

# Polynomial

## Polynomial

An expression of the form  $p(x) = a_0 + a_1x + a_2x^2 + \dots + a_nx^n$  where  $a_n \neq 0$  is called a polynomial in one variable  $x$  of degree  $n$ , where;  $a_0, a_1, a_2, \dots, a_n$  are constants and they are called the coefficients of  $x_0, x, x^2, \dots, x^n$ . Each power of  $x$  is a non-negative integer.

Eg:  $-2x^2 - 5x + 1$  is a polynomial of degree 2

Note:  $\sqrt{x} + 3$  is not a polynomial

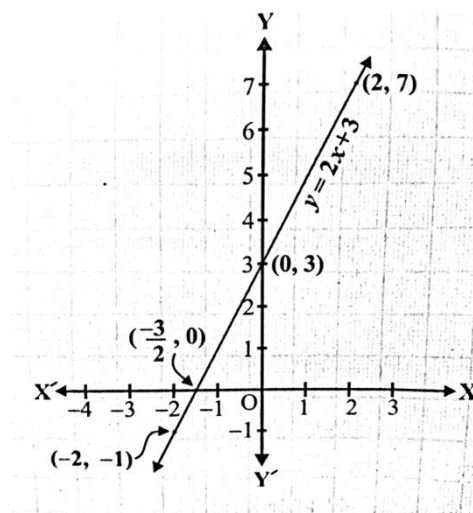
- A polynomial  $p(x) = ax + b$  of degree 1 is called a linear polynomial Eg:  $5x - 3, 2x$  etc
- A polynomial  $p(x) = ax^2 + bx + c$  of degree 2 is called a quadratic polynomial Eg:  $2x^2 + x - 1$
- A polynomial  $p(x) = ax^3 + bx^2 + cx + d$  of degree 3 is called a cubic polynomial.  
Eg:  $\sqrt{3}x^3 - x + \sqrt{5}, x^3 - 1$  etc

Zeros of a polynomial: A real number  $k$  is called a zero of polynomial  $p(x)$  if  $p(k)=0$ . If the graph of  $y= p(x)$  intersects the  $X$ -axis at  $n$  times, the number of zeroes of  $y= p(x)$  is  $n$ .

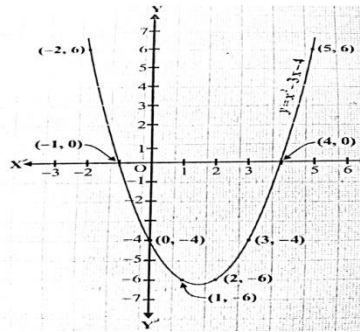
- A linear polynomial has only one zero.
- A quadratic polynomial has two zeroes.
- A cubic polynomial has three zeroes.

### Graphs of different types of polynomials:

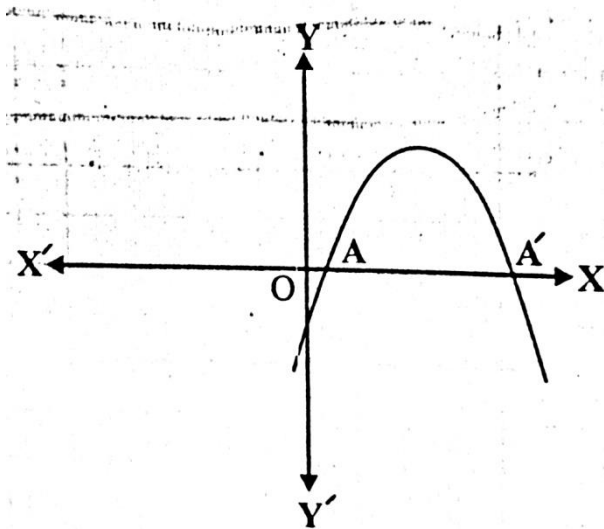
- Linear polynomial:- The graph of a linear polynomial  $ax+b$  is a straight line, intersecting  $X$ - axis at one point



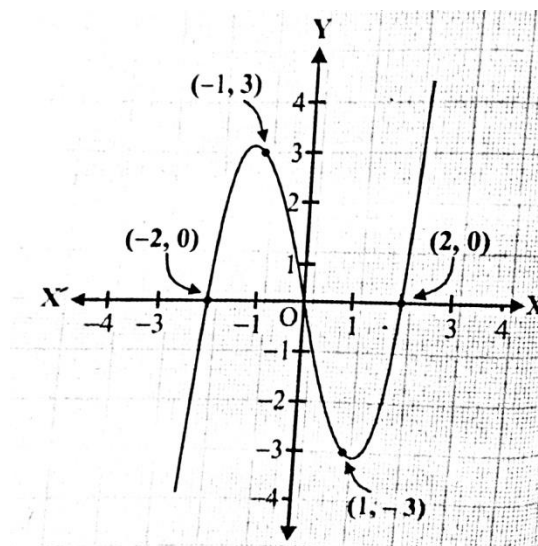
- Quadratic polynomial:-  
(i) Graph of a quadratic polynomial  $p(x) = ax^2 + bx + c$  is a parabola open upwards like U, if  $a > 0$  & intersects  $x$ -axis at maximum two distinct points.



(ii) Graph of a quadratic polynomial  $p(x) = ax^2 + bx + c$  is a parabola open downwards like  $\cap$  if  $a < 0$  & intersects x-axis at maximum two distinct points



- Cubic polynomial and its graph:- in general a polynomial  $p(x)$  of degree  $n$  crosses the x-axis at most  $n$  points.



For a quadratic polynomial:- If  $\alpha, \beta$  are zeroes of  $p(x) = ax^2 + bx + c$  then,

1. Sum of zeroes  $= \alpha + \beta = -\frac{b}{a} = \frac{\text{-coefficients of } x}{\text{coefficient of } x^2}$
2. Product of zeroes  $= \alpha \cdot \beta = \frac{c}{a} = \frac{\text{constant term}}{\text{coefficient of } x^2}$

- A quadratic polynomial whose zeroes are  $\alpha$  and  $\beta$ , is given by:

$$p(x) = x^2 - (\alpha + \beta)x + \alpha\beta$$

- If  $\alpha, \beta$  and  $\gamma$  are zeroes of the cubic polynomial  $ax^3 + bx^2 + cx + d$  then:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$$

$$\alpha\beta\gamma = -\frac{d}{a}$$

- If  $\alpha, \beta, \gamma$  are zeroes of a cubic polynomial  $p(x)$ ,

$$p(x) = x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma$$

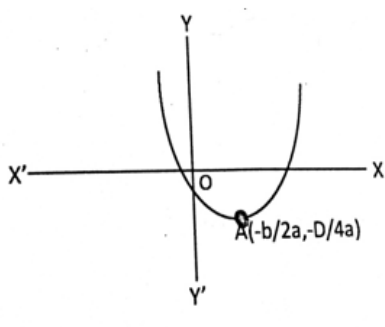
**Division algorithm for polynomials:** If  $p(x)$  and  $g(x)$  are any two polynomials with  $g(x) \neq 0$ , then we have polynomials  $q(x)$  and  $r(x)$  such that

$$P(x) = g(x) \times q(x) + r(x), \quad \text{where } r(x) = 0 \text{ or degree of } r(x) < \text{degree of } g(x).$$

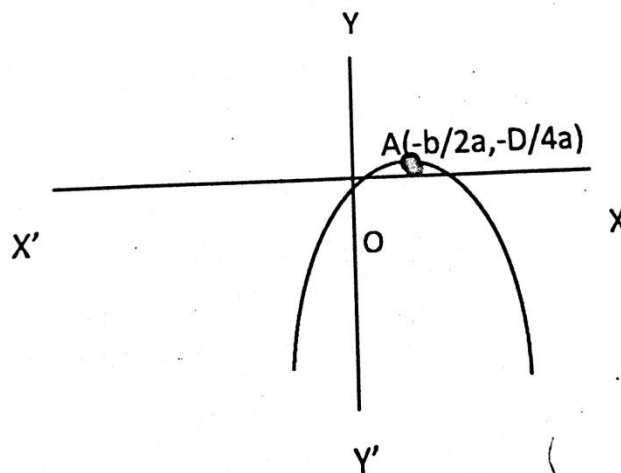
**Nature of graph of polynomial  $P(x) = ax^2 + bx + c$ :-**

Case-1 When polynomial  $ax^2 + bx + c$  is factorable in two distinct linear factors.

In this case, curve cuts X-axis at two distinct points. The co-ordinate of the vertex of parabola are  $(-b/2a, -D/4a)$  where  $D = b^2 - 4ac$ . The x co-ordinates of these points are the two zeroes of the polynomial.



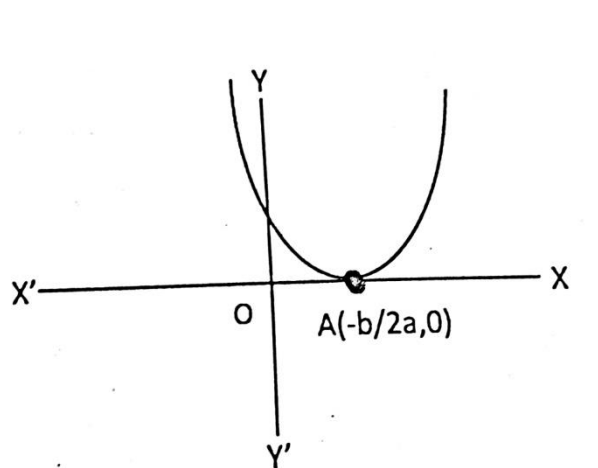
(i)  $a > 0$



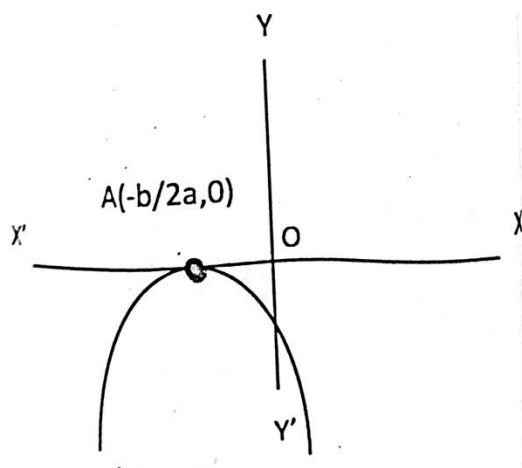
(ii)  $a < 0$

Case 2:- When Polynomial  $ax^2 + bx + c$  is factorisable into two equal factors.

In this case, curve touches X-axis at the point  $(-b/2a, 0)$ . The x- Co-ordinates of the point gives two equal zeroes of the polynomial.

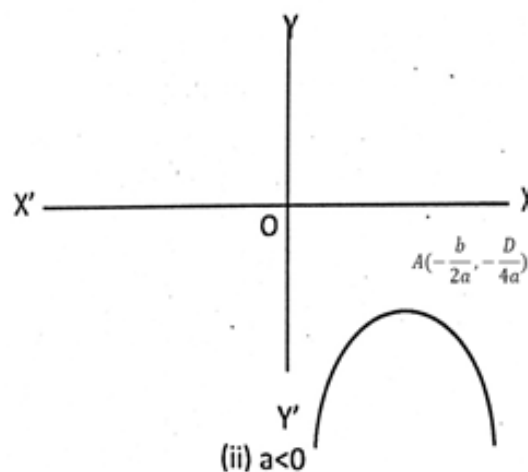
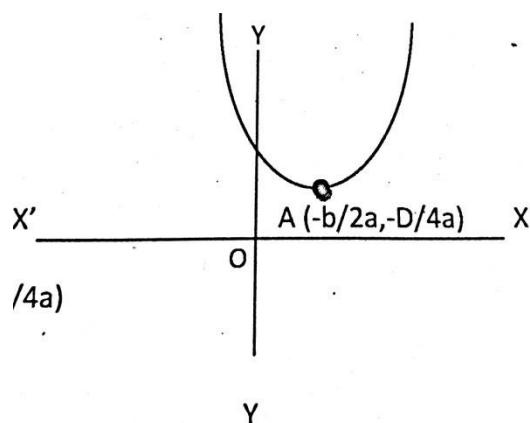


(i)  $a > 0$



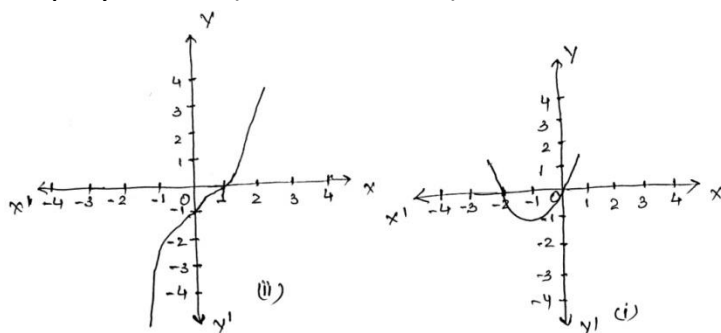
(ii)  $a < 0$

Case- 3 When Polynomial  $ax^2 + bx + c$  is not factorizable. In this case, the curve doesn't cut or touches X-axis



## Level – I

- Find the value of zeroes of the polynomials  $p(x)$  as shown in the graph and hence find the polynomial. (CBSE 2014-15).



- Let  $\alpha$  and  $\beta$  are the zeroes of a quadratic polynomial  $2x^2 - 5x - 6$  then form a quadratic polynomial whose zeroes are  $\alpha + \beta$  and  $\alpha\beta$ . (CBSE 2011)

3. Check whether  $x^2 + 3x + 1$  is a factor of  $3x^4 + 5x^3 - 7x^2 + 2x + 2$ ?  
(CBSE 2010)
4. Can  $(x-7)$  be the remainder on division of a polynomial  $p(x)$  by  $(7x + 2)$ ? Justify your answer(CBSE 2010)
5. What must be subtracted from the polynomial  $f(x) = x^4 + 2x^3 - 13x^2 - 12x + 21$ , so that the resulting polynomial is exactly divisible by  $x^2 - 4x + 3$ ? (CBSE 2013)
6. Write the degree of zero polynomial?
7. Find the zeroes of a quadratic polynomial  $6x^2 - 7x - 3$  and verify the relationship between the zeroes and the coefficients? (CBSE 2014-15)
8. Find the quadratic polynomial sum of whose zeroes is  $2\sqrt{3}$  and their product is 2?(CBSE 2008)

## Level II

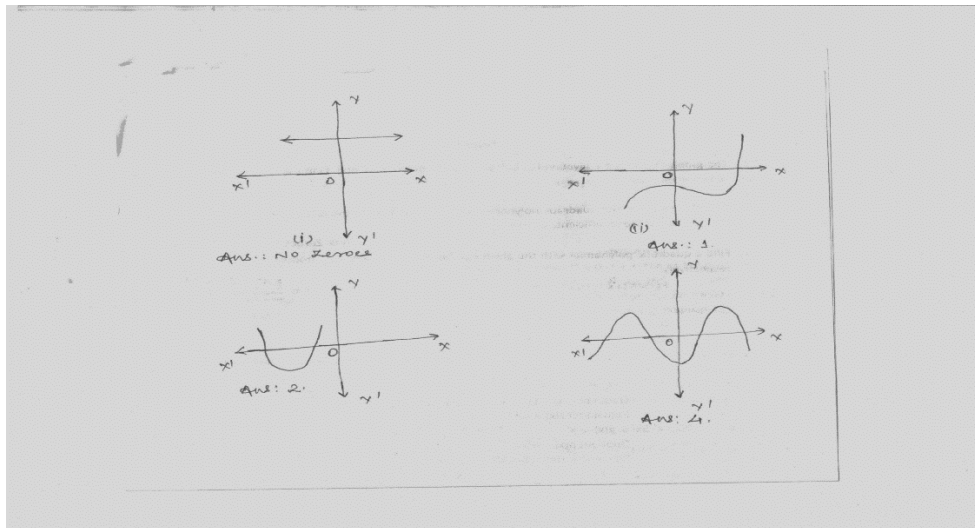
9. If the sum of squares of the zeroes of the polynomials  $6x^2 + x + k$  is  $\frac{25}{36}$ , find the value of k?( CBSE 2014-15)
10. If one zero of the quadratic polynomial  $f(x) = 4x^2 - 8kx - 9$  is negative of the other, then find the value of k?(CBSE 2014-15)
11. Find the values of k for which the quadratic equation  $9x^2 - 3kx + k = 0$  has equal roots. (CBSE 2014)
12. On dividing  $3x^3 - 2x^2 + 5x + 5$  by the polynomial  $p(x)$ , the quotient and remainder are  $x^2 - x + 2$  and  $-7$  respectively. Find  $p(x)$ ? (CBSE 2013)
13. Find all the zeroes of the polynomial  $x^4 + x^3 - 9x^2 - 3x + 18$ , if two of its zeroes are  $\sqrt{3}$  and  $\sqrt{-3}$ . (CBSE 2010,13)
14. If  $\alpha, \beta$  are zeroes of the quadratic polynomial  $p(x) = x^2 - (k - 6)x + (2k + 1)$ . Find the value of k if  $\alpha + \beta = \alpha\beta$ . (CBSE 2010)
15. If the zeroes of the polynomial  $x^2 - 5x + k$  are the reciprocal of each other, then find the value of K? (CBSE 2011)
16. If  $\alpha$  and  $\beta$  are zeroes of the quadratic polynomial  $x^2 - 6x + a$ , find the value of  $a'$ . If  $3\alpha + 2\beta = 20$ .(CBSE 2010)

### LEVEL III

17. On dividing  $3x^3 + 4x^2 + 5x - 13$  by a polynomial  $g(x)$ , the quotient and remainder are  $3x + 10$  and  $16x - 43$  respectively. Find the polynomial  $g(x)$ . (CBSE 14-15)
18. If  $-5$  is a root of quadratic equation  $2x^2 + px - 15 = 0$  and the quadratic equation  $p(x^2 + x)k = 0$  has equal roots, find the value of  $k$ . (CBSE 2106)
19. If  $\alpha, \beta$  and  $\gamma$  are zeroes of the polynomial  $6x^3 + 3x^2 - 5x + 1$ , then find the values of  $\alpha^{-1} + \beta^{-1} + \gamma^{-1}$ . (CBSE 2010)
20. Form a cubic polynomial whose zeroes are 3, 2 and  $-1$ . Hence find
- Sum of its zeroes
  - Sum of the product, taken two at a time
  - Product of its zero.

### (SELF EVALUATION QUESTIONS)

21. Find the number of zeroes of  $p(x)$  in each case, for some polynomials  $p(x)$ .



22. If  $\alpha$  and  $\beta$  are the zeroes of the equation  $6x^2 + x - 2 = 0$ , find  $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$
23. If one of the zeroes of the polynomial  $2x^2 + px + 4 = 0$  is 2, find the other zero, also find the value of  $p$
24. If one zero of the polynomial  $(a^2 + 9)x^2 + 13x + 6a$  is reciprocal of the other. Find the value of  $a$ . (All India)

## **Value Based Questions**

25. If  $\alpha$  be the number of person who take junk food,  $\beta$  be the person who take food at home and  $\alpha$  and  $\beta$  be the zeroes of quadratic polynomial  $f(x) = x^2 - 3x + 2$ , then find a quadratic polynomial whose zeroes are  $\frac{1}{2\alpha+\beta}$  and  $\frac{1}{2\beta+\alpha}$ , which way of taking food you prefer and why?
26. If the number of apples and mangoes are the zeroes of the polynomial  $3x^2 = 8x - 2k + 1$  and the number of apples is 7 times the number of mangoes, then find the number of zeroes and value of  $k$ . What are benefits of fruits in our daily life?