CHAPTER 06

Beams

6.1 Major Consideration in Beam Design

(a) Beams must be adequately proportioned for strength in bending taking into consideration the local and lateral instability of the compression flange and also the selected beam section must develop the necessary shear strength and the bearing resistance.

(b) The beam section must be proportioned for stiffness keeping in view that their deflections and deformations are within the limits as specified under service load conditions.

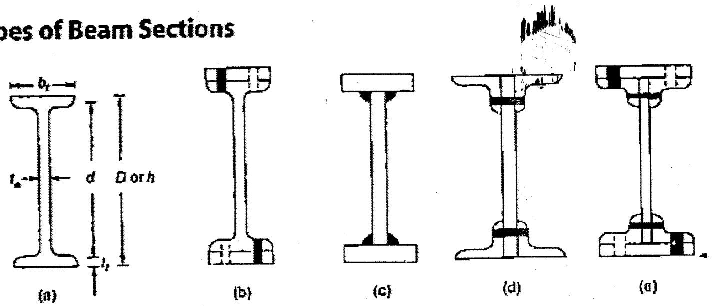
(c) The beam section must be proportioned for economy also by paying due attention to the grade of steel to yield.

- **Achieving the optimum bending resistance:** The beam material must be displaced off from the neutral axis as far as possible to have large moment of inertia of the beam section. The web area must be sufficient enough to have adequate shearing resistance.

NOTE: The maximum moment and the maximum shear in a beam occur at different sections. However, even if they are found at the same section, their interaction is neglected.

- Beam design is made complicated by the phenomenon of lateral buckling of the beam or buckling of compression flange or its web portion.
- Another problem with beam design is the depth of the beam section. Increasing the depth of beam increases the flexural resistance but reduces the resistance to lateral or web buckling.

6.2 Types of Beam Sections



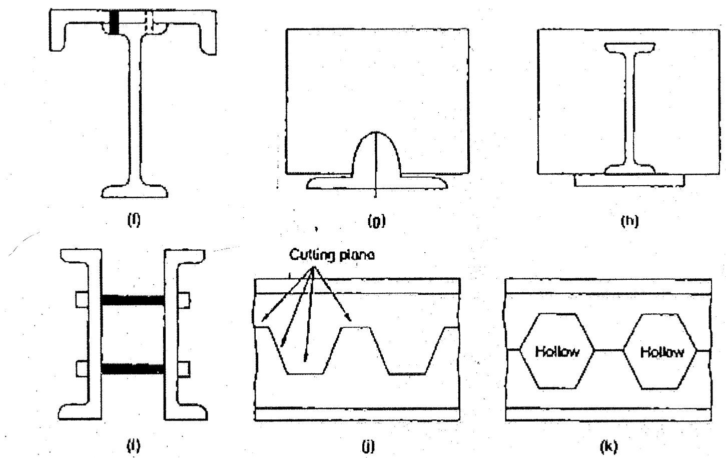


Fig. 6.1 *Different types of open beam sections*

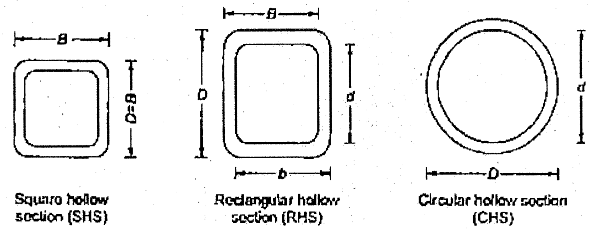


Fig. 6.2 Some typical types of closed beam sections

- Normally used beam sections are of open type wherein the majority of the material is distributed in the flanges away from the center of gravity of the beam section. This enhances the in-plane flexural resistance of the beam section. Also open sections are very convenient in making the connections with other members and alike.
- **Major drawback of open sections:** Open section are quite weak in flexural strength about minor axis and also the torsional resistance.
- **Angles and T-sections** are quite weak in bending and **channel sections** can only be used for light loads.
- **Rolled steel I-section** is generally adopted as beam section.
- **Closed sections** have very high torsional strength but are difficult to make connections with other members.
- **I-section** being the most preferred beam section can either be a **rolled section** or a **built up section**. An I-section with cover plates is required where large section modulus is required which cannot be furnished by the available rolled I-sections. This built up I-section may either be welded or riveted/bolted.

- For beams subjected to lateral loads at compression flange along with transverse loads, there,an I-section (preferably ISWB) along with a channel section in the compression flange is used. For example: 'Beam section of gantry gliders'.
- **Channel and angle sections as beam:** The use of channel section and angle section as beam must be done with due care because load is not likely to be in the plane which eliminates torsional eccentricities. Also, it is quite difficult to predict the lateral bucking behaviour of these sections.

6.3 Plastic Moment Carrying Capacity of a Section

Consider a section of a simply supported beam where the flexural moment is maximum for the given loading. Within elastic limit, the flexural stress varies linearly from compression side to tension side as shown in Fig. 6.3(b).

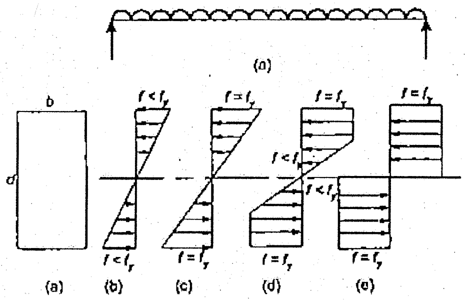


Fig. 6.3 Plastic moment capacity of a beam section

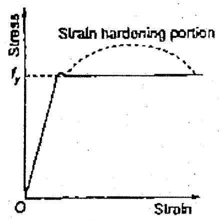


Fig. 6.4 Stress strain curve for steel

- Now the load is gradually increased which results in the increase in the stresses in the fibers with extreme fibers being highly stressed. The loading is increased till the extreme fibers reach yield stress (**Fig. 6.3(c)**). The stress-strain relationship for steel is assumed as shown in Fig. 6.4 wherein the strain hardening part of the stress-strain curve is ignored and it is assumed that once the yield point is reached, fibers go on yielding without resisting any additional load.
- As per the theory of plastic analysis, the highly stressed fiber once yields is not capable of resisting any load (or moment). But since the inner fibers has not yielded yet, and thus additional load is resisted by the un-yielded portion of the beam section.
- As the loading is increased further gradually, the inner fibers will also yield and will cease to take any additional load [**Fig. 6.3(d)**].
- With still further increase of load, resistance to load is offered by inner fibers which are not yielded yet. This resistance to load increases till all the fibers get yielded (**Fig. 6.3(e)**).
- Once all the fibers get yielded, no more resistance to the load is offered by the beam section. This condition i.e. when all the fibers have yielded at a section is called as formation of **plastic hinge.**
- Once plastic hinge condition has reached then, infinite rotation can take place at constant load and without resisting any additional load.This moment carrying capacity of the section (at this ultimate load) is called as **plastic moment capacity of the section (M_p)**.

6.3.1 Expression for Plastic Moment Carrying Capacity of a Section

For the beam section as shown in **Fig. 6.3.**

Let A_c = Area of beam section in compression

A_t = Area of beam section in tension

∴ A = Total area of beam section = $A_c + A_t$

Now, total compressive force (C) = $A_c f_y$

and total tensile force (T) = $A_t f_y$

For static equilibrium, $C = T$

$$A_c f_y = A_t f_y$$

$$A_c = A_t = \frac{A}{2} \qquad ...(6.1)$$

- The stress in the beam section varies from compression (taken as negative) to tension (taken as positive).
- At the location where the stress changes its sign i.e. becomes zero is referred to as **plastic neutral axis** of the beam section.
- From Eq. (6.1), it can be inferred that plastic neutral axis divides the beam section into two equal parts. For symmetric sections, this plastic neutral axis lies at mid-depth of the section but for unsymmetrical sections (like T- section, L-section etc.) it has to be located from the condition as derived in Eq. (6.1)
- Plastic moment capacity of the section can be found out by taking the moment of horizontal forces about the plastic neutral axis. The moments due to compressive and tensile forces are additive.

  Let M_p = Plastic moment capacity of a section

  Then from the bending equation,

  $$M = f.Z$$

  it can be deduced that,

  $$M_p = f_y Z_p$$

  where f_y = Yield stress of the material of the beam section

  Z_p = Plastic section modulus of the beam section
- For standard rolled sections, Z_p is about 1.125 to 1.14 times Z_e for I-sections and about 1.7 to 1.8 times for channel sections.

NOTE

1. Indian rolled steel sections consist of sloping flanges from inside, fillets at the junctions and rounded edges.
2. For determinate structures, formation of a single hinge causes collapse of the whole structure since it goes on rotating without resisting any additional load but this is not so for indeterminate structures.

- Collapse mechanism for propped cantilever and fixed beam are shown in **Fig. 6.5** and **6.6** respectively.

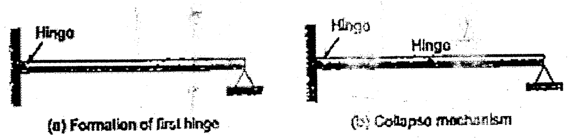


Fig. 6.5 Collapse mechanism for a propped cantilever

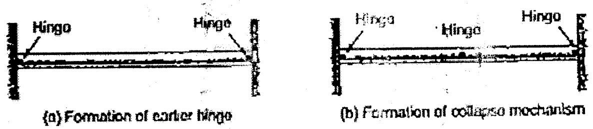


Fig. 6.6 Collapse mechanism for a fixed beam

6.4 Classification of Beam Cross-sections

- Limit state design of steel beams require that sections must be classified since the ultimate moment carrying capacity of the beam depends on the type of section being used i.e. plastic, compact, semi-compact or the slender.
- These four classes of sections have been defined based on yield and plastic moments and the rotation capacities.

NOTE: Rolled I-sections and channel sections as given in IS Handbook No. 1 are either plastic or compact.

- In India, the various rolled I-sections are at least semi-compact but in case of built up welded sections, they may have semi-compact or slender webs.

Remember

1. In case of indeterminate structures, it is preferable to use plastic sections.
2. For simply supported beams, it is preferable to use compact sections which fail after reaching the plastic moment at one section.
3. Semi-compact sections are preferred for elastic design wherein the section fails after reaching the first yield moment (M_y) at the extreme fibers of the beam section.
4. For plate girders, slender sections are used.

- The various types of sections are discussed in Section 1.4 and Table 1.2.

6.5 Different Types of Elements of a Cross-section

Cl.3.7.3 of IS 800:2007 classifies different types of elements as:

(a) Internal elements: These are those elements that are attached along both the edges to longitudinal stiffeners connected at adequate intervals to transverse stiffeners or other elements. e.g. Web of I-section, web and flange of box section etc.

(b) Outside elements or outstands: These elements are attached only to one of the longitudinal edges to an adjacent element and the other edge is free to get displace out of the plane. e.g. Flange overhang of I-section, legs of angle section, stem of T-section etc.

(c) Tapered elements: These are in fact flat elements with average thickness as given in SP: 6 (Part I).

6.6 Lateral Stability of Beams

- When a beam is loaded, one of the flanges of the beam comes in compression and other in tension. For economy in beam design, I_z is made considerably larger than I_y
- Such beam sections are quite weak in bending in the plane normal to the web and thus compression flange of the beam is liable to buckle in the direction in which it is free to move i.e. in the horizontal direction.
- This buckling tendency increases as the ratio I_z / I_y increases. However the bottom flange of the beam remains in tension and thus remains straight. But the bottom flange, web and the compression flange acts a unit and thus the whole section rotates as shown in Fig. 6.7.

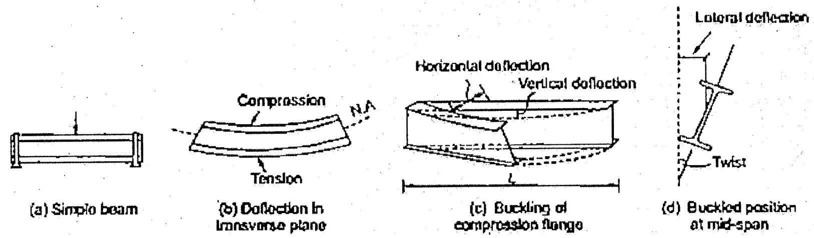


Fig. 6.7 Lateral torsional buckling of I-beam section

- **Elastic Critical Moment:** It is the bending moment at which the beam fails by lateral buckling when subjected to uniform moment.
- If however, the lateral buckling of compression flange can be prevented then flexural strength of the beam can be used to its full value.
- This lateral buckling of compression flange can be prevented by any of the methods as shown in Fig. 6.8 and briefly described as below:
  - (a) By embedding the compression flange of the beam in the slab concrete which gives full lateral restraint to the beam.
  - (b) Shear connectors welded to the compression flange of the beam and embedded in concrete slab.
  - (c) Most of the non-continuous supports provide sufficient lateral support at their points of joining to the beam. When the cross beams join the main beams and are connected to compression

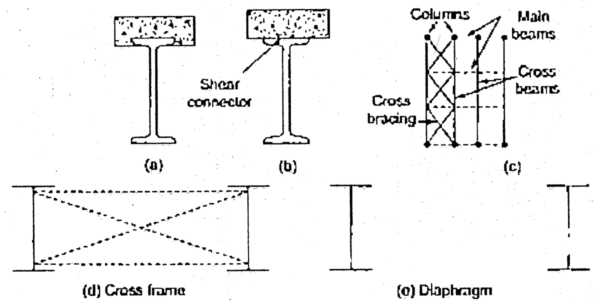


Fig. 6.8 Lateral support in beams

flange then full lateral support is assumed to exist at the connections. In case cross beams are connected to tension flange of the main beam then cross bracing is done to have lateral restraint.

(d) Torsional bracing in the form of cross frames or diaphragms are used to prevent twisting.

6.7 Elastic Critical Moment

- Bending of beam in the plane of its strong axis and buckling about the weak axis is accompanied by twisting/torsion and is characterized as **lateral torsional buckling** and flanges wrap.
- This warping of flanges is characterized by out of plane deformation due to rotation of the beam section about the longitudinal axis of the beam i.e., x-axis.
- The compression flange of the beam is acts as a column only with no tendency of lateral torsional buckling until the moment reaches a certain critical value called as **critical moment** (M_{cr}).
- The load at which such a beam buckles is much less than the moment capacity of the beam section.
- The torsional behavior of different structural steel shapes is described by:
  (a) Saint Venant torsion
  (b) Warping torsion
- From the theory of elastic stability, the expression for the stability limit state for a steel beam subjected to a uniform moment, the critical moment (M_{cr}) is given by,

$$M_{cr} = \sqrt{\left(\frac{\pi^2 EI_y GI_t}{L^2} + \frac{\pi^4 E^2 I_w I_y}{L^4}\right)} \quad ...(6.2)$$

- In Eq. 6.2 $\left(\frac{\pi^2 EI_y GI_t}{L^2}\right)$ term is Saint Venant stiffness and $\left(\frac{\pi^4 E^2 I_w I_y}{L^4}\right)$ term is flange bending stiffness or warping stiffness.
- Thus the IS definition of lateral torsional stability limit state is expressed as:

$$M_{cr} = \text{A constant} \times (\text{Saint Venant stiffness} + \text{Warping stiffness})^{1/2} \quad ...(6.3)$$

- The elastic critical moment (M_{cr}) given by Eq. 6.2 can be rewritten as:

$$M_{cr} = \frac{\pi}{L}\sqrt{EI_y GI_t + \left(\frac{\pi E}{L}\right)^2 I_w I_y} \quad ...(6.4)$$

where EI_y = Flexural rigidity about minor axis i.e., y-y axis
GI_t = Torsional rigidity
I_t and I_w = Saint Venant torsional and warping constant respectively
L = Unbraced length of the beam subjected to constant moment in the plane

Remember

1. For symmetrical I-section with flange width b_f, flange thickness t_f and overall depth of the section h,

$$I_y = \frac{b_f t_f^3}{6}, \quad I_t = 0.9 b_f t_f^3, \quad I_w = \frac{I_y h^2}{4}$$

2. Eq. 6.4 is applicable only for compact doubly symmetric I-sections, I-sections loaded in the plane of their webs, singly symmetric I-sections with compression flange larger than tension flange.
3. It is not possible for lateral torsional buckling to occur if the second moment of area of the section about the bending axis is equal to or less than the second moment of area of the section out of plane. Thus limit state of lateral torsional buckling is not applicable for shapes bent about their minor axis with $I_z \le I_y$ or for circular or square shapes (for which $I_z = I_y$).

Eq. 6.4 can also be written as,

$$M_{cr} = \frac{\pi}{L}\sqrt{EI_y GI_t}\sqrt{1 + \frac{\pi^2 EI_w}{L^2 GI_t}}$$

$$\Rightarrow \quad M_{cr} = \frac{\pi}{L}\sqrt{EI_y GI_t}\sqrt{1 + \frac{EI_y}{GI_t}\left(\frac{\pi}{2}\frac{h}{L}\right)^2} \quad ...(6.5)$$

- In the term I_y, the contribution of web is quite negligible and thus for symmetrical I-sections, the second moment of area of each flange of I-section is practically equal to $I_y/2$. Further, the warping resistance will be greater when $EI_y/2$ and h are large. From Eq. 6.5, it is evident that critical flexural moment (M_{cr}) can be increased by increasing I_y and / or I_t.
- Short and deep beams have very large warping resistance while long and shallow beams have low warping resistance (i.e. $I_w \simeq 0$).

  When $I_w \simeq 0$, Eq. 6.5 reduces to,

$$M_{cr} = \frac{\pi}{L}\sqrt{EI_y GI_t} \qquad \left(\because I_w = \frac{I_y h^2}{4}\right) \quad ...(6.6)$$

- If torsional rigidity GI_t of a section is very large in comparison to its warping rigidity, then the section will be in uniform torsion (called as **Saint Venant torsion**) and warping moment will be quite negligible. Hot rolled rectangular and square hollow sections, Tee and angle sections possess this type of behavior.
- In angle and Tee sections, warping deformations are large but warping moment is small since warping moment can develop only when the warping deformations are restrained.

Assumptions Involved in deriving the expression for elastic critical moment (M_{cr}):

(a) The beam is initially undistorted and without any imperfections.
(b) The behavior of the beam is truly elastic.
(c) The beam is acted upon by equal and opposite moments at the ends in the plane of the web.
(d) The load acts in the plane of web only.
(e) The ends of the beam are simply supported laterally and vertically.
(f) The beam does not have any residual stresses.

- Eq. 6.5 is applicable provided the applied moment is uniform over the unbraced length of the beam else there will occur a moment gradient and the equation has to be modified by introducing a suitable modification factor.

- A modification factor c_1 is introduced in Eq. 6.5 to account for different loading and support conditions and Eq. 6.5 is modified as,

$$M_{cr} = c_1 \frac{\pi}{L}\sqrt{EI_yGI_t}\sqrt{1+\frac{\pi^2 EI_w}{L^2 GI_t}}$$

$$\Rightarrow \quad M_{cr} = c_1\sqrt{EI_yGI_t}\left[\frac{\pi}{L}\sqrt{1+\frac{\pi^2 EI_w}{L^2 GI_t}}\right]$$

Put, $$\frac{L^2 GI_t}{EI_w} = B^2$$

Thus $$M_{cr} = c_1\sqrt{EI_yGI_t}\left[\frac{\pi}{L}\sqrt{1+\frac{\pi^2}{B^2}}\right] = \left(c_1\sqrt{EI_yGI_t}\right)\gamma \qquad \ldots(6.7)$$

where, $$\gamma = \frac{\pi}{L}\sqrt{1+\frac{\pi^2}{B^2}}$$

Eq. 6.7 is basic equation for lateral torsional buckling of beams. The three terms in this basic equation are c_1, $\sqrt{EI_yGI_t}$ and the γ term.

6.7.1 The c_1 Term

This factor takes into account the moment gradient on lateral torsional buckling and is denoted by c_1 called as **equivalent uniform moment factor** or **moment coefficient**. For torsionally simple supports, the factor c_1 is equal to or greater than one depending on the type of loading.

Remember: Beams with unequal moments are less prone to instability and the equivalent uniform moment factor caters for this beneficial effect on the beam.

6.7.2 The $\sqrt{EI_yGI_t}$ Term

The value of this term varies with the shape and material of the beam. Provision of intermediate lateral supports capable of preventing the lateral defection and twisting have an effect to increase M_{cr}

6.7.3 The γ Term

This value varies with the length of the beam. For beams with no intermediate lateral supports, the effective length of the beam is equal to the actual length of the beam between the supports. The warping restraint is usually represented by an effective length factor K at the support or the point of bracing. This effective length factor K is used to represent rational freedom about the vertical axis (warping) at the support or the brace point.

While developing IS 800 : 2007, it was presumed that flange is free to warp at brace points and thus K is unity. Thus for unrestrained flanges against warping, K = 1 and for fully restrained flanges, theoretically, K = 0.5 (but IS 800:2007 gives this value as 0.7 for design purposes).

In situations, where possibility of translation of one end is also there, there K can be greater than unity. For example, in cantilever beam, with no support at the free end.

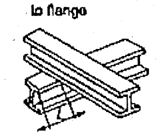
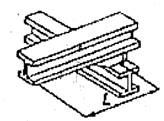
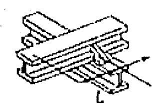
Table 6.1 Effective length for simply supported beams

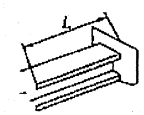
Sl. No.	Conditions of Restraints at Supports		Loading Condition	
	Torsional Restraint	Warping Restraint	Normal*	Destabilizing#
(i)	Fully restrained	Both flanges fully restrained	0.70 L	0.85 L
(ii)	Fully restrained	Compression flange fully restrained	0.75 L	0.90 L
(iii)	Fully restrained	Both flanges partially restrained	0.80 L	0.95 L
(iv)	Fully restrained	Compression flange partially restrained	0.85 L	1.00 L
(v)	Fully restrained	Warping not restrained in both flanges	1.00 L	1.20 L
(vi)	Partially restrained by bottom flange support connection	Warping not restrained in both flanges	1.0 L + 2D	1.2 L + 2D
(vii)	Partially restrained by bottom flange bearing supprt	Warping not restrained in both flanges	1.2 L + 2D	1.4 L + 2D

* Normal loading: Load is acting through shear centre

\# Destabilizing Loading: Load is not acting through shear centre causing destabilization.

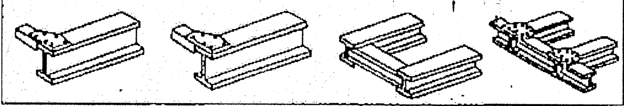
Table 6.2 Effective length for cantilever beams

Restraint Condition		Loading Condition	
At Support	At Top	Normal	Destabilizing
(a) Continuous, with lateral restraint to flange	(i) Free	3.0 L	7.5 L
	(ii) Lateral restraint to top flange	2.7 L	7.5 L
	(iii) Torsional restraint	2.4 L	4.5 L
	(iv) Lateral and torsional restraint	2.1 L	3.6 L
(b) Continuous, with partial torsional restraint	(i) Free	2.0 L	5.0 L
	(ii) Lateral restraint to top flange	1.8 L	5.0 L
	(iii) Torsional restraint	1.6 L	3.0 L
	(iv) Lateral and torsional restraint	1.4 L	2.4 L
(c) Continuous, with lateral and torsional restraint	(i) Free	1.0 L	2.5 L
	(ii) Lateral restraint to top flange	0.9 L	2.5 L
	(iii) Torsional restraint	0.8 L	1.5 L
	(iv) Lateral and torsional restraint	0.7 L	1.2 L

(d) Restrained laterally, torsionally and against rotation on plan	(i) Free	0.8 L	1.4 L
	(ii) Lateral restraint to top flange	0.7 L	1.4 L
	(iii) Torsional restraint	0.6 L	0.6 L
	(iv) Lateral and torsional restraint	0.5 L	0.5 L

Top restraint conditions

(i) Free (ii) Lateral restraint to top flange (iii) Torsional restraint (iv) Lateral and torsional restraint



- The lateral restraint provided by the simply supported conditions as assumed in Eq. 6.4 is the lowest and therefore the critical moment M_{cr} is also the lowest.
- The equation for computing the elastic critical moment corresponding to lateral torsional buckling of doubly symmetric torsionally restrained prismatic beam subjected to uniform moment in the unsupported length as recommended by IS 800 : 2007 is same as Eq. 6.4 but in slightly different form as given below.

$$M_{cr} = \frac{\pi}{L}\sqrt{EI_yGI_t + \left(\frac{\pi E}{L}\right)^2 I_wI_y} = \sqrt{\frac{\pi^2}{L^2}EI_yGI_t + \frac{\pi^2}{L^2}\left(\frac{\pi E}{L}\right)^2 I_wI_y}$$

$$= \sqrt{\frac{\pi^2}{L^2}EI_y\left(GI_t + \frac{\pi^2EI_w}{L^2}\right)} = \sqrt{\frac{\pi^2}{L_{LT}^2}EI_y\left(GI_t + \frac{\pi^2EI_w}{L_{LT}^2}\right)} \quad ...(6.8)$$

where L_{LT} = Effective length of the beam subjected to lateral torsional buckling

Eq. 6.4 can also be modified as below.

$$M_{cr} = \frac{\pi}{L}\sqrt{EI_yGI_t + \left(\frac{\pi E}{L}\right)^2 I_wI_y} = \frac{\pi}{L}\sqrt{\left(\frac{\pi E}{L}\right)^2 I_wI_y + EI_yGI_t}$$

$$= \frac{\pi}{L}\cdot\frac{\pi E}{L}\sqrt{I_wI_y + \frac{EI_yGI_t}{(\pi E/L)^2}} = \frac{\pi^2 E}{L^2}\cdot I_y\sqrt{\frac{I_wI_y}{I_y^2} + \frac{EI_yGI_t}{(\pi E/L)^2 I_y^2}}$$

$$= \frac{\pi^2EI_y}{L^2}\sqrt{\frac{I_w}{I_y} + \frac{GI_tL^2}{\pi^2EI_y}} = \frac{\pi^2EI_y}{L_{LT}^2}\sqrt{\frac{I_w}{I_y} + \frac{GI_tL_{LT}^2}{\pi^2EI_y}} \quad ...(6.9)$$

where I_y, I_w, I_t = Moment of inertia about the minor axis, warping constant and Saint Venant's torsion constant, respectively

G = Modulus of rigidity

L_{LT} = Effective length against lateral torsional buckling

6.8 Strength of Beams in Bending

- **Laterally Restrained Beam:** A beam can fail by reaching the plastic moment M_p if it can remain stable up to the fully plastic condition.
- This situation is possible where compression flange buckling of the beam is restrained and constituent elements (flanges and web) do not buckle locally and are referred to as **laterally restrained beams**. With increasing transverse loads, these laterally restrained beams will attain full plastic moment capacity.
- **Laterally Unrestrained Beam:** If failure occurs by lateral torsional buckling, the whole beam may buckle laterally between the supports, local buckling of flange may occur, or local buckling of web may occur.
- Web may buckle in longitudinal or diagonal waves or under points of concentrated loads in vertical waves.
- Beam fails well before it attains its full plastic moment capacity.
- These type of beams which fail by lateral buckling are referred to as **laterally unrestrained beams.**
- Also any of these type of failure due to instability, can be either in elastic or the plastic range. When maximum bending stress is less than the proportional limit when buckling occurs then failure is said to be elastic else it is inelastic.
- A laterally unrestrained beam will fail by
  - (a) Yielding if beam is short
  - (b) Elastic buckling if beam is long
  - (c) Inelastic lateral buckling if beam is of intermediate length.

Remember Web buckling is a less critical problem than flange buckling because:

1. Buckling stresses even for simply supported beams are generally much higher than for flanges.
2. Edge conditions are generally more favorable for web.
3. Even if web buckling occurs, it is less critical than flange buckling since there exists several ways in which post buckling strength may get developed.

- The important considerations in beam design are.

(a) Moment	(b) Shear	(c) Deflection
(d) Crippling	(e) Buckling	(f) Lateral support

NOTE: For laterally supported beams, the design bending strength is governed by yield stress while for laterally unsupported beams; the design bending strength is governed by lateral torsional buckling.

6.8.1 Laterally Restrained Beams

- Beams of plastic or compact shapes with lateral support throughout have nominal flexural strength equal to plastic moment capacity of the beam section.
- For a beam adequately supported against lateral torsional buckling, the factored design moment (M) at any section due to external loading must be less than or equal to the design flexural strength (M_d) of the section.
- Here the assumption made is that the web of the beam is stocky (i.e. $d/t_w \le 67\varepsilon$) where d is the depth of the web and t_w is the thickness of the web.

- In beams with plastic, compact and semi-compact flanges and slender web ($d/t_w > 67\varepsilon$), the web is prone to shear buckling before yielding. When there is no shear buckling ($d/t_w \leq 67\varepsilon$), the nominal shear resistance (V_n) equals plastic shear strength (V_p) of the beam section.

IMPORTANT: The plastic shear resistance is equal to shear area times ($f_y/\sqrt{3}$). Theoretically a pure shear stress of $f_y/\sqrt{3}$ ($=0.577 f_y \simeq 0.6 f_y$) will cause the material to yield. Hence for a factored design shear force $V \leq 0.6 V_d$, the web will be fully effective and entire cross section of the beam will be effective in resisting the moment M_d. But when shear exceeds the limit of $0.6 V_d$ then web area will be ineffective and entire flange will resist the moment. Because of this high shear, the moment capacity of the beam is reduced.

6.9 Flexural Strength of Laterally Supported Beam

- When $d/t_w < 67\varepsilon$, IS 800:2007 takes into account two cases viz. one with design shear strength less than $0.6V_d$ and other with design shear strength more than $0.6V_d$ where V_d is the design shear force.
- When $d/t_w > 67\varepsilon$, web is likely to undergo shear buckling. This aspect is covered in chapter on Plate Girders.
- When $d/t_w \leq 67\varepsilon$, the flexural strength of laterally supported beam is computed as:

Case -1 Low Shear Case ($V \leq 0.6V_d$)

(i) For plastic, compact and semicompact sections

The design flexural strength is given by,

$$M_d = \beta_b \frac{Z_p f_y}{\gamma_{m0}} \quad \text{...(6.10)}$$

$$\leq \begin{cases} 1.2\dfrac{Z_e f_y}{\gamma_{m0}} & \text{for simply supported beam} \\ 1.5\dfrac{Z_e f_y}{\gamma_{m0}} & \text{for cantilever beam} \end{cases}$$

where β_b = 1.0 for plastic and compact sections

$= \dfrac{Z_e}{Z_p}$ for semi-compact sections

Z_p = Plastic section modulus of the beam section

Z_e = Elastic section modulus of the beam section

γ_{m0} = Partial factor of safety for material governed by yield stress

Here the upper limit $1.2Z_e f_y/\gamma_{m0}$ for simply supported beams and $1.5Z_e f_y/\gamma_{m0}$ for cantilever beams has been imposed so as to avoid the commencement of plasticity under unfactored loads that is to say excessive deformations are prevented.

(ii) For slender sections, the design flexural strength for this low shear case is given by,

$$M_d = Z_{ev} f_y \quad \text{...(6.11)}$$

where Z_{ev} = Effective elastic section modulus

(a) Effect of bolt holes: When beams are connected to other beams on columns, then rivets/bolts are used for making the connections. For this, bolt holes are punched or drilled through either the flanges or web of the beam. Experimentally, it was seen that strength of section depends on strength of the compression flange in spite of the fact that there are bolt holes in tension flange also. IS 800:2007 allows to neglect the bolt holes in the tension flanges of the beam when tensile fracture strength of the flange is at least equal to tensile yield strength i.e.

$$\frac{0.9A_n f_u}{\gamma_{m1}} = \frac{A_g f_y}{\gamma_{m0}} \quad \text{...(6.12)}$$

Remember: For bolts in bolt holes in compression flange, no reduction for bolt holes is made since it is presumed that bolts can transmit compression through hole by means of bolts.

(b) Effect of shear lag: High stresses occur near the junction of web and flange elements. This stress distribution is shown in Fig. 6.9.

- In case of rolled I-sections, this effect of shear lag is small but for built up sections and for wide flange beams, the effect of shear lag is quite considerable.
- This shear lag effect depends on width to span ratio, end restraints at the beam ends and the type of load.
- Point load gives rise to more shear lag than the uniformly distributed load.

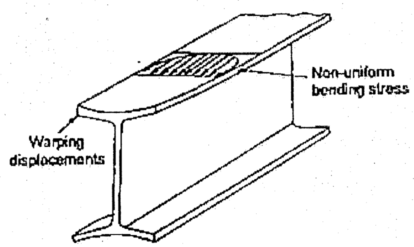


Fig. 6.9 Effect of shear lag

- As per IS 800:2007, the effect of shear lag can be ignored provided:

$$b_o \leq \frac{L_o}{20} \quad \text{(for outstand elements)}$$

$$b_i \leq \frac{L_o}{10} \quad \text{(for internal elements)}$$

where b_o = Width of the outstand of the flange

b_i = Width of internal element

L_o = Length between the points of zero moment (i.e. contraflexure points)

Case-2 High shear case ($V > 0.6V_d$)

- The design flexural strength for this case is given by,

$$M_d = M_{dv} \quad \text{...(6.13)}$$

where M_{dv} is the design flexural strength under high shear force.

- A reduced value is recommended to account for the higher shear force on flexural strength of the sections.
- Cl. 9.2.2 of IS 800:2007 gives expressions for computing M_{dv} for different types of sections as:

(a) Plastic and compact sections

$$M_{dv} = M_d - \beta_o (M_d - M_{fd})$$

$$\leq 1.2\frac{Z_o f_y}{\gamma_{m0}} \quad ...(6.14)$$

where $\beta = \left(\frac{2V}{V_d}-1\right)^2$

M_d = Plastic design moment of the whole section

V = Factored shear force on the beam section

V_d = Design shear strength of the beam section

M_{fd} = Plastic design strength of the area of cross section excluding the shear area taking into account the partial factor of safety γ_{m0}.

(b) **Semi-compact sections:** Flexural strength of semi-compact sections is given by,

$$M_{d} = \frac{Z_e f_y}{\gamma_{m0}} \quad ...(6.15)$$

6.10 Shear Strength of Laterally Supported Beam

- A typical beam is subjected to two types of shear viz. the transverse (or vertical) shear and the longitudinal shear. In general, transverse shear failure does not usually occur in beams because prior to that web crippling will occur thereby making the beam to undergo failure.
- But transverse shear failure may do occur in case of short and heavily loaded beam or deep coped beam as shown in Fig. 6.10.

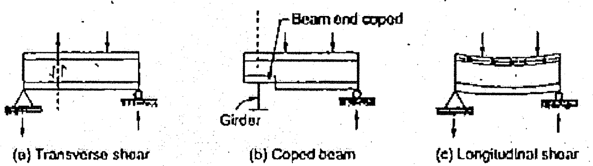


Fig. 6.10 Longitudinal and transverse (vertical) shear in beams

- Bending of beam gives rise to longitudinal shear making the fibers to elongate in the lower side and contracting the fibers in upper side.
- These fibers tend to slip over each other and this effect is maximum at the neutral axis. This tendency of slip is being resisted by shear strength of the material.
- The longitudinal shear is given as:

$$\tau = \frac{VAy}{I_z t} \quad ...(6.16)$$

where V = Transverse (vertical) shear force

Ay = Moment of area (A) of the cross section (above the section under consideration) about neutral axis.

I_z = Moment of inertia of the beam cross section about neutral axis.

The mean or the average shear stress is given by,

$$\tau_{avg} = \frac{V}{A_v} = \frac{V}{dt_w} \quad ...(6.17)$$

where A_v = Shear area, t_w = Thickness of the web, d = Depth of the web

- The nominal shear yield strength which is arrived at on the basis of Von-Mises yield condition is given as,

$$\tau_y = \frac{V_n}{A_v} = \frac{f_{yw}}{\sqrt{3}}; \quad V_n = \frac{A_v f_{yw}}{\sqrt{3}} \quad ...(6.18)$$

where f_{yw} = Yield strength of the web

Now here arise the following two cases:

(a) Nominal shear resistance of the cross section governed by plastic shear resistance

(b) Strength of web that is governed by shear buckling

6.10.1 Plastic Shear Resistance

- When there [illegible] shear buckling, the nominal shear resistance under pure shear is given by,

$$V_n = V_p = \frac{A_v f_{yw}}{\sqrt{3}} \quad ...(6.19)$$

IS 800 : 2007 defines shear area for different sections and are also given in Section 6.11.3.

- Also check for the block shear failure at the end connection. The design shear strength is given by,

$$V_d = \frac{V_n}{\gamma_{m0}} \quad ...(6.20)$$

where γ_{m0} = Partial factor of safety for the material governed by yield = 1.1

6.10.2 Resistance to Shear Buckling

- Under concentrated loads and support reactions, the web of the I-section behaves like a column. Web being thin, is prone to vertical buckling.
- With rolled beam sections, buckling of web is not a problem since for rolled sections, $d/t_w < 67\varepsilon$ but this problem occurs in built up sections with thin webs like the plate girders. The section is checked for shear strength when $d/t_w > 67\varepsilon$ for web without web stiffeners and when $d/t_w > 67\varepsilon\sqrt{k_v/5.35}$ for web with web stiffeners.
- Cl. 8.4 of IS 800:2007 specifies design shear strength of a section as:

$$V_d = \frac{V_n}{\gamma_{m0}} = \frac{A_v f_{yw}}{\sqrt{3}\gamma_{m0}} \quad ...(6.21)$$

where A_v = Shear area, f_{yw} = Yield strength of the web

6.10.3 Computation of Shear area

(a) I and channel sections

(i) Bending about major axis

Hot rolled $A_v = ht_w$

Welded $A_v = dt_w$

(ii) Bending about minor axis

Hot rolled or welded $A_v = 2bt_f$

(b) Rectangular hollow sections of uniform thickness

(i) Loading parallel to depth (h) $A_v = \frac{Ah}{b+h}$

(ii) Loading parallel to width (b) $A_v = \frac{Ab}{b+h}$

(c) Circular hollow tubes of uniform thickness $A_v = \frac{2A}{\pi}$

(d) Plates and solid bars $A_v = A$

where, A = Gross cross sectional area, b = Overall width of flange of I-section, overall breadth of tubular section. d = Clear depth of web between the flanges, h = Overall depth of the section, t_f = Thickness of the flange, t_w = Thickness of the web

6.11 Limits on Deflection of Beam

- The deflection is calculated by elastic theory for working load conditions i.e. without factored loads.
- Table 6 of IS 800:2007 gives the maximum permissible deflection limits and the same are also given here is Table 6.3.

Table 6.3 Permissible limits of deflection

Type of Building	Deflection	Design load	Member	Supporting	Maximum Deflection
Industrial Building	Vertical	Live load/Wind load	Purlins and Girts	Elastic cladding	Span/150
				Brittle cladding	Span/180
		Live load	Simple span	Elastic cladding	Span/240
				Brittle cladding	Span/300
		Live load	Cantilever span	Elastic cladding	Span/120
				Brittle cladding	Span/150
		Live load/Wind load	Rafter supporting	Profiled Metal Sheeting	Span/180
				Plastered Sheeting	Span/240
		Crane load (Manual operation)	Gantry	Crane	Span/500
		Crane load (Electric operation up to 50 t)	Gantry	Crane	Span/750
		Crane load (Electric operation over 50 t)	Gantry	Crane	Span/1000
	Lateral	No cranes	Column	Elastic cladding	Height/150
				Masonry/brittle cladding	Height/240
		Crane + wind	Gantry (lateral)	Crane (absolute)	Span/400
				Relative displacement between rails supporting crane	10 mm
				Gantry (Elastic cladding pendant operated)	Height/200
		Crane + wind	Column/frame	Gantry (Brittle cladding; cab operated)	Height/400
Other Buildings	Vertical	Live load	Floor and roof	Elements not susceptible to cracking	Span/300
				Elements susceptible to cracking	Span/360
		Live load	Cantilever	Elements not susceptible to cracking	Span/150
				Elements susceptible to cracking	Span/180
	Lateral	Wind	Building	Elastic cladding	Height/300
				Brittle cladding	Height/500
		Wind	Inter storey drift		Storey Height/300

6.12 Buckling Strength of Web

- A certain portion of web acts as column to transfer the load from beam to the supports.
- Under this compressive force, the web is liable to buckle as shown in Fig. 6.11.

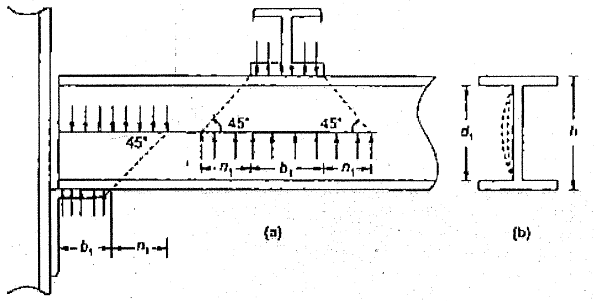


Fig. 6.11 Web buckling in beams

- The load dispersion angle is usually taken as 45°.
- Rolled sections are provided with suitable web thickness so that web buckling is avoided.
- In case of built up sections, it is essential to check the buckling possibility of the web and web stiffeners are provided if required.
- As per IS 800:2007, effective web buckling strength is to be determined based on the cross section of the web which is given by,

$$A_w = (b_1 + 2n_1)\, t_w \qquad ...(6.22)$$

where, b_1 = Width of stiffened bearing on the web

$n_1 = \frac{h}{2}$, h = Depth of the section

- If f_c is the allowable compressive stress corresponding to the assumed web column, then web buckling strength is,

$$F_{cdw} = (b_1 + 2n_1)\, t_w\, f_c \qquad ...(6.23)$$

where $F_{cd,w}$ = Web buckling strength

Let effective length of web column = $0.7d_1$

Then, the radius of gyration of web is given by,

$$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{\frac{1}{12}(b_1+2n_1)t_w^3}{(b_1+2n_1)t_w}} = \frac{t_w}{2\sqrt{3}}$$

Thus slenderness ratio is,

$$\lambda = \frac{\text{Effective length}}{r_y} = \frac{0.7d_1}{t_w / 2\sqrt{3}} \simeq 2.5\frac{d_1}{t_w} \qquad ....(6.24)$$

For the above computed slenderness ratio, permissible compressive stress (f_c) can be determined from Table 9 of IS 800:2007 and from Table 5.8 of this book.

6.13 Web Crippling

- At supports, the web of the beam may cripple due to insufficient bearing capacity as shown in Fig. 6.12.

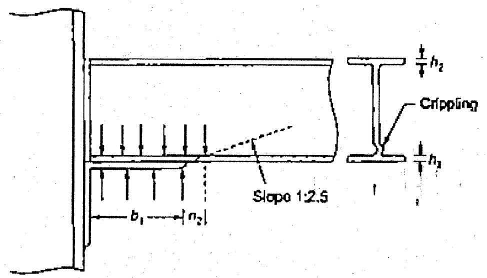


Fig. 6.12 Web crippling in beams

- Crippling usually occurs at the root of the section. IS 800:2007 provides an expression for computing the crippling strength of web as:

$$F_w = (b_1 + n_2)\frac{t_w f_{yw}}{\gamma_{m0}} \qquad \text{...(6.25)}$$

where b_1 = Length of stiffened bearing

n_2 = Length obtained by load dispersion through the flange to the web junction at a slope of 1 : 2.5 to the plane of the flange as shown in Fig. 6.12.

f_{yw} = Yield stress of the web

- The load transferred by bearing must not exceed web crippling strength (F_w).
- For rolled steel sections, this provision is already accounted for and thus for rolled steel sections being used as beam, there is no need to check for this failure.
- When built up sections are used as a beam then web must be checked for web crippling.

6.14 Procedure for the Design of Beams

Step-1 Select a suitable beam section.
Step-2 Check the section for the class it belongs.
Step-3 Check the section for its bending strength.
Step-4 Check the section for its shear strength.
Step-5 Check for deflection of the beam.
In case any of the above check fails, then section has to be revised.

6.15 Design of Built-up Beams

When loading on the beam is so large that available rolled steel sections are not adequate to take up the moment then built up beam sections are used. In case of bolted and riveted beams, the area of tension flange gets reduced due to the bolt/rivet holes and thus actual neutral axis moves upward towards the compression side. However this shift in neutral axis is neglected in usual design process.

6.16 Design of Laterally Unsupported Beams

- For I-section beams, $I_y < I_z$
- Thus when a beam is loaded, it has the tendency to deflect sideways along with twisting.
- To reduce the possibility of lateral buckling, beams are provided with restraints at the ends and at some intermediate points.
- This restraint against torsional rotation at supports is provided by either of the following ways:
  - (a) Building the beams into the walls.
  - (b) Providing web and flange cleats.
  - (c) Bearing stiffeners acting in conjunction with bearing of the beams
  - (d) Lateral end supports or frames which provide lateral restraint to the compression flange at the ends.
- Intermediate lateral support is provided by lateral bracing or by connecting to an independent heavy post of the structure. Such restraint must be capable of resisting 2.5% of the maximum force in the compression flange and should be connected closer to the compression flange.

NOTE IS 800:2007 specifies that a member may be treated as laterally supported for the following cases:
(a) Bending is about the minor axis
(b) Section is hollow (rectangular, circular etc.) or a solid bar
(c) Non-dimensionalised slenderness ratio is less than 0.4 i.e. $\lambda_{LT} = \sqrt{f_y / f_{crb}} < 0.4$

- Taking into account, the imperfection factors $\alpha_{LT} = 0.21$ for rolled steel sections and $\alpha_{LT} = 0.49$ for welded steel sections, IS 800:2007 has presented the **Tables 13(a)** and **13(b)** to determine the reduced bending strength f_{bd} of the section.
- Thus the designed bending strength of laterally unsupported beam as governed by torsional buckling is given by,

$$M_d = \beta Z_p f_{bd}$$

where β = 1.0 for plastic and compact sections

$= \frac{Z_e}{Z_p}$ for semi-compact sections

- The design of laterally unsupported beam is a trial and error procedure.
- First assume f_{bd} and determine the required Z_p of the section. Select a trial section and determine the moment carrying capacity of the section. If section selected gives inadequate section modulus then section has to be revised. For the selected section, all usual checks are applied.

6.17 Effective Length of Laterally Unsupported Beams

- For simply supported beams and girders of span length L where there is no lateral restraint to the compression flanges is provided but end of the beam is restrained against torsion, the effective length L_{LT} shall be as per **Table 15** in **IS 800:2007** and is given in **Table 6.1** wherein normal loading means load is acting through shear center and destabilizing loading means load is not acting through the shear center and it creates destabilizing effect.

- In between the lateral restraints, the effective length of the segment is taken as 1.2 times the length of the segment.
- For cantilever beam of clear length L, effective length L_{LT} is taken as given in Table 16 of IS 800:2007 for different support conditions and is given in Table 6.2.

6.18 Design of Laterally Unsupported Beam : Purlin

For simply supported purlins, maximum flexural moment is $wL^2/8$ and for continuous purlins, maximum flexural moment is $wL^2/10$ where w is the uniformly distributed load on the purlin.

6.18.1 Rigorous procedure for the design of purlins

Step-1 Resolve the factored loads normal and parallel to the sheeting.

Step-2 Determine the moments and shear forces about the two in-plane principal axes z-z and y-y.

Step-3 In case of bi-axial bending, the required section modulus about z-z axis is given by,

$$Z_{pz} = \frac{M_z}{f_y}\gamma_{m0} + 2.5\frac{d}{b}\frac{M_y}{f_y}\gamma_{m0}$$

where γ_{m0} = Partial factor of safety for material = 1.1

d = Depth of the section

b = Width of the section

The second term in the above expression makes an approximate relationship between Z_{py} and Z_{pz}

Step-4 Check the section for shear capacity as:

$$V_{dz} = \frac{f_y A_{vz}}{\sqrt{3}\gamma_{m0}}$$

$$V_{dy} = \frac{f_y A_{vy}}{\sqrt{3}\gamma_{m0}}$$

where, $A_{vz} = ht_w$, $A_{vy} = 2b_f t_f$

Step-5 Compute the flexural capacity of the section about both the axes as:

$$M_{dz} = \frac{Z_{pz} f_y}{\gamma_{m0}} \le 1.2\frac{Z_{ez} f_y}{\gamma_{m0}}$$

$$M_{dy} = \frac{Z_{py} f_y}{\gamma_{m0}} \le 1.5\frac{Z_{ey} f_y}{\gamma_{m0}}$$

Step-6 The section so selected and the consequent design must satisfy the following relation:

$$\frac{M_z}{M_{dz}} + \frac{M_y}{M_{dy}} \le 1$$

Step-7 Check the purlin for deflection.

Step-8 Check for wind suction. When dead and live loads are predominant, the top flange of the purlin is under compression which is restrained laterally by roof sheeting. Thus for dead and live loads, analysis is done assuming the purlin to be laterally restrained. But when wind suction acts, top flange of the purlin comes under tension and bottom under compression which is not laterally restrained. For this, wind suction along with dead load alone is considered as critical load. Taking into account the torsional buckling, M_{dz} and M_{dy} are found out and interaction formula is checked.

6.18.2 Simplified Procedure for the Design of Purlins

- As per IS 800:2007, the angle purlins should be designed for biaxial bending. But IS 800:1984 permits the design of purlins by assuming that load normal to the sheeting is resisted by purlin and load parallel to the purlin is sheeting is resisted by sheeting provided the following conditions are satisfied:
  - (a) Roof slope is less than 30°.
  - (b) Width of angle leg normal to sheeting ≥ $L/45$.
  - (c) Width of angle parallel to sheeting ≥ $L/60$.
- For this case, bending moment about z-z axis is taken as $W_z L/10$ where W_z is the total load in the direction normal to the sheeting and L is the spacing of trusses.
- Thus the design effectively gets reduce to uniaxial bending. The limits on leg sizes are based on limiting deflections. Thus there is no need to check the deflections. This simplified method is NOT covered by IS 800:2007.

6.19 Design of Laterally Unsupported Beam: Grillage Beams

- A minimum clear spacing of 75 mm is required to be maintained between the flanges of adjacent grillage beams so as to have effective penetration of concrete in between the beams.
- To maintain the required spacing, tie rods are used.
- The column load is transferred to top tier of grillage beams through a base plate.
- Let L be the length of the grillage beam and length of base plate in this direction is 'a' as shown in Fig. 6.13.

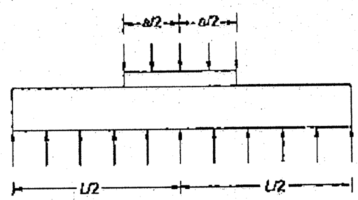


Fig. 6.13 Grillage beam

- If the beam carries a total load P then uniformly distributed load under the beam is P/L and on the length occupied by base plate is P/a.
- Maximum moment occurs at the center and is given as:

$$M = \frac{P}{L}\times\frac{L}{2}\times\frac{L}{4} - \frac{P}{a}\times\frac{a}{2}\times\frac{a}{4} = \frac{P}{8}(L-a) \quad \text{...(6.26)}$$

- Maximum shear force occurs at a distance 'a' from the center of the beam and is given by,

$$F = \frac{P}{L}\left(\frac{L}{2}-\frac{a}{2}\right) = \frac{P(L-a)}{2L} \qquad ...(6.27)$$

- The grillage beams are thus designed for the above computed moment and shear. It should be checked for web crippling also.

6.20 Design of Built-up beams/plated beams

If a single beam section is not capable of withstanding the applied load, then built-up beams are used. General design procedure for design of built-up beams is as follows:

Step-1: Calculate Z_{req} from given beam by $Z_{req} = \frac{M}{f_{bc}}$

$$\sigma_{bc} = 0.66\, f_y \text{ (for WSM)}$$

$$= \frac{f_y}{1.1} \text{ (for LSM)}$$

Step-2: $Z_{req} = Z_{beam} + Z_{plates}$

Ignoring t^2, t^3 terms in Z_{plates} i.e., considering it as a line area

$$Z_p = A_p d$$

$$\therefore \quad A_p = \frac{Z_{req} - Z_{beam}}{d}$$

Note: If bolting/rivetting is done then increase plate area by 40-50% (to take care of area of bolt holes).

Step-3: $\sigma_{bc,cal}$ = Calculated bending compressive stress

$$= \frac{M}{I_{xx}} \times y \le \sigma_{bc}$$

Step-4: From equilibrium considerations

$$\sigma_{bt,cal} = \sigma_{bc,cal} \times \frac{A_f}{A_{nf}}$$

Step-5: Check for shear

$$\tau_{va,cal} = \frac{V}{D \times t_w} \le \tau_{va} = 0.4 f_y \text{ (WSM)}$$

$$= \frac{f_y}{\sqrt{3} \times 1.1} \text{ (LSM)}$$

Where D = overall depth of beam excluding cover plates.

Illustrative Examples

Example 6.1 Design a simply supported steel beam of effective span 1.5 m carrying a concentrated load of 240 kN at mid-span.

Solution:

Concentrated load (at mid-span) = 240 kN

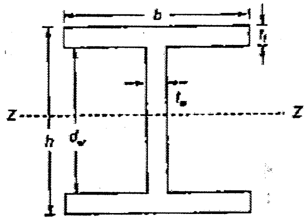
$\therefore$ Factored load $(P) = 1.5 \times 240 = 360$ kN

$\therefore$ Factored maximum moment $(M) = \frac{PL}{4} = 360 \times \frac{1.5}{4} = 135$ kNm

But $\quad M = \frac{f_y Z_e}{\gamma_{m0}}$

$$\Rightarrow \quad Z_e = \frac{M\gamma_{m0}}{f_y} = \frac{135 \times 10^6 \times 1.1}{250} = 594 \times 10^3 \text{ mm}^3$$

Try ISMB 300 with section properties as follows:



Overall depth (h) = 300 mm

Flange width (b) = 140 mm

Flange thickness (t_f) = 12.4 mm

Web depth $(d_w) = h - 2t_f = 300 - 2(12.4)$
$= 275.2$ mm

$\therefore \quad I_{zz} = 8603 \times 10^4 \text{ mm}^4$

$$\therefore \quad Z_{zz} = \frac{I_{xx}}{(h/2)} = \frac{8603 \times 10^4}{150} = 573.5 \times 10^3 \text{ mm}^3$$

Self weight of beam = 0.4336 kN/m

$\therefore$ Factored self weight = 1.5 × 0.4336 = 0.6504 kN/m

$\therefore$ Total factored moment = 135 + 0.1829 = 135.1829 kNm

$$\text{Factored moment due to} = \frac{0.6504 \times 1.5^2}{8} = 0.1829 \text{ kNm}$$

$$\text{Factored shear force } (V) = 0.6504 \times \frac{1.5}{2} + \frac{360}{2} = 180.4878 \text{ kN}$$

Classification of the section

$$\epsilon = \sqrt{\frac{250}{f_y}} = \sqrt{\frac{250}{250}} = 1$$

$$\text{Flange overhang (b)} = \frac{140}{2} = 70 \text{ mm}$$

$$\therefore \quad \frac{b}{t_f} = \frac{70}{12.4} = 5.645 < 9.4\,\epsilon$$

$$\frac{d}{t_w} = \frac{275.2}{7.5} = 36.693 < 84\,\epsilon$$

Section in plastic.

Shear capacity of the section

shear area $(A_v) = ht_w = 300 \times 7.5$ mm²

$$V_d = \frac{f_{yw} A_v}{\gamma_{mo}\sqrt{3}} = \frac{250 \times (300 \times 7.5)}{1.1 \times \sqrt{3}} = 295235.93 \text{ N} = 295.235 \text{ kN}$$

$\therefore$ $0.6\,V_d = 0.6 \times 295.235 = 177.141$ kN

$\therefore$ $V > 0.6\,V_d$

$\therefore$ $M_d = M_{dv}$

$$M_{dv} = M_d - \beta(M_d - M_{fd}) \le 1.2\frac{Z_e f_y}{\gamma_{mo}}$$

$$M_d = \frac{Z_p f_y}{\gamma_{mo}} \le 1.2\frac{Z_e f_y}{\gamma_{mo}}$$

Now,

$$M_d = \frac{Z_p f_y}{\gamma_{mo}} = \frac{651.73 \times 10^3 \times 250}{1.1} = 148.12 \times 10^6 \text{ N/mm}$$

$$1.2\frac{Z_e f_y}{\gamma_{mo}} = \frac{1.2 \times 573.5 \times 10^3 \times 250}{1.1} = 156.41 \times 10^6 \text{ N/mm}$$

$\therefore$ $M_d = 148.12$ kNm

$$\beta = \left(\frac{2V}{V_d} - 1\right)^2 = \left(\frac{2 \times 180.4878}{295.235} - 1\right)^2 = 0.0496$$

For doubly symmetric beam section, M_{fd} is obtained from table 14 of IS 800:2007 as,

$$\frac{kL}{N} = \frac{1500}{28.4} = 52.817$$

$$\frac{h}{t_f} = \frac{300}{12.4} = 24.194$$

From table 14 of IS 800:2007,

$f_{cb} = 916$ N/mm²

$\therefore$ Design compressive stress

$$f_{cd} = 204.5 + \frac{16}{100}(209.1 - 204.5) = 205.24 \text{ N/mm}^2$$

$$M_{dv} = M_d - b(M_d - M_{fd})$$
$$= 148.12 - 0.0496(148.12 - 1.155) = 140.83 \text{ kNm} > 135.1829 \text{ kNm}$$

Check for deflection

$$\delta = \frac{WL^3}{48EI} = \frac{(360 \times 1000)(1500)^3}{48 \times 2 \times 10^5 \times 8603 \times 10^4} = 1.47 \text{ mm}$$

$$\frac{\text{span}}{300} = \frac{1500}{300} = 5 \text{ mm}$$

$\therefore$ $\delta < \frac{\text{span}}{300}$

$\therefore$ provide ISMB 300 @ 0.4336 kN/m as beam section.

Example 6.2 The roof of a hall has the following data:

Clear span = 6.2 m

Support width = 150 mm

c/c spacing of beams = 3.2 m

Imposed load on beam = 10 kN/m²

Dead load including self weight of beam = 3.5 kN/m²

Assuming beam to be laterally supported throughout, design the beam section using steel of grade Fe410. The depth is restricted to 450 mm.

Solution:

For steel of grade Fe410, $f_u = 410$ N/mm², $f_y = 250$ N/mm²

Factored dead load = 1.5 × 3.5 = 5.25 kN/m²

Factored live load = 1.5 × 10 = 15 kN/m²

$\therefore$ cc spacing of beam = 3.2 mm

$\therefore$ Uniformly distributed total load on beam

= (5.25 + 15) 3.2 kN/m = 64.8 kN/m

$$\text{Effective span } (l) = 6.2 + \frac{0.15}{2} + \frac{0.15}{2} = 6.35 \text{ mm}$$

$$\text{Factored bending moment } (M) = 64.8 \times \frac{6.35^2}{8} = 326.61 \text{ kNm}$$

$$\text{Factored shear force } (V) = 64.8 \times \frac{6.35}{2} = 205.74 \text{ kN}$$

Plastic section modulus required,

$$Z_{pz,reqd} = \frac{M\gamma_{mo}}{f_y} = \frac{326.61 \times 10^6 \times 1.1}{250} \text{ mm}^3$$
$$= 1437.084 \times 10^3 \text{ mm}^3$$

$\because$ Depth is resisted to 450 mm.

$\therefore$ Try ISMB 400@ 604.3 N/m with $z_{pz} = 1176.18 \times 10^3$ mm³ and balance of plastic section modulus will be furnished by cover plates.

$\therefore$ Plastic section modulus required from cover plates

$$Z_a = Z_{pz,reqd} - Z_{pz}$$
$$= 1437.084 \times 10^3 - 1176.18 \times 10^3$$
$$= 260.904 \times 10^3 \text{ mm}^3$$

$\therefore$ Area of cover plate $A_a = \frac{Z_a}{h} = \frac{260.904 \times 10^3}{400} = 652.26 \text{ mm}^2$

Let width of cover plate = width of flange of I-section = $b_f = 140$ mm

$\therefore$ Thickness of cover plate $= \frac{652.26}{140} = 4.659 \text{ mm} < 6 \text{ mm}$

$\therefore$ Provide cover plate of thickness 6 mm

∴ Plastic section modulus provided.

$$Z_{pz,prov.} = 1176.18 \times 10^3 + (140 \times 6)(400 + 6)$$
$$= 1517.22 \times 10^3 \text{ mm}^3$$

Total depth of section = 400 + (6 + 6) = 412 mm < 450 mm (OK)

For ISMB 400 @ 604.3 N/mm, the relevant section properties are

Depth of section, h = 400 mm
Flange width, b_f = 140 mm
Flange thickness, t_f = 16 mm
Web thickness, t_w = 8.9 mm
Radius of root, R_1 = 14 mm
Moment of ineria, $I_z = 20458.4 \times 10^4$ mm^4
$I_y = 622.1 \times 10^4$ mm^4
Elastic section modulus, $Z_{ez} = 1020 \times 10^3$ mm^3
Web depth, $d = h - 2(t_f + R_1) = 400 - 2(16 + 14) = 340$ mm

Classification of section

$$\epsilon = \sqrt{\frac{250}{f_y}} = \sqrt{\frac{250}{250}} = 1$$

Outstand of flange, $b = \frac{b_f}{2} = \frac{140}{2} = 70$ mm

∴ $$\frac{b}{t_f} = \frac{70}{16} = 4.375 < 9.4\epsilon (= 9.4 \times 1 = 9.4)$$

$$\frac{d}{t_w} = \frac{340}{8.9} = 38.2 < 85.96 (= 83.9 \times 1 = 83.9)$$

Section in plastic.

Now since $\frac{d}{t_w} = 3.82 < 67\epsilon (= 67 \times 1 = 67)$

∴ Web of beam is stocky and shear buckling check of web is not required.

Shear capcity check of the section

Design shear force, V = 205.74 kN

Design shear strenght of the seciton

$$V_n = \frac{A_v f_y}{\sqrt{3}\gamma_{m0}} = \frac{(ht_w)f_y}{\sqrt{3}\gamma_{m0}} = \frac{(400 \times 8.9)250}{\sqrt{3} \times 1.1} \text{ N}$$
$$= 467.13 \text{ kN} > V (= 205.74 \text{ kN})$$

Check for high/low shear

$$0.6\,V_d = 0.6 \times 467.13 = 280.28 \text{ kN} > V (= 205.74 \text{ kN})$$

Thus it is a case of low shear and shear force will not effect the moment capcity of the beam section.

Flexural capcity check of the section.

∵ Section is plastic.

∴ $\beta_b = 1$

∴ Design bending strength of the section,

$$M_d = \beta_b \frac{Z_{pz} f_y}{\gamma_{m0}} = \frac{1 \times (1517.22 \times 10^3) \times 250}{1.1} \text{ Nm} = 344.82 \text{ kNm}$$
$$> M (= 326.61 \text{ kNm})$$

Here Z_{pz} is the $Z_{pz,prov.}$

$$I_{z,prov} = 20458.4 \times 10^4 + 2\left[\frac{140 \times 6^3}{12} + 140 \times 6\left(\frac{400+6}{2}\right)^2\right]$$
$$= 27382.016 \times 10^4 \text{ mm}^4$$

∴ $$Z_{ez} = \frac{I_{z,prov}}{D/2} = \frac{27382.016 \times 10^4}{(200+6)} = 1329.224 \times 10^3 \text{ mm}^3$$

∴ $$\frac{1.2 Z_{ez} f_y}{\gamma_{m0}} = \frac{1.2 \times 1329.224 \times 10^3 \times 250}{17} \text{ Nm} = 362.52 \text{ kNm}$$

Thus, $$M_d < \frac{1.2 Z_{ez} f_y}{\gamma_{m0}}$$ (OK)

Deflection check

∵ Factored uniformly distributed load = 64.8 kN/m

∴ Service uniformly distributed load = $\frac{64.8}{1.5}$ = 43.2 kN/m = 43.2 N/mm

Maximum deflection, $$\delta_{max} = \frac{5 \times 43.2 \times (6200)^4}{384 \times 2 \times 10^5 \times 27382.016 \times 10^4} = 15.18 \text{ mm}$$

Maximum permissible deflection,

$$\delta_{Allowable} = \frac{l}{300} = \frac{6200}{300} = 20.67 \text{ mm}$$

Thus $\delta_{max} < \delta_{Allowable}$

∴ Section satisfies the deflection check.

Check for web-bearing

Let stiff bearing length (b_1) = 75 mm

Also $n_1 = 2.5(t_f + R_1) = 2.5(16 + 14) = 75$ mm

∴ Strength of section in bearing,

$$F_w = \frac{A_b f_{yw}}{\gamma_{m0}} = \frac{(b_1 + n_1) t_w f_{yw}}{\gamma_{m0}} = (75+75)8.9 \times \frac{250}{1.1} \text{ N}$$
$$= 303.41 \text{ kN} > V (= 205.74 \text{ kN})$$

∴ Section is safe in bearing.

Example 6.3 Design a purlin for an industrial roof truss for the following data:

c/c spacing of truss = 5 m
span of truss = 10 m
slope of truss = 20°

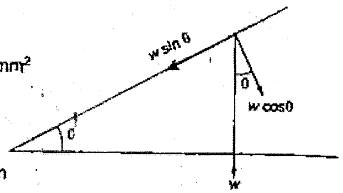
c/c spacing of purlins = 1.5 m

Intensity of wind pressure = 1.5 kN/m^2

weight of roofing material = 125 N/m^2

Use steel of grade Fe410.

Solution:



For steel of grade Fe410, f_u = 410 N/mm^2, f_y = 250 N/mm^2

Weight of roofing material coming on one purlin

= 125 × 1.5 = 187.5 N/m

Let dead weight of purlin = 100 N/m

∴ Total dead load = 100 + 187.5 = 287.5 N/m

Component of dead load along the principal rafter of roof truss

= $W\sin\theta$

= 287.5 sin 20° = 98.33 N/m

Component of dead load normal to principal rafter of roof truss

= $W\sin\theta$

= 287.5 cos 20° = 270.16 N/m

For maxinum wind load, it is assume to act normal to roof

∴ Maximum wind load = (1.5 × 1000) 1.5 × 1 = 2250 N/m

∴ Total load normal to purlin = 270.16 + 2250 = 2520.16 N/m

∴Factored load normal to purlin (P)

= 1.5 × 2520.16 = 3780.24 N/m

Factored load parallel to roof (H)

= 1.5 × 98.33 = 147.5 N/m

∴ Maximum moment in the direction normal to purlin

$$M_z = \frac{PL}{10} = (3780.24 \times 5) \times \frac{5}{10} = 9450.6 \text{ Nm} = 9.45 \text{ kNm}$$

Maximum moment along the direction of principal rafter

$$= M_y = \frac{ML}{10} = \frac{147.54 \times 5 \times 5}{10} = 368.75 \text{ N/m} = 0.37 \text{ kNm}$$

Plasto section modulus required,

$$Z_{pz,reqd} = \frac{M_z}{f_y}\gamma_{m0} + 2.5\left(\frac{d}{b}\right)\left(M_y \frac{\gamma_{m0}}{f_y}\right)$$

Here, since b and d are not known and thus has to be assumed.

Let ISMB 125 @ 127.53 N/m is used.

∴ b_f = 75 mm

and d = 125 mm

$$\therefore \quad Z_{pz,reqd} = \frac{9.45 \times 10^6 \times 1.1}{250} + 2.5\left(\frac{125}{75}\right)\left(\frac{0.37 \times 10^6 \times 1.1}{250}\right)$$

$$= 0.04158 \times 10^6 + 0.006783 \times 10^6$$

$$= 48.36 \times 10^3 \text{ mm}^3 = 48.36 \text{ cm}^3$$

For ISMB 125, Z_{pz} = 81.85 × 10^3 mm^3 > 48.36 × 10^3 mm^3 (= $Z_{pz\ reqd}$)

Z_{ez} = 71.8 × 10^3 mm^3

∴ ISMB 125 @ 127.53 N/m furnishes the required section modulus

Section properties are

h = 125 mm, b_f = 75 mm

t_f = 7.6 mm, t_w = 4.4 mm

R_1 = 9 mm, I_z = 449 × 10^4 mm^4

I_y = 43.7 × 10^4 mm^4, d = 91.8 mm

Classification of section

$$\epsilon = \sqrt{\frac{250}{f_y}} = \sqrt{\frac{250}{250}} = 1$$

Outstand of flange,

$$b = \frac{b_f}{2} = \frac{75}{2} = 37.5 \text{ mm}$$

$$\frac{b}{t_f} = \frac{37.5}{76} = 4.934 < 9.4\epsilon (= 9.4 \times 1 = 9.4)$$

$$\frac{d}{t_w} = \frac{91.8}{4.4} = 20.86 < 83.9\epsilon (= 83.9 \times 1 = 83.9)$$

∴ Section is plastic.

Check the flexural capcity of the section

Design flexural capacity about strong axis,

$$M_{dz} = Z_{pz} \times \frac{f_y}{\gamma_{m0}} = 81.85 \times 10^3 \times \frac{250}{1.1} \text{ Nmm} = 18.6 \text{ kNm}$$

$$\frac{1.2 Z_{ez} f_y}{\gamma_{m0}} = 1.2 \times 71.8 \times 10^3 \times \frac{250}{1.1} \text{ Nmm} = 19.58 \text{ kNm}$$

$$\therefore \quad M_z = 9.45 \text{ kNm} < M_{dz} < \frac{1.2 Z_{ez} f_y}{\gamma_{M0}} \qquad \text{(OK)}$$

Design flexural capacity about weak axis,

$$M_{dy} = \frac{Z_{py} f_y}{\gamma_{M0}}$$

$$Z_{py} = 2 \times \left(125 \times \frac{4.4}{2} \times \frac{4.4}{4}\right) + 2\left(\frac{75 - 4.4}{2}\right)\left(\frac{75 - 4.4}{2} + \frac{4.4}{2}\right) 7.6$$

$$= 605 + 20121 = 20726 \text{ mm}^3$$

$$\therefore \quad M_{dy} = \frac{Z_{py} f_y}{\gamma_{M0}} = 20726 \times \frac{250}{1.1} \text{ Nmm} = 4.71 \text{ kNm} > M_y (= 0.37 \text{ kNm})$$

$$Z_{ey} = \frac{20.726 \times 10^3}{(75/2)} = 11.653 \times 10^3 \text{ mm}^3$$

$$\therefore \quad \frac{Z_{py}}{Z_{ey}} = \frac{20.726 \times 10^3}{11.653 \times 10^3} = 1.78 > 1.2$$

⇒ 1.2 has to be replace by 1.5 in $\frac{1.2Z_e f_y}{\gamma_{M0}}$

$$\therefore \quad \frac{1.5Z_{ey}f_y}{\gamma_{M0}} = 1.5\times11.653\times10^3\times\frac{250}{1.1}\text{ Nm} = 3.97\text{ kNm}$$

Thus,
$$M_{dy} = \frac{Z_{py}f_y}{\gamma_{M0}} = \frac{20726\times250}{1.1}\text{ Nmm}=4.71\text{ kNm} > M_y\ (= 0.37\text{ kNm})$$

$$Z_{ey} = \frac{I_y}{(b_f/2)} = \frac{43.7\times10^4}{(75/2)} = 11.653\times10^3\text{ mm}^3$$

$$\therefore \quad \frac{Z_{py}}{Z_{ey}} = \frac{20.726\times10^3}{11.653\times10^3} = 1.78 > 1.2$$

⇒ 1.2 has to be replaced by1.5 in $\frac{1.2Z_e f_y}{\gamma_{M0}}$

$$\therefore \quad \frac{1.5Z_{ey}f_y}{\gamma_{M0}} = 1.5\times11.653\times10^3\times\frac{250}{1.1}\text{ Nmm} = 3.97\text{ kNm}$$

Thus,
$$M_{dy} > 1.5\frac{Z_{ey}f_y}{\gamma_{M0}}$$

But ideally,
$$M_{dy} = \frac{1.5Z_{ey}f_y}{\gamma_{M0}}$$

$$\therefore \quad M_{dy} = 3.97\text{ kNm} > M_y\ (= 0.37\text{ kNm}) \quad \text{(OK)}$$

Check for local capacity of the section

$$\frac{M_z}{M_{dz}} + \frac{M_y}{M_{dy}} \le 1$$

$$\text{LHS} = \frac{M_z}{M_{dz}} + \frac{M_y}{M_{dy}} = \frac{9.45}{18.6} + \frac{0.37}{3.97} = 0.603 < 1 \quad \text{(OK)}$$

Check for deflection

$$\delta_{max} = \frac{5}{384}\frac{wl^4}{EI} = \frac{5}{384}\times\frac{2520.16\times10^{-3}\times(5000)^4}{2\times10^5\times449\times10^4} = 22.84\text{ mm}$$

$$\delta_{allowable} = \frac{\text{Span}}{180} = \frac{5000}{180} = 27.79\text{ mm} > 22.84\text{ mm}$$

∴ Section satisfies the deflection check.

Example 6.4 Design a bearing plate for ISMB 500 @ 852.49 N/m resting on concrete support 250 mm thick for a reaction of 190 kN. Use concrete of grade M30.

Solution:

Let Fe 410 steel is used

∴ For Fe410, $f_u = 410\text{ N/mm}^2$, $f_y = 250\text{ N/mm}^2$

Permissible bearing pressure on concrete

$$f_b = 0.45 f_{ck} = 0.45\times30 = 13.5\text{ N/mm}^2$$

Section properties of ISMB 500 @ 852.49 N/m are as follows:

h = 500 mm, b_f = 180 mm

t_f = 17.2 mm, t_w = 10.2 mm

R_1 = 17 mm, I_z = $45218.3\times10^4\text{ mm}^4$

I_y = $1369.8\times10^4\text{ mm}^4$, Z_{ez} = $1808.7\times10^3\text{ mm}^3$

Z_{pz} = $2074.67\times10^3\text{ mm}^3$, d = 431.6 mm

Area of bearing plate required,

$$A = \frac{R}{f_b} = \frac{190\times10^3}{13.5} = 14074.07\text{ mm}^2$$

Let width of bearing plate = Width of support = 250 mm

∴ Height of bearing plate required = $\frac{14074.07}{250} = 56.3\text{ mm} = 190\text{ mm (say)}$

∴ Provide a bearing plate of size 190 mm × 250 mm

Thickness of bearing plate required,

$$t = \sqrt{\frac{2.75R_n^2}{Af_y}}$$

where, $n = \frac{b_f}{2} - \frac{t_w}{2} - R_1 = \frac{180}{2} - \frac{102}{2} - 17 = 67.9\text{ mm}$

$$\therefore \quad t = \sqrt{\frac{2.75\times190\times1000\times67.9^2}{190\times250\times250}} = 14.24\text{ mm}=16\text{mm}$$

∴ Provide a bearing plate of size 190 × 250 × 16 mm.

Example 6.5 A cantilever beam of length 2 m carries a factored concentrated load of 210 kN at free end. If section available is ISMB 300 @ 402 N/m, design the flange plates if required.

Solution:

Factored moment (M) = 210 × 2 = 420 kNm

Factored shear force (V) = 210 kN

For Fe 410 steel, $f_u = 410\text{ N/mm}^2$

$f_y = 250\text{ N/mm}^2$

$$\epsilon = \sqrt{\frac{250}{f_y}} = \sqrt{\frac{50}{250}} = 1$$

$$M = \frac{\beta_b f_y Z_p}{\gamma_{m0}}$$

For plastic section, $\beta_b = 1$

$$\therefore \quad Z_{p,reqd} = \frac{M\gamma_{m0}}{\beta_b f_y} = \frac{420\times10^6\times1.1}{1\times250} = 1848\times10^3\text{ mm}^3$$

Z_p available with ISMB 300 @ 402 N/m,

$$= 651.7 \times 10^3 \text{ mm}^3$$

$\therefore$ Cover plates are required to provide additional plastic section modulus

$$= (1848 \times 10^3 - 651.7 \times 10^3) \text{ mm}^3 = 1196.3 \times 10^3 \text{ mm}^3$$

$\therefore$ Area of each cover plate $= \dfrac{1196.3 \times 10^3}{300} = 3987.67 \text{ mm}^2$

Width of flange of ISMB 300 @ 402 N/m,

$$b_f = 3987.67 \text{ mm}^2$$

Let width of flange plate = 300 mm

$\therefore$ Thickness of flange plate $= \dfrac{3987.67}{300} = 13.29 \text{ mm} \simeq 20 \text{ mm (say)}$

Thus provide flange plates of size 300 × 20 mm.

Thus plastic section modulus of built up section,

$$Z_p = Z_p \text{ of ISMB} + Z_p \text{ provided by cover plates}$$

$$= 651.7 \times 10^3 + 300 \times 20 \times 320 = 2571.7 \times 10^3 \text{ mm}^3$$

$\therefore$ Design flexural strength of built-up section.

$$M_d = \frac{\beta_b f_y Z_p}{\gamma_{m0}} = \frac{1 \times 250 \times 2571.7 \times 10^3}{1.1} \text{ Nmm}$$

$$= 584.48 \text{ kNm}$$

$$> 420 \text{ kNm} \qquad \text{(OK)}$$

Design shear strength of built-up section,

$$V_d = \frac{A_v f_y}{\sqrt{3}\gamma_{mV}} = \frac{300 \times 7.5 \times 250}{\sqrt{3} \times 1.1} \text{ N}$$

$$= 295.24 \text{ kN} > 210 \text{ kN} \qquad \text{(OK)}$$

Also $\quad 0.6\,V_d = 0.6 \times 295.24 = 177.14 \text{ kN}$

Now $\quad V > 0.6\,V_d$

Thus it is a case of high shear and design flexural strength has to be reduced.

$$\therefore \quad M_{dv} = M_d - \beta(M_d - M_{fd})$$

$$\beta = \left(\frac{2V}{V_d} - 1\right)^2 = \left(\frac{2 \times 210}{295.24} - 1\right)^2 = 0.17857$$

M_{fd} = Moment to be resisted by flange alone.

$$= \frac{300 \times 20 \times 250 \times 320 + 140 \times 12.4 \times 250 \times 287.6}{1.1} \text{ Nmm}$$

$$= 549.83 \text{ kNm}$$

$$\therefore \quad M_{dv} = 854.48 - 0.17857\,(584.48 - 549.83)$$

$$= 578.29 \text{ kNm} > 420 \text{ kNm} \qquad \text{(OK)}$$

Example 6.6 A simply supported beam of span 4 m (effective) is laterally supported throughout. It carries a total uniformly distributed load of 35 kN including its self weight. Using steel of grade Fe410, design the beam.

Solution:

For steel of grade Fe410, $\quad f_u = 410 \text{ N/mm}^2,\ f_y = 250 \text{ N/mm}^2$

Factored load = 1.5 × 35 = 52.5 kN

$\therefore$ Factored moment $(M) = \dfrac{(wl)l}{8} = 52.5 \times \dfrac{4}{8} = 26.25 \text{ kNm}$

Factored shear force $(V) = \dfrac{wl}{2} = \dfrac{52.5}{2} = 26.25 \text{ kN}$

Plastic section modulus required,

$$Z_{pz,\text{req.}} = \frac{M\gamma_{m0}}{f_y} = \frac{26.25 \times 10^6 \times 1.1}{250} = 115.5 \times 10^3 \text{ mm}^3 = 115.5 \text{ cm}^3$$

Try ISLB 200 @ 184.24 N/m with section properties as follows:

h = 200 mm $\qquad b_f$ = 100 mm

t_f = 7.3 mm $\qquad t_w$ = 5.4 mm

R_1 = 9.5 mm

$\therefore$ Depth of web, $\quad d = h - 2(t_f + R_1) = 200 - 2(7.3 + 9.5) = 166.4$ m

Moment of inertia about strong axis,

$$I_z = 1696.6 \times 10^4 \text{ mm}^4$$

Plastic section modulus about strong axis,

$$Z_{pz} = 184.34 \times 10^3 \text{ mm}^3$$

Elastic section modulus about strong axis,

$$Z_{ez} = 169.7 \times 10^3 \text{ mm}^3$$

Classification of section

$$\epsilon = \sqrt{\frac{250}{f_y}} = \sqrt{\frac{250}{250}} = 1$$

Outstand of flange, $\quad b = \dfrac{b_f}{2} = \dfrac{100}{2} = 50 \text{ mm}$

$$\frac{b}{t_f} = \frac{50}{7.3} = 6.8493 < 9.4\epsilon (= 9.4 \times 1 = 9.4)$$

$$\frac{d}{t_w} = \frac{166.4}{5.4} = 30.8148 < 83.9\epsilon (= 83.9 \times 1 = 83.9)$$

Section is plastic.

Now, $\quad \dfrac{d}{t_w} = 30.8148 < 67\epsilon \; (= 67 \times 1 = 67)$

$\therefore$ Web is stocky and shear buckling of web is not required.

Shear capacity check of the section

Design shear force, $V = 26.25$ kN

Design shear strength of section,

$$V_d = \frac{A_v f_y}{\sqrt{3}\gamma_{m0}} = \frac{(200\times5.4)\times250}{\sqrt{3}\times1.1} N = 141.71 \text{ kN} > V(= 26.25 \text{ kN})$$

∴ Section is safe in shear.

Check for high/low shear

$$0.6\,V_d = 0.6\times141.71 = 85.03 \text{ kN} > V(= 26.25 \text{ kN})$$

∴ $V < 0.6\,V_d$

Thus shear force will not effect the design moment capacity of the beam section.

Flexural capacity check of the section

∵ The section is plastic.

∴ $\beta_p = 1$

Design bending strength of the section,

$$M_d = \frac{\beta_p Z_{pz} f_y}{\gamma_{m0}} = 1(184.34\times10^3)\frac{250}{1.1} \text{ Nmm}$$

$= 41.895$ kNm

$< M(= 26.25 \text{ kNm})$

Also, $$\frac{1.2 Z_{ez} f_y}{\gamma_{m0}} = 1.2\times169.7\times10^3\times\frac{250}{1.1}\text{Nmm} = 46.282 \text{ kNm}$$

Thus $$M_d < \frac{1.2 Z_{ez} f_y}{\gamma_{m0}}$$ (OK)

Deflection check

Maximum deflection, $$\delta_{max} = \frac{5}{384}\frac{wl^4}{EI} = \frac{5}{384}\frac{(35\times1000)(4000)^4}{2\times10^5\times1696.6\times10^4} \text{ mm} = 8.596 \text{ mm}$$

Maximum permissible deflection,

$$\delta_{allowable} = \frac{l}{300} = \frac{4000}{300} \text{ mm} = 13.33 \text{ mm}$$

Thus, $\delta_{max} < \delta_{allowable}$

Thus section is satisfying the deflection check.

Check for web bearing

Let stiff bearing length $(b_1) = 75$ mm

Also, $n_1 = 2.5(t_f + R_1) = 2.5\,(7.3 + 9.5) = 42$ mm

∴ Strength of section in bearing,

$$F_w = \frac{A_b f_{yw}}{\gamma_{m0}} = (b_1 + n_1)\frac{t_w f_{yw}}{\gamma_0} = (75+41)\frac{5.4\times250}{1.1} \text{ N}$$

$= 143.59 \text{ kN} > V(= 30 \text{ kN})$

∴ Section is safe bearing also.

Example 6.7 **A beam is carrying the following forces:**

Maximum flexural moment (M) = 155 kNm

Maximum shear force (V) = 220 kN

The span of the beam is 5.5 m. Using steel of grade Fe410, design a suitable section for the beam.

Solution:

For steel of grade Fe410, $f_u = 410$ N/mm², $f_y = 250$ N/mm²

Plastic section modulus required,

$$Z_{pz,req} = \frac{M\gamma_{m0}}{f_y} = (155\times10^6)\frac{1.1}{250} \text{ mm}^3$$

$= 682\times10^3$ mm³ $= 682$ cm³

Try ISLB 325 @ 422.81 N/m with section properties as follows:

$h = 325$ mm $\quad b_f = 165$ mm

$t_f = 9.8$ mm $\quad t_w = 7.0$ mm

$R_1 = 16$ mm

Elastic section modulus about strong axis

$$Z_{ez} = 607.7\times10^3 \text{ mm}^3$$

Plastic section modulus about strong axis,

$$Z_{pz} = 687.76\times10^3 \text{ mm}^3$$

Depth of web, $d = h - (t_f - R_1) = 325 - 2\,(5.8 + 16) = 273.4$ mm

Moment of inertia about strong axis,

$$I_z = 9874.6\times10^4 \text{ mm}^4$$

Classification of section

$$\epsilon = \sqrt{\frac{250}{f_y}} = \sqrt{\frac{250}{250}} = 1$$

Outstand of flange, $$b = \frac{b_f}{2} = \frac{165}{2} = 82.5 \text{ mm}$$

∴ $$\frac{b}{t_f} = \frac{82.5}{9.8} = 8.4184 < 9.4\epsilon(= 9.4\times1 = 9.4)$$

$$\frac{d}{t_w} = \frac{273.4}{7.0} = 30.0571 > 83.9\epsilon(= 83.9\times1 = 83.9)$$

∴ As per Table (Table 1.4 of SKD), section is plastic.

Now, $$\frac{d}{t_w} = 39.571 < 67\epsilon\ (= 67\times1 = 67)$$

∴ Web is stocky and shear buckling check of web is not required.

Shear capacity check of the section

Design shear force, $V = 220$ kN

Design shear strength of the section,

$$V_d = \frac{A_v f_y}{\sqrt{3}\gamma_{m0}} = \frac{h t_w f_y}{\sqrt{3}\gamma_{m0}} = \frac{(325\times7)250}{\sqrt{3}\times1.1}\text{ N}=298.52\text{ kN} > V(=220\text{ kN})$$

∴ Section is safe in shear.

Check for high/low shear

$$0.6\,V_d = 0.6\times298.52 = 179.11\text{ kN} < V(=220\text{ kN})$$

∴ $V > 0.6\,V_d$

Thus it is a case of high shear and design flexural strength will be less than the moment capacity of the section subjected to moment and shear.

Flexural capacity check of the section

∵ It is a case of high shear and reduced bending capacity M_{dv} is,

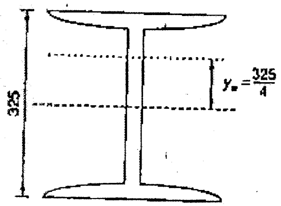
$$M_{dv} = M_d - \beta(M_d - M_{fd})$$

$$\beta = \left(\frac{2V}{V_d}-1\right)^2 = \left(\frac{2\times220}{298.52}-1\right)^2 = 0.2246$$

Plastic moment capacity of the whole section disregarding the effect of high shear but considering the web buckling effect,

$$M_d = \frac{Z_{pz}f_y}{\gamma_{m0}} = \frac{687.76\times10^3\times250}{1.1}$$

$$= 156.31\text{ kNm}$$



Section modulus of flange $Z_{fd} = Z_{pz} - A_w y_w$

$$= 687.76\times10^3 - (325\times7)\frac{325}{4}$$

$$= 502.92\times10^3\text{ mm}^3$$

∴ Plastic design moment capacity of the section excluding the shear area,

$$M_{fd} = \frac{Z_{fd}f_y}{\gamma_{m0}} = 502.92\times10^3\times\frac{250}{1.1}\text{Nmm}=114.3\text{ kNm}$$

∴

$$M_{dv} = M_d - \beta(M_d - M_{fd})$$

$$= 156.31 - 0.2246(156.31 - 114.3)$$

$$= 146.87\text{ kNm} < M(=155\text{ kNm})$$

∴ Section is not safe in bending.

Thus section needs to be revised.

Try ISMB 350 @ 514.04 N/m with section properties as follows:

h = 350 mm	b_f = 140 mm
t_f = 14.2 mm	t_w = 8.1 mm
R_1 = 14 mm	I_z = 13630.3 × 10⁴ mm⁴
Z_{ez} = 779 × 10³ mm³	

Depth of web, $d = h - 2(t_f + R_1) = 350 - 2(14.2 + 14) = 293.6$ mm

Classification of section

Outstand of flange, $b = \frac{b_f}{2} = \frac{140}{2} = 70$ mm

$$\frac{b}{t_f} = \frac{70}{14.2} = 4.9296 < 9.4\epsilon(=9.4\times1=9.4)$$

$$\frac{d}{t_w} = \frac{293.6}{8.1} = 36.247 < 83.9\epsilon(=83.9\times1=83.9)$$

∴ Section is plastic.

Also, $\frac{d}{t_w} = 36.247 < 67\epsilon\ (=67\times1=67)$

Thus web of the section is stocky and buckling check of web is not required.

Shear capacity check of the section

Design shear force, $V = 220$ kN

Design shear strength of the section,

$$V_d = \frac{A_v f_y}{\sqrt{3}\gamma_{m0}} = \frac{h t_w f_y}{\sqrt{3}\gamma_{m0}} = \frac{(350\times8.1)\times250}{\sqrt{3}\times1.1}\text{ N}$$

$$= 371.99\text{ kN} > V(=220\text{ kN})$$

∴ Section is safe in shear.

Check for high/low shear

$$0.6\,V_d = 0.6\times371.99 = 223.19\text{ kN} > V(=220\text{ kN})$$

∴ $V < 0.6\,V_d$

∴ It is a case of low shear.

⇒ Shear will not effect the design moment capacity of the section.

Flexural capacity check of the section

∵ Section is plastic.

∴ $\beta_b = 1$

∴

$$M_d = \frac{\beta_b Z_{pz} f_y}{\gamma_{m0}} = \frac{1\times889.57\times10^3\times250}{1.1}\text{ Nmm}$$

$$= 202.18\text{ kNm} > M(=155\text{ kNm})$$

$$\frac{1.2Z_{ez}f_y}{\gamma_{m0}} = \frac{1.2\times779\times10^3\times250}{1.1}\text{ Nmm} = 212.45\text{ kNm}$$

∴ $M_d = \frac{1.2Z_{ez}f_y}{\gamma_{m0}}$ (OK)

∴ Section is safe in bending.

Check for web bearing

Let stiff bearing length, $b_1 = 75$ mm

$$n_1 = 2.5(t_f + R_1) = 2.5(14.2 + 14) = 70.5\text{ mm}$$

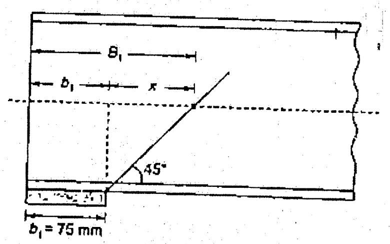
∴ Bearing strength, $F_w = (b_1 + n_1)\frac{t_w f_{yw}}{\gamma_{mo}} = (75+70.5)\frac{8.1\times 250}{1.1}$ N

$= 267.85 \text{ kN} > V (= 220 \text{ kN})$

∴ Section is safe in bearing.

Check for web buckling

As stated earlier, $\frac{d}{t_w} < 67\epsilon$ and thus check for web buckling is not required. Here this check is made of verify this.



Stiff bearing length, $b_1 = 75$ mm

$x = \frac{h}{2}\tan 45° = 175$ mm

∴ Bearing area, $A_b = (b_1 + x)\, t_w = (75 + 175)\, 8.1 = 2025 \text{ mm}^2$

Effective length (i.e., depth) of web

$= kL = 0.7d = 0.7 \times 293.6$

$= 205.52$ mm

Effective moment of inertia of web,

$$I_{eff} = \frac{b_1 t_w^3}{12} = \frac{75\times 8.1^3}{12} = 3321.51 \text{ mm}^4$$

Effective web area, $A_w = bt_w = 75 \times 8.1 = 607.5 \text{ mm}^2$

∴ Radius of gyration, $r = \sqrt{\frac{I_{eff}}{A_{eff}}} = \sqrt{\frac{3321.51}{607.5}} = 2.338$ mm

∴ Slenderness ratio, $\lambda = \frac{kL}{r} = \frac{205.52}{2.338} = 87.904$

∴ For $f_y = 250 \text{ N/mm}^2$

and $\lambda = 87.904$ and buckling curve 'c' design compressive stress

$f_{cd} = 124.14 \text{ N/mm}^2$

∴ Web capacity of section,

$F_{wb} = A_b f_{cd} = 2025 \times 124.14 \text{ N} = 251.38 \text{ kN} > V (= 220 \text{ kN})$

∴ Section is safe is web buckling.

Objective Brain Teasers

Q.1 Purlins as per IS : 800 are designed as
(a) Continuous beams
(b) Simply supported beams
(c) Cantilever beam
(d) None of these

Q.2 The maximum permissible deflection of steel beams in buildings other than industrial buildings is limited to
(a) span/250
(b) span/300
(c) span/180
(d) span/500

Q.3 Match List-I with List-II and select the correct option using the codes given below:

List-I
A. B_b
B. Imperfection factor (α)
C. Non-dimensional slenderness ratio
D. Tortional constant

1. $\sqrt{\frac{f_y}{f_{ab}}}$
2. $\frac{Z_e}{Z_p}$
3. 0.21
4. $\frac{\Sigma b_i t_i^3}{3}$

	A	B	C	D
(a)	4	2	1	3
(b)	1	2	4	3
(c)	3	4	2	1
(d)	2	3	1	4

Q.4 A section is required to be designed as high shear case in case factored shear force is
(a) less than 0.6 V_d
(b) more than 0.6 V_d
(c) less than 0.4 V_d
(d) less than 0.8 V_d

Q.5 For an *I*-beam, web-buckling is less critical than flange buckling because
(a) web can develop part bucking strength
(b) edge conditions of flange are more favorable for buckling
(c) web is thicker than flange
(d) flange is thickner than web

Q.6 In beams of low shear case, the design flexural strength determined by $B_b\frac{Z_p f_y}{\gamma_{mo}}$ is limited to $\frac{1.2 Z_e f_y}{\gamma_{mo}}$ for simple beams because
(i) commencement of plastically under working loads is prevented
(ii) Yield does not occur at working loads
(iii) Lateral-torsional bucking is prevented
Of the above statements, the correct one(s) is/are:
(a) (i) and (iii)
(b) (i) and (ii)
(c) (ii) and (iii)
(d) (ii) only

Q.7 The lateral restraint provided at the support for a laterally supported beam can be provided in the form of
(a) web cleats
(b) partial depth end plates
(c) contnuity with adjacent span i.e. continuous beam
(d) any of the above

Q.8 The problem of web crippling in beams is significant when
(a) web is weak under concentrated loads
(b) there is too much flexural moment
(c) compression flange is weak
(d) all of the above

Q.9 As per Indian standard codal requirements, beam should be
(a) rolled to have maximum sectional modulus
(b) plastic or atleast compact
(c) atleast symmetrical about one axis
(d) all of the above

Q.10 The imperfection factor (α) for welded steel section is

(a) 0.21 (b) 0.42
(c) 0.49 (d) 0.35

Answers

1. (a)	2. (b)	3. (d)	4. (b)	5. (a)
6. (b)	7. (d)	8. (a)	9. (d)	10. (c)

Conventional Practice Questions

Q.1 Design a beam of span 9 m carrying a uniformly distributed load of 15 kN/m. The depth of the beam is limited to 400 mm. Use steel of grade Fe410.

Q.2 Design a channel section purlin for an industrial roof with following data:
C/C distance between the trusses = 6 m
C/C distance between the purlins = 1.4 m
Inclination of roof surface with the horizontal = 20°
Weight of GI sheets = 130 N/mm²
Windload normal to the roof = 1200 kN/m²
Use steal of grade Fe410.

Q.3 Design a two span continuous beam 10 m long with each span being 5 m. It supports a uniform load of 50 kN/m. Use steel of grade Fe410.

Q.4 ISMB 600 @ 1202 N/m transmits an end reaction of 200 kN on 230 mm thick masonry walls. The allowable bearing stress in the masonry is 9 N/mm². Design is to table bearing plate.

Q.5 Design a fixed end beam of span 5.5 m loaded uniformly with a service load of 12 kN/m.

■■■■