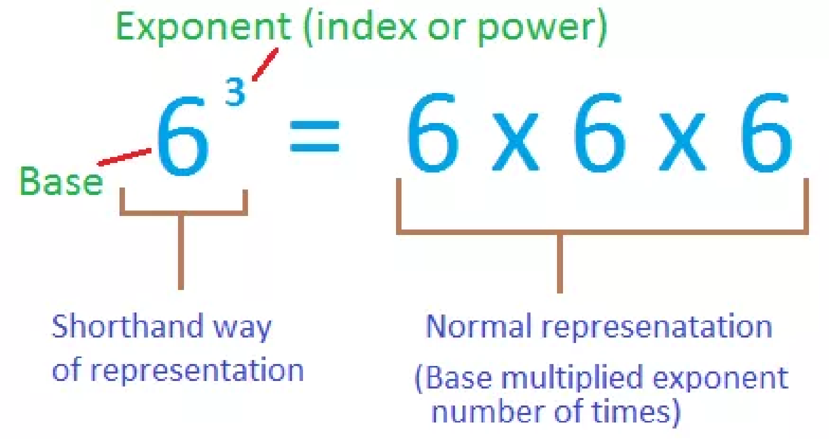
4. Exponents

What is an Exponent?

Exponents:

The repeated addition of numbers can be written in short form (product form).



Examples:

S.No.	Statements	Repeated Addition	Products Form
(i)	4 times 2	$2+2+2+2$	4×2
(ii)	5 time – 1	$(-1)+(-1)+(-1)+(-1)+(-1)$	$5 \times (-1)$
(iii)	3 times $\frac{-2}{3}$	$\left(\frac{-2}{3}\right)+\left(\frac{-2}{3}\right)+\left(\frac{-2}{3}\right)$	$3 \times \left(\frac{-2}{3}\right)$
(iv)	2 times 1	$1+1$	2×1

Also, we can write the repeated multiplication of numbers in a short form known as exponential form. For example, when 5 is multiplied by itself for two times, we write the product 5 × 5 in exponential form as 5^2 which is read as 5 raised to the power two.

Similarly, if we multiply 5 by itself for 6 times, the product 5 × 5 × 5 × 5 × 5 × 5 is written in exponential form as 5^6 which is read as 5 raised to the power 6.
In 5^6, the number 5 is called the base of 5^6 and 6 is called the **exponent of the base**.

In general, we write,
An exponential number as b^a, where b is the base and a is the exponent.
The notation of writing the multiplication of a number by itself several times is called the **exponential notation** or **power notation**.

Thus, in general we find that :
If 'a' is a rational number then 'n' times the product of 'a' by itself is given as a × a × a × a ….. , n times and is denoted by a^n, where 'a' is called the base and n is called the exponent of a^n.

Examples

1. Write the following statements as repeated multiplication and complete the table:

S.No.	Statements	Repeated Multiplication	Short form
(i)	3 multiplied by 3 for 6 times	3 × 3 × 3 × 3 × 3 × 3 = 729	3^6
(ii)	2 multiplied by 2 for 3 times	2 × 2 × 2	2^3
(iii)	1 multiplied by 1 for 7 times	1 × 1 × 1 × 1 × 1 × 1 × 1	1^7

2. Write the base and exponent of following numbers. And also write in expanded form:

S.No.	Numbers	Base	Exponent	Expanded Form	Value
(i)	3^4	3	4	3 × 3 × 3 × 3	81
(ii)	2^5	2	5	2 × 2 × 2 × 2 × 2	32
(iii)	3^3	3	3	3 × 3 × 3	27
(iv)	2^2	2	2	2 × 2	4
(v)	1^7	1	7	1 × 1 × 1 × 1 × 1 × 1 × 1	1

Exponents of Negative Integers

When the exponent of a negative integer is odd, the resultant is a negative number, and when the power of a negative number is even, the resultant is a positive number.When the exponent of a negative integer is odd, the resultant is a negative number, and when the power of a negative number is even, the resultant is a positive number.
or (a negative integer) an odd number = a negative integer.
(a negative integer) an even number = a positive integer.

Examples:

Ex.1 Express 144 in the powers of prime factors.

Solution:
144 = 16 × 9 = 2 × 2 × 2 × 2 × 3 × 3
Here 2 is multiplied four times and 3 is multiplied 2 times to get 144.
∴ $144 = 2^4 \times 3^2$

Ex.2 Which one is greater : 3^5 or 5^3 ?

Solution:
$3^5 = 3 \times 3 \times 3 \times 3 \times 3 = 9 \times 9 \times 3$
= 81 × 3 = 243
and $5^3 = 5 \times 5 \times 5 = 25 \times 5 = 125$
Clearly, 243 > 125
∴ $3^5 > 5^3$

What are Laws of Exponents?

Laws of Exponents

Law-1:

If a is any non-zero integer and m and n are whole numbers, then

$$a^m \times a^n = a^{m+n}$$

Eg :

$$\text{(i)}\quad 3^4 \times 3^2 = \underbrace{(3\times3\times3\times3)}_{\substack{\text{4 times multiplication}\\ \text{of 3 by itself}}} \times \underbrace{(3\times3)}_{\substack{\text{2 times multiplication}\\ \text{of 3 by itself}}}$$

$$= \underbrace{3\times3\times3\times3\times3\times3}_{\substack{\text{6 times multiplication}\\ \text{of 3 by itself}}} = 3^6 = 3^{4+2}$$

$$\text{Thus, } 3^4 \times 3^2 = 3^{4+2}$$

$$\text{(ii)}\quad 2^3 \times 2^5 = \underbrace{(2\times2\times2)}_{\substack{\text{3 times multiplication}\\ \text{of 2 by itself}}} \times \underbrace{(2\times2\times2\times2\times2)}_{\substack{\text{5 times multiplication}\\ \text{of 2 by itself}}}$$

$$= \underbrace{2\times2\times2\times2\times2\times2\times2\times2}_{\substack{\text{8 times multiplication}\\ \text{of 2 by itself}}}$$

$$= 2^8 = 2^{3+5}$$

$$\text{Thus, } 2^3 \times 2^5 = 2^{3+5}$$

Therefore, in general, we write,

$$a^m \times a^n = \underbrace{(a\times a\times a\times a\times \ldots)}_{\substack{\text{m times multiplication}\\ \text{of `a' by itself}}} \times \underbrace{(a\times a\times a\times a\times \ldots)}_{\substack{\text{n times multiplication}\\ \text{of `a' by itself}}}$$

$$= a \times a \times a \times a \times a \times \ldots, (m+n) \text{ times} = a^{m+n}.$$

Law-2:

If a and b are non-zero integers and m is a positive integer, then

$a^m \times b^m = (a \times b)^m$

Eg :
$5^3 \times 3^3 = (5 \times 5 \times 5) \times (3 \times 3 \times 3)$
$= (5 \times 3) \times (5 \times 3) \times (5 \times 3)$
$= 15 \times 15 \times 15 = (15)^3$
So, $5^3 \times 3^3 = (5 \times 3)^3 = (15)^3$
Here, we find that 15 is the product of bases 5 and 3.
Also, if a and b are non-zero integers, then
$a^5 \times b^5 = (a \times a \times a \times a \times a) \times (b \times b \times b \times b \times b)$
$= (a \times b) (a \times b) (a \times b) (a \times b) (a \times b) = (ab)^5$

Law-3:
If a is a non-zero integer and m and n are two whole numbers such that m > n, then

$a^m \div a^n = a^{m-n}$ and for $m < n$ For example, $2^5 \div 2^7 = \dfrac{2\times2\times2\times2\times2}{2\times2\times2\times2\times2\times2\times2}$

$a^m \div a^n = (a)^{m-n} = \dfrac{1}{a^{n-m}}$ $= \dfrac{1}{2\times2} = \dfrac{1}{2^2} = \dfrac{1}{2^{7-5}}$

When an exponential form is divided by another exponential form whose bases are same, then the resultant is an exponential form with same base but the exponent is the difference of the exponent of the divisor from the exponent of the dividend.

Law-4:

Division of exponential forms with the same exponents and different base:
If a and b are any two non-zero integers, have same exponent m then for $a^m \div b^m$, we write

$$\frac{a^m}{b^m} = \frac{a\times a\times a\times ..., \text{m times}}{b\times b\times b\times ..., \text{m times}}$$

$$= \left(\frac{a}{b}\right)\times\left(\frac{a}{b}\right)\times\left(\frac{a}{b}\right)\times ..., \text{m times} = \left(\frac{a}{b}\right)^m$$

For examples

(i) $$2^6 \div 3^6 = \frac{2^6}{3^6} = \frac{2\times2\times2\times2\times2\times2}{3\times3\times3\times3\times3\times3}$$

$$= \frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3} = \left(\frac{2}{3}\right)^6$$

Hence, $2^6 \div 3^6 = \left(\dfrac{2}{3}\right)^6$

(ii) $$(-2)^4 \div b^4 = \frac{(-2)^4}{b^4} = \frac{-2\times-2\times-2\times-2}{b\times b\times b\times b}$$

$$= \frac{-2}{b}\times\frac{-2}{b}\times\frac{-2}{b}\times\frac{-2}{b} = \left(-\frac{2}{b}\right)^4$$

Hence, $(-2)^4 \div b^4 = \left(-\dfrac{2}{b}\right)^4$

Law-5:
If ‘a’ be any non-zero integer and m and n any two positive integers then

$[(a)^m]^n = a^{mn}$

Eg :
$(2^2)^3 = 2^2 \times 2^2 \times 2^2 = 2^{2+2+2} = 2^6 = 2^{2\times3}$
$(2^7)^2 = 2^7 \times 2^7 = 2^{7+7} = 2^{14} = 2^{7\times2}$

Law-6:

Law of zero Exponent:
We know that

$$2^6 \div 2^6 = \frac{2^6}{2^6} = \frac{2\times2\times2\times2\times2\times2}{2\times2\times2\times2\times2\times2} = 1$$

By using Law-3 of exponents, we have
$2^6 \div 2^6 = 2^{6-6} = 2^0$
Thus, $2^0 = 1$
In general $a^m \div a^m = a^{m-m} = a^0$ and also

$$\frac{a^m}{a^m} = \frac{a\times a\times a\times a\times a\times a\times\text{...., m times}}{a\times a\times a\times a\times a\times a\times\text{...., m times}} = 1$$

Hence, $a^0 = 1$
Any non-zero integer raised to the power 0 always results into 1.

Use Of Exponents In Expressing Large Numbers

We know that
$100 = 10 \times 10 = 10^2$,
$1000 = 10 \times 10 \times 10 = 10^3$,
$10000 = 10 \times 10 \times 10 \times 10 = 10^4$
We can write a number followed by large number of zeroes in powers of 10.
For example, we can write the speed of light in vacuum = 300,000,000 m/s
$= 3 \times 1,00,000,000$ m/s $= 3 \times 10^8$ m/s
$= 30 \times 10^7$ m/s $= 300 \times 10^6$ m/s
Similarly,
the age of universe = 8,000,000,000 years (app.)
$= 8 \times 10^9$ years (app.)
We can also express the age of universe as
80×10^8 years or 800×10^7 years, etc.
But generally the number which preceded the power of 10 should be less than 10. Such a notation is called **standard or scientific** notation.
So 8×10^9 years is the standard form of the age of the universe.
Similarly, the standard form of the speed of light is 3×10^8 m/s.

Eg: Write the following numbers in standard form :

(i) 4340000
(ii) 173000
(iii) 140000

Solution:

(i) It is clear that $4340000 = 434 \times 10000$
Also, $4340000 = 4.34 \times 10^6$
$\because$ $434 = 4.34 \times 100 = 4.34 \times 10^2$
(ii) Also, $173000 = 1.73 \times 10^5$
(iii) Also, $140000 = 1.4 \times 10^5$

Laws of Exponents Problems with Solutions

1. Write in exponential form :
(i) $(5 \times 7)^6$ (ii) $(-7n)^5$

Solution:

(i) $(5 \times 7)^6$
$= (5 \times 7)(5 \times 7)(5 \times 7)(5 \times 7)(5 \times 7)(5 \times 7)$
$= (5 \times 5 \times 5 \times 5 \times 5 \times 5)(7 \times 7 \times 7 \times 7 \times 7 \times 7) = 5^6 \times 7^6$
Hence, $(5 \times 7)^6 = 5^6 \times 7^6$
(ii) $(-7n)^5 = (-7n)(-7n)(-7n)(-7n)(-7n)$
$= (-7 \times -7 \times -7 \times -7 \times -7)(n \times n \times n \times n \times n)$
$= (-7)^5 \times (n)^5$

2. Write the following in expanded form :

(i) $\left(-\frac{7}{9}\right)^3$ (ii) $\left(\frac{5}{8}\right)^6$

Solution:

$$\text{(i) } \left(-\frac{7}{9}\right)^3 = \frac{-7}{9} \times \frac{-7}{9} \times \frac{-7}{9}$$

$$= \frac{-7 \times -7 \times -7}{9 \times 9 \times 9} = \frac{(-7)^3}{9^3}$$

$$\text{(ii) } \left(\frac{5}{8}\right)^6 = \frac{5}{8} \times \frac{5}{8} \times \frac{5}{8} \times \frac{5}{8} \times \frac{5}{8} \times \frac{5}{8}$$

$$= \frac{5 \times 5 \times 5 \times 5 \times 5 \times 5}{8 \times 8 \times 8 \times 8 \times 8 \times 8} = \frac{5^6}{8^6}$$

3. Find the value of :

(i) $(3^0 - 2^0) \times 5^0$
(ii) $2^0 \times 3^0 \times 4^0$
(iii) $(6^0 - 2^0) \times (6^0 + 2^0)$

Solution:

(i) We have, $(3^0 - 2^0) \times 5^0$
Therefore, $(1 - 1) \times 1 = 0 \times 1 = 0$
[Since $3^0 = 1$, $2^0 = 1$]
(ii) We have, $2^0 \times 3^0 \times 4^0 = (1 \times 1 \times 1) = 1$
(iii) We have, $(6^0 - 2^0) \times (6^0 + 2^0)$
$= (1 - 1) \times (1 + 1)$
$= 0 \times 2 = 0.$

Review of Exponents

1. Exponents are the mathematician's shorthand.
2. In general, the format for using exponents is:
   $(\text{base})^{\text{exponent}}$
   where the exponent tells you how many of the **base** are being multiplied together.
3. Consider: 2 • 2 • 2 is the same as 2^3, since there are **three 2's** being multiplied together.
   Likewise, 5 • 5 • 5 • 5 = 54, because there are **four 5's** being multiplied together.
4. Exponents are also referred to as "powers".
   For example, 2^3 can be read as "two cubed" or as "two raised to the third power".

Laws of Exponents		
product	$a^m \cdot a^n = a^{m+n}$	$2^2 \cdot 2^3 = (2\cdot2)(2\cdot2\cdot2) = 2^5$
quotient	$\frac{a^m}{a^n} = a^{m-n}$	$\frac{2^3}{2^2} = \frac{2\cdot\not{2}\cdot\not{2}}{\not{2}\cdot\not{2}} = 2^{3-1} = 2$
power	$(a^m)^n = a^{m\cdot n}$	$(2^2)^3 = (2\cdot2\cdot2)(2\cdot2\cdot2)(2\cdot2\cdot2) = 2^6$
inverse	$a^{-1} = \frac{1}{a}$	$2^{-1} = \frac{1}{2}$ (this is a definition)
zero power	$a^0 = 1$	Why? We need $a^m a^n = a^{m+n}$ when m = 0. In order for this law to be satisfied when m = 0, we have $a^n = a^{m+n} = a^{0+n} = a^0 a^n$, so a^0 must be 1.

Exponents of Negative Values

When we multiply **negative** numbers together, we must utilize parentheses to switch to exponent notation.

$$(-3)(-3)(-3)(-3)(-3)(-3) = (-3)^6$$

BEWARE!! -3^6 is NOT the same as $(-3)^6$

The missing parentheses mean that **-3^6** will multiply **six 3's** together first (by order of operations), and then **take the negative of that answer.**

$(-3)^6 = 729$ but $-3^6 = -729$

so be careful with negative values and exponents !

Note: Even powers of negative numbers allow for the negative values to be arranged in pairs. This pairing guarantees that the answer will always be **positive.**

$$\begin{aligned}(-5)^6 &= (-5)\bullet(-5) \bullet (-5)\bullet(-5) \bullet (-5)\bullet(-5) \quad \leftarrow \text{All pairs.}\\ &= 25 \bullet 25 \bullet 25\\ &= 15625 \quad \text{(a positive answer)}\end{aligned}$$

Odd powers of negative numbers, however, always leave one factor of the negative number not paired. This one lone negative term guarantees that the answer will always be **negative.**

$$\begin{aligned}(-5)^5 &= (-5)\bullet(-5) \bullet (-5)\bullet(-5) \bullet (-5) \quad \longleftarrow \text{One lone, un-paired, negative.}\\ &= 25 \bullet 25 \bullet (-5)\\ &= -3125 \quad \text{(a negative answer)}\end{aligned}$$

Zero and Negative Exponents

- Zero Exponent: any nonzero expression to the zero power is 1. $a^0 = 1, a \neq 0$ $\quad 5^0 = 1$

- Negative Exponent: any term to a negative power is the reciprocal of that term with a positive power

$$a^{-n} = \frac{1}{a^n}, a \neq 0 \qquad 2^{-3} = \frac{1}{2^3}$$

$$\frac{1}{a^{-n}} = a^n, a \neq 0 \qquad \frac{1}{2^{-4}} = 2^4$$

Zero Exponents

The number zero may be used as an exponent.
The value of any expression raised to the zero power is 1.
(Except zero raised to the zero power is undefined.)

Base^0	Value
$2^0 =$	1
$(-6)^0 =$	1
$4^0 =$	1
$-8^0 =$	-1 Raise to the zero power first: $8^0=1$ then take the negative.
$0^0 =$	undefined

Negative Exponents

Negative numbers as exponents have a special meaning.
The rule is as follows:

$$\text{base}^{\text{negative exponent}} = \frac{1}{\text{base}^{\text{positive exponent}}}$$

For example:

Negative Exponent	Positive Exponent
$4^{-1} =$	$\frac{1}{4^1}$
$7^{-3} =$	$\frac{1}{7^3}$
$(-5)^{-2} =$	$\frac{1}{(-5)^2}$

Exponents and Units

When working with units and exponents (or powers), remember to adjust the units appropriately.
$(36 \text{ ft})^3 = (36 \text{ ft}) \bullet (36 \text{ ft}) \bullet (36 \text{ ft})$
$= (36 \bullet 36 \bullet 36) (\text{ft} \bullet \text{ft} \bullet \text{ft})$
$= 46656 \text{ ft}^3$