divisible by 9
0 and 9	8 x 7!
1 and 8	7 x 7!
2 and 7	7 x 7!
3 and 6	7 x 7!
4 and 5	7 x 7!

Hence total number of 8 digits numbers which are divisible by 9
= 8 × (7!) + 7 × (7!) + 7 × (7!) + 7 × (7!) + 7 × (7!)
= 36 × (7!)

Question145

8-digit numbers are formed using the digits 1,1,2,2,2,3,4 4. The number of such numbers in which the odd digits do no occupy odd places, is:
[Online April 12, 2014]

Options:

A. 160

B. 120

C. 60

D. 48

Answer: B

Solution:

Solution:
In 8 digits numbers, 4 places are odd places.
Also, in the given 8 digits, there are three odd digits 1, 1 and 3
No. of ways three odd digits arranged at four even places $= \frac{4P_3}{2!} = \frac{4!}{2!}$
No. of ways the remaining five digits 2,2,2,4and 4 arranged at remaining five places $= \frac{5!}{3!2!}$
Hence, required number of 8 digits number
$= \frac{4!}{2!} \times \frac{5!}{3!2!} = 120$

Question146

Two women and some men participated in a chess tournament in which every participant played two games with each of the other participants. If the number of games that the men played between themselves exceeds the number of games that the men played with the women by 66, then the number of men who participated in the tournament lies in the interval: [Online April 19, 2014]

Options:

A. [8,9]

B. [10,12)

C. (11,13]

D. (14,17)

Answer: B

Solution:

Solution:
Let no. of men $= n$
No. of women $= 2$
Total participants $= n + 2$
No. of games that M_1 plays with all other men $= 2(n-1)$
These games are played by all men
$M_2, M_3, \ldots\ldots, M_n$
So, total no. of games among men $= n \cdot 2(n-1)$.
However, we must divide it by '2', since each game is counted twice (for both players).

So, total no. of games among all men
$= n(n-1)$ .... (i)
Now, no. of games M_1 plays with W_1 and $W_2 = 4$
(2 games with each)
Total no. of games that $M_1, M_2, \ldots, M_n$ play with W_1 and $W_2 = 4n$ ......(ii)
...... (ii)
Given :$n(n-1) - 4n = 66 \Rightarrow n = 11, -6$
As the number of men can't be negative.
So, $n = 11$

Question147

A committee of 4 persons is to be formed from 2 ladies, 2 old men and 4 young men such that it includes at least 1 lady, at least 1 old man and at most 2 young men. Then the total number of ways in which this committee can be formed is : [Online April 9, 2013]

Options:

A. 40

B. 41

C. 16

D. 32

Answer: B

Solution:

Solution:

L	O	Y
2	2	4
≥ 1	≥ 1	$2 \leq$

$\Rightarrow$

L	O	Y
1	1	2
1	2	1
2	1	1
2	2	0

Required number of ways
$= {}^2C_1 \times {}^2C_1 \times {}^2C_2 + {}^2C_1 \times {}^2C_2 \times {}^4C_1 + {}^2C_2 \times {}^2C_1 \times {}^4C_1 + {}^2C_2 \times {}^2C_2 \times {}^4C_0$
$= 2 \times 2 \times \dfrac{4 \times 3}{2} + 2 \times 1 \times 4 + 1 \times 2 \times 4 + 1 \times 1 \times 1$
$= 24 + 8 + 8 + 1 = 41$

Question148

The number of ways in which an examiner can assign 30 marks to 8 questions, giving not less than 2 marks to any question, is: [Online April 22, 2013]

Options:

A. ${}^{30}C_7$

B. ${}^{21}C_8$

C. ${}^{21}C_7$

D. ${}^{30}C_8$

Answer: C

Solution:

Solution:
30 marks to be alloted to 8 questions. Each question has to be given ≥2 marks
Let questions be a, b, c, d , e, f , g, h
and $a + b + c + d + e + f + g + h = 30$
Let $a = a_1 + 2$ so, $a_1 \geq 0$
$b = a_2 + 2$ so, $a_2 \geq 0, \ldots\ldots, a_8 \geq 0$
So, $a_1 + a_2 + \ldots\ldots + a_8 + 2 + 2 + \ldots\ldots + 2\} = 30$
$\Rightarrow a_1 + a_2 + \ldots\ldots + a_8 = 30 - 16 = 14$
So, this is a problem of distributing 14 articles in 8 groups.
Number of ways $= {}^{14+8-1}C_{8-1} = {}^{21}C_7$

Question149

On the sides AB, BC, CA of a ΔABC, 3, 4, 5 distinct points (excluding vertices A, B, C) are respectively chosen. The number of triangles that can be constructed using these chosen points as vertices are : [Online April 23, 2013]

Options:

A. 210

B. 205

C. 215

D. 220

Answer: B

Solution:

Solution:
Required number of triangles
$= {}^{12}C_3 - ({}^{3}C_3 + {}^{4}C_3 + {}^{5}C_3) = 205$

Question150

5 - digit numbers are to be formed using 2,3,5,7,9 without repeating the digits. If p be the number of such numbers that exceed 20000 and q be the number of those that lie between 30000 and 90000, then p : q is : [Online April 25, 2013]

Options:

A. 6: 5

B. 3: 2

C. 4: 3

D. 5: 3

Answer: D

Solution:

Solution:
p: 0 0 0 0 0 place
$\quad$ 5 4 3 2 1 $\quad$ ways
Total no. of ways $= 5! = 120$
Since all numbers are $> 20,000$
$\therefore$ all numbers 2,3,5,7,9 can come at first place.
q: 0 0 0 0 0 place
$\quad$ 3 4 3 2 1 $\quad$ ways
Total no. of ways $= 3 \times 4! = 72$
($\because$ 2 and 9 can not be put at first place)
So, $p : q = 120 : 72 = 5 : 3$

Question151

Let A and B two sets containing 2 elements and 4 elements respectively. The number of subsets of A × B having 3 or more elements is [2013]

Options:

A. 256

B. 220

C. 219

D. 211

Answer: C

Solution:

Solution:
Given
$n(A) = 2, n(B) = 4, n(A \times B) = 8$
Required number of subsets
$= {}^8C_3 + {}^8C_4 + \ldots + {}^8C_8 = 2^8 - {}^8C_0 - {}^8C_1 - {}^8C_2$
$= 256 - 1 - 8 - 28 = 219$

Question152

Let T_n be the number of all possible triangles formed by joining vertices

of an n -sided regular polygon. If $T_{n+1} - T_n = 10$, then the value of n is : [2013]

Options:

A. 7

B. 5

C. 10

D. 8

Answer: B

Solution:

Solution:
We know,
$T_n = {}^nC_3$, $T_{n+1} = {}^{n+1}C_3$
ATQ, $T_{n+1} - T_n = {}^{n+1}C_3 - {}^nC_3 = 10$
$\Rightarrow {}^nC_2 = 10$
$\Rightarrow n = 5$

Question153

If the number of 5 -element subsets of the set $A = \{a_1, a_2, ..., a_{20}\}$ of 20 distinct elements is k times the number of 5 -element subsets containing a_4, then k is [Online May 7, 2012]

Options:

A. 5

B. $\frac{20}{7}$

C. 4

D. $\frac{10}{3}$

Answer: C

Solution:

Solution:
Set $A = \{a_1, a_2, ..., a_{20}\}$ has 20 distinct elements.
We have to select 5 -element subset.
$\therefore$ Number of 5 -element subsets $= {}^{20}C_5$
According to question
${}^{20}C_5 = ({}^{19}C_4) \cdot k$
$\Rightarrow \frac{20!}{5!15!} = k \cdot \left(\frac{19!}{4!15!} \right)$

$\Rightarrow \quad \frac{20}{5} = k \Rightarrow k = 4$

Question154

Statement 1: If A and B be two sets having p and q elements respectively, where q > p. Then the total number of functions from set A to set B is q^p
Statement 2 : The total number of selections of p different objects out of q objects is ${}^qC_p$
[Online May 12, 2012]

Options:

A. Statement 1 is true, Statement 2 is false.

B. Statement 1 is true, Statement 2 is true, Statement 2 is not a correct explanation of Statement 1.

C. Statement 1 is false, Statement 2 is true

D. Statement 1 is true, Statement 2 is true, Statement 2 is a correct explanation of Statement 1

Answer: D

Solution:

Solution:
Statement -1: $n(A) = p, n(B) = q, q > p$
Total number of functions from $A \rightarrow B = q^p$
It is a true statement.
Statement -2: The total number of selections of p different objects out of q objects is ${}^qC_p$
It is also a true statement and it is a correct explanation for statement - 1 also.

Question155

The number of arrangements that can be formed from the letters a, b, c, d , e, f taken 3 at a time without repetition and each arrangement containing at least one vowel, is
[Online May 19, 2012]

Options:

A. 96

B. 128

C. 24

D. 72

Answer: A

Solution:

Solution:
There are 2 vowels and 4 consonants in the letters a, b, c, d, e, f
If we select one vowel, then number of arrangements
$= {}^2C_1 \times {}^4C_2 \times 3! = 2 \times \frac{4 \times 3}{2} \times 3 \times 2 = 72$
If we select two vowels, then number of arrangements
$= {}^2C_2 \times {}^4C_1 \times 3! = 1 \times 4 \times 6 = 24$
Hence, total number of arrangements
$= 72 + 24 = 96$

Question156

If n = ${}^mC_2$, then the value of ${}^nC_2$ is given by [Online May 19, 2012]

Options:

A. $3({}^{m+1}C_4)$

B. ${}^{m-1}C_4$

C. ${}^{m+1}C_4$

D. $2({}^{m+2}C_4)$

Answer: A

Solution:

Solution:
$n = {}^mC_2 = \frac{m(m-1)}{2}$
Since m and $(m-1)$ are two consecutive natural numbers, therefore their product is an even natural number. So $\frac{m(m-1)}{2}$ is also a natural number.
Now $\frac{m(m-1)}{2} = \frac{m^2 - m}{2}$
$\therefore {}^{\frac{m(m-1)}{2}}C_2 = \frac{\left(\frac{m^2-m}{2}\right)\left(\frac{m^2-m}{2} - 1\right)}{2}$
$= \frac{m(m-1)(m^2-m-2)}{8}$
$= \frac{m(m-1)[m^2-2m+m-2]}{8}$
$= \frac{m(m-1)[m(m-2)+1(m-2)]}{8}$
$= \frac{m(m-1)(m-2)(m+1)}{8}$
$= \frac{3 \times (m+1)m(m-1)(m-2)}{4 \times 3 \times 2 \times 1} = 3({}^{m+1}C_4)$

Question157

If seven women and seven men are to be seated around a circular table such that there is a man on either side of every woman, then the number of seating arrangements is
[Online May 26, 2012]

Options:

A. $6!7!$

B. $(6!)^2$

C. $(7!)^2$

D. $7!$

Answer: A

Solution:

Solution:
7 women can be arranged around a circular table in $6!$ ways.
Among these 7 men can sit in $7!$ ways.
Hence, number of seating arrangement $= 7! \times 6!$

Question158

Assuming the balls to be identical except for difference in colours, the number of ways in which one or more balls can be selected from 10 white, 9 green and 7 black balls is:
[2012]

Options:

A. 880

B. 629

C. 630

D. 879

Answer: D

Solution:

Solution:
Given that number of white balls $= 10$
Number of green balls $= 9$
and Number of black balls $= 7$
∴ Required probability
$= (10+1)(9+1)(7+1) - 1$
$= 11.10.8 - 1 = 879$
[∵ The total number of ways of selecting one or more items from p identical items of one kind, q identical items of second kind; r identical items of third kind is
$(p+1)(q+1)(r+1) - 1$]

Question159

There are 10 points in a plane, out of these 6 are collinear. If N is the number of triangles formed by joining these points. Then : [2011RS]

Options:

A. N ≤ 100

B. $100 <$ N ≤ 140

C. $140 <$ N ≤ 190

D. N > 190

Answer: A

Solution:

Solution:
Number of required triangles
$= {}^{10}C_3 - {}^{6}C_3$
$= \frac{10 \times 9 \times 8}{6} - \frac{6 \times 5 \times 4}{6} = 120 - 20 = 100$

Question160

Statement-1: The number of ways of distributing 10 identical balls in 4 distinct boxes such that no box is empty is ${}^{9}C_{3}$ Statement-2: The number of ways of choosing any 3 places from 9 different places is ${}^{9}C_{3}$ [2011]

Options:

A. Statement- 1 is true, Statement- 2 is true; Statement- 2 is not a correct explanation for Statement- 1.

B. Statement- 1 is true, Statement- 2 is false.

C. Statement- 1 is false, Statement- 2 is true.

D. Statement- 1 is true, Statement- 2 is true; Statement- 2 is a correct explanation for Statement- 1.

Answer: A

Solution:

Solution:
The number of ways of distributing 10 identical balls in 4 distinct boxes
$= {}^{10-1}C_{4-1} = {}^{9}C_{3}$

Question161

There are two urns. Urn A has 3 distinct red balls and urn B has 9 distinct blue balls. From each urn two balls are taken out at random and then transferred to the other. The number of ways in which this can be done is [2010]

Options:

A. 36

B. 66

C. 108

D. 3

Answer: C

Solution:

Solution:
Two balls are taken from each urn Total number of ways
$= {}^3C_2 \times {}^9C_2$
$= 3 \times \dfrac{9 \times 8}{2} = 3 \times 36 = 108$

Question162

From 6 different novels and 3 different dictionaries, 4 novels and 1 dictionary are to be selected and arranged in a row on a shelf so that the dictionary is always in the middle. Then the number of such arrangement is: [2009]

Options:

A. at least 500 but less than 750

B. at least 750 but less than 1000

C. at least 1000

D. less than 500

Answer: C

Solution:

Solution:
4 novels, out of 6 novels and 1 dictionary out of 3 can be selected in ${}^6C_4 \times {}^3C_1$ ways
Then 4 novels with one dictionary in the middle can be arranged in 4! ways.

∴ Total ways of arrangement
$= {}^6C_4 \times {}^3C_1 \times 4! = 1080$

Question163

How many different words can be formed by jumbling the letters in the word MISSISSIPPI in which no two S are adjacent?
[2008]

Options:

A. $8 \cdot {}^6C_4 \cdot {}^7C_4$

B. $6.7 \cdot {}^8C_4$

C. $6.8 \cdot {}^7C_4$

D. $7 \cdot {}^6C_4 \cdot {}^8C_4$

Answer: D

Solution:

Solution:
First let us arrange M, I, I, I, I, P, P
Which can be done in $\frac{7!}{4!2!}$ ways
$*M\ *I\ *I\ *I\ *I\ *P*P*$
Now 4S can be kept at any of the * places in ${}^8C_4$ ways so that no two S are adjacent.
Total required ways
$= \frac{7!}{4!2!}{}^8C_4 = \frac{7!}{4!2!}{}^8C_4 = 7 \times {}^6C_4 \times {}^8C_4$

Question164

The set S = {1, 2, 3, ..., 12} is to be partitioned into three sets A, B, C of equal size.
Thus A ∪ B ∪ C = S, A ∩ B = B ∩ C = A ∩ C = φ. The number of ways to partition S is
[2007]

Options:

A. $\frac{12!}{(4!)^3}$

B. $\frac{12!}{(4!)^4}$

C. $\frac{12!}{3!(4!)^3}$

D. $\frac{12!}{3!(4!)^4}$

Answer: A

Solution:

Solution:
Set $S = \{1, 2, 3, \ldots\ldots 12\}$
$A \cup B \cup C = S, A \cap B = B \cap C = A \cap C = \varphi$
∴ Each sets contain 4 elements.
∴ The number of ways to partition
$= {}^{12}C_4 \times {}^{8}C_4 \times {}^{4}C_4$
$= \frac{12!}{4!8!} \times \frac{8!}{4!4!} \times \frac{4!}{4!0!} = \frac{12!}{(4!)^3}$

Question165

At an election, a voter may vote for any number of candidates, not greater than the number to be elected. There are 10 candidates and 4 are of be selected, if a voter votes for at least one candidate, then the number of ways in which he can vote is [2006]

Options:

A. 5040

B. 6210

C. 385

D. 1110

Answer: C

Solution:

Solution:
The number of ways can vote
$= {}^{10}C_1 + {}^{10}C_2 + {}^{10}C_3 + {}^{10}C_4$
$= 10 + 45 + 120 + 210 = 385$

Question166

If the letters of the word SACHIN are arranged in all possible ways and these words are written out as in dictionary, then the word SACHIN appears at serial number [2005]

Options:

A. 601

B. 600

C. 603

D. 602

Answer: A

Solution:

Solution:
Alphabetical order is
A, C, H , I , N , S
No. of words starting with A = 5! = 120
No. of words starting with C = 5! = 120
No. of words starting with H = 5! = 120
No. of words starting with I = 5! = 120
No. of words starting with N = 5! = 120
SACHIN -1
∴ Sachin appears at serial no. 601

Question167

The value of ${}^{50}C_4 + \sum_{r=1}^{6} {}^{56-r}C_3$ is [2005]

Options:

A. ${}^{55}C_4$

B. ${}^{55}C_3$

C. ${}^{56}C_3$

D. ${}^{56}C_4$

Answer: D

Solution:

Solution:

$${}^{50}C_4 + \sum_{r=1}^{6} {}^{56-r}C_3$$

$$= {}^{50}C_4 + \left[\begin{array}{c} {}^{55}C_3 + {}^{54}C_3 + {}^{53}C_3 + {}^{52}C_3 \\ + {}^{51}C_3 + {}^{50}C_3 \end{array} \right]$$

We know
$[{}^{n}C_r + {}^{n}C_{r-1} = {}^{n+1}C_r]$
$= ({}^{50}C_4 + {}^{50}C_3) + {}^{51}C_3 + {}^{52}C_3 + {}^{53}C_3 + {}^{54}C_3 + {}^{55}C_3$
$= ({}^{51}C_4 + {}^{51}C_3) + {}^{52}C_3 + {}^{53}C_3 + {}^{54}C_3 + {}^{55}C_3$
Proceeding in the same way, we get
$= {}^{55}C_4 + {}^{55}C_3 = {}^{56}C_4$

Question168

How many ways are there to arrange the letters in the word GARDEN with vowels in alphabetical order
[2004]

Options:

A. 480

B. 240

C. 360

D. 120

Answer: C

Solution:

Solution:
Total number of arrangements of letters in the word GARDEN $= 6! = 720$ there are two vowels A and E, in halfof the arrangements A preceeds E and other half A follows E. So, numbers of word with vowels in alphabetical order in
$\frac{1}{2} \times 720 = 360$

Question169

The range of the function $f(x) = {}^{7-x}P_{x-3}$ is
[2004]

Options:

A. $\{1, 2, 3, 4, 5\}$

B. $\{1, 2, 3, 4, 5, 6\}$

C. $\{1, 2, 3, 4,\}$

D. $\{1, 2, 3,\}$

Answer: D

Solution:

Solution:
${}^{7-x}P_{x-3}$ is defined if
$7 - x \geq 0, x - 3 \geq 0$ and $7 - x \geq x - 3$
$\Rightarrow 3 \leq x \leq 5$ and $x \in I$
$\therefore x = 3, 4, 5$
$\therefore f(3) = {}^{7-3}P_{3-3} = {}^{4}P_{0} = 1$
$\therefore f(4) = {}^{7-4}P_{4-3} = {}^{3}P_{1} = 3$
$\therefore f(5) = {}^{7-5}P_{5-3} = {}^{2}P_{2} = 2$
Hence range $= \{1, 2, 3\}$

Question170

The number of ways of distributing 8 identical balls in 3 distinct boxes so that none of the boxes is empty is
[2004]

Options:

A. ${}^{8}C_{3}$

B. 21

C. 3^{8}

D. 5

Answer: B

Solution:

Solution:
We know that the number of ways of distributing n identical items among r persons, when each one of them receives at least one item is ${}^{n-1}C_{r-1}$
$\therefore$ The required number of ways
$= {}^{8-1}C_{3-1} = {}^{7}C_{2} = \frac{7!}{2!5!} = \frac{7 \times 6}{2 \times 1} = 21$

Question171

The number of ways in which 6 men and 5 women can dine at a round table if no two women are to sit together is given by
[2003]

Options:

A. $6! \times 5!$

B. 6×5

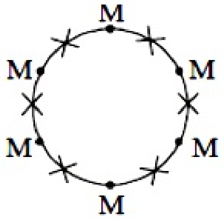
C. 30

D. 5×4

Answer: A

Solution:

Solution:
No. of ways in which 6 men can be arranged at a round table $= (6-1)! = 5!$



Now women can be arranged in ${}^{6}P_{5}$
$= 6!$ ways
Total Number of ways $= 6! \times 5!$

Question172

**A student is to answer 10 out of 13 questions in an examination such that he must choose at least 4 from the first five questions. The number of choices available to him is
[2003]**

Options:

A. 346

B. 140

C. 196

D. 280

Answer: C

Solution:

According to given question two cases are possible.
(i) Selecting 4 out of first five question and 6 out of remaining question
$= {}^5C_4 \times {}^8C_6 = 140$ ways
(ii) Selecting 5 out of first five question and 5 out of remaining
8 questions $= {}^5C_5 \times {}^8C_5 = 56$ ways
Therefore, total number of choices
$= 140 + 56 = 196$

Question173

**If ${}^nC_r$ denotes the number of combination of n things taken r at a time, then the expression ${}^nC_{r+1} + {}^nC_{r-1} + 2 \times {}^nC_r$ equals
[2003]**

Options:

A. ${}^{n+1}C_{r+1}$

B. ${}^{n+2}C_r$

C. ${}^{n+2}C_{r+1}$

D. ${}^{n+1}C_r$

Answer: C

Solution:

${}^nC_{r+1} + {}^nC_{r-1} + 2\,{}^nC_r$
$= {}^nC_{r-1} + {}^nC_r + {}^nC_r + {}^nC_{r+1}$
$[\because {}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r]$
$= {}^{n+1}C_r + {}^{n+1}C_{r+1} = {}^{n+2}C_{r+1}$

Question174

The sum of integers from 1 to 100 that are divisible by 2 or 5 is [2002]

Options:

A. 3000

B. 3050

C. 3600

D. 3250

Answer: B

Solution:

Required sum
$= (2+4+6+\ldots+100)+(5+10+15+\ldots+100)$
$-(10+20+\ldots+100)$
$= 2(1+2+3\ldots+50)+5(1+2+3+\ldots+50)$
$= 2550+1050-530=3050.$

Question175

Number greater than 1000 but less than 4000 is formed using the digits 0,1,2,3,4 (repetition allowed). Their number is [2002]

Options:

A. 125

B. 105

C. 374

D. 625

Answer: C

Solution:

Solution:
Total number of numbers
$= 3 \times 5 \times 5 \times 5 - 1 = 374$

Question176

Total number of four digit odd numbers that can be formed using 0,1,2,3,5,7 (using repetition allowed) are
[2002]

Options:

A. 216

B. 375

C. 400

D. 720

Answer: D

Solution:

Solution:
Total number of numbers formed using 0,1,2,3,5,7
$= 5 \times 6 \times 6 \times 4 = 36 \times 20 = 720$

Question177

Five digit number divisible by 3 is formed using 0,1,2,3,4 6 and 7 without repetition. Total number of such numbers are
[2002]

Options:

A. 312

B. 3125

C. 120

D. 216

Answer: D

Solution:

Solution:
We know that a number is divisible by 3 only when the sum of the digits is divisible by 3 . The given digits are 0,1,2,3,4,5
Here the possible number of combinations of 5 digits out of 6 are ${}^5C_4 = 5$, which are as follows -
$1 + 2 + 3 + 4 + 5 = 15 = 3 \times 5$(divisible by 3)
$0 + 2 + 3 + 4 + 5 = 14$ (not divisible by 3)
$0 + 1 + 3 + 4 + 5 = 13$ (not divisible by 3)
$0 + 1 + 2 + 4 + 5 = 12 = 3 \times 4$(divisible by 3)
$0 + 1 + 2 + 3 + 5 = 11$ (not divisible by 3)
$0 + 1 + 2 + 3 + 4 = 10$ (not divisible by 3)
Thus the number should contain the digits 1,2,3,4,5 or the digits 0,1,2,4,5

Taking $1, 2, 3, 4, 5$, the 5 digit numbers are $= 5! = 120$
Taking $0, 1, 2, 4, 5$, the 5 digit numbers are $= 5! - 4! = 96$
$\therefore$ Total number of numbers $= 120 + 96 = 216$
