Coordinate Geometry

Exercise-8.1

Question 1:

Find the distance between the following pairs of points :

1. (2, -3), (7, 9)
2. (-3, 4), (0, 0)
3. (a + b, a – b), (b – a, a + b)

Solution :

1. Here, $A(x_1, y_1) = A(2, -3)$ and
$B(x_2, y_2) = (7, 9)$
$AB^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2$
$= (2-7)^2 + (-3-9)^2$
$= 25 + 144$
$= 169$
$\therefore AB = \sqrt{169} = 13$
$\therefore$ The distance between the given points is 13.

2. Here, $A(x_1, y_1) = A(-3, 4)$ and
$B(x_2, y_2) = (0, 0)$
$AB^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2$
$= (-3-0)^2 + (4-0)^2$
$= 9 + 16$
$= 25$
$\therefore AB = \sqrt{25} = 5$
$\therefore$ The distance between the given points is 5.

3. Here, $A(x_1, y_1) = A(a + b, b - a)$ and
$B(x_2, y_2) = B(a - b, a + b)$
$AB^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2$
$= [(a + b) - (a - b)]^2 + [(b - a) - (a + b)]^2$
$= (2b)^2 + (-2a)^2$
$= 4b^2 + 4a^2$
$= 4(a^2 + b^2)$
$\therefore AB = \sqrt{4(a^2 + b^2)} = 2\sqrt{(a^2 + b^2)}$
$\therefore$ The distance between the given points is $2\sqrt{(a^2 + b^2)}$.

Question 2:

A point P on X-axis is at distance 13 from A(11, 12). Find the coordinates of P.

Solution :

The y-coordinate of any point on the x-axis is zero.

Let, P(x, 0) be a point on the x-axis such that the distance of it from A(11, 12) is 13 units.

PA = 13

$\therefore PA^2 = 169$

$\therefore (x - 11)^2 + (0 - 12)^2 = 169$

$\therefore x^2 - 22x + 121 + 144 = 169$

$\therefore x^2 - 22x + 265 = 169$

$\therefore x^2 - 22x + 96 = 0$

$\therefore x^2 - 16x - 6x + 96 = 0$

$\therefore (x - 16)(x - 6) = 0$

$\therefore x - 16 = 0$ or $x - 6 = 0$

$\therefore x = 16$ or $x = 6$

$\therefore$ The required point on x-axis is P(x, 0) = P(16, 0) or P(x, 0) = P(6, 0).

Question 3:

Using distance formula, show that A(2, 3), B(4, 7) and C(0, -1) are collinear points.

Solution :

Let us find AB, BC, AC.

$AB^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2$

$AB^2 = (2 - 4)^2 + (3 - 7)^2$

$= 4 + 16$

$= 20$

$\therefore AB = \sqrt{20} = 2\sqrt{5}$

$BC^2 = (4 - 0)^2 + [7 - (-1)]^2$

$= 16 + 64$

$= 80$

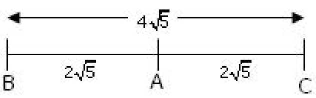
$\therefore BC = \sqrt{80} = 4\sqrt{5}$

$AC^2 = (2 - 0)^2 + [3 - (-1)]^2$

$= 4 + 16$

$= 20$

$\therefore AC = \sqrt{20} = 2\sqrt{5}$



$\therefore AB + AC = BC$

$\therefore$ A, B, and C are collinear and A lies between B and C.

Question 4:

Find a point on Y-axis which is equidistant from P(-6, 4) and Q(2, -8).

Solution :

The x-coordinate of any point on the y-axis is zero.
Let A(0, y) be a point on the y-axis which is equidistant from P(-6, 4) and Q(2, -8).

$\therefore AP = AQ$

$\therefore AP^2 = AQ^2$

$\therefore [0-(-6)]^2 + (y-4)^2 = (0-2)^2 + [y-(-8)]^2$

$\therefore 36 + y^2 - 8y + 16 = 4 + y^2 + 16y + 64$

$\therefore 24y = -16$

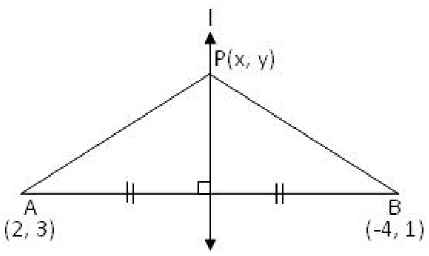
$\therefore y = -\frac{16}{24} = -\frac{2}{3}$

$\therefore$ The required point on the y-axis is $A\left(0, -\frac{2}{3}\right)$.

Question 5:

P(x, y) is a point on the perpendicular bisector of the segment $\overline{AB}$ joining A(2, 3) and B(-4, 1). Find the relation between x and y.

Solution :



P(x, y) is a point on the perpendicular bisector of the line segment joining A(2, 3) and B(− 4, 1).
So P is equidistant from A and B.

$\therefore PA = PB$

$\therefore (x-2)^2 + (y-3)^2 = (x-(-4))^2 + (y-1)^2$

$\therefore x^2 - 4x + 4 + y^2 - 6y + 9 = x^2 + 8x + 16y^2 + y^2 - 2y + 1$

$\therefore 12x + 4y + 4 = 0$

$\therefore 3x + y + 1 = 0 \qquad \ldots\ldots(1)$

Hence, if P(x, y) is any point on the perpendicular bisector of $\overline{AB}$ then the relation between x and y is $3x + y + 1 = 0$.

Question 6:

If the distance between A(10, 8) and B(a, 2) is 10, find a.

Solution :

Here, AB =10

AB = 100

$\therefore (10 - a)^2 + (8 - 2)^2 = 100$

$\therefore 100 - 20a + a^2 + 36 = 100$

$\therefore a^2 - 20a + 36 = 0$

$\therefore a(a - 18)(a - 2) = 0$

$\therefore (a - 18)(a - 2) = 0$

$\therefore a - 18 = 0$ or $a - 2 = 0$

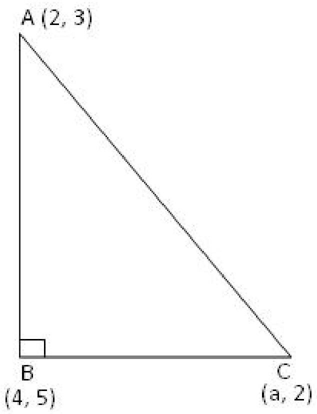
$\therefore a = 18$ or $a = 2$

So if the distance between A(10, 8) and B(a, 2) is 10, then a = 18 or a = 2.

Question 7:

$m\angle B = 90$ in the triangle whose vertices are A(2, 3), B(4, 5) and C(a, 2). Find a.

Solution :



In ΔABC, $m\angle B = 90°$

$\therefore AB^2 + BC^2 = AC^2$

$\therefore (2 - 4)^2 + (3 - 5)^2 + (4 - a)^2 + (5 - 2)^2 = (2 - a)^2 + (3 - 2)^2$

$\therefore 4 + 4 + 16 - 8a + a^2 + 9 = 4 - 4a + a^2 + 1$

$\therefore 4a = 28$

$\therefore a = \frac{28}{4} = 7$

Hence, if $m\angle B = 90°$, then a = 7.

Question 8:

A(-3, O) and B(3, O) are the vertices of an equilateral ΔABC. Find coordinates of C.

Solution :

Let the coordinates of C in equilateral ΔABC be (x, y).

$\therefore$ AB = BC = AC (Equilateral triangle)

$\therefore$ $AB^2 = BC^2 = AC^2$

$AB^2 = BC^2$

$\therefore$ $(-3-3)^2 + (0)^2 = (x-3)^2 + (y-0)^2$

$\therefore$ $36 = x^2 - 6x + 9 + y^2$

$\therefore$ $x^2 - 6x + y^2 = 27$ .....(1)

$BC^2 = AC^2$

$\therefore$ $(x-3)^2 + (y-0)^2 = (x+3)^2 + (y-0)^2$

$\therefore$ $x^2 - 6x + 9 + y^2 = x^2 + 6x + 9 + y^2$

$\therefore$ $12x = 0$

$\therefore$ $x = 0$

Substituting x = 0 from equation (2) in equation (1),

$(0)^2 - 6(0) + y^2 = 27$

$\therefore$ $y^2 = 27$

$\therefore$ $y^2 = \pm\sqrt{27}$

$\therefore$ $y = \pm 3\sqrt{3}$

$\therefore$ The coordinates of C are $(0, 3\sqrt{3})$ or $(0, -3\sqrt{3})$.

Question 9:

A(1, 7), B(2, 4), C(k, 5) are the vertices of a right angled ΔABC,

1. Find k, if ∠A is a right angle
2. Find k, if ∠B is a right angle
3. Find k, if ∠C is a right angle

Solution :

Here, A(1, 7), B(2, 4), C(k, 5) are the vertices of a right angled triangle.

$AB^2 = (1 - 2)^2 + (7 - 4)^2 = 1 + 9 = 10$

$BC^2 = (2 - k)^2 + (4 - 5)^2$

$= 4 - 4k + k^2 + 1$

$= k^2 - 4k + 5$

$AC^2 = (1 - k)^2 + (7 - 5)^2$

$= 1 - 2k + k^2 + 4$

$= k^2 - 2k + 5$

1. $\angle A$ is a right angle.

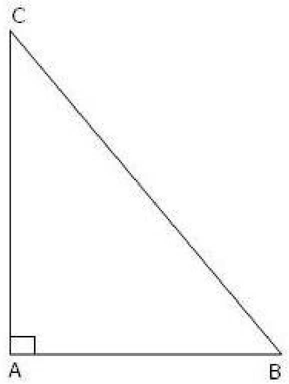
$\therefore BC^2 = AB^2 + AC^2$

$\therefore k^2 - 4k + 5 = 10 + k^2 - 2k + 5$

$\therefore -2k = 10$

$\therefore k = -5$

Hence, if $\angle A$ is a right angle in ΔABC, then k = -5.



2. $\angle B$ is a right angle.

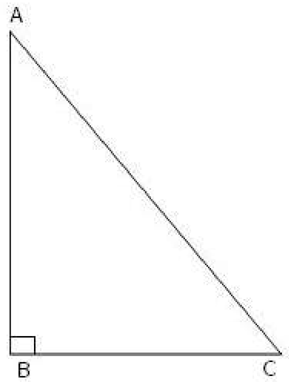
$\therefore AC^2 = AB^2 + BC^2$

$\therefore k^2 - 2k + 5 = 10 + k^2 - 4k + 5$

$\therefore 2k = 10$

$\therefore k = 5$

Hence, if $\angle B$ is a right angle in ΔABC, then k = 5.



3. $\angle C$ is a right angle.

$\therefore AB^2 = BC^2 + AC^2$

$\therefore 10 = k^2 - 4k + 5 + k^2 - 2k + 5$

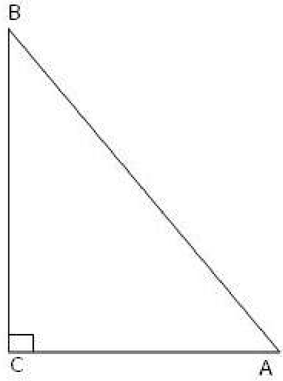
$\therefore 2k^2 - 6k = 0$

$\therefore k^2 - 3k = 0$

$\therefore k(k - 3) = 0$

$\therefore k = 0$ or $k = 3$

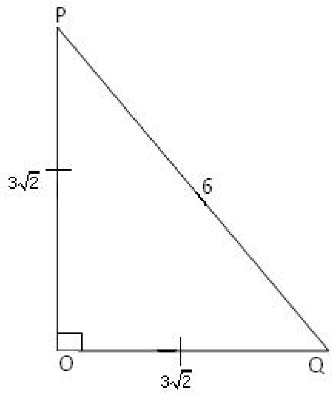
Hence, if $\angle C$ is a right angle in ΔABC, then k = 0 or k = 3



Question 10:

Show that the points P(3, -3), Q(-3, -3) and O(0, 0) are the vertices of an isosceles right angled triangle.

Solution :



P(3, −3), Q(−3, −3) and O(0, 0) are the given points.

$PQ^2 = (3+3)^2 + (-3-3)^2 = 36 + 0 = 36$

$OQ^2 = (0+3)^2 + (0+3)^2$

$= 9 + 9$

$= 18$

$OP^2 = (0-3)^2 + (0+3)^2$

$= 9 + 9$

$= 18$

We get, $OQ^2 = OP^2 = 18$

$OP = OQ = \sqrt{18} = 3\sqrt{2}$

Further, $OP^2 + OQ^2 = 18 + 18 = 36 = PQ^2$

O is a right angle in ΔPOQ and $OP = OQ$.

$\therefore$ ΔPOQ is an isosceles right angled triangle.

Exercise-8.2

Question 1:

Find the coordinates of the point which divides the line-segment joining A(-1, 7) and B(4, 2) in the ratio 3 : 2 from A.

Solution :

$A(x_1, y_1) = A(-1, 7)$ and $B(x_2, y_2) = B(4, 2)$ are the given points.

Let $P(x, y)$ be the point on $\overline{AB}$ such that $\frac{AP}{PB} = \frac{3}{2}$

$\therefore \frac{m}{n} = \frac{3}{2}$

$\therefore P(x, y)$ divides $\overline{AB}$ from A in the ratio 3 : 2

$$\therefore x = \frac{mx_2 + nx_1}{m + n}$$
$$= \frac{3(4) + 2(-1)}{3 + 2}$$
$$= \frac{10}{5}$$
$$= 2$$
$$y = \frac{my_2 + ny_1}{m + n}$$
$$= \frac{3(2) + 2(7)}{3 + 2}$$
$$= \frac{20}{5}$$
$$= 4$$

$\therefore x = 2$ and $y = 4$

$\therefore$ The coordinates of the point which divides $\overline{AB}$ in the ratio 3 : 2 from A are $(2, 4)$.

Question 2:

$P(1, y)$ is a point on $\overline{BC}$. (0, 2) and (3, 5) are the coordinates of A and B respectively. Find the ratio $\frac{AP}{PB}$ and the value of y.

Solution :

Let $P(1, y)$ divide $\overline{AB}$ in the ratio $\frac{m}{n}$ from A.

$A(x_1, y_1) = A(0, 2)$, $B(x_2, y_2) = B(3, 5)$, $P(x, y) = P(1, y)$

Let $\frac{AP}{PB} = \frac{m}{n}$

$$x = \frac{mx_2 + nx_1}{m + n}$$
$$\therefore 1 = \frac{m(3) + n(0)}{m + n}$$
$$\therefore m + n = 3m$$
$$\therefore 2m = n$$
$$\therefore \frac{m}{n} = \frac{1}{2}$$
$$y = \frac{my_2 + ny_1}{m + n}$$
$$\therefore y = \frac{1(5) + 2(2)}{1 + 2} \qquad \left(\because \frac{m}{n} = \frac{1}{2}\text{. So, taking } m = 1 \text{ and } n = 2\right)$$
$$\therefore y = 3$$

$\therefore P(1, y) = P(1, 3)$ divides $\overline{AB}$ in ratio $\frac{AP}{PB} = \frac{1}{2}$ from A and the value of y is 3.

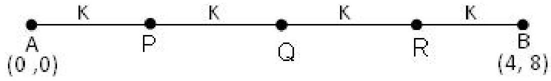
$\therefore$ The required ratio is 1 : 2 and $y = 3$.

Question 3:

Find the coordinates of points which divide the segment joining A(0, 0) and B(4, 8) in four congruent segments.

Solution :

There will be three points, which divide $\overline{AB}$ in four congruent segments.
Let P, Q and R be the points dividing $\overline{AB}$ in four congruent segments (each of length K, where K>0).



P divides $\overline{AB}$ from A in the ratio

$$\frac{AP}{PB} = \frac{K}{3K} = \frac{1}{3} = \frac{m}{n}.$$

$\therefore$ m = 1, n = 3

Here $A(x_1, y_1) = A(0,0)$, $B(x_2, y_2) = B(4,8)$ and $P(x, y)$.

$$x = \frac{mx_2 + nx_1}{m+n}$$

$$\therefore x = \frac{1(4) + 3(0)}{1+3} = 1$$

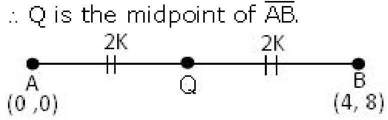
$$y = \frac{my_2 + ny_1}{m+n}$$

$$\therefore y = \frac{1(8) + 3(0)}{1+3} = 2$$

$\therefore P(x, y) = P(1, 2)$

Now, A – Q – B and AQ = QB = 2K

$\therefore$ Q is the midpoint of $\overline{AB}$.

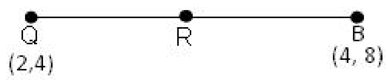


$$\therefore Q(x, y) = \left(\frac{0+4}{2}, \frac{0+8}{2}\right)$$
$$= (2, 4)$$

$\therefore Q(x, y) = (2, 4)$ ... ...(2)

Also, Q – R – B and QR = RB = K.

$\therefore$ R is midpoint of $\overline{QB}$.



$$\therefore R(x, y) = \left(\frac{2+4}{2}, \frac{4+8}{2}\right)$$
$$= (3, 6)$$

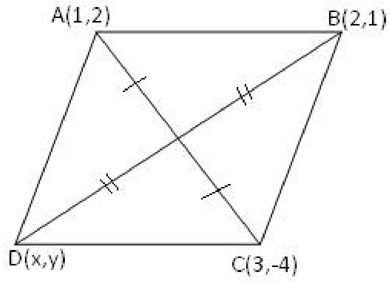
$\therefore R(x, y) = R(3, 6)$

From equations (1), (2) and (3), we get the coordinates of points which divide $\overline{AB}$ in four congruent segments as (1,2), (2, 4) and (3,6).

Question 4:

If A(1, 2), B(2, 1), C(3, -4) are the vertices of $\square^{m}$ABCD, find the coordinates of D.

Solution :



A(1, 2), B(2, 1), C(3, − 4) are the vertices of ▱ABCD.
So midpoints of its diagonals $\overline{AC}$ and $\overline{BD}$ are the same.
∴ The coordinates of midpoint $\overline{BD}$
= The coordinates of midpoint $\overline{AC}$

$$\therefore \left(\frac{x+2}{2}, \frac{y+1}{2}\right) = \left(\frac{1+3}{2}, \frac{2-4}{2}\right)$$

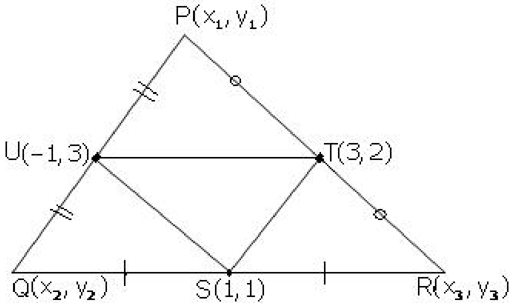
$$\therefore \frac{x+2}{2} = \frac{1+3}{2} \text{ and } \frac{y+1}{2} = \frac{2-4}{2}$$

∴ $x = 2$ and $y = -3$
∴ D(x, y) = D(2, −3).

Question 5:

(1, 1), (3, 2), (-1, 3) are the coordinates of mid-points of a triangle. Find the coordinates of the vertices of the triangle.

Solution :



Let $P(x_1, y_1)$, $Q(x_2, y_2)$ and $R(x_3, y_3)$ be the vertices of the triangle.
S(1, 1), T(3, 2), U(− 1, 3) are the midpoints of $\overline{QR}$, $\overline{RP}$, $\overline{PQ}$ respectively.

$$\therefore \left(\frac{x_2+x_3}{2}, \frac{y_2+y_3}{2}\right) = (1, 1)$$

$$\therefore \frac{x_2+x_3}{2} = 1 \text{ and } \frac{y_2+y_3}{2} = 1$$

$\therefore x_2 + x_3 = 2$ ... ...(1)
$y_2 + y_3 = 2$ ... ...(2)
Next, T(3, 2) is a midpoint of $\overline{PR}$.

$$\therefore \left(\frac{x_1+x_3}{2}, \frac{y_1+y_3}{2}\right) = (3, 2)$$

$$\therefore \frac{x_1+x_3}{2} = 3 \text{ and } \frac{y_1+y_3}{2} = 2$$

$\therefore x_1 + x_3 = 6$ ... ...(3)
and $y_1 + y_3 = 4$ ... ...(4)
Also, U(−1, 3) is a midpoint of $\overline{PQ}$

$\therefore \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = (-1, 3)$

$\therefore \frac{x_1 + x_2}{2} = -1$ and $\frac{y_1 + y_2}{2} = 3$

$\therefore x_1 + x_2 = -2 \qquad \ldots\ldots(5)$

and $y_1 + y_2 = 6 \qquad \ldots\ldots(6)$

Adding equations (1), (3) and (5),

$2(x_1 + x_2 + x_3) = 2 + 6 - 2 = 6$

$\therefore x_1 + x_2 + x_3 = 3 \qquad \ldots\ldots(7)$

Subtracting eq. (1), (3) and (5) from equation

(7) one at a time, we get,

$x_1 = 1,\ x_2 = -3$ and $x_3 = 5$

Adding equations (2), (4) and (6),

$2(y_1 + y_2 + y_3) = 2 + 4 + 6 = 12$

$\therefore y_1 + y_2 + y_3 = 6 \qquad \ldots\ldots(8)$

Subtracting eq. (2), (4) and (6) from (8) one at a time, we get,

$y_1 = 4,\ y_2 = 2$ and $y_3 = 0.$

$\therefore$ The vertices of ΔABC are

$P(x_1, y_1) = (1, 4)$

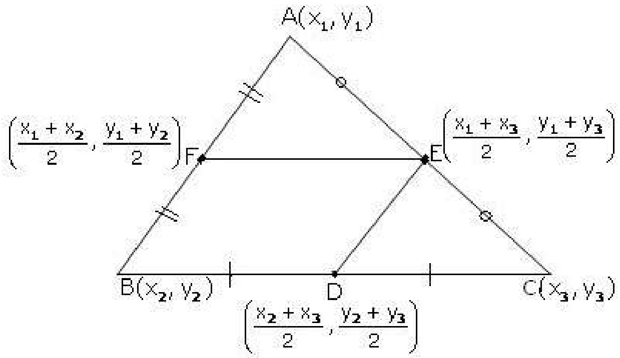
$Q(x_2, y_2) = (-3, 2)$

$R(x_3, y_3) = (5, 0).$

Question 6:

$A(X_1, Y_1)$, $B(X_2, Y_2)$, (X_3, Y_3) are the vertices of ΔABC and D, E, F are the mid-points of the sides $\overline{BC}, \overline{CA}, \overline{AB}$ respectively. Prove that $\square$ DEFB is a parallelogram.

Solution :



D is the midpoint of $\overline{BC}$.

$\therefore D\left(\frac{x_2 + x_3}{2}, \frac{y_2 + y_3}{2}\right)$.

E is the midpoint of $\overline{CA}$.

$\therefore E\left(\frac{x_1 + x_3}{2}, \frac{y_1 + y_3}{2}\right)$.

F is the midpoint of $\overline{AB}$.

$\therefore F\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

To prove □DEFB as a paralleogram, we show that,
the midpoint of its diagonals $\overline{DF}$ and $\overline{BE}$ are the same.

The midpoint of diagonal $\overline{DF}$

$$= \left(\frac{\frac{x_1 + x_2}{2} + \frac{x_2 + x_3}{2}}{2}, \frac{\frac{y_1 + y_2}{2} + \frac{y_2 + y_3}{2}}{2}\right)$$

$$= \left(\frac{x_1 + x_2 + x_2 + x_3}{4}, \frac{y_1 + y_2 + y_2 + y_3}{4}\right)$$

$$= \left(\frac{x_1 + 2x_2 + x_3}{4}, \frac{y_1 + 2y_2 + y_3}{4}\right) \qquad \ldots\ \ldots(1)$$

The midpoint of diagonal $\overline{BE}$

$$= \left(\frac{x_2 + \frac{x_1 + x_3}{2}}{2}, \frac{y_2 + \frac{y_1 + y_3}{2}}{2}\right)$$

$$= \left(\frac{2x_2 + x_1 + x_3}{4}, \frac{2y_2 + y_1 + y_3}{4}\right)$$

$$= \left(\frac{x_1 + 2x_2 + x_3}{4}, \frac{y_1 + 2y_2 + y_3}{4}\right) \qquad \ldots\ \ldots(2)$$

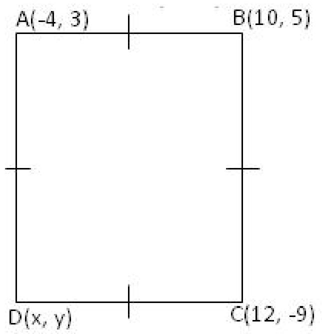
From (1) and (2), we can say that midpoint of diagonals $\overline{DF}$ and $\overline{BE}$
are the same.

$\therefore$ □DEFB is a paralleogram.

Question 7:

If A(-4, 3), B(10, 5), C(12, -9) are the vertices of a square, then find the coordinates of the fourth vertex.

Solution :



$A(-4, 3)$, $B(10, 5)$ and $C(12, -9)$ are the given points.

$$AB^2 = (-4-10)^2 + (3-5)^2$$
$$= 196 + 4$$
$$= 200$$
$$BC^2 = (10-12)^2 + (5+9)^2$$
$$= 4 + 196$$
$$= 200$$
$$AC^2 = (-4-12)^2 + (3+9)^2$$
$$= 256 + 144$$
$$= 400$$
$$AB^2 = BC^2 = 200$$
$$\therefore AB = BC = \sqrt{200}$$
$$AB^2 + BC^2 = AC^2$$

$\therefore \angle B$ is right angle in ΔABC.

Hence, $AB = BC$ and $\angle B$ is a right angle

$\therefore \Delta ABC$ is an isoceles right angled triangle.

Now, we can find the fourth point D opposite to $\angle B$ such that $\square ABCD$ becomes a square.

The diagonals of a square bisect each other.

$\therefore$ The midpoint of a diagonal $\overline{BD}$ = The midpoint of a diagonal $\overline{AC}$

$$\therefore \left(\frac{x+10}{2}, \frac{y+5}{2}\right) = \left(\frac{-4+12}{2}, \frac{3-9}{2}\right)$$

$$\therefore \frac{x+10}{2} = \frac{-4+12}{2} \text{ and } \frac{y+5}{2} = \frac{3-9}{2}$$

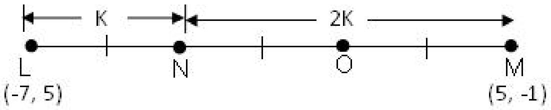
$\therefore x = -2$ and $y = -11$

$\therefore$ The fourth vertex of the square is $D(x, y) = D(-2, -11)$.

Question 8:

Find the coordinates of points which divide the segment joining A(-7, 5) and B(5, -1) into three congruent segments. (Such points are called the points of trisection of the segment).

Solution :



L(−7, 5) and M(5, − 1) are the given points.

The two points divide $\overline{LM}$ in three congruent segments.

Let N and O trisect $\overline{LM}$

N divides $\overline{LM}$ from L in ratio $\frac{LN}{NM} = \frac{K}{2K} = \frac{1}{2} = \frac{m}{n}$; where, K>0.

$\therefore$ m = 1, n = 2

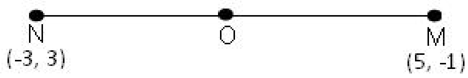
$L(x_1, y_1) = L(-7, 5)$, $M(x_2, y_2) = M(5, -1)$ and $N(x, y)$.

$$x = \frac{mx_2 + nx_1}{m+n}$$
$$= \frac{1(5) + 2(-7)}{1+2}$$
$$= -3$$
$$y = \frac{my_2 + ny_1}{m+n}$$
$$= \frac{1(-1) + 2(5)}{1+2}$$
$$= 3$$

$\therefore N(x, y) = N(-3, 3) \qquad \ldots \ldots (1)$

Now, N − O − M and NO = OM = K

$\therefore$ O is the midpoint of $\overline{NM}$



$$\therefore O(x, y) = \left(\frac{-3+5}{2}, \frac{3-1}{2}\right)$$
$$= (1, 1)$$

$\therefore O(x, y) = (1, 1) \qquad \ldots \ldots (2)$

$\therefore$ From (1) and (2), the trisection points of $\overline{LM}$ are (− 3, 3) and (1, 1)

Question 9:

A(-4, 2), B(-2, 1), C(4, -2) are collinear points. Find the ratio in which B divides $\overline{AC}$ from A.

Solution :

Let B(-2, 1) divide $\overline{AC}$ from A in the ratio $\frac{AB}{BC} = \frac{m}{n}$.

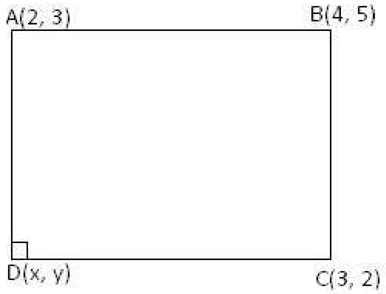
$A(x_1, y_1) = A(-4, 2)$, $C(x_2, y_2) = C(4, -2)$ and $B(x, y) = B(-2, 1)$

$$x = \frac{mx_2 + nx_1}{m+n}$$
$$\therefore -2 = \frac{m(4) + n(-4)}{m+n}$$
$$\therefore -2m - 2n = 4m - 4n$$
$$\therefore 6m = 2n$$
$$\therefore \frac{m}{n} = \frac{1}{3}$$
$$\therefore \frac{AB}{BC} = \frac{m}{n} = \frac{1}{3}$$

Question 10:

Show that A(2, 3), B(4, 5) and C(3, 2) can be the vertices of a rectangle. Find the coordinates of the fourth vertex.

Solution :



A(2, 3), B(4, 5) and C(3, 2) are the given points.

$AB^2 = (2 - 4)^2 + (3 - 5)^2$

$= 4 + 4$

$= 8$

$BC^2 = (4 - 3)^2 + (5 - 2)^2$

$= 1 + 9$

$= 10$

$AC^2 = (2 - 3)^2 + (3 - 2)^2$

$= 1 + 1$

$= 2$

$AB^2 + AC^2 = 8 + 2$

$=10$

$\therefore\ AB^2 + AC^2 = BC^2$

∴ In ΔABC, ∠A is a right angle,

∴ ΔABC is a right angled triangle. So, we can say that A, B and C can be the vertices of a rectangle whose fourth vertex is D(x, y) opposite to vertex A,

Now, the diagonals of a rectangle bisect each other

$\therefore$ The midpoint of $\overline{AD}$ = the midpoint of $\overline{BC}$

$\therefore \left(\dfrac{x+2}{2}, \dfrac{y+3}{2}\right) = \left(\dfrac{3+4}{2}, \dfrac{2+5}{2}\right)$

$\therefore \dfrac{x+2}{2} = \dfrac{3+4}{2}$ and $\dfrac{y+3}{2} = \dfrac{2+5}{2}$

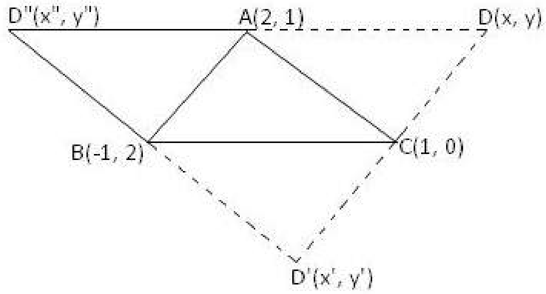
$\therefore$ x = 5 and y = 4

$\therefore$ The fourth vertex of the rectangle D(x, y) = D(5, 4).

Question 11:

(2, 1), (-1, 2) and (1, 0) are the coordinates of three vertices of a parallelogram. Find the fourth vertex.

Solution :



Here (2, 1), (-1, 2) and (1, 0) are the three vertices of a parallelogram and the fourth vertex needs to be found.

The fourth vertex can be obtained in three different ways.

1. □ABCD is a parallelogram with vertices A(2, 1), B(-1, 2) and C(1, 0).
Let the coordinates of D be (x,y).
The midpoints of the diagonals $\overline{BD}$ and $\overline{AC}$ are the same.

$$\therefore \frac{x-1}{2} = \frac{2+1}{2} \text{ and } \frac{y+2}{2} = \frac{1+0}{2}$$

$\therefore$ x = 4 and y = − 1

$\therefore$ D(x, y) = D(4, −1)

2. □ABD'C is a parallelogram with vertices A(2, 1), B(-1, 2) and C(1, 0).
Let the coordinates of D' be (x', y').
The midpoints of the diagonals $\overline{AD}'$ and $\overline{BC}$ are the same.

$$\therefore \frac{x'+2}{2} = \frac{-1+1}{2} \text{ and } \frac{y'+1}{2} = \frac{2+0}{2}$$

$\therefore$ x' = − 2 and y' = 1

$\therefore$ D'(x', y') = D'(−2, 1)

3. □ACBD" is a parallelogram with vertices A(2, 1), B(-1, 2) and C(1, 0).
Let the coordinates of D" be (x", y").
The midpoints of the diagonals $\overline{CD}$" and $\overline{AB}$ are the same.

$$\therefore \frac{x''+2}{2} = \frac{-2+1}{2} \text{ and } \frac{y''+0}{2} = \frac{1+2}{2}$$

$\therefore$ x" = 0 and y" = 3

$\therefore$ D"(x", y") = D"(0, 3)

The coordinates of the fourth vertex of the parallelogram are (4, − 1) or (− 2, 1) or (0, 3).

Exercise-8.3

Question 1:

Find the area of ΔABC whose vertices are A(4, 2), B(3, 9) and C(10, 10).

Solution :

If $A(x_1, y_1), B(x_2, y_2)$, and $C(x_3, y_3)$ are the vertices of ΔABC, then

$$\text{Area of } \Delta ABC = \frac{1}{2}\left| x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) \right|$$

$$\therefore \text{Area of } \Delta ABC = \frac{1}{2}\left| 4(9-10) + 3(10-2) + 10(2-9) \right|$$

$$= \frac{1}{2}\left| -4 + 24 - 70 \right|$$

$$= \frac{50}{2}$$

$$= 25$$

$\therefore$ Area of ΔABC is 25.

Question 2:

Find a if the area of ΔABC is 5. The coordinates A, B, C are (2, 3), (4, 5) and (a, 3).

Solution :

A(2, 3), B(4, 5) and C(a, 3) are the vertices of ΔABC
Given that the area of ΔABC is 5 sq. units.
$\therefore$ A(ΔABC) = 5

$$\therefore \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)| = 5$$

$$\therefore \frac{1}{2}|2(5-3) + 4(3-3) + a(3-5)| = 5$$

$$\therefore \frac{1}{2}|4 + 0 - 2a| = 5$$

$$\therefore |4 - 2a| = 10$$

$$\therefore 4 - 2a = 10 \text{ or } 4 - 2a = -10$$

$$\therefore 2a = 4 - 10 \text{ or } 2a = 4 + 10$$

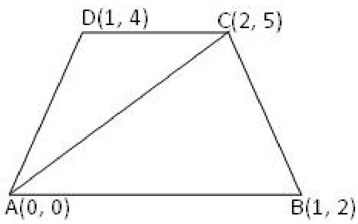
$$\therefore 2a = -6 \text{ or } 2a = 14$$

$$\therefore a = -3 \text{ or } a = 7$$

Question 3:

Find the area of □ ABCD where coordinates of A, B, C, D are (0, 0), (1, 2), (2, 5) and (1, 4)respectively.

Solution :



A(0, 0), B(1, 2), C(2, 5) and D(1, 4) are the vertices of □ABCD.
From the figure, it is clear that the area of □ABCD is equal to the sum of the areas of ΔABC and ΔACD

$\therefore$ Area of □ABCD = Area of ΔABC + Area of ΔACD ... ...(1)

$$\therefore A(\Delta ABC) = \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

$$= \frac{1}{2}|0(2-5) + 1(5-0) + 2(0-2)|$$

$$= \frac{1}{2}|0 + 5 - 4|$$

$$= \frac{1}{2} \qquad \text{... ...(2)}$$

$$A(\Delta ACD) = \frac{1}{2}|0(5-4) + 2(4-0) + 1(0-5)|$$

$$= \frac{1}{2}|0 + 8 - 5|$$

$$= \frac{3}{2} \qquad \text{... ...(3)}$$

From the results, (1), (2) and (3), we get,

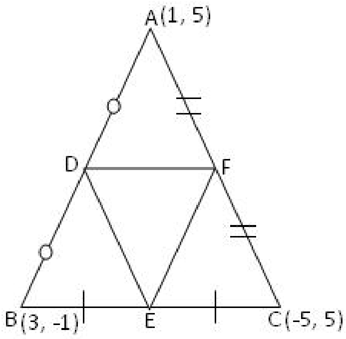
$$\text{Area of □ABCD} = \frac{1}{2} + \frac{3}{2} = 2$$

$\therefore$ Area of □ABCD is 2 sq. units.

Question 4:

A(1, 5), B(3, -1), C(-5, 5) are the vertices of ΔABC. Find the area of ΔDEF, if D, E, F are the mid-points of the sides of ΔABC.

Solution :



$A(1,\ 5),\ B(3,\ -1)$ and $C(-5,\ 5)$ are the vertices of ΔABC.

The midpoint of $\overline{AB}$ is D.

$\therefore D(x_1,\ y_1) = D\left(\frac{1+3}{2},\ \frac{5-1}{2}\right) = D(2,\ 2)$

The midpoint of $\overline{BC}$ is E.

$\therefore E(x_2, y_2) = E\left(\frac{3-5}{2},\ \frac{-1+5}{2}\right) = E(-1,\ 2)$

The midpoint of $\overline{AC}$ is F.

$\therefore F(x_3,\ y_3) = F\left(\frac{1-5}{2},\ \frac{5+5}{2}\right) = F(-2,\ 5)$

$\therefore$ Area of ΔDEF

$= \frac{1}{2}|\ x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$

$= \frac{1}{2}|\ 2(2-5) + (-1)(5-2) + (-2)(2-2)|$

$= \frac{1}{2}|-6 - 3 + 0|$

$= \frac{9}{2}$

Question 5:

(9, a), (6, 7), (2, 3) are the coordinates of the vertices of a triangle. If the area of the triangle is 10, find a.

Solution :

$A(9,\ a),\ B(6,\ 7)$ and $C(2,\ 3)$ are the vertices of the ΔABC

Area of ΔABC is 10.

$\therefore A(\Delta ABC) = 10$

$\therefore \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)| = 10$

$\therefore \frac{1}{2}|9(7-3) + 6(3-a) + 2(a-7)| = 10$

$\therefore \frac{1}{2}|36 + 18 - 6a + 2a - 14| = 10$

$\therefore \frac{1}{2}|40 - 4a| = 10$

$\therefore |40 - 4a| = 20$

$\therefore 40 - 4a = -20$ or $40 - 4a = 20$

$\therefore 4a = 60$ or $4a = 20$

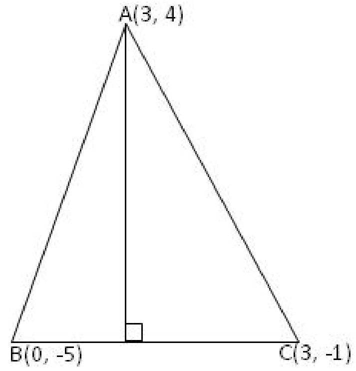
$\therefore a = 15$ or $a = 5$

Exercise-8

Question 1:

Can A(3, 4), B(0, -5), C(3, -1) be the vertices of a triangle ? If your answers is yes, find the area of the triangle. Hence find the length of the altitude on $\overline{BC}$.

Solution :



$A(x_1, y_1) = A(3, 4)$, $B(x_2, y_2) = B(0, -5)$ and $C(x_3, y_3) = C(3, -1)$

$AB^2 = (3-0)^2 + (4+5)^2$

$= 9 + 81$

$= 90$

$\therefore AB = \sqrt{90} = 3\sqrt{10}$ ... ...(1)

$BC^2 = (0-3)^2 + (-5+1)^2$

$= 9 + 16$

$= 25$

$\therefore BC = \sqrt{25} = 5$ ... ...(2)

$AC^2 = (3-3)^2 + (4+1)^2$

$= 0 + 25$

$= 25$

$\therefore AC = \sqrt{25} = 5$ ... ...(3)

Hence, from (1), (2) and (3), the sum of the lengths of any two segments from $\overline{AB}$, $\overline{BC}$ and $\overline{AC}$ is not equal to the length of the remaining line segments. Hence, the given points A, B and C are non-collinear.

$\therefore$ A,B and C are the vertices of a triangle.

Now, the area of ΔABC

$= \frac{1}{2} | x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) |$

$= \frac{1}{2} | 3(-5 + 1) + 0(-1 - 4) + 3(4 + 5) |$

$= \frac{1}{2} | -12 + 0 + 27 |$

$= \frac{15}{2}$

Also, altitude $\overline{AM} \perp \overline{BC}$ in ΔABC.

$\therefore$ Area of ΔABC

$= \frac{1}{2}$ base x altitude

$= \frac{1}{2} BC \times AM$

$\therefore \frac{15}{2} = \frac{1}{2} \times 5 \times AM$

(Substituting the values of area of ΔABC and BC from the results (2) and (4) respectively.)

$\therefore AM = 3$

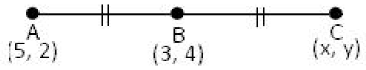
$\therefore$ The length of altitude is 3.

$\therefore$ The area of a triangle is $\frac{15}{2}$ and the length of altitude on $\overline{BC}$ is 3.

Question 2:

If A(5, 2), B(3, 4), C(x, y) are collinear and AB = BC, then find (x, y).

Solution :



A(5, 2), B(3, 4) and C(x, y) are collinear and AB = BC

$\therefore$ A – B – C and B is a midpoint of $\overline{AC}$.

$\therefore \left(\frac{x+5}{2}, \frac{y+2}{2}\right) = (3, 4)$

$\therefore \frac{x+5}{2} = 3$ and $\frac{y+2}{2} = 4$

$\therefore x = 1$ and $y = 6$

$\therefore (x, y) = (1, 6)$

Question 3:

Show that A(0, 1), B(2, 9) C$\left(\frac{2}{3}, \frac{11}{3}\right)$ are collinear. Find which point is between the remaining two. Also find the ratio in which $\overline{AB}$ is divided by C from A.

Solution :

$A(x_1, y_1) = A(0, 1), B(x_2, y_2) = B(2, 9), C(x_3, y_3) = C\left(\frac{2}{3}, \frac{11}{3}\right)$

$$AB^2 = (0-2)^2 + (1-9)^2$$
$$= 4 + 64$$
$$= 68$$
$$\therefore AB = \sqrt{68} = 2\sqrt{17}$$
$$\text{Next, } BC^2 = \left(2-\frac{2}{3}\right)^2 + \left(9-\frac{11}{3}\right)^2$$
$$= \left(\frac{4}{3}\right)^2 + \left(\frac{16}{3}\right)^2$$
$$= \frac{16+256}{9}$$
$$= \frac{272}{9}$$
$$\therefore BC = \sqrt{\frac{272}{9}} = \frac{4}{3}\sqrt{17}$$
$$\text{Next, } AC^2 = \left(\frac{2}{3}-0\right)^2 + \left(\frac{11}{3}-1\right)^2$$
$$= \left(\frac{2}{3}\right)^2 + \left(\frac{8}{3}\right)^2$$
$$= \frac{4+64}{9}$$
$$= \frac{68}{9}$$
$$\therefore AC = \sqrt{\frac{68}{9}} = \frac{2}{3}\sqrt{17}$$
$$\text{Here, } AC + BC = \frac{2}{3}\sqrt{17} + \frac{4}{3}\sqrt{17}$$
$$\left(\frac{2+4}{3}\right)\sqrt{17} = 2\sqrt{17}$$

So, $AC + BC = AB$.

$\therefore$ A, B and C are collinear points.

Also A – C – B. So C lies between A and B.

$\therefore$ C divides $\overline{AB}$ from A in ratio $\frac{AC}{CB}$.

$$\therefore \frac{AC}{CB} = \frac{\frac{2}{3}\sqrt{17}}{\frac{4}{3}\sqrt{17}} = \frac{1}{2}$$

$\therefore$ C divides $\overline{AB}$ from A in ratio 1:2.

Question 4:

Find k if A(k, 2), B(3, 1), C(4, 2) are the vertices of a right angle triangle.

Solution :

A(k, 2), B(3, 1), C(4, 2) are the vertices of a right angles triangle.

$AB^2 = (k - 3)^2 + (2 - 1)^2$

$= k^2 - 6k + 9 + 1$

$= k^2 - 6k + 10$

$BC^2 = (3 - 4)^2 + (1 - 2)^2$

$= 1 + 1$

$= 2$

$AC^2 = (k - 4)^2 + (2 - 2)^2$

$= k^2 - 8k + 16$

There are three possible cases:

Case 1:

$m\angle A = 90°$

$\therefore BC^2 = AB^2 + AC^2$

$\therefore 2 = k^2 - 6k + 10 + k^2 - 8k + 16$

$\therefore 2 = 2k^2 - 14k + 24$

$\therefore 2k^2 - 14k + 24 = 0$

$\therefore k^2 - 7k + 12 = 0$

$\therefore k(k - 4) - 3(k - 4) = 0$

$\therefore (k - 4)(k - 3) = 0$

$\therefore$ k = 4 or k = 3. But for k = 4, the points A and C are equal so we cannot take k = 4. Thus, in this case we get k = 3.

Case 2:

$m\angle B = 90°$

$\therefore AC^2 = AB^2 + BC^2$

$\therefore k^2 - 8k + 16 = k^2 - 6k + 10 + 2$

$\therefore 4 = 2k$

$\therefore k = 2$

Case 3:

$m\angle C = 90°$

$\therefore AB^2 = BC^2 + AC^2$

$\therefore k^2 - 6k + 10 = 2 + k^2 - 8k + 16$

$\therefore 2k = 8 \therefore k = 4$

But for k = 4, A and C are equal. So $k \neq 4$

i.e., For the vertices A(k, 2), B(3, 1) and C(4, 2),

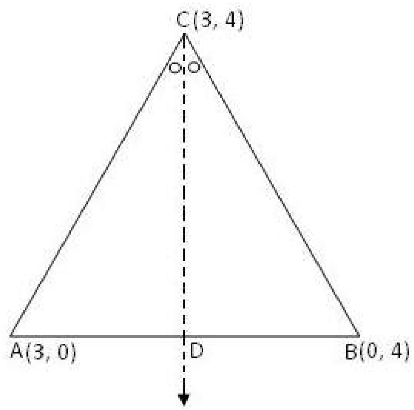
$\angle C = 90°$ is not possible.

Thus, for $m\angle A = 90°$, we get, k = 3, for $m\angle B = 90°$, we get, k = 2 and $m\angle C \neq 90$.

Question 5:

A(3, 0), B(0, 4) and C(3, 4) are the vertices of ΔABC. The bisector of ∠C intersects $\overline{AB}$ in D. Hint : The result $\frac{AD}{DB} = \frac{CQ}{DA}$

Solution :



A(3, 0), B(0, 4) and C(3, 4) are the vertices of ΔABC.

$$AC^2 = (3-3)^2+(0-4)^2$$
$$= 0 + 16$$
$$\therefore AC = \sqrt{16} = 4$$
$$BC^2 = (3-0)^2 + (4+4)^2$$
$$= 9 + 0$$
$$\therefore BC = \sqrt{9}$$
$$= 3$$

In ΔABC, the bisector of ∠C intersects $\overline{AB}$ in D.

$$\therefore \frac{AD}{DB} = \frac{AC}{CB}$$

D divides $\overline{AB}$ from A in ratio $\frac{AD}{DB} = \frac{AC}{BC} = \frac{4}{3} = \frac{m}{n}$ (From Hint)

$\therefore m = 4,\ n = 3,$

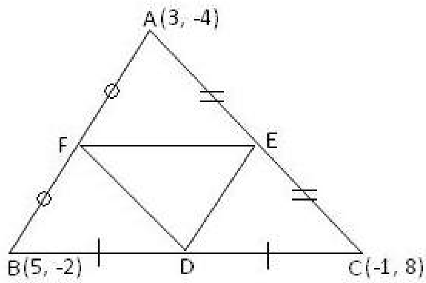
$A(x_1, y_1) = A(3, 0)$, $B(x_2, y_2) = B(0, 4)$ and $D(x, y)$.

$$x = \frac{mx_2 + nx_1}{m+n}$$
$$= \frac{4(0) + 3(3)}{4+3}$$
$$= \frac{9}{7}$$
$$y = \frac{my_2 + ny_1}{m+n}$$
$$= \frac{4(4) + 3(0)}{4+3}$$
$$= \frac{16}{7}$$
$$\therefore D(x, y) = D\left(\frac{9}{7}, \frac{16}{7}\right).$$

Question 6:

A(3, -4), B(5, -2), C(-1, 8) are the vertices of ΔABC. D, E, F are the mid-points of side $\overline{BC}$, $\overline{CA}$ and $\overline{AB}$ respectively. Find area of ΔABC. Using coordinates of D, E, F, find area of ΔDEF. Hence show that the ABC = 4(DEF).

Solution :



$A(x_1, y_1) = A(3, -4)$,
$B(x_2, y_2) = B(5, -2)$
$C(x_3, y_3) = C(-1, 8)$.
If $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$ are the vertices of ΔABC, then

Area of $\Delta ABC = \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$

$\therefore A(\Delta ABC) = \frac{1}{2}|3(-2-8) + 5(8+4) - 1(-4+2)|$

$= \frac{1}{2}|3(-10) + 5(12) - 1(-2)|$

$= \frac{1}{2}|-30 + 60 + 2|$

$= \frac{32}{2}$

$= 16 \qquad \ldots \ldots (1)$

$\therefore$ Area of ΔABC is 16.

The midpoint of $\overline{BC}$ is D.

$\therefore D\left(\frac{5-1}{2}, \frac{-2+8}{2}\right) = D(2, 3)$

The midpoint of $\overline{CA}$ is E.

$\therefore E\left(\frac{3-1}{2}, \frac{-4+8}{2}\right) = E(1, 2)$

The midpoint of $\overline{AB}$ is F.

$\therefore F\left(\frac{3+5}{2}, \frac{-4-2}{2}\right) = F(4, -3)$

Area of ΔDEF

$= \frac{1}{2}|2(2+3) + 1(-3-3) + 4(3-2)|$

$\therefore A(\Delta DEF) = \frac{1}{2}|10 - 6 + 4|$

$= \frac{1}{2} \times 8$

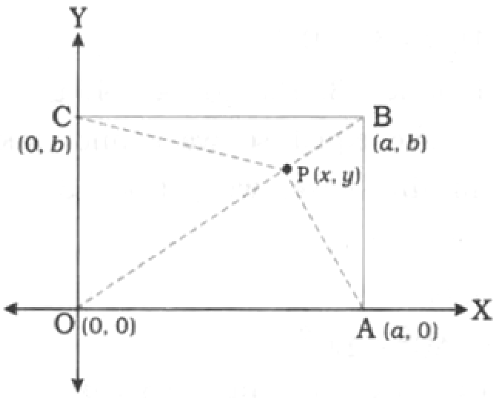
$= 4 \qquad \ldots \ldots (2)$

From the results (1) and (2),
$A(\Delta ABC) = 16$ and $A(\Delta DEF) = 4$
$\therefore A(\Delta ABC) = 16 = 4 \times 4 = 4A(\Delta DEF)$
$\therefore$ The area of ΔABC is 16 and the area of ΔDEF is 4,
So, $\Delta ABC = 4(\Delta DEF)$.

Question 7:

Show that O(0, 0), A(a, 0), B(a, b), C(0, b) are the vertices of a rectangle. If P(x, y) is a point in the coordinate plane, then prove using distance formula that $PO^2 + PB^2 = PA^2 + PC^2$.

Solution :



$OA^2 = (0 - a)^2 + (0 - 0)^2 = a^2 \quad \therefore OA = a$

$AB^2 = (a - a)^2 + (0 - b)^2 = b^2 \quad \therefore AB = b$

$BC^2 = (a - 0)^2 + (b - b)^2 = a^2 \quad \therefore BC = a$

$CO^2 = (0 - 0)^2 + (b - 0)^2 = b^2 \quad \therefore CO = b$

$\therefore OA = BC$ and $AB = CO$ (Opposite sides are equal.)

$OB^2 = (0 - a)^2 + (0 - b)^2 = a^2 + b^2$

$\therefore OB = \sqrt{a^2 + b^2}$

$AC^2 = (a - 0)^2 + (0 - b)^2 = a^2 + b^2$

$\therefore AC = \sqrt{a^2 + b^2}$

$\therefore OB = AC$ (Diagonals are equal.)

Hence, opposite sides are equal so O(0, 0), A(a, 0), B(a, b) and C(0, b) are the vertices of a rectangle.

P(x, y) be any point in the plane containig rectangle.

L.H.S.

$= PO^2 + PB^2$

$= (x - 0)^2 + (y - 0)^2 + (x - a)^2 + (y - b)^2$

$= x^2 + y^2 + (x - a)^2 + (y - b)^2$

R.H.S

$= PA^2 + PC^2$

$= (x - a)^2 + (y - a)^2 + (x - 0)^2 + (y - b)^2$

$= x^2 + y^2 + (x - a)^2 + (y - b)^2$

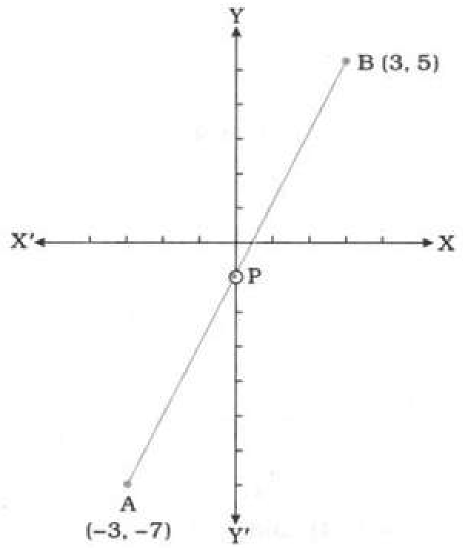
$\therefore$ L.H.S. = R.H.S.

$\therefore PO^2 + PB^2 = PA^2 + PC^2$

Question 8:

Find the coordinates of the points of intersection of the segment joining A(-3, -7) and B(3, 5) with Y-axis.

Solution :



The point A is in the third quadrant and the point B is in the first quadrant, so the y-axis divides $\overline{AB}$.
The x-coordinate of any point on the y-axis is zero.
Assume the y-axis divides $\overline{AB}$ at point P(0, y) and P divides $\overline{AB}$ from A in ratio λ.

$\therefore \frac{AP}{PB} = \lambda$, $A(x_1, y_1) = A(-3, -7)$ and $B(x_2, y_2) = B(3, 5)$.

$x = \frac{\lambda x_2 + x_1}{\lambda + 1}$

$\therefore 0 = \frac{\lambda(3) + (-3)}{\lambda + 1}$

$\therefore 3\lambda = 3$

$\therefore \lambda = 1$

$y = \frac{\lambda y_2 + y_1}{\lambda + 1}$

$= \frac{1(5) + (-7)}{1 + 1} \quad (\because \lambda = 1)$

$= \frac{-2}{2}$

$= -1$

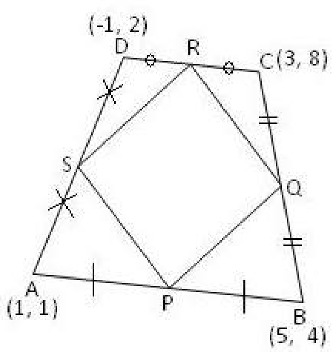
$\therefore P(0, y) = P(0, -1)$

$\therefore$ The coordinates of the intersecting point of the segment joining A and B with the y-axis are $(0, -1)$.

Question 9:

A(1, 1), B(5, 4), C(3, 8), D(-1, 2) are the vertices of□ABCD. If P, Q, R, S are the mid-points of $\overline{AB}, \overline{BC}, \overline{CD}, \overline{DA}$ respectively, show that□PQRS is parallelogram.

Solution :



A(1, 1), B(5, 4), C(3, 8), D(− 1, 2) are the vertices of ▭ABCD.

The midpoint of $\overline{AB}$ is P.

$\therefore P\left(\frac{1+5}{2}, \frac{1+4}{2}\right) = P\left(3, \frac{5}{2}\right)$

The midpoint of $\overline{BC}$ is Q.

$\therefore Q\left(\frac{5+3}{2}, \frac{4+8}{2}\right) = Q(4, 6)$

The midpoint of $\overline{CD}$ is R.

$\therefore R\left(\frac{3-1}{2}, \frac{8+2}{2}\right) = R(1, 5)$

The midpoint of $\overline{DA}$ is S.

$\therefore S\left(\frac{-1+1}{2}, \frac{2+1}{2}\right) = S\left(0, \frac{3}{2}\right)$

For showing ▭PQRS is a parallelogram we can show that the diagonals bisect each other.

The midpoint of diagonal $\overline{PR}$

$= \left(\frac{3+1}{2}, \frac{\frac{5}{2}+5}{2}\right)$

$= \left(2, \frac{15}{4}\right) \qquad \ldots\ldots(1)$

The midpoint of diagonal $\overline{SQ}$

$= \left(\frac{4+0}{2}, \frac{6+\frac{3}{2}}{2}\right)$

$= \left(2, \frac{15}{4}\right) \qquad \ldots\ldots(2)$

From the results (1) and (2),

Midpoint of diagonal $\overline{PR}$ = Midpoint of diagonal $\overline{SQ}$.

∴▭PQRS is a parallelogram.

Question 10:

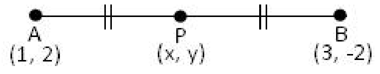
Select a proper option (a), (b), (c) or (d) from given options :

Question 10(1):

If A(1, 2) and B(3, -2) are given points, then …… is the mid-point o$\overline{AB}$.

Solution :

c. P(2, 0)



Let P(x, y) be the midpoint of line segment joining A(1, 2) and B(3, −2).

$\therefore x = \frac{1+3}{2}$ and $y = \frac{2-2}{2}$

$\therefore x = \frac{4}{2}$ and $y = \frac{0}{2}$

$\therefore x = 2$ and $y = 0$

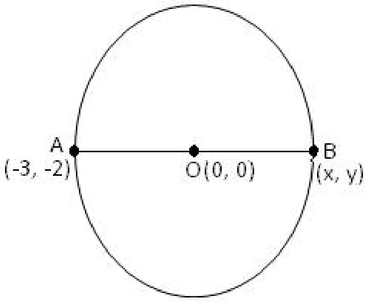
$\therefore P(x, y) = P(2, 0)$

Question 10(2):

One of the end-points of a circle having centre at origin is A(3, -2), then the other end-point of the diameter has the coordinates ……

Solution :

a. $(-3, 2)$



Let, the other end point of a diagonal be B(x, y).
$\overline{AB}$ is a dimeter of a circle so the center O(0, 0)
becomes the midpoint of the line segment joining A(3, − 2)
and B(x, y).

$\therefore 0 = \dfrac{3 + x}{2}$ and $0 = \dfrac{-2 + y}{2}$

$\therefore 3 + x = 0$ and $-2 + y = 0$

$\therefore x = -3$ and $y = 2$

$\therefore B(x, y) = B(-3, 2)$

Question 10(3):

The distance of A(x, y) from origin is………

Solution :

d. $\sqrt{x^2+y^2}$

For the distance between A(x, y) and the origin O(0, 0),

$OA^2 = (x - 0)^2 + (y - 0)^2$

$= x^2 + y^2$

$\therefore OA = \sqrt{x^2 + y^2}$

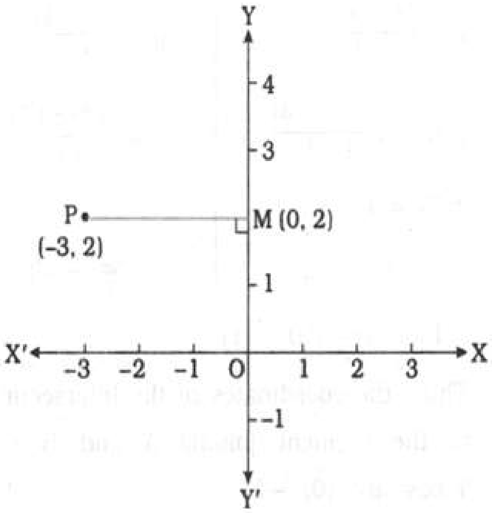
Question 10(4):

The foot of the perpendicular from P(-3, 2) to Y-axis is M. Coordinates of M are …..

Solution :

b. (0, 2)

It is clear from the following figure.

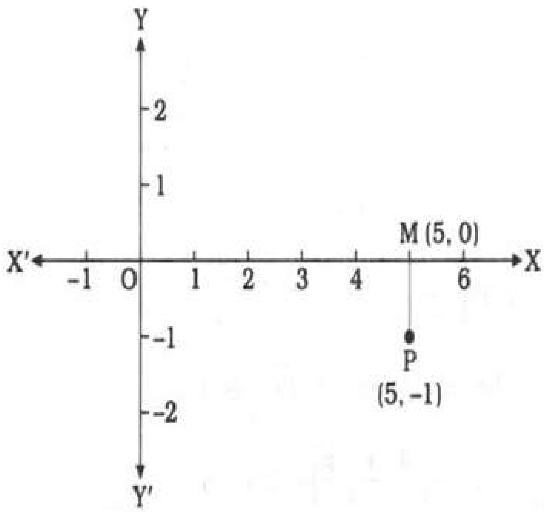


Question 10(5):

The coordinates of the foot of the perpendicular from P(5, -1) to X-axis are …..

Solution :

d. (5, 0)
This is clear from the following figure.



Question 10(6):

A(0, 0), B(3, 0), C(3, 4) are the vertices of a …… triangle.

Solution :

a. right angled
A(0, 0), B(3, 0) and C(3, 4)
$AB^2 = (0 – 3)^2 + (0 – 0)^2$
= 9 + 0
= 9
$BC^2 = (3 – 3)^2 + (0 – 4)^2$
= 0 + 16
=16
$CA^2 = (3 – 0)^2 + (4 – 0)^2$

$= 9 + 16$

$=25$

Now,

$AB^2 + BC^2$

$= 9 + 16$

$= 25$

$\therefore AB^2 + BC^2 = CA^2$

$\therefore \Delta ABC$ is a right angled triangle and $\angle B$ is a right angle.

Question 10(7):

(1, 0), (0, 1), (1, 1) are the coordinates of vertices of a triangle. The triangle is …… triangle.

Solution :

c. Isosceles

Let A(1, 0), B(0, 1) and C(1, 1).

$AB^2 = (1 - 0)^2 + (0 - 1)^2$

$= 1 + 1$

$= 2$

$\therefore AB = \sqrt{2}$

$BC^2 = (0 - 1)^2 + (1 - 1)^2$

$= 1 + 0$

$= 1$

$\therefore BC = \sqrt{1} = 1$

$CA^2 = (1 - 1)^2 + (1 - 0)^2$

$= 0 + 1$

$= 1$

$\therefore CA = \sqrt{1} = 1$

$\therefore BC = CA = 1$

$\therefore$ The triangle is isosceles.

Question 10(8):

A(1, 2), B(2, 3), C(3, 4) are given points ……. of the following is true.

Solution :

a. AB + BC = AC

A(1, 2), B(2, 3) and C(3, 4).

$AB^2 = (1-2)^2 + (2-3)^2$
$= 1 + 1$
$= 2$

$\therefore AB = \sqrt{2}$

$BC^2 = (2-3)^2 + (3-4)^2$
$= 1 + 1$
$= 2$

$\therefore BC = \sqrt{2}$

$AC^2 = (1-3)^2 + (2-4)^2$
$= 4 + 4$
$= 8$

$\therefore AC = \sqrt{8} = 2\sqrt{2}$

Hence, $AB + BC = \sqrt{2} + \sqrt{2} = 2\sqrt{2}$

$\therefore AB + BC = AC$

Note : Also, AB = BC

$\therefore$ B is the midpoint of $\overline{AC}$.

Hence, (c) is also true.

*The answer given at the back of the textbook is (b) and we have shown that (c) is also correct.Hence the question has two answers.