

Operations On Algebraic Expressions

Ex 6A

Q1

Answer :

Writing the terms of the given expressions (in the same order) in the form of rows with like terms below each other and adding column-wise, we get:

$$\begin{array}{r} 8ab \\ - 5ab \\ 3ab \\ - ab \\ \hline 5ab \end{array}$$

Q2

Answer :

Writing the terms of the given expressions (in the same order) in the form of rows with like terms below each other and adding column-wise, we get:

$$\begin{array}{r} 7x \\ - 3x \\ 5x \\ - x \\ - 2x \\ \hline 6x \end{array}$$

Q3

Answer :

Writing the terms of the given expressions (in the same order) in the form of rows with like terms below each other and adding column-wise, we get:

$$\begin{array}{r} 3a - 4b + 4c \\ 2a + 3b - 8c \\ a - 6b + c \\ \hline 6a - 7b - 3c \end{array}$$

Q4

Answer :

Writing the terms of the given expressions (in the same order) in the form of rows with like terms below each other and adding column-wise, we get:

$$\begin{array}{r} 5x - 8y + 2z \\ - 2x - 4y + 3z \\ - x + 6y - z \\ 3x - 3y - 2z \\ \hline 5x - 9y + 2z \end{array}$$

Q5

Answer :

Writing the terms of the given expressions (in the same order) in the form of rows with like terms below each other and adding column-wise, we get:

$$\begin{array}{r} 6ax - 2by + 3cz \\ - 11ax + 6by - cz \\ - 2ax - 3by + 10cz \\ \hline - 7ax + by + 12cz \end{array}$$

Q6

Answer :

On arranging the terms of the given expressions in the descending powers of x and adding column-wise:

$$\begin{array}{r} 2x^3 - 9x^2 + 0x + 8 \\ 0x^3 + 3x^2 - 6x - 5 \\ 7x^3 + 0x^2 - 10x + 1 \\ - 4x^3 - 5x^2 + 2x + 3 \\ \hline 5x^3 - 11x^2 - 14x + 7 \end{array}$$

Q7

Answer :

Writing the terms of the given expressions (in the same order) in the form of rows with like terms below each other and adding column-wise:

$$\begin{array}{r} 6p + 4q - r + 3 \\ - 5p + 0q + 2r - 6 \\ - 7p + 11q + 2r - 1 \\ 0p + 2q - 3r + 4 \\ \hline - 6p + 17q + 0r + 0 \\ = -6p + 17q \end{array}$$

Q8

Answer :

On arranging the terms of the given expressions in the descending powers of x and adding column-wise:

$$\begin{array}{r} 4x^2 + 4y^2 - 7xy - 3 \\ x^2 + 6y^2 - 8xy + 0 \\ 2x^2 - 5y^2 - 2xy + 6 \\ \hline 7x^2 + 5y^2 - 17xy + 3 \end{array}$$

Q9

Answer :

On arranging the terms of the given expressions in the descending powers of x and subtracting:

$$\begin{array}{r} -5a^2b \\ 3a^2b \\ - \\ -8a^2b \end{array}$$

Q10

Answer :

Writing the terms of the given expressions (in the same order) in the form of rows with like terms below each other and subtracting column-wise:

$$\begin{array}{r} 6pq \\ -8pq \\ + \\ \hline 14pq \end{array}$$

Q11

Answer :

Writing the terms of the given expressions (in the same order) in the form of rows with like terms below each other and subtracting column-wise:

$$\begin{array}{r} -8abc \\ -2abc \\ + \\ \hline -6abc \end{array}$$

Q12

Answer :

Writing the terms of the given expressions (in the same order) in the form of rows with like terms below each other and subtracting column-wise:

$$\begin{array}{r} -11p \\ -16p \\ + \\ \hline 5p \end{array}$$

Q13

Answer :

Writing the terms of the given expressions (in the same order) in the form of rows with like terms below each other and subtracting column-wise:

$$\begin{array}{r} 3a - 4b - c + 6 \\ 2a - 5b + 2c - 9 \\ - \quad + \quad - \quad + \\ \hline a + b - 3c + 15 \end{array}$$

Q14

Answer :

Writing the terms of the given expressions (in the same order) in the form of rows with like terms below each other and subtracting column-wise:

$$\begin{array}{r} p - 2q - 5r - 8 \\ -6p + q + 3r + 8 \\ + \quad - \quad - \quad - \\ \hline 7p - 3q - 8r - 16 \end{array}$$

Q15

Answer :

On arranging the terms of the given expressions in the descending powers of x and subtracting column-wise:

$$\begin{array}{r} 3x^3 - x^2 + 2x - 4 \\ x^3 + 3x^2 - 5x + 4 \\ - \quad - \quad + \quad - \\ \hline 2x^3 - 4x^2 + 7x - 8 \end{array}$$

Q16

Answer :

Arranging the terms of the given expressions in the descending powers of x and subtracting column-wise:

$$\begin{array}{r} 4y^4 - 2y^3 - 6y^2 - y + 5 \\ 5y^4 - 3y^3 + 2y^2 + y - 1 \\ - \quad + \quad - \quad - \quad + \\ \hline -y^4 + y^3 - 8y^2 - 2y + 6 \end{array}$$

Q17

Answer :

Writing the terms of the given expressions (in the same order) in the form of rows with like terms below each other and subtracting column-wise:

$$\begin{array}{r} 3p^2 - 4q^2 - 5r^2 - 6 \\ 4p^2 + 5q^2 - 6r^2 + 7 \\ - \quad - \quad + \quad - \\ \hline -p^2 - 9q^2 + r^2 - 13 \end{array}$$

Q18

Answer :

Let the required number be x .

$$\begin{aligned} (3a^2 - 6ab - 3b^2 - 1) - x &= 4a^2 - 7ab - 4b^2 + 1 \\ (3a^2 - 6ab - 3b^2 - 1) - (4a^2 - 7ab - 4b^2 + 1) &= x \end{aligned}$$

$$\begin{array}{r} 3a^2 - 6ab - 3b^2 - 1 \\ 4a^2 - 7ab - 4b^2 + 1 \\ - \quad + \quad + \quad - \\ \hline -a^2 + ab + b^2 - 2 \end{array}$$

$$\therefore \text{Required number} = -a^2 + ab + b^2 - 2$$

Q19

Answer :

Sides of the rectangle are l and b .

$$l = 5x^2 - 3y^2$$

$$b = x^2 + 2xy$$

Perimeter of the rectangle is $(2l + 2b)$.

$$\begin{aligned} \text{Perimeter} &= 2 \left(5x^2 - 3y^2 \right) + 2 \left(x^2 + 2xy \right) \\ &= 10x^2 - 6y^2 + 2x^2 + 4xy \\ &\quad 10x^2 - 6y^2 \\ &\quad \underline{2x^2 \quad + \quad 4xy} \\ &= 12x^2 - 6y^2 + 4xy \end{aligned}$$

Hence, the perimeter of the rectangle is $12x^2 - 6y^2 + 4xy$.

Q20

Answer :

Let ***a***, ***b*** and ***c*** be the three sides of the triangle.

∴ Perimeter of the triangle = ***(a + b + c)***

Given perimeter of the triangle = $6p^2 - 4p + 9$

One side (***a***) = $p^2 - 2p + 1$

Other side (***b***) = $3p^2 - 5p + 3$

Perimeter = ***(a + b + c)***

$$(6p^2 - 4p + 9) = (p^2 - 2p + 1) + (3p^2 - 5p + 3) + c$$

$$6p^2 - 4p + 9 - p^2 + 2p - 1 - 3p^2 + 5p - 3 = c$$

$$(6p^2 - p^2 - 3p^2) + (-4p + 2p + 5p) + (9 - 1 - 3) = c$$

$$2p^2 + 3p + 5 = c$$

Thus, the third side is $2p^2 + 3p + 5$.

Operations On Algebraic Expressions

Ex 6B

Q1

Answer :

By horizontal method:

$$\begin{aligned}(5x + 7) \times (3x + 4) \\&= 5x(3x + 4) + 7(3x + 4) \\&= 15x^2 + 20x + 21x + 28 \\&= 15x^2 + 41x + 28\end{aligned}$$

Q2

Answer :

By horizontal method:

$$\begin{aligned}(4x + 9) \times (x - 6) \\&= 4x(x - 6) + 9(x - 6) \\&= 4x^2 - 24x + 9x - 54 \\&= 4x^2 - 15x - 54\end{aligned}$$

Q3

Answer :

By horizontal method:

$$\begin{aligned}(2x + 5) \times (4x - 3) \\&= 2x(4x - 3) + 5(4x - 3) \\&= 8x^2 - 6x + 20x - 15 \\&= 8x^2 + 14x - 15\end{aligned}$$

Q4

Answer :

By horizontal method:

$$\begin{aligned}(3y - 8) \times (5y - 1) \\&= 3y(5y - 1) - 8(5y - 1) \\&= 15y^2 - 3y - 40y + 8 \\&= 15y^2 - 43y + 8\end{aligned}$$

Q5

Answer :

By horizontal method:

$$\begin{aligned}(7x + 2y) \times (x + 4y) \\&= 7x(x + 4y) + 2y(x + 4y) \\&= 7x^2 + 28xy + 2xy + 8y^2 \\&= 7x^2 + 30xy + 8y^2\end{aligned}$$

Q6

Answer :

By horizontal method:

$$\begin{aligned}(9x + 5y) \times (4x + 3y) \\&= 9x(4x + 3y) + 5y(4x + 3y) \\&= 36x^2 + 27xy + 20xy + 15y^2 \\&= 36x^2 + 47xy + 15y^2\end{aligned}$$

Q7

Answer :

By horizontal method:

$$\begin{aligned}(3m - 4n) \times (2m - 3n) \\&= 3m(2m - 3n) - 4n(2m - 3n) \\&= 6m^2 - 9mn - 8mn + 12n^2 \\&= 6m^2 - 17mn + 12n^2\end{aligned}$$

Q8

Answer :

By horizontal method:

$$\begin{aligned}(x^2 - a^2) \times (x - a) \\&= x^2(x - a) - a^2(x - a) \\&= x^3 - ax^2 - a^2x + a^3 \\&\text{i.e. } (x^3 + a^3) - ax(x - a)\end{aligned}$$

Q9

Answer :

By horizontal method:

$$\begin{aligned}(x^2 - y^2) \times (x + 2y) \\&= x^2(x + 2y) - y^2(x + 2y) \\&= x^3 + 2x^2y - xy^2 - 2y^3 \\&\text{i.e. } (x^3 - 2y^3) + xy(2x - y)\end{aligned}$$

Q10

Answer :

By horizontal method:

$$\begin{aligned} & (3p^2 + q^2) \times (2p^2 - 3q^2) \\ &= 3p^2(2p^2 - 3q^2) + q^2(2p^2 - 3q^2) \\ &= 6p^4 - 9p^2q^2 + 2p^2q^2 - 3q^4 \\ & i.e. 6p^4 - 7p^2q^2 - 3q^4 \end{aligned}$$

Q11

Answer :

By horizontal method:

$$\begin{aligned} & (2x^2 - 5y^2) \times (x^2 + 3y^2) \\ &= 2x^2(x^2 + 3y^2) - 5y^2(x^2 + 3y^2) \\ &= 2x^4 + 6x^2y^2 - 5x^2y^2 - 15y^4 \\ &= 2x^4 + x^2y^2 - 15y^4 \end{aligned}$$

Q12

Answer :

By horizontal method:

$$\begin{aligned} & (x^3 - y^3) \times (x^2 + y^2) \\ &= x^3(x^2 + y^2) - y^3(x^2 + y^2) \\ &= x^5 + x^3y^2 - x^2y^3 - y^5 \\ &= (x^5 - y^5) + x^2y^2(x - y) \end{aligned}$$

Q13

Answer :

By horizontal method:

$$\begin{aligned} & (x^4 + y^4) \times (x^2 - y^2) \\ &= x^4(x^2 - y^2) + y^4(x^2 - y^2) \\ &= x^6 - x^4y^2 + y^4x^2 - y^6 \\ &= (x^6 - y^6) - x^2y^2(x^2 - y^2) \end{aligned}$$

Q14

Answer :

By horizontal method:

$$\begin{aligned} & \left(x^4 + \frac{1}{x^4}\right) \times \left(x + \frac{1}{x}\right) \\ &= x^4\left(x + \frac{1}{x}\right) + \frac{1}{x^4}\left(x + \frac{1}{x}\right) \\ &= x^5 + x^3 + \frac{1}{x^3} + \frac{1}{x^5} \\ & i.e. x^3\left(x^2 + 1\right) + \frac{1}{x^3}\left(1 + \frac{1}{x^2}\right) \end{aligned}$$

Q15

Answer :

By horizontal method:

$$\begin{aligned} & (x^2 - 3x + 7) \times (2x + 3) \\ &= 2x(x^2 - 3x + 7) + 3(x^2 - 3x + 7) \\ &= 2x^3 - 6x^2 + 14x + 3x^2 - 9x + 21 \\ &= 2x^3 - 3x^2 + 5x + 21 \end{aligned}$$

Q16

Answer :

By horizontal method:

$$\begin{aligned} & (3x^2 + 5x - 9) \times (3x - 5) \\ &= 3x(3x^2 + 5x - 9) - 5(3x^2 + 5x - 9) \\ &= 9x^3 + 15x^2 - 27x - 15x^2 - 25x + 45 \\ &= 9x^3 - 52x + 45 \end{aligned}$$

Q17

Answer :

By horizontal method:

$$\begin{aligned} & (x^2 - xy + y^2) \times (x + y) \\ &= x(x^2 - xy + y^2) + y(x^2 - xy + y^2) \\ &= x^3 - x^2y + y^2x + x^2y - xy^2 + y^3 \\ &= x^3 + y^3 \end{aligned}$$

Q18

Answer :

By horizontal method:

$$\begin{aligned} & (x^2 + xy + y^2) \times (x - y) \\ &= x(x^2 + xy + y^2) - y(x^2 + xy + y^2) \\ &= x^3 + x^2y + xy^2 - x^2y - xy^2 - y^3 \\ &= x^3 - y^3 \end{aligned}$$

Q19

Answer :

By horizontal method:

$$\begin{aligned} & (x^3 - 2x^2 + 5) \times (4x - 1) \\ &= 4x(x^3 - 2x^2 + 5) - 1(x^3 - 2x^2 + 5) \\ &= 4x^4 - 8x^3 + 20x - x^3 + 2x^2 - 5 \\ &= 4x^4 - 9x^3 + 2x^2 + 20x - 5 \end{aligned}$$

Q20

Answer :

By horizontal method:

$$\begin{aligned} & (9x^2 - x + 15) \times (x^2 - 3) \\ &= x^2(9x^2 - x + 15) - 3(9x^2 - x + 15) \\ &= 9x^4 - x^3 + 15x^2 - 27x^2 + 3x - 45 \\ &= 9x^4 - x^3 - 12x^2 + 3x - 45 \end{aligned}$$

Q21

Answer :

By horizontal method:

$$\begin{aligned} & (x^2 - 5x + 8) \times (x^2 + 2) \\ &= x^2(x^2 - 5x + 8) + 2(x^2 - 5x + 8) \\ &= x^4 - 5x^3 + 8x^2 + 2x^2 - 10x + 16 \\ &= x^4 - 5x^3 + 10x^2 - 10x + 16 \end{aligned}$$

Q22

Answer :

By horizontal method:

$$\begin{aligned} & (x^3 - 5x^2 + 3x + 1) \times (x^2 - 3) \\ &= x^2(x^3 - 5x^2 + 3x + 1) - 3(x^3 - 5x^2 + 3x + 1) \\ &= x^5 - 5x^4 + 3x^3 + x^2 - 3x^3 + 15x^2 - 9x - 3 \\ &= x^5 - 5x^4 + 16x^2 - 9x - 3 \end{aligned}$$

Q23

Answer :

By horizontal method:

$$\begin{aligned}& (3x + 2y - 4) \times (x - y + 2) \\& x(3x + 2y - 4) - y(3x + 2y - 4) + 2(3x + 2y - 4) \\& = 3x^2 + 2xy - 4x - 3xy - 2y^2 + 4y + 6x + 4y - 8 \\& = 3x^2 - 2y^2 - xy + 2x + 8y - 8\end{aligned}$$

Q24

Answer :

By horizontal method:

$$\begin{aligned}& (x^2 - 5x + 8) \times (x^2 + 2x - 3) \\& = x^2(x^2 - 5x + 8) + 2x(x^2 - 5x + 8) - 3(x^2 - 5x + 8) \\& = x^4 - 5x^3 + 8x^2 + 2x^3 - 10x^2 + 16x - 3x^2 + 15x - 24 \\& = x^4 - 3x^3 - 5x^2 + 31x - 24\end{aligned}$$

Q25

Answer :

By horizontal method:

$$\begin{aligned}& (2x^2 + 3x - 7) \times (3x^2 - 5x + 4) \\& = 2x^2(3x^2 - 5x + 4) + 3x(3x^2 - 5x + 4) - 7(3x^2 - 5x + 4) \\& = 6x^4 - 10x^3 + 8x^2 + 9x^3 - 15x^2 + 12x - 21x^2 + 35x - 28 \\& = 6x^4 - x^3 - 28x^2 + 47x - 28\end{aligned}$$

Q26

Answer :

By horizontal method:

$$\begin{aligned}& (9x^2 - x + 15) \times (x^2 - x - 1) \\& = x^2(9x^2 - x + 15) - x(9x^2 - x + 15) - 1(9x^2 - x + 15) \\& = 9x^4 - x^3 + 15x^2 - 9x^3 + x^2 - 15x - 9x^2 + x - 15 \\& = 9x^4 - 10x^3 + 7x^2 - 14x - 15\end{aligned}$$

Operations On Algebraic Expressions

Ex 6C

Q1

Answer :

(i) $24x^2y^3$ by $3xy$

$$\begin{aligned} & \frac{24x^2y^3}{3xy} \\ & \Rightarrow \left(\frac{24}{3}\right)(x^{2-1})(y^{3-1}) \\ & \Rightarrow 8xy^2. \end{aligned}$$

Therefore, the quotient is $8xy^2$.

(ii) $36xyz^2$ by $-9xz$

$$\begin{aligned} & \frac{36xyz^2}{-9xz} \\ & \Rightarrow \left(\frac{36}{-9}\right)(x^{1-1})(y^{1-0})(z^{2-1}) \\ & \Rightarrow -4yz \end{aligned}$$

Therefore, the quotient is $-4yz$.

(iii)

$-72x^2y^2z$ by $-12xyz$

$$\begin{aligned} & \frac{-72x^2y^2z}{-12xyz} \\ & \Rightarrow \left(\frac{-72}{-12}\right)(x^{2-1})(y^{2-1})(z^{1-1}) \\ & \Rightarrow 6xy \end{aligned}$$

Therefore, the quotient is $6xy$.

(iv) $-56mnp^2$ by $7mnp$

$$\begin{aligned} & \frac{-56mnp^2}{7mnp} \\ & \Rightarrow \left(\frac{-56}{7}\right)(m^{1-1})(n^{1-1})(p^{2-1}) \\ & \Rightarrow -8p \end{aligned}$$

Therefore, the quotient is $-8p$.

Q2

Answer :

(i) $5m^3 - 30m^2 + 45m$ by $5m$

$$\begin{aligned} & (5m^3 - 30m^2 + 45m) \div 5m \\ & \Rightarrow \frac{5m^3}{5m} - \frac{30m^2}{5m} + \frac{45m}{5m} \\ & \Rightarrow m^2 - 6m + 9 \end{aligned}$$

Therefore, the quotient is $m^2 - 6m + 9$.

(ii) $8x^2y^2 - 6xy^2 + 10x^2y^3$ by $2xy$

$$\begin{aligned} & (8x^2y^2 - 6xy^2 + 10x^2y^3) \div 2xy \\ & \Rightarrow \frac{8x^2y^2}{2xy} - \frac{6xy^2}{2xy} + \frac{10x^2y^3}{2xy} \\ & \Rightarrow 4xy - 3y + 5xy^2 \end{aligned}$$

Therefore, the quotient is $4xy - 3y + 5xy^2$.

(iii) $9x^2y - 6xy + 12xy^2$ by $-3xy$

$$\begin{aligned} & \left(9x^2y - 6xy + 12xy^2 \right) \div -3xy \\ & \Rightarrow \frac{9x^2y}{-3xy} - \frac{6xy}{-3xy} + \frac{12xy^2}{-3xy} \\ & \Rightarrow -3x + 2 - 4y \end{aligned}$$

Therefore, the quotient is $-3x + 2 - 4y$.

(iv) $12x^4 + 8x^3 - 6x^2$ by $-2x^2$

Q3

Answer :

$(x^2 - 4x + 4) \div (x - 2)$

$$\begin{array}{r} x-2 \overline{) x^2 - 4x + 4} \\ \underline{x^2 - 2x} \\ -2x + 4 \\ \underline{-2x + 4} \\ 0 \end{array}$$

$$\begin{aligned} & \left(12x^4 + 8x^3 - 6x^2 \right) \div -2x^2 \\ & \Rightarrow \frac{12x^4}{-2x^2} + \frac{8x^3}{-2x^2} - \frac{6x^2}{-2x^2} \\ & \Rightarrow -6x^2 - 4x + 3 \end{aligned}$$

Therefore the quotient is $-6x^2 - 4x + 3$.

Therefore, the quotient is $(x - 2)$ and the remainder is 0.

Q4

Answer :

$$\begin{array}{r} x+2 \overline{) x^2 - 4} \\ \underline{x^2 + 2x} \\ -2x - 4 \\ \underline{-2x - 4} \\ 0 \end{array}$$

Therefore, the quotient is $x+2$ and the remainder is 0.

Q5

Answer :

$(x^2 + 12x + 35)$ by $(x + 7)$

$$\begin{array}{r} x+7 \overline{) x^2 + 12x + 35} \\ \underline{x^2 + 7x} \\ 5x + 35 \\ \underline{5x + 35} \\ 0 \end{array}$$

Therefore, the quotient is $(x + 5)$ and the remainder is 0.

Answer :

$$\begin{array}{r} 3x+2 \overline{) 15x^2 + 10x - 6} \\ \underline{15x^2 + 10x} \\ -9x - 6 \\ \underline{-9x - 6} \\ 0 \end{array}$$

Therefore, the quotient is $(5x - 3)$ and the remainder is 0.

Q7

Answer :

$$\begin{array}{r} 7x-9 \overline{) 14x^2 - 53x + 45} \\ \underline{14x^2 - 18x} \\ -35x + 45 \\ \underline{-35x + 45} \\ 0 \end{array}$$

Therefore, the quotient is $(2x - 5)$ and the remainder is 0.

Q8

Answer :

$$\begin{array}{r}
 2x-5 \overline{) \begin{array}{r} 6x^2-31x+47 \\ 6x^2-15x \\ \hline -16x+47 \\ -16x+40 \\ \hline +7 \end{array}} (3x-8
 \end{array}$$

Therefore, the quotient is $(3x - 8)$ and the remainder is 7.

Q9

Answer :

$$\begin{array}{r}
 2x+3 \overline{) \begin{array}{r} 2x^3+x^2-5x-2 \\ 2x^3+3x^2 \\ \hline -2x^2-5x \\ -2x^2-5x \\ \hline +2 \\ -2x-2 \\ -2x-3 \\ \hline +1 \end{array}} (x^2-x-1
 \end{array}$$

Therefore, the quotient is $(x^2 - x - 1)$ and the remainder is 1.

Q10

Answer :

$$\begin{array}{r}
 x+1 \overline{) \begin{array}{r} x^3+1 \\ x^3 \quad + \quad x^2 \\ \hline -x^2+1 \\ -x^2 \quad -x \\ \hline + \quad + \\ x+1 \\ x+1 \\ \hline 0 \end{array}} (x^2-x+1
 \end{array}$$

Therefore, the quotient is x^2-x+1 and the remainder is 0.

Q11

Answer :

$$\begin{array}{r}
 x^2+x+1 \overline{) \begin{array}{r} x^4-2x^3+2x^2+x+4 \\ x^4+x^3+x^2 \\ \hline -3x^3+x^2+x \\ -3x^3-3x^2-3x \\ \hline 4x^2+4x+4 \\ 4x^2+4x+4 \\ \hline 0 \end{array}} (x^2-3x+4
 \end{array}$$

Therefore, the quotient is $(x^2 - 3x + 4)$ and remainder is 0.

Q12

Answer :

$$\begin{array}{r}
 x^2-5x+6 \overline{) \begin{array}{r} x^3-6x^2+11x-6 \\ x^3-5x^2+6x \\ \hline -1x^2+5x-6 \\ -1x^2+5x-6 \\ \hline 0 \end{array}} (x-1
 \end{array}$$

Therefore, the quotient is $(x-1)$ and the remainder is 0.

Q13

Answer :

$$\begin{array}{r}
 x^2 - 3x + 4 \overline{) 5x^3 - 12x^2 + 12x + 13} \left(5x + 3 \right. \\
 \underline{x^3 - 15x^2 + 20x} \\
 3x^2 - 8x + 13 \\
 \underline{3x^2 - 9x + 12} \\
 x + 1
 \end{array}$$

Therefore, the quotient is (5x+ 3) and the remainder is (x + 1).

Q14

Answer :

$$\begin{array}{r}
 2x^2 - 3x + 5 \overline{) 2x^3 - 5x^2 + 8x - 5} \left(x - 1 \right. \\
 \underline{2x^3 - 3x^2 + 5x} \\
 -2x^2 + 3x - 5 \\
 \underline{-2x^2 + 3x - 5} \\
 0
 \end{array}$$

Therefore, the quotient is (x-1) and the remainder is 0.

Q15

Answer :

$$\begin{array}{r}
 2x^2 + x - 1 \overline{) 8x^4 + 10x^3 - 5x^2 - 4x + 1} \left(4x^2 + 3x - 2 \right. \\
 \underline{8x^4 + 4x^3 - 4x^2} \\
 6x^3 - x^2 - 4x + 1 \\
 \underline{6x^3 + 3x^2 - 3x} \\
 -4x^2 - x + 1 \\
 \underline{-4x^2 - 2x + 2} \\
 x - 1
 \end{array}$$

Therefore, the quotient is (4x²+ 3x -2) and the remainder is (x-1).

Operations On Algebraic Expressions

Ex 6D

1. $(a + b)^2 = a^2 + 2ab + b^2 = (-a - b)^2$
2. $(a - b)^2 = a^2 - 2ab + b^2$
3. $(a - b)(a + b) = a^2 - b^2$
4. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$
5. $(a + b - c)^2 = a^2 + b^2 + c^2 + 2ab - 2bc - 2ca$
6. $(a - b + c)^2 = a^2 + b^2 + c^2 - 2ab - 2bc + 2ca$
7. $(-a + b + c)^2 = a^2 + b^2 + c^2 - 2ab + 2bc - 2ca$
8. $(a - b - c)^2 = a^2 + b^2 + c^2 - 2ab + 2bc - 2ca$
9. $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$
10. $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$
11. $a^3 + b^3 = (a + b)^3 - 3ab(a + b)$
 $= (a + b)(a^2 - ab + b^2)$
12. $a^3 - b^3 = (a - b)^3 + 3ab(a - b)$
 $= (a - b)(a^2 + ab + b^2)$
13. $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$
 if $a + b + c = 0$ then $a^3 + b^3 + c^3 = 3abc$

Q1

Answer :

(i) We have:

$$\begin{aligned}
 & (x + 6)(x + 6) \\
 &= (x + 6)^2 \\
 &= x^2 + 6^2 + 2 \times x \times 6 \quad \left[\text{using } (a + b)^2 = a^2 + b^2 + 2ab \right] \\
 &= x^2 + 36 + 12x
 \end{aligned}$$

(ii) We have:

$$\begin{aligned}
 & (4x + 5y)(4x + 5y) \\
 &= (4x + 5y)^2 \\
 &= (4x)^2 + (5y)^2 + 2 \times 4x \times 5y \quad \left[\text{using } (a + b)^2 = a^2 + b^2 + 2ab \right] \\
 &= 16x^2 + 25y^2 + 40xy
 \end{aligned}$$

(iii) We have:

$$\begin{aligned}
 & (7a + 9b)(7a + 9b) \\
 &= (7a + 9b)^2 \\
 &= (7a)^2 + (9b)^2 + 2 \times 7a \times 9b \quad \left[\text{using } (a + b)^2 = a^2 + b^2 + 2ab \right] \\
 &= 49a^2 + 81b^2 + 126ab
 \end{aligned}$$

(iv) We have:

$$\begin{aligned}
 & \left(\frac{2}{3}x + \frac{4}{5}y \right) \left(\frac{2}{3}x + \frac{4}{5}y \right) \\
 &= \left(\frac{2}{3}x + \frac{4}{5}y \right)^2 \\
 &= \left(\frac{2}{3}x \right)^2 + \left(\frac{4}{5}y \right)^2 + 2 \times \frac{2}{3}x \times \frac{4}{5}y \quad \left[\text{using } (a + b)^2 = a^2 + b^2 + 2ab \right] \\
 &= \frac{4}{9}x^2 + \frac{16}{25}y^2 + \frac{16}{15}xy
 \end{aligned}$$

(v) We have:

$$\begin{aligned}& (x^2 + 7)(x^2 + 7) \\&= (x^2 + 7)^2 \\&= (x^2)^2 + 7^2 + 2 \times x^2 \times 7 \quad \left[\text{using } (a + b)^2 = a^2 + b^2 + 2ab \right] \\&= x^4 + 49 + 14x^2\end{aligned}$$

(vi) We have:

$$\begin{aligned}& \left(\frac{5}{6}a^2 + 2\right)\left(\frac{5}{6}a^2 + 2\right) \\&= \left(\frac{5}{6}a^2 + 2\right)^2 \\&= \left(\frac{5}{6}a^2\right)^2 + (2)^2 + 2 \times \frac{5}{6}a^2 \times 2 \quad \left[\text{using } (a + b)^2 = a^2 + b^2 + 2ab \right] \\&= \frac{25}{36}a^4 + 4 + \frac{10}{3}a^2\end{aligned}$$

Q2

Answer :

(i) We have:

$$\begin{aligned}& (x - 4)(x - 4) \\&= (x - 4)^2 \\&= x^2 - 2 \times x \times 4 + 4^2 \quad \left[\text{using } (a - b)^2 = a^2 - 2ab + b^2 \right] \\&= x^2 - 8x + 16\end{aligned}$$

(ii) We have:

$$\begin{aligned}& (2x - 3y)(2x - 3y) \\&= (2x - 3y)^2 \\&= (2x)^2 - 2 \times 2x \times 3y + (3y)^2 \quad \left[\text{using } (a - b)^2 = a^2 - 2ab + b^2 \right] \\&= 4x^2 - 12xy + 9y^2\end{aligned}$$

(iii) We have:

$$\begin{aligned}& \left(\frac{3}{4}x - \frac{5}{6}y\right)\left(\frac{3}{4}x - \frac{5}{6}y\right) \\&= \left(\frac{3}{4}x - \frac{5}{6}y\right)^2 \\&= \left(\frac{3}{4}x\right)^2 - 2 \times \frac{3}{4}x \times \frac{5}{6}y + \left(\frac{5}{6}y\right)^2 \quad \left[\text{using } (a - b)^2 = a^2 - 2ab + b^2 \right] \\&= \frac{9}{16}x^2 - \frac{15}{12}xy + \frac{25}{36}y^2\end{aligned}$$

(iv) We have:

$$\begin{aligned}& \left(x - \frac{3}{x}\right)\left(x - \frac{3}{x}\right) \\&= \left(x - \frac{3}{x}\right)^2 \\&= (x)^2 - 2 \times x \times \frac{3}{x} + \left(\frac{3}{x}\right)^2 \quad \left[\text{using } (a - b)^2 = a^2 - 2ab + b^2 \right] \\&= x^2 - 6 + \frac{9}{x^2}\end{aligned}$$

(v) We have:

$$\begin{aligned}& \left(\frac{1}{3}x^2 - 9\right)\left(\frac{1}{3}x^2 - 9\right) \\&= \left(\frac{1}{3}x^2 - 9\right)^2 \\&= \left(\frac{1}{3}x^2\right)^2 - 2 \times \frac{1}{3}x^2 \times 9 + (9)^2 \quad \left[\text{using } (a - b)^2 = a^2 - 2ab + b^2 \right] \\&= \frac{1}{9}x^4 - 6x^2 + 81\end{aligned}$$

(vi) We have:

$$\begin{aligned}& \left(\frac{1}{2}y^2 - \frac{1}{3}y\right)\left(\frac{1}{2}y^2 - \frac{1}{3}y\right) \\&= \left(\frac{1}{2}y^2 - \frac{1}{3}y\right)^2 \\&= \left(\frac{1}{2}y^2\right)^2 - 2 \times \frac{1}{2}y^2 \times \frac{1}{3}y + \left(\frac{1}{3}y\right)^2 \quad \left[\text{using } (a-b)^2 = a^2 - 2ab + b^2\right] \\&= \frac{1}{4}y^4 - \frac{1}{3}y^3 + \frac{1}{9}y^2\end{aligned}$$

Q3

Answer :

We shall use the identities $(a+b)^2 = a^2 + b^2 + 2ab$ and $(a-b)^2 = a^2 + b^2 - 2ab$.

(i) We have:

$$\begin{aligned}& (8a + 3b)^2 \\&= (8a)^2 + 2 \times 8a \times 3b + (3b)^2 \\&= 64a^2 + 48ab + 9b^2\end{aligned}$$

(ii) We have:

$$\begin{aligned}& (7x + 2y)^2 \\&= (7x)^2 + 2 \times 7x \times 2y + (2y)^2 \\&= 49x^2 + 28xy + 4y^2\end{aligned}$$

(iii) We have :

$$\begin{aligned}& (5x + 11)^2 \\&= (5x)^2 + 2 \times 5x \times 11 + (11)^2 \\&= 25x^2 + 110x + 121\end{aligned}$$

(iv) We have:

$$\begin{aligned}& \left(\frac{a}{2} + \frac{2}{a}\right)^2 \\&= \left(\frac{a}{2}\right)^2 + 2 \times \frac{a}{2} \times \frac{2}{a} + \left(\frac{2}{a}\right)^2 \\&= \frac{a^2}{4} + 2 + \frac{4}{a^2}\end{aligned}$$

(v) We have:

$$\begin{aligned}& \left(\frac{3x}{4} + \frac{2y}{9}\right)^2 \\&= \left(\frac{3x}{4}\right)^2 + 2 \times \frac{3x}{4} \times \frac{2y}{9} + \left(\frac{2y}{9}\right)^2 \\&= \frac{9x^2}{16} + \frac{1}{3}xy + \frac{4y^2}{81}\end{aligned}$$

(vi) We have:

$$\begin{aligned}& (9x - 10)^2 \\&= (9x)^2 - 2 \times 9x \times 10 + (10)^2 \\&= 81x^2 - 180x + 100\end{aligned}$$

(vii) We have:

$$\begin{aligned} & \left(x^2y - yz^2\right)^2 \\ & \left(x^2y\right)^2 - 2 \times x^2y \times yz^2 + \left(yz^2\right)^2 \\ & = x^4y^2 - 2x^2y^2z^2 + y^2z^4 \end{aligned}$$

(viii) We have:

$$\begin{aligned} & \left(\frac{x}{y} - \frac{y}{x}\right)^2 \\ & = \left(\frac{x}{y}\right)^2 - 2 \times \frac{x}{y} \times \frac{y}{x} + \left(\frac{y}{x}\right)^2 \\ & = \frac{x^2}{y^2} - 2 + \frac{y^2}{x^2} \end{aligned}$$

(ix) We have:

$$\begin{aligned} & \left(3m - \frac{4}{5}n\right)^2 \\ & = (3m)^2 - 2 \times 3m \times \frac{4}{5}n + \left(\frac{4}{5}n\right)^2 \\ & = 9m^2 - \frac{24mn}{5} + \frac{16}{25}n^2 \end{aligned}$$

Q4

Answer :

(i) We have:

$$\begin{aligned} & (x+3)(x-3) \\ & = x^2 - 9 \quad \left[\text{using } (a+b)(a-b) = a^2 - b^2\right] \end{aligned}$$

(ii) We have:

$$\begin{aligned} & (2x+5)(2x-5) \\ & = 4x^2 - 25 \quad \left[\text{using } (a+b)(a-b) = a^2 - b^2\right] \end{aligned}$$

(iii) We have:

$$\begin{aligned} & (8+x)(8-x) \\ & = 64 - x^2 \quad \left[\text{using } (a+b)(a-b) = a^2 - b^2\right] \end{aligned}$$

(iv) We have:

$$\begin{aligned} & (7x+11y)(7x-11y) \\ & = 49x^2 - 121y^2 \quad \left[\text{using } (a+b)(a-b) = a^2 - b^2\right] \end{aligned}$$

(v) We have:

$$\begin{aligned} & \left(5x^2 + \frac{3}{4}y^2\right)\left(5x^2 - \frac{3}{4}y^2\right) \\ & = 25x^4 - \frac{9}{16}y^4 \quad \left[\text{using } (a+b)(a-b) = a^2 - b^2\right] \end{aligned}$$

(vi) We have:

$$\begin{aligned} & \left(\frac{4x}{5} - \frac{5y}{3}\right)\left(\frac{4x}{5} + \frac{5y}{3}\right) \\ & = \frac{16x^2}{25} - \frac{25y^2}{9} \quad \left[\text{using } (a+b)(a-b) = a^2 - b^2\right] \end{aligned}$$

(vii) We have:

$$\begin{aligned} & \left(x + \frac{1}{x}\right) \left(x - \frac{1}{x}\right) \\ &= x^2 - \frac{1}{x^2} \qquad \left[\text{using } (a+b)(a-b) = a^2 - b^2\right] \end{aligned}$$

(viii) We have:

$$\begin{aligned} & \left(\frac{1}{x} + \frac{1}{y}\right) \left(\frac{1}{x} - \frac{1}{y}\right) \\ &= \frac{1}{x^2} - \frac{1}{y^2} \qquad \left[\text{using } (a+b)(a-b) = a^2 - b^2\right] \end{aligned}$$

(ix) We have:

$$\begin{aligned} & \left(2a + \frac{3}{b}\right) \left(2a - \frac{3}{b}\right) \\ &= 4a^2 - \frac{9}{b^2} \qquad \left[\text{using } (a+b)(a-b) = a^2 - b^2\right] \end{aligned}$$

Q5

Answer :

We shall use the identity $(a+b)^2 = a^2 + b^2 + 2ab$.

(i)

$$\begin{aligned} & (54)^2 \\ &= (50 + 4)^2 \\ &= (50)^2 + 2 \times 50 \times 4 + (4)^2 \\ &= 2500 + 400 + 16 \\ &= 2916 \end{aligned}$$

(ii)

$$\begin{aligned} & (82)^2 \\ &= (80 + 2)^2 \\ &= (80)^2 + 2 \times 80 \times 2 + (2)^2 \\ &= 6400 + 320 + 4 \\ &= 6724 \end{aligned}$$

(iii)

$$\begin{aligned} & (103)^2 \\ &= (100 + 3)^2 \\ &= (100)^2 + 2 \times 100 \times 3 + (3)^2 \\ &= 10000 + 600 + 9 \\ &= 10609 \end{aligned}$$

(iv)

$$\begin{aligned} & (704)^2 \\ &= (700 + 4)^2 \\ &= (700)^2 + 2 \times 700 \times 4 + (4)^2 \\ &= 490000 + 5600 + 16 \\ &= 495616 \end{aligned}$$

Q6

Answer :

We shall use the identity $(a-b)^2 = a^2 + b^2 - 2ab$.

(i)

$$\begin{aligned}(69)^2 &= (70 - 1)^2 \\ &= (70)^2 - 2 \times 70 \times 1 + 1 \\ &= 4900 - 140 + 1 \\ &= 4761\end{aligned}$$

(ii)

$$\begin{aligned}(78)^2 &= (80 - 2)^2 \\ &= (80)^2 - 2 \times 80 \times 2 + 4 \\ &= 6400 - 320 + 4 \\ &= 6084\end{aligned}$$

(iii)

$$\begin{aligned}(197)^2 &= (200 - 3)^2 \\ &= (200)^2 - 2 \times 200 \times 3 + 9 \\ &= 40000 - 1200 + 9 \\ &= 38809\end{aligned}$$

(iv)

$$\begin{aligned}(999)^2 &= (1000 - 1)^2 \\ &= (1000)^2 - 2 \times 1000 \times 1 + 1 \\ &= 1000000 - 2000 + 1 \\ &= 998001\end{aligned}$$

Q7

Answer :

We shall use the identity $(a-b)(a+b) = a^2 - b^2$.

(i)

$$\begin{aligned}(82)^2 - (18)^2 &= (82 - 18)(82 + 18) \\ &= (64)(100) \\ &= 6400\end{aligned}$$

(ii)

$$\begin{aligned}(128)^2 - (72)^2 &= (128 - 72)(128 + 72) \\ &= (56)(200) \\ &= 11200\end{aligned}$$

(iii)

$$\begin{aligned}197 \times 203 &= (200 - 3)(200 + 3) \\ &= (200)^2 - (3)^2 \\ &= 40000 - 9 \\ &= 39991\end{aligned}$$

$$\begin{aligned}
 & \text{(iv)} \\
 & \frac{198 \times 198 - 102 \times 102}{96} \\
 & = \frac{(198)^2 - (102)^2}{96} \\
 & = \frac{(198 - 102)(198 + 102)}{96} \\
 & = \frac{(96)(300)}{96} \\
 & = 300
 \end{aligned}$$

$$\begin{aligned}
 & \text{(v)} \\
 & (14.7 \times 15.3) \\
 & = (15 - 0.3) \times (15 + 0.3) \\
 & = (15)^2 - (0.3)^2 \\
 & = 225 - 0.09 \\
 & = 224.91
 \end{aligned}$$

$$\begin{aligned}
 & \text{(vi)} \\
 & (8.63)^2 - (1.37)^2 \\
 & = (8.63 - 1.37)(8.63 + 1.37) \\
 & = (7.26)(10) \\
 & = 72.6
 \end{aligned}$$

Q8

Answer :

$$\begin{aligned}
 & (9x^2 + 24x + 16) \\
 & \text{Given, } x = 12 \\
 & \Rightarrow (3x)^2 + 2(3x)(4) + (4)^2 \\
 & \Rightarrow (3x + 4)^2 \\
 & \Rightarrow (3(12) + 4)^2 \\
 & \Rightarrow (36 + 4)^2 \\
 & \Rightarrow (40)^2 = 1600
 \end{aligned}$$

Therefore, the value of the expression $(9x^2 + 24x + 16)$, when $x = 12$, is 1600.

Q9

Answer :

$$\begin{aligned}
 & (64x^2 + 81y^2 + 144xy) \\
 & \text{Given :} \\
 & x = 11 \\
 & y = \frac{4}{3} \\
 & \Rightarrow (8x)^2 + (9y)^2 + 2(8x)(9y) \\
 & \Rightarrow (8x + 9y)^2 \\
 & \Rightarrow \left(8\left(11\right) + 9\left(\frac{4}{3}\right)\right)^2 \\
 & \Rightarrow (88 + 12)^2 \\
 & \Rightarrow (100)^2 \\
 & \Rightarrow 10000
 \end{aligned}$$

Therefore, the value of the expression $(64x^2 + 81y^2 + 144xy)$, when $x = 11$ and $y = \frac{4}{3}$, is 10000. ^σ

Q10

Answer :

$$\begin{aligned}& (36x^2 + 25y^2 - 60xy) \\& \Rightarrow x = \frac{2}{3}, y = \frac{1}{5} \\& = (6x)^2 + (5y)^2 - 2(6x)(5y) \\& = (6x - 5y)^2 \\& = \left(6\left(\frac{2}{3}\right) - 5\left(\frac{1}{5}\right)\right)^2 \\& = (4 - 1)^2 \\& = (3)^2 \\& \Rightarrow 9\end{aligned}$$

Q11

Answer :

$$(i) \left(x + \frac{1}{x}\right) = 4$$

Squaring both the sides :

$$\begin{aligned}\Rightarrow \left(x + \frac{1}{x}\right)^2 &= (4)^2 \\ \Rightarrow \left(x^2 + \frac{1}{x^2} + 2(x)\left(\frac{1}{x}\right)\right) &= 16 \\ \Rightarrow \left(x^2 + \frac{1}{x^2}\right) + 2 &= 16 \\ \Rightarrow \left(x^2 + \frac{1}{x^2}\right) &= 16 - 2 \\ \Rightarrow \left(x^2 + \frac{1}{x^2}\right) &= 14\end{aligned}$$

Therefore, the value of $x^2 + \frac{1}{x^2}$ is 14.

$$\left(x^2 + \frac{1}{x^2}\right) = 14$$

Squaring both the sides :

$$\begin{aligned}\Rightarrow \left(x^4 + \frac{1}{x^4} + 2(x^2)\left(\frac{1}{x^2}\right)\right) &= (14)^2 \\ \Rightarrow \left(x^4 + \frac{1}{x^4}\right) + 2 &= 196 \\ \Rightarrow \left(x^4 + \frac{1}{x^4}\right) &= 196 - 2 \\ \Rightarrow \left(x^4 + \frac{1}{x^4}\right) &= 194\end{aligned}$$

Therefore, the value of $x^4 + \frac{1}{x^4}$ is 194.

Q12

Answer :

$$(i) \left(x - \frac{1}{x}\right) = 5$$

$\Rightarrow$ Squaring both the sides :

$$\begin{aligned}\Rightarrow \left(x - \frac{1}{x}\right)^2 &= (5)^2 \\ \Rightarrow \left(x^2 + \frac{1}{x^2} - 2(x)\left(\frac{1}{x}\right)\right) &= 25 \\ \Rightarrow \left(x^2 + \frac{1}{x^2}\right) - 2 &= 25 \\ \Rightarrow \left(x^2 + \frac{1}{x^2}\right) &= 25 + 2 \\ \Rightarrow \left(x^2 + \frac{1}{x^2}\right) &= 27\end{aligned}$$

Therefore, the value of $\left(x^2 + \frac{1}{x^2}\right)$ is 27.

$$\left(x^2 + \frac{1}{x^2}\right) = 27$$

$\Rightarrow$ Squaring both the sides :

$$\Rightarrow \left(x^4 + \frac{1}{x^4} - 2\left(x^2\right)\left(\frac{1}{x^2}\right)\right) = (27)^2$$

$$\Rightarrow \left(x^4 + \frac{1}{x^4}\right) - 2 = 729$$

$$\Rightarrow \left(x^4 + \frac{1}{x^4}\right) = 729 + 2$$

$$\Rightarrow \left(x^4 + \frac{1}{x^4}\right) = 731$$

Therefore, the value of $\left(x^4 + \frac{1}{x^4}\right)$ is 731.

Q13

Answer :

$$(i) (x+1)(x-1)(x^2+1)$$

$$\Rightarrow (x^2 - x + x - 1)(x^2 + 1)$$

$$\Rightarrow (x^2 - 1)(x^2 + 1)$$

$$\Rightarrow (x^2)^2 - (1^2)^2 \quad \left[\text{according to the formula } a^2 - b^2 = (a+b)(a-b) \right]$$

$$\Rightarrow x^4 - 1.$$

Therefore, the product of $(x+1)(x-1)(x^2+1)$ is $x^4 - 1$.

$$(ii) (x-3)(x+3)(x^2+9)$$

$$\Rightarrow ((x)^2 - (3)^2)(x^2 + 9) \quad \left[\text{according to the formula } a^2 - b^2 = (a+b)(a-b) \right]$$

$$\Rightarrow (x^2 - 9)(x^2 + 9)$$

$$\Rightarrow (x^2)^2 - (9)^2 \quad \left[\text{according to the formula } a^2 - b^2 = (a+b)(a-b) \right]$$

$$\Rightarrow x^4 - 81$$

Therefore, the product of $(x-3)(x+3)(x^2+9)$ is $x^4 - 81$.

$$(iii) (3x-2y)(3x+2y)(9x^2+4y^2)$$

$$\Rightarrow ((3x)^2 - (2y)^2)(9x^2 + 4y^2)$$

$$\left[\text{according to the formula } a^2 - b^2 = (a+b)(a-b) \right]$$

$$\Rightarrow (9x^2 - 4y^2)(9x^2 + 4y^2)$$

$$\Rightarrow (9x^2)^2 - (4y^2)^2 \quad \left[\text{according to the formula } a^2 - b^2 = (a+b)(a-b) \right]$$

$$\Rightarrow 81x^4 - 16y^4.$$

Therefore, the product of $(3x-2y)(3x+2y)(9x^2+4y^2)$ is $81x^4 - 16y^4$.

$$(iv) (2p+3)(2p-3)(4p^2+9)$$

$$\Rightarrow ((2p)^2 - (3)^2)(4p^2 + 9) \quad \left[\text{according to the formula } a^2 - b^2 = (a+b)(a-b) \right]$$

$$\Rightarrow (4p^2 - 9)(4p^2 + 9)$$

$$\Rightarrow (4p^2)^2 - (9)^2 \quad \left[\text{according to the formula } a^2 - b^2 = (a+b)(a-b) \right]$$

$$\Rightarrow 16p^4 - 81.$$

Therefore, the product of $(2p+3)(2p-3)(4p^2+9)$ is $16p^4 - 81$.

Q14

Answer :

$$x + y = 12$$

On squaring both the sides :

$$\Rightarrow (x + y)^2 = (12)^2$$

$$\Rightarrow x^2 + y^2 + 2xy = 144$$

$$\Rightarrow x^2 + y^2 = 144 - 2xy$$

Given :

$$xy = 14$$

$$\Rightarrow x^2 + y^2 = 144 - 2(14)$$

$$\Rightarrow x^2 + y^2 = 144 - 28$$

$$\Rightarrow x^2 + y^2 = 116$$

Therefore, the value of $x^2 + y^2$ is 116.

Q15

Answer :

$$x - y = 7$$

$\Rightarrow$ On squaring both the sides :

$$\Rightarrow (x - y)^2 = (7)^2$$

$$\Rightarrow x^2 + y^2 - 2xy = 49$$

$$\Rightarrow x^2 + y^2 = 49 + 2xy$$

Given :

$$xy = 9$$

$$\Rightarrow x^2 + y^2 = 49 + 2(9)$$

$$\Rightarrow x^2 + y^2 = 49 + 18$$

$$\Rightarrow x^2 + y^2 = 67.$$

Therefore, the value of $x^2 + y^2$ is 67.

Operations On Algebraic Expressions

Ex 6E

Q1

Answer :

(c) $(-6a + 17b)$

$$\begin{array}{r} 6a + 4b - c + 3 \\ + 2b - 3c + 4 \\ - 7a + 11b + 2c - 1 \\ - 5a \quad \quad + 2c - 6 \\ \hline -6a + 17b + 0c + 0 \end{array}$$

Q2

Answer :

(d) $(3p^2 + 5q - 9r^3 + 7)$

$$\begin{array}{r} 7p^2 + 3q - 2r^3 + 4 \\ 4p^2 - 2q + 7r^3 - 3 \\ - \quad + \quad - \quad + \\ \hline 3p^2 + 5q - 9r^3 + 7 \end{array}$$

Q3

Answer :

(d) $x^2 + 2x - 15$

$$\begin{aligned} & (x+5)(x-3) \\ & \Rightarrow (x)(x-3) + (5)(x-3) \\ & \Rightarrow x^2 - 3x + 5x - 15 \\ & \Rightarrow x^2 + 2x - 15 \end{aligned}$$

Q4

Answer :

(b) $(6x^2 + 7x - 3)$

$$\begin{aligned} & (2x + 3)(3x - 1) \\ & \Rightarrow (2x)(3x - 1) + (3)(3x - 1) \\ & \Rightarrow 6x^2 - 2x + 9x - 3 \\ & \Rightarrow 6x^2 + 7x - 3 \end{aligned}$$

Q5

Answer :

(c) $(x^2 + 8x + 16)$

$$\begin{aligned} & (x + 4)(x + 4) \\ & \Rightarrow (x + 4)^2 \quad \left(\text{according to the formula } (a + b)^2 = a^2 + 2ab + b^2 \right) \\ & \Rightarrow (x^2) + 2(x)(4) + (4)^2 \\ & \Rightarrow x^2 + 8x + 16 \end{aligned}$$

Q6

Answer :

(d) $(x^2 - 12x + 36)$

$$\begin{aligned} & (x - 6)(x - 6) \\ & \Rightarrow (x - 6)^2 \quad \left(\text{according to the formula } (a - b)^2 = a^2 - 2ab + b^2 \right) \\ & \Rightarrow (x^2) - 2(x)(6) + (6)^2 \\ & \Rightarrow x^2 - 12x + 36 \end{aligned}$$

Q7

Answer :

(b) $(4x^2 - 25)$

$$\begin{aligned} & (2x + 5)(2x - 5) \\ & \Rightarrow (2x)^2 - (5)^2 \quad \left(\text{according to the formula } (a + b)(a - b) = a^2 - b^2 \right) \\ & \Rightarrow 4x^2 - 25 \end{aligned}$$

Q8

Answer :

(c) $-4ab^2$

$$\begin{aligned} & 8a^2b^3 \div (-2ab) \\ & \Rightarrow \left(\frac{8}{-2} \right) (a^{2-1}) (b^{3-1}) \\ & \Rightarrow -4ab^2 \end{aligned}$$

Q9

Answer :

(b) $(2x + 1)$

$$\begin{array}{r} x + 1 \overline{) 2x^2 + 3x + 1} \left(\begin{array}{l} 2x + 1 \\ 2x^2 + 2x \\ \hline + 1x + 1 \\ + 1x + 1 \\ \hline - \\ \hline \times \end{array} \right) \end{array}$$

Q10

Answer :

(a) $(x - 2)$

$$\begin{array}{r} x-2 \overline{) \begin{array}{r} x^2-4x+4 \\ -x^2-2x \\ \hline -2x+4 \\ -2x+4 \\ \hline 0 \end{array} } (x-2 \end{array}$$

Q11

Answer :

(c) $(a^4 - 1)$

$$\begin{aligned} & (i) (a+1)(a-1)(a^2+1) \\ & \Rightarrow \left((a)^2 - (1)^2\right)(a^2+1) \quad \left[\text{according to the formula } a^2 - b^2 = (a+b)(a-b)\right] \\ & \Rightarrow (a^2-1)(a^2+1) \\ & \Rightarrow (a^2)^2 - (1^2)^2 \quad \left[\text{according to the formula } a^2 - b^2 = (a+b)(a-b)\right] \\ & \Rightarrow a^4 - 1 \end{aligned}$$

Q12

Answer :

a) $\left(\frac{1}{x^2} - \frac{1}{y^2}\right)$

$$\begin{aligned} & \left(\frac{1}{x} + \frac{1}{y}\right)\left(\frac{1}{x} - \frac{1}{y}\right) \\ & \Rightarrow \text{According to the formula } (a+b)(a-b) = (a)^2 - (b)^2 : \\ & \Rightarrow \left(\frac{1}{x^2} - \frac{1}{y^2}\right) \end{aligned}$$

Q13

Answer :

(c) 23

$$\begin{aligned} & \left(x + \frac{1}{x}\right) = 5 \\ & \Rightarrow \text{Squaring both the sides :} \\ & \Rightarrow \left(x + \frac{1}{x}\right)^2 = (5)^2 \\ & \Rightarrow \left(x^2 + \frac{1}{x^2} + 2(x)\left(\frac{1}{x}\right)\right) = 25 \\ & \Rightarrow \left(x^2 + \frac{1}{x^2}\right) + 2 = 25 \\ & \Rightarrow \left(x^2 + \frac{1}{x^2}\right) = 25 - 2 \\ & \Rightarrow \left(x^2 + \frac{1}{x^2}\right) = 23 \end{aligned}$$

Q14

Answer :

(b) 38

$$\begin{aligned} & \left(x - \frac{1}{x}\right) = 6 \\ & \Rightarrow \text{Squaring both the sides :} \\ & \Rightarrow \left(x - \frac{1}{x}\right)^2 = (6)^2 \\ & \Rightarrow \left(x^2 + \frac{1}{x^2} - 2(x)\left(\frac{1}{x}\right)\right) = 36 \\ & \Rightarrow \left(x^2 + \frac{1}{x^2}\right) - 2 = 36 \\ & \Rightarrow \left(x^2 + \frac{1}{x^2}\right) = 36 + 2 \\ & \Rightarrow \left(x^2 + \frac{1}{x^2}\right) = 38 \end{aligned}$$

Q15

Answer :

(c) 6400

$$\begin{aligned}(82)^2 - (18)^2 & \quad \text{[using the identity } (a-b)(a+b)=a^2 -b^2\text{]} \\&= (82 + 18)(82 - 18) \\&= (100)(64) \\&= 6400\end{aligned}$$

Q16

Answer :

(a) 39991

$$\begin{aligned}(197) \times (203) & \quad \text{[using the identity } (a+b)(a-b) = a^2 -b^2\text{]} \\&\Rightarrow (200 - 3)(200 + 3) \\&\Rightarrow (200)^2 - (3)^2 \\&\Rightarrow 40000 - 9 \\&\Rightarrow 39991\end{aligned}$$

Q17

Answer :

(b) 116

$$\begin{aligned}(a + b) &= 12 \\&\Rightarrow \text{Squaring both the sides :} \\&\Rightarrow (a + b)^2 = (12)^2 \\&\Rightarrow (a^2 + b^2 + 2ab) = 144 \\&\Rightarrow (a^2 + b^2) = 144 - 2ab \\&\Rightarrow (a^2 + b^2) = 144 - 2(14) \\&\Rightarrow (a^2 + b^2) = 144 - 28 \\&\Rightarrow (a^2 + b^2) = 116\end{aligned}$$

Q18

Answer :

(a) 67

$$\begin{aligned}(a - b) &= 7 \\&\Rightarrow \text{Squaring both the sides :} \\&\Rightarrow (a - b)^2 = (7)^2 \\&\Rightarrow (a^2 + b^2 - 2ab) = 49 \\&\Rightarrow (a^2 + b^2) = 49 + 2ab \\&\Rightarrow (a^2 + b^2) = 49 + 2(9) \\&\Rightarrow (a^2 + b^2) = 49 + 18 \\&\Rightarrow (a^2 + b^2) = 67\end{aligned}$$

Q19 **Answer :**

(c) 625

$$\begin{aligned}(4x^2 + 20x + 25) \\&\Rightarrow (2x)^2 + 2(2x)(5) + (5)^2 \\&\Rightarrow (2x + 5)^2 \\&\Rightarrow (2(10) + 5)^2 \\&\Rightarrow (20 + 5)^2 \\&\Rightarrow (25)^2 \\&\Rightarrow 625\end{aligned}$$