

5. HYPERBOLA

1. **Standard Equation:** Standard equation of the hyperbola is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, where $b^2 = a^2(e^2 - 1)$.

Foci: $S \equiv (\pm ae, 0)$ **Directrices:** $x = \pm \frac{a}{e}$

Vertices: $A \equiv (\pm a, 0)$

Latus Rectum (ℓ): $\ell = \frac{2b^2}{a} = 2a(e^2 - 1)$.

2. **Conjugate Hyperbola:** $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ & $-\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ are conjugate hyperbolas of each.

3. **Auxiliary Circle:** $x^2 + y^2 = a^2$.

4. **Parametric Representation:** $x = a \sec \theta$ & $y = b \tan \theta$

5. **Position of A Point 'P' w.r.t. A Hyperbola:**

$S_1 \equiv \frac{x_1^2}{a^2} - \frac{y_1^2}{b^2} - 1$ $>$, $=$ or $<$ 0 according as the point (x_1, y_1) lies inside, on or outside the curve.

6. **Tangents:**

(i) **Slope Form:** $y = m x \pm \sqrt{a^2 m^2 - b^2}$

(ii) **Point Form:** at the point (x_1, y_1) is $\frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1$.

(iii) **Parametric Form:** $\frac{x \sec \theta}{a} - \frac{y \tan \theta}{b} = 1$.

7. **Normals:**

(a) at the point $P(x_1, y_1)$ is $\frac{a^2 x}{x_1} + \frac{b^2 y}{y_1} = a^2 + b^2 = a^2 e^2$.

(b) at the point $P(a \sec \theta, b \tan \theta)$ is $\frac{ax}{\sec \theta} + \frac{by}{\tan \theta} = a^2 + b^2 = a^2 e^2$.

(c) Equation of normals in terms of its slope 'm' are $y = mx \pm \frac{(a^2 + b^2)m}{\sqrt{a^2 - b^2 m^2}}$.

8. **Asymptotes:** $\frac{x}{a} + \frac{y}{b} = 0$ and $\frac{x}{a} - \frac{y}{b} = 0$. Pair of asymptotes: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 0$.

9. **Rectangular Or Equilateral Hyperbola:** $xy = c^2$, eccentricity is $\sqrt{2}$.

Vertices: $(\pm c, \pm c)$; **Foci:** $(\pm \sqrt{2}c, \pm \sqrt{2}c)$. **Directrices:** $x + y = \pm \sqrt{2} c$

Latus Rectum (ℓ): $\ell = 2\sqrt{2} c = T.A. = C.A.$

Parametric equation $x = ct$, $y = c/t$, $t \in \mathbb{R} - \{0\}$

Equation of the tangent at $P(x_1, y_1)$ is $\frac{x}{x_1} + \frac{y}{y_1} = 2$ & at $P(t)$ is $\frac{x}{t} + ty = 2c$.

Equation of the normal at $P(t)$ is $xt^3 - yt = c(t^4 - 1)$.

Chord with a given middle point as (h, k) is $kx + hy = 2hk$.