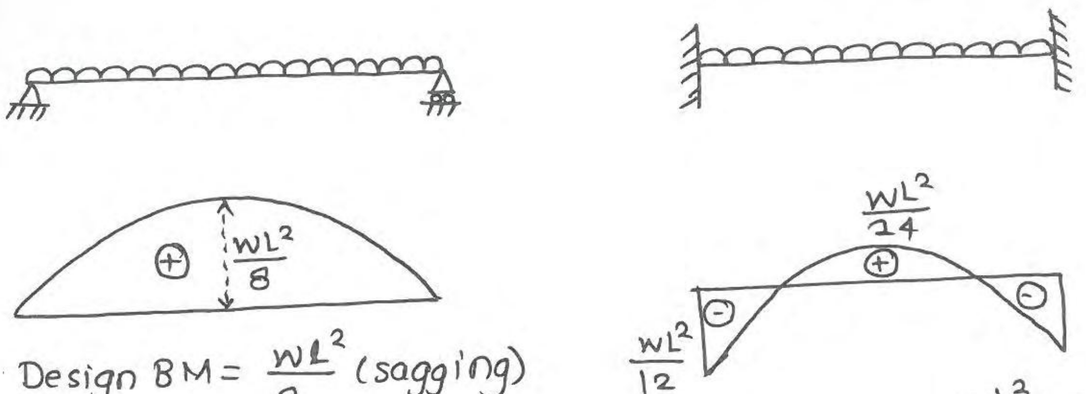
5 Statically Indeterminate Beams and Frames.

5.1 Advantages and Disadvantages:

5.1.1 Advantages:

1) Less design bending moment.



Design BM = $\frac{wL^2}{8}$ (sagging)

Design BM = $\frac{wL^2}{24}$ sagging

= $\frac{wL^2}{12}$ Hogging

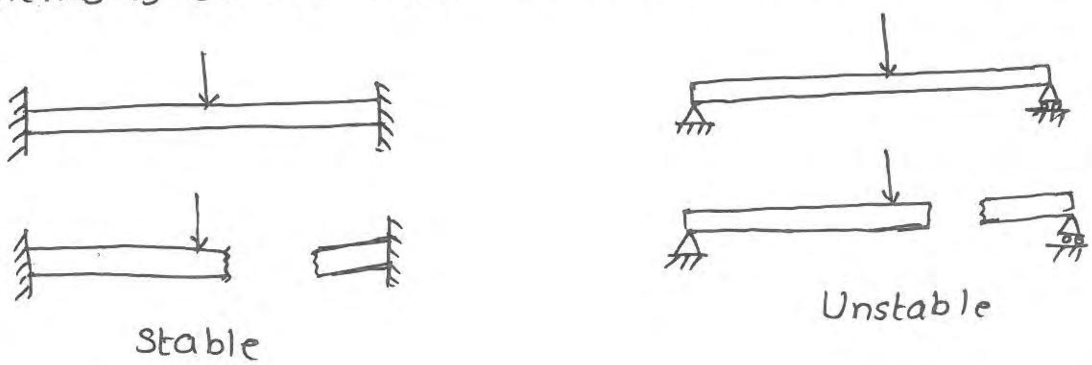
2) Less Material Required:

Since Design B.M. is less so requirement of section is also less.

3) Less Deflection:

$\Delta > \delta$

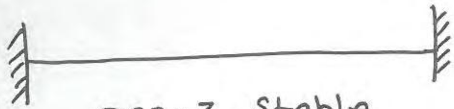
4) Multiple load paths are possible so failure of few members doesn't affect overall stability of a structure.



Stable

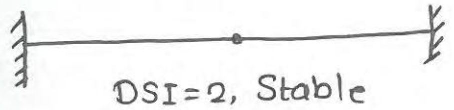
Unstable

5) Static indeterminacy increases stability of structure.



DSI = 3, Stable

DSI = 0 , Stable



DSI = 2, Stable

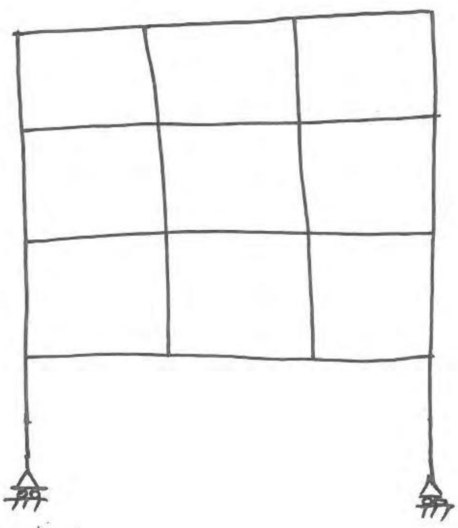
DSI = -1 , Unstable

DSI = 1 , Stable

DSI = 0, Stable

* Note:

Static indeterminacy increases stability of structure but it doesn't mean that all statically indeterminate structures are stable.



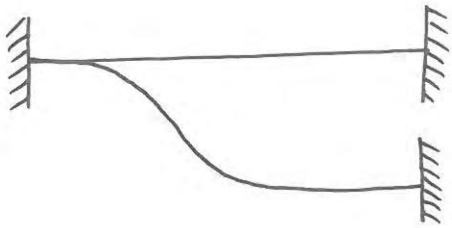
DSI > 0

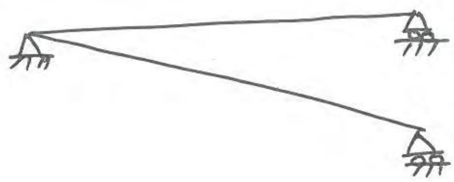
but still unstable.

5.1.2 Disadvantages:

1) Construction / Fabrication of joints is costly.

2) Internal stresses due to support settlement.

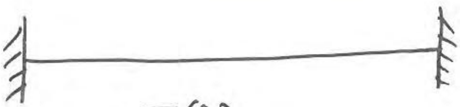




No stress.

Internal stress

3) Internal stress due to temperature change.



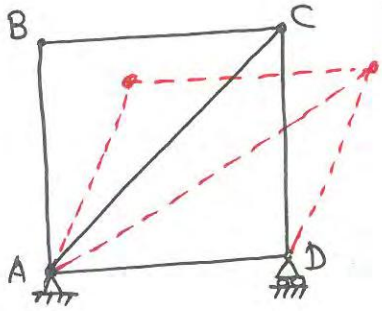
T(↑)

Internal Stress

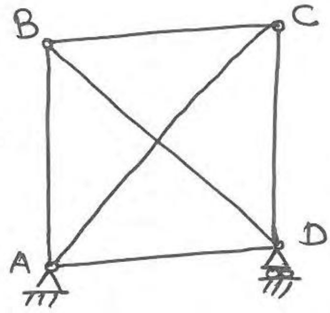
T(↑)

No stress.

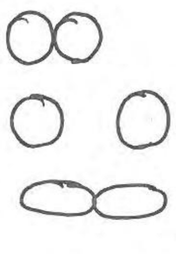
Temp. of AC(↑)



Temp. of AC(↑)



Increase of length of memble AC due to temp change will try to reduce distance b/w BD. Since joint B and D are connected by a member so BD will be subjected to comp. force which will produce stress in entire truss.



expansion — Temp (↑) (No stress)

Elongation — Force (stress).

5.2 Difficulty in Analysis:

Equations of equilibrium are not sufficient to analyze the structure so extra equations from compatibility conditions are formulated. This makes analysis of statically indeterminate structure relatively difficult.

5.3 Methods of Analysis:

Analysis of indeterminate structure (static and kinematic both) is done by following methods:

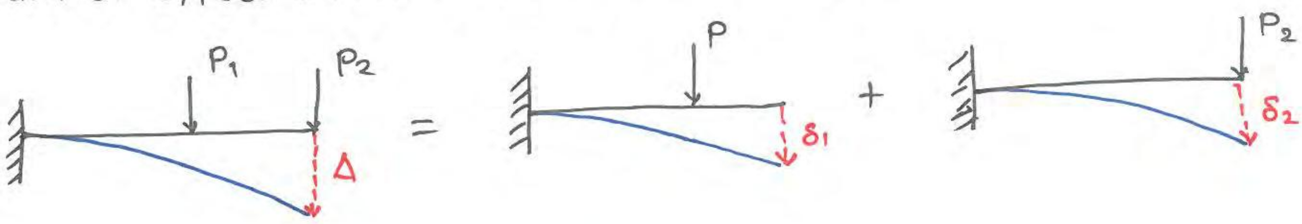
1) Force Method / Compatibility Method / Flexibility Method
2) Displacement Method / Equilibrium Method / Stiffness Method.

5.4 Difference between

Force Method	Displacement Method.
i) Forces are taken as unknown.	i) Displacements are taken as unknown.
ii) Compatibility conditions are used to calculate unknown forces.	ii) Equilibrium conditions are used to calculate unknown displacements.
iii) If DSI < KI then force method is preferable.	iii) If KI < DSI then displacement method is preferable.
iv) Methods: • Method of consistent Deformation • Strain Energy Method • 3-Moment Method / Clapeyron's Method. • Flexibility Matrix Method. • Column Analogy Method	iv) Methods: • Slope Deflection Method • Moment Distribution Method • Stiffness Matrix Method • Kani's Method.

5.5 Principle of Superposition:

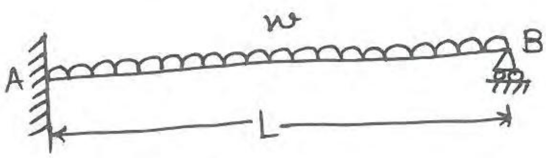
For linearly elastic structure, resultant BM, SF, Deflection, stress, strain etc due to multiple loadings is the algebraic sum of effect due to individual loading.



$$\Delta = \delta_1 + \delta_2$$

5.6 Method of Consistent Deformation:

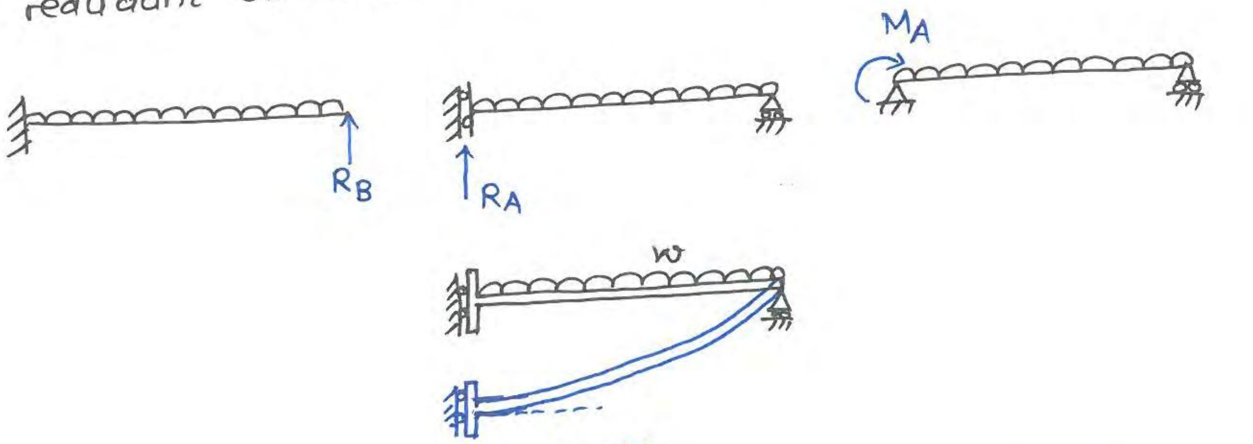
Considering a propped cantilever subjected to udl as given below,



StepI: Calculate DSI.

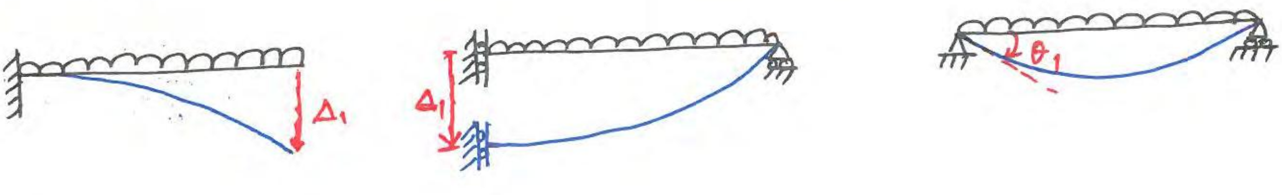
DSI = 1

StepII: Identify Redundants of the Structure. Redundant should be selected in such a way that after removal of redudant structure must remain stable.

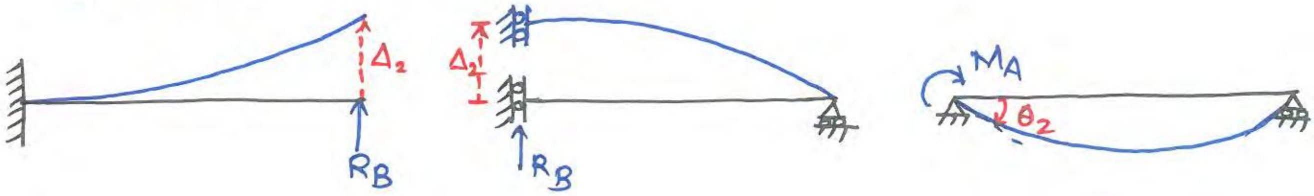


stable.

StepIII: Remove all Redundants and make primary structure.



Step IV: Apply redundants on primary structure (without loading), one at a time.



Step V: Write compatibility equations corresponding to each redundant.

$\Delta_B = 0$

$\Rightarrow -\Delta_1 + \Delta_2 = 0$

$\Rightarrow -\frac{wL^4}{8EI} + \frac{R_B L^3}{3EI} = 0$

$\Rightarrow R_B = \frac{3}{8} wL$

$\Delta_A = 0$

$\Rightarrow -\Delta_1 + \Delta_2 = 0$

No standard formula for Δ_1 & Δ_2 so it is not preferable to consider R_A as redundant.

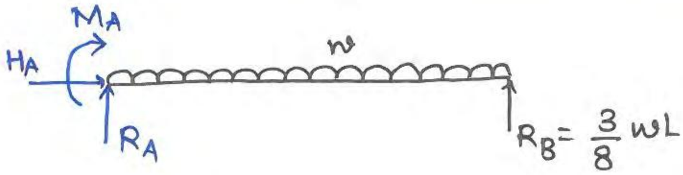
$\theta_A = 0$

$\Rightarrow \theta_1 + \theta_2 = 0$

$\Rightarrow \frac{wL^3}{24EI} + \frac{M_A L}{3EI} = 0$

$\Rightarrow M_A = -\frac{wL^2}{8}$

Step VI: Calculate other reactions using equations of equilibrium.



$\Sigma F_x = 0$

$\Rightarrow H_A = 0$ ---- (i)

$\Sigma F_y = 0$

$\Rightarrow R_A + R_B - wL = 0$

$\Rightarrow R_A + \frac{3}{8} wL - wL = 0$

$\Rightarrow R_A = \frac{5}{8} wL$ --- (ii)

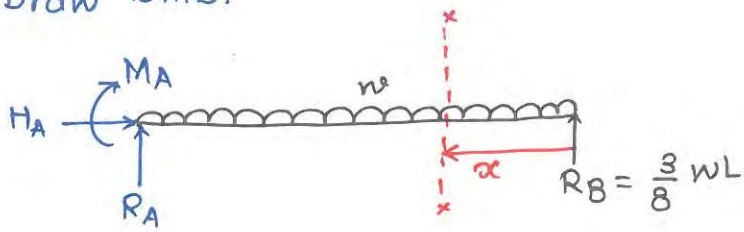
$\Sigma M_z = 0$

$\Rightarrow \Sigma M_A = 0$

$\Rightarrow M_A + w \times L \times \frac{L}{2} - R_B \times L = 0$

$\Rightarrow M_A = -\frac{wL^2}{8}$

Step VII: Draw BMD:

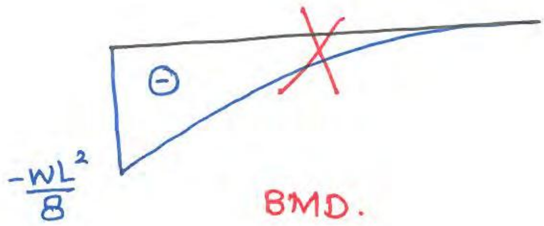


$$BM_x = R_B \cdot x - \frac{wx^2}{2}$$

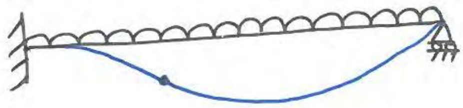
$$= \frac{3}{8} wL \cdot x - \frac{wx^2}{2}$$

At $x=0$, $BM = 0$

$x = L$, $BM = -\frac{wL^2}{8}$



BMD.



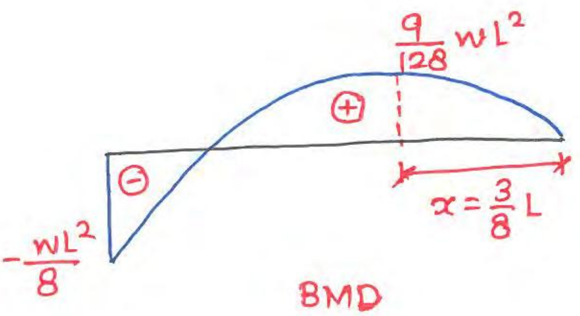
For maximum BM:-

$$\frac{d(BM_x)}{dx} = 0$$

$$\Rightarrow \frac{3}{8} wL - wx = 0$$

$$\Rightarrow x = \frac{3}{8} L$$

At $x = \frac{3}{8} L$, $BM = \frac{9}{128} wL^2$



BMD

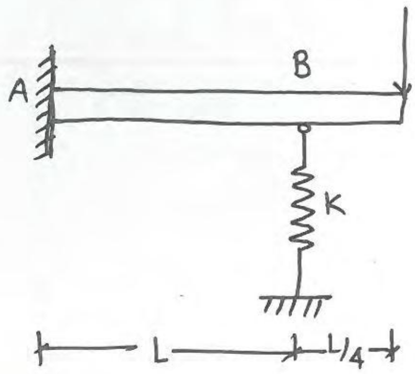
*** Note:**

In above procedure, after writing compatibility equations, problem is converted into calculation of displacements of a structure. This is done by any one of the following methods.

1) Double Integration Method.
2) Moment Area Method
3) Conjugate Beam Method.
4) Strain Energy Method.
5) Unit load Method.

First three methods are applicable for beams only and remaining two methods can be applied to any type of structure.

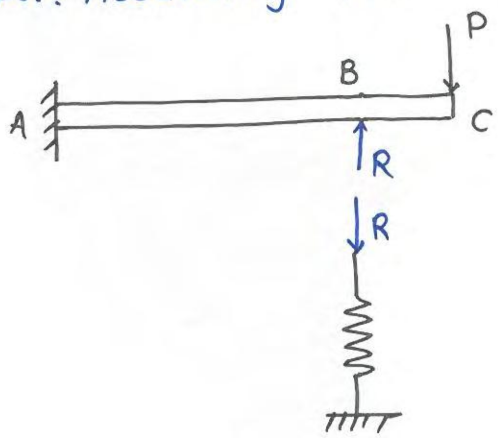
Ex.

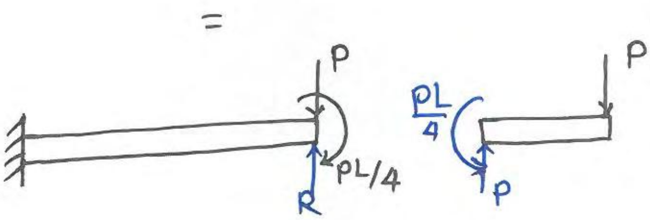


Calculate force in spring.

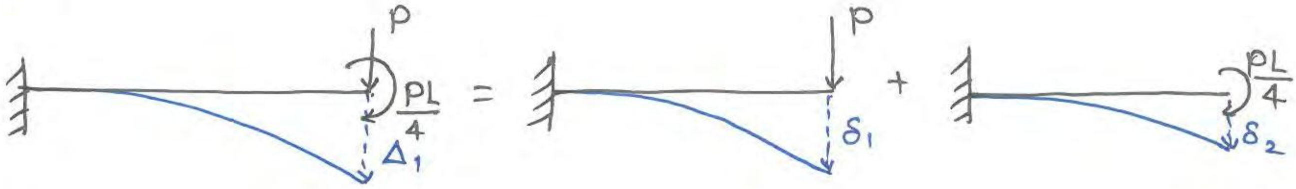
Step I: DSI = 1

Step II: Assuming force of spring as redundant.

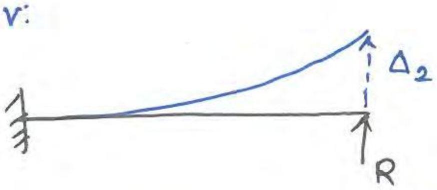




StepIII:



StepIV:



Stepv: Compatibility Equation.

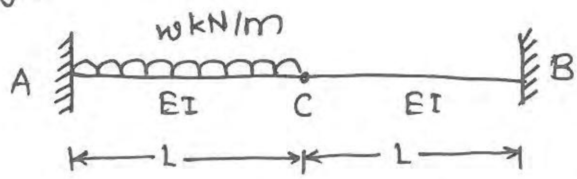
$$-\Delta_1 + \Delta_2 = -\Delta$$

$$\Rightarrow \quad -(\delta_1 + \delta_2) + \Delta_2 = -\Delta$$

$$\Rightarrow \quad -\left(\frac{PL^3}{3EI} + \frac{\left(\frac{PL}{4}\right)L^2}{2EI}\right) + \frac{RL^3}{3EI} = -\frac{R}{K}$$

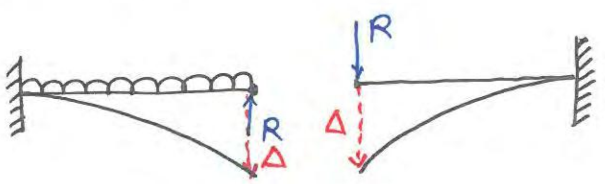
$$\Rightarrow \quad R = \frac{11PL^3K}{8(3EI + KL^3)}$$

Ex. What is the reaction on the hinge C for a beam as shown in the figure?



$\Rightarrow$ DSI = 1 for vertical loading.

Assuming SF at hinge as redundant



Compatibility equation:-

$$\Delta = \Delta$$

$$\Rightarrow \quad \frac{wL^4}{8EI} - \frac{RL^3}{3EI} = \frac{RL^3}{3EI}$$

$$\Rightarrow \quad \boxed{R = \frac{3}{16}wL}$$

5.6.1 How to make Primary Structure?

DSI = 2

Total number of reactions = 5

Number of possible primary structure = $\frac{\underline{|5}}{\underline{|2}\,\underline{|5-2}} = 10$

1. V_A, H_A
2. V_A, M_A
3. V_A, V_B
4. V_A, H_B
5. H_A, M_A
6. H_A, V_B
7. H_A, H_B
8. M_A, V_B
9. M_A, H_B
10. V_B, H_B

① Not possible

②

③

④

⑤

⑥

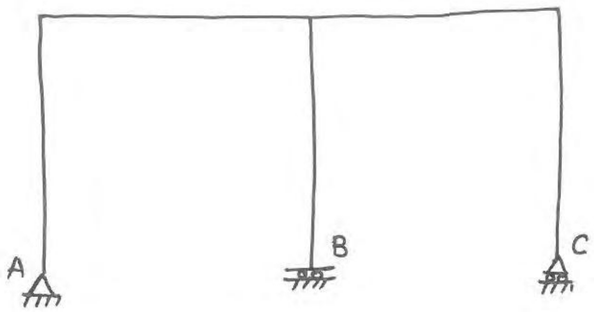
⑦

⑧

⑨

⑩

Ex. 2.



DSI = 2

Total no. of reactions = 5

No. of possible primary structure = ${}^5C_2 = 10$

① V_A, H_A

② V_A, V_B

③ V_A, M_B

④ V_A, V_C

⑤ H_A, V_B

⑥ H_A, M_B

⑦ H_A, V_C

⑧ V_B, M_B

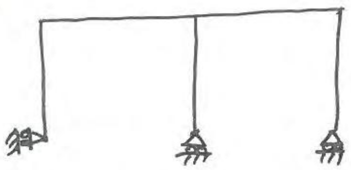
⑨ V_B, V_C

⑩ M_B, V_C

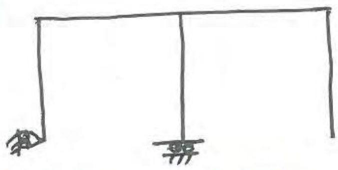
①

② Not possible

③



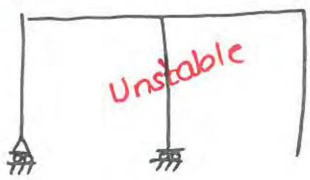
④



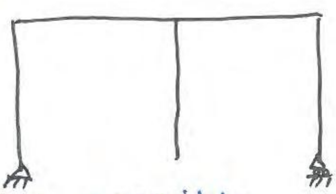
⑤ Not possible

⑥

⑦

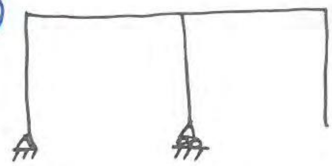


⑧



⑨ Not possible

⑩



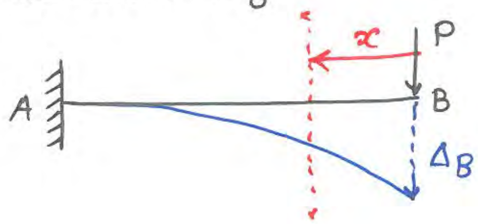
5.7 Principle of Virtual Work:

5.7.1 Derivation:

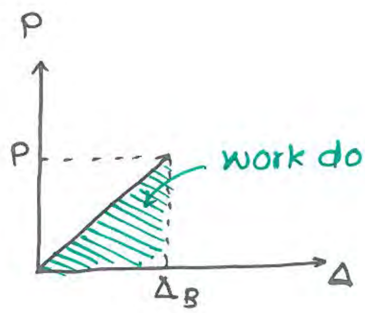
From Work-Energy Theorem

- External Workdone = Internal Workdone
- External Work done = Strain Energy.

Ex. Calculate Δ_B



External workdone = $\frac{1}{2} \times P \times \Delta_B$



$$\text{Strain Energy} = U$$

$$= \int \frac{M_x^2\,dx}{2EI}$$

$$= \int_0^L \frac{(P.x)^2\,dx}{2EI}$$

$$= \frac{P^2}{2EI}\left[\frac{x^3}{3}\right]_0^L$$

$$= \frac{P^2L^3}{6EI}$$

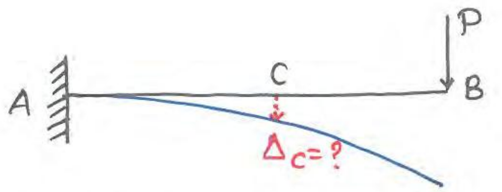
from Strain Energy Method:-

External Workdone = Strain Energy

$$\frac{1}{2} \cdot P \cdot \Delta_B = \frac{P^2L^3}{6EI}$$

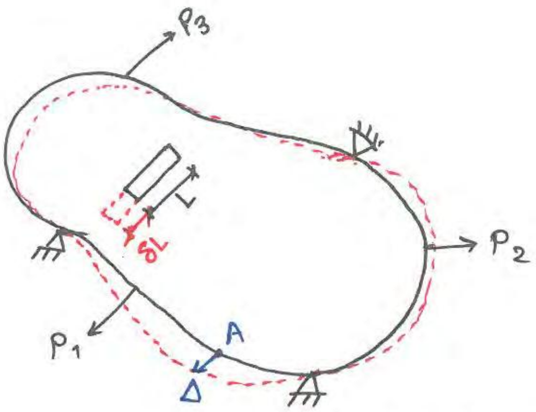
$$\Rightarrow \boxed{\Delta_B = \frac{PL^3}{3EI}}$$

Above procedure canbe used if deflection is need to be calculated in the direction of applied force. It means, following example can not be solved by above procedure.



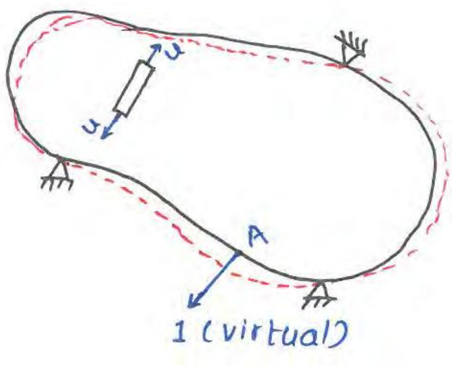
So this problem is solved using virtual work principle:-

Considering an arbitary elastic body subjected to P force system as given below



Work energy thereom cannot be used to calculate Δ because no term of external workdone comprises Δ.

To solve this problem, unit virtual load is applied on the arbitary body in the direction of Δ.



Now apply P force system on orbitary body which is already subjected to unit virtual load.

From principle of virtual work:

External virtual workdone = Internal virtual workdone

real

$$\boxed{1 \cdot \Delta = \sum u \, \delta L}$$

virtual

• Case I: Statically stable Elastic Body:

External virtual workdone = Internal Virtual Workdone.

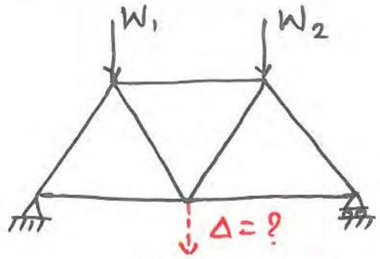
• Case II: Statically stable Rigid Body:

External virtual workdone = Internal virtual workdone = 0

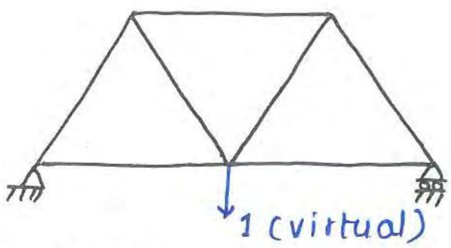
• Case III: Statically Unstable Elastic / Rigid Body:

External workdone ≠ Internal Workdon.

5.7.2 Principle of Virtual Work for Truss:-



Member force = P



Member force = k

from principle of virtual work.

$$\Rightarrow \quad 1 \cdot \Delta = \sum u \cdot \delta L$$

$$\Rightarrow \quad 1 \cdot \Delta = \sum k \, \frac{PL}{AE}$$

$$\Rightarrow \quad \boxed{\Delta = \sum \frac{kPL}{AE}}$$

Considering effect of temperature change and fabrication defect also.

$$\boxed{\Delta = \sum \frac{kPL}{AE} + \sum k(L\alpha T) + \sum k \cdot \delta}$$

T = +ve if temp. increases

δ = +ve if member is too long.

Procedure:

StepI: Calculate force in each member due to applied loading.(P

StepII: Apply unit load on the truss (without applied loading) in the direction of displacement need to be calculated and calculate force in each member. (k).

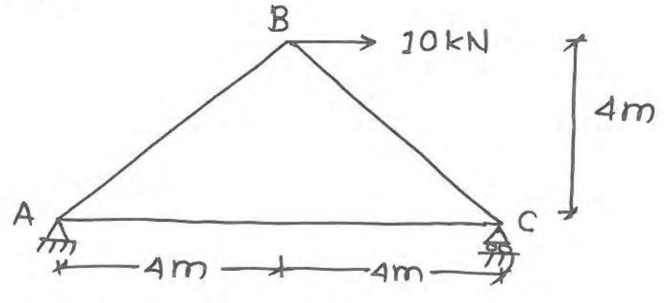
StepIII: Arrange all calculated values in tabular format as given below.

Member	P (kN)	K (kN)	L (m)	AE (kN)	$\frac{KPL}{AE}$	$K(L\alpha T)$	$K.\delta$
					$\Sigma = ?$	$\Sigma = ?$	$\Sigma = ?$

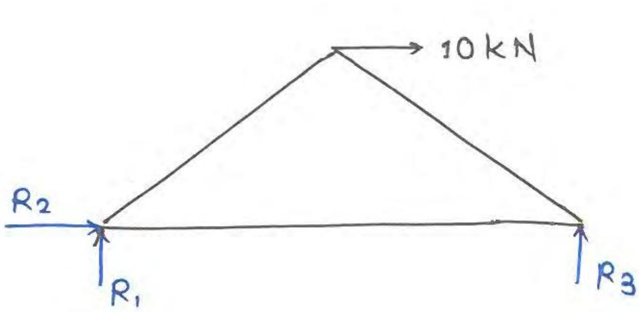
StepIV: Calculate deflection using expression given below.

$$\Delta = \Sigma \frac{KPL}{AE} + \Sigma k(L\alpha T) + \Sigma k.\delta$$

Ex. Calculate vertical deflection of joint B. $A = 300\ mm^2$, $E = 200\ kN/mm^2$, $\alpha = 12 \times 10^{-6}/°C$, member AC is 5mm too short, increase in temp. of AB is 40°C.



⇒ StepI: Calculation of P

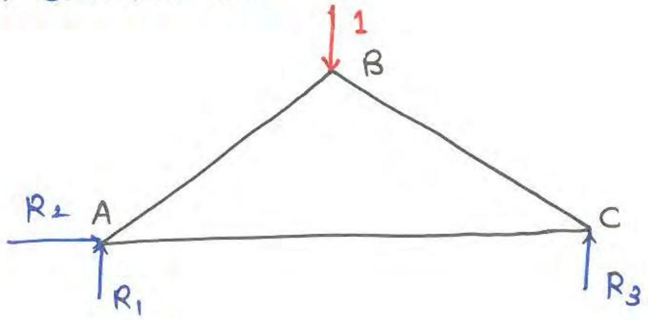


$F_{AB} = 7.07$ kN

$F_{BC} = -7.07$ kN

$F_{CA} = 5$ kN

Step II: Calculation of k.



$F_{AB} = -0.707$ kN

$F_{BC} = -0.707$ kN

$F_{CA} = 0.5$ kN.

Step III:

Member	P(kN)	k(kN	L(m)	AE(kN)	$\frac{kPL}{AE}$	$k(L\alpha T)$	$k \cdot \delta$
AB	7.07	-0.707	$4\sqrt{2}$		-4.71×10^{-4}	-1.91×10^{-3}	0
BC	-7.07	-0.707	$4\sqrt{2}$	6×10^{4}	4.7×10^{-4}	0	0
CA	5	0.5	8		3.33×10^{-4}	0	0.5(-0.005)
				$\Sigma =$	3.33×10^{-4}	-1.91×10^{-3}	2.5×10^{-3}

$$\Delta = \Sigma \frac{kPL}{AE} + \Sigma k(L\alpha T) + \Sigma k \cdot \delta.$$

$$= 3.33 \times 10^{-4} - 1.91 \times 10^{-3} - 2.5 \times 10^{-3}$$

$$= -4.07 \times 10^{-3} \text{ m}$$

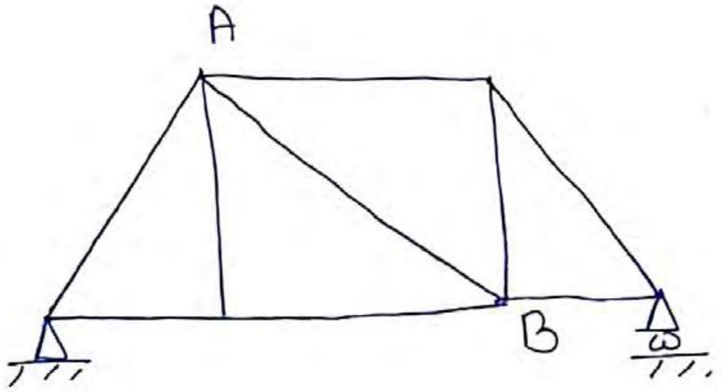
$$\boxed{\Delta = -4.07 \text{ mm}}$$

-ve value shows deflection opposite to the applied unit load. It means upward.

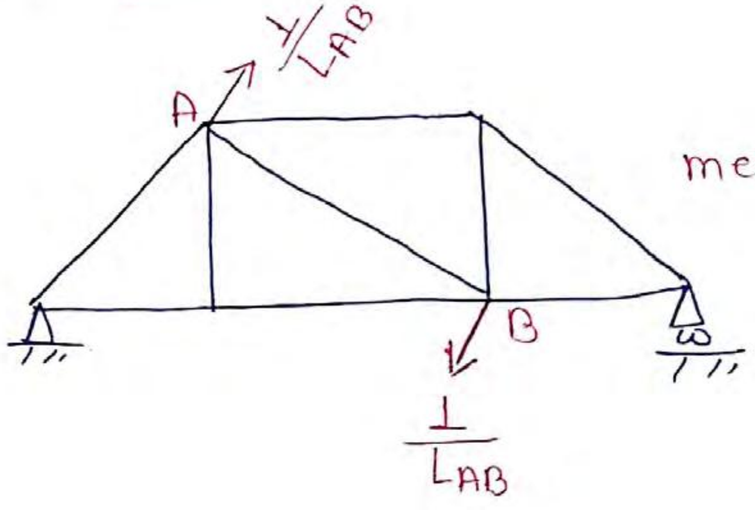
Case III :- If rotation of member is to be calculated then apply unit load divided by length of member, perpendicular to member as ~~calculated~~ given below.

~~Calculation~~

Calculate rotation of member AB :



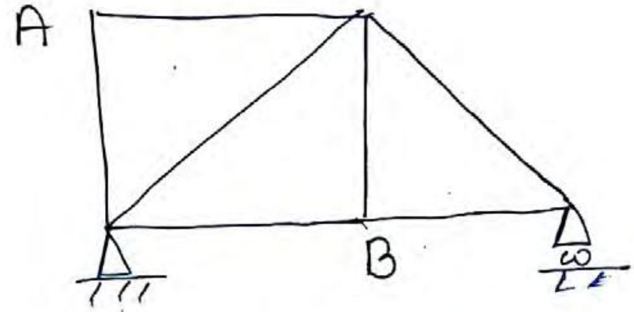
Applying unit load for member force (k).



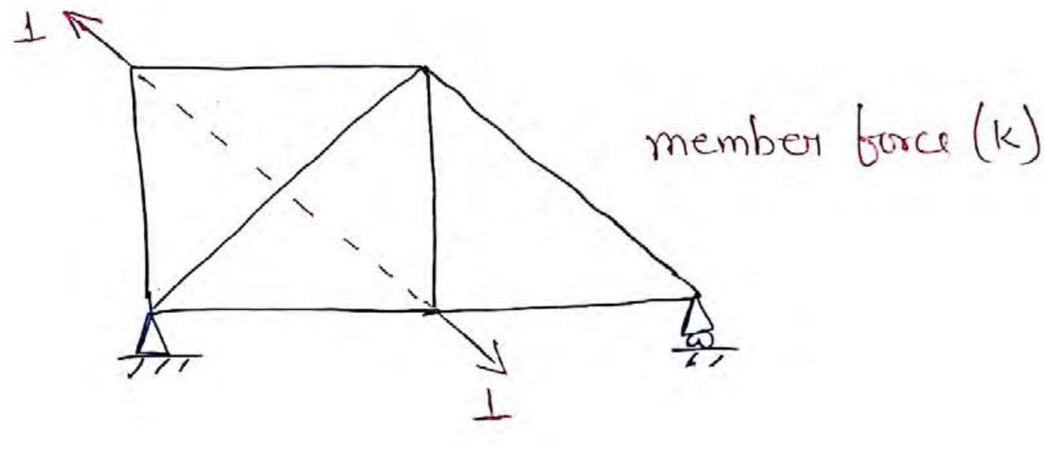
member force (k).

Case II :- If relative movement between two joints need to be calculated. then apply unit load at both joints together.

For example, calculate relative movement of joint AB in the direction of AB.

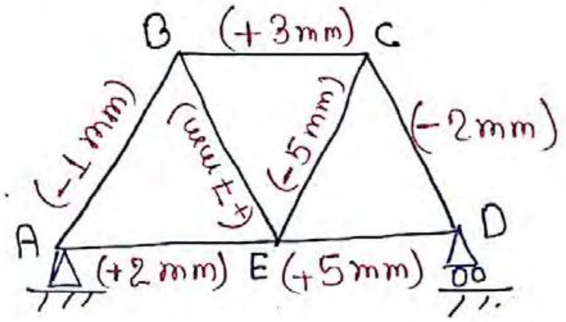


Applying unit virtual load as follows to Calculate force in member (K).



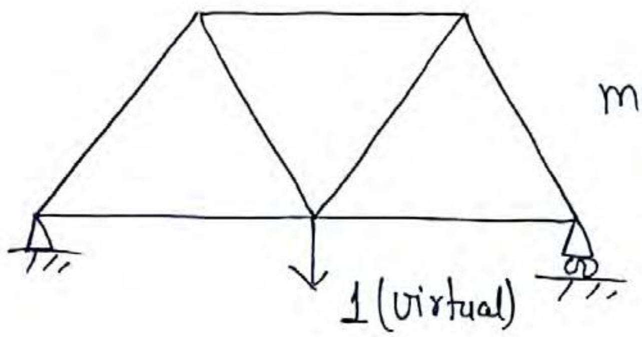
Calculation of Deflection of truss for different cases.

Case I :- If elongation of members due to applied loading is directly given.



Calculate vertical deflection at E.

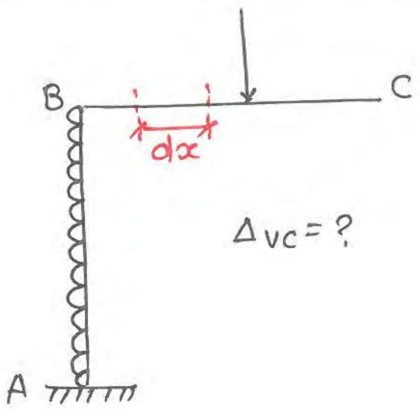
Apply unit load at E and Calculate force in each member. (K)

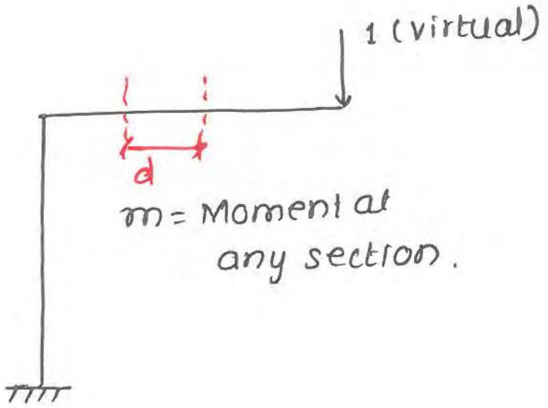


$$\Delta = \Sigma K \left(\frac{PL}{AE}\right)$$

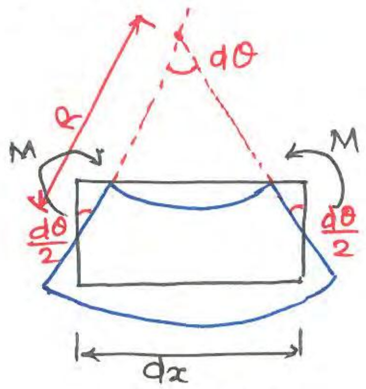
This is elongation and comp of member due to applied loading.

5.7.3 Principle of Virtual Work for Beams and Frames.





For $d\theta$:–



$$d\theta = \frac{dx}{R}$$

$$\Rightarrow \frac{1}{R} = \frac{d\theta}{dx}$$

Now from flexure formula :-

$$\frac{M}{I} = \frac{E}{R}$$

$$\Rightarrow \frac{M}{I} = E\left(\frac{d\theta}{dx}\right)$$

$$\Rightarrow d\theta = \frac{M}{EI}\,dx$$

from principle of virtual work :-

$$1 \cdot \Delta = \Sigma u \cdot \delta L$$

$$\Rightarrow 1 \cdot \Delta = \int m \cdot d\theta$$

$$\boxed{\Delta = \int \frac{m\,M\,dx}{EI}}$$

For Rotation:-

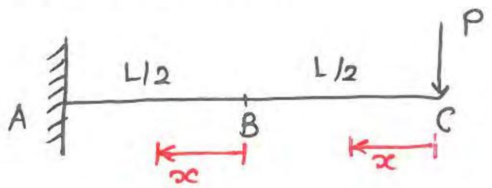
$$\boxed{\theta = \int \frac{mMdx}{EI}}$$

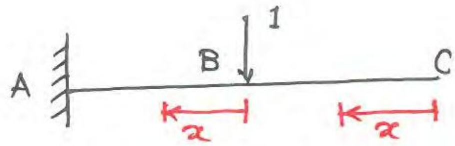
m= Moment at any section due to unit force/moment

M= Moment at any section due to applied loading.

m & M= +ve if clockwise/sagging/comp. on ref. face.

Ex. Calculate Δ_B and θ_B





⇒

For CB:-

M= Px (+ve becoz clockwise)

For BA:-

M= P (L/2 + x) (+ve becoz clockwise)

For CB:-

m=0

For BA:-

m= 1·x (+ve becoz clockwise)

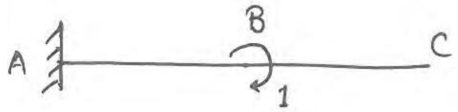
$$\Delta = \int \frac{mM\,dx}{EI}$$

$$= \underbrace{\int_0^{L/2} \frac{mM\,dx}{EI}}_{CB} + \underbrace{\int_0^{L/2} \frac{mMdx}{EI}}_{BA}$$

$$= \int_0^{L/2} \frac{0\,(Px)dx}{EI} + \int_0^{L/2} \frac{x\left[P\left(\frac{L}{2}+x\right)\right]dx}{EI}$$

$$\boxed{\Delta_B = \frac{5PL^3}{48EI}}$$

For θ_B :-



For CB :-

$$m = 0$$

For BA :-

$$m = 1$$

$$\theta = \int \frac{m M dx}{EI}$$

$$= \int_0^{L/2} \frac{m M dx}{EI} + \int_0^{L/2} \frac{m M dx}{EI}$$

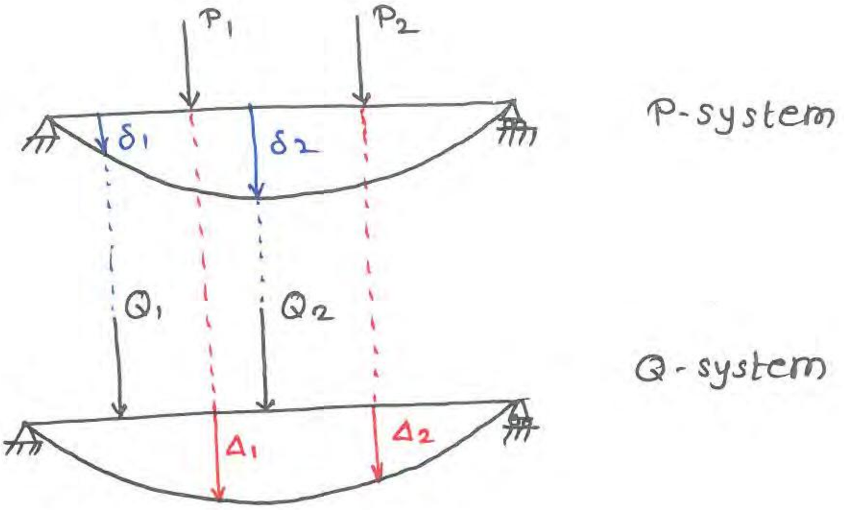
$$= \int_0^{L/2} \frac{0 \times (Px) dx}{EI} + \int_0^{L/2} \frac{1 \cdot P\left(\frac{L}{2} + x\right) dx}{EI}$$

$$= \frac{P}{EI}\left[\frac{L}{2}x + \frac{x^2}{2}\right]_0^{L/2}$$

$$\boxed{\theta_B = \frac{3PL^2}{8EI}}$$

5.8 Betti's Law:

Virtual workdone by P force system in going through deformation of Q force system is equal to virtual workdone by Q force system in going through deformation of P force system.



from Betti's Law:-

$$P_1\Delta_1 + P_2\Delta_2 = Q_1\delta_1 + Q_2\delta_2$$

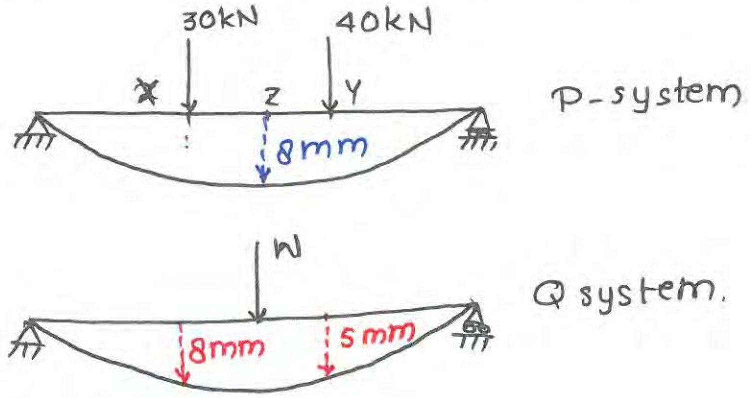
$$\Rightarrow \quad \Sigma P\Delta = \Sigma Q\delta$$

Δ = Displacement by Q-system corresponding to P-system

δ = Displacement by P-system corresponding to Q-system.

Ex. The beam given below produces deflection of 8 mm at Z. To produce deflection of 8 mm and 5 mm at x and y respectively, load required at z would be

a) 20 kN b) 40 kN c) 55 kN d) 80 kN.



from Betti's Law :-

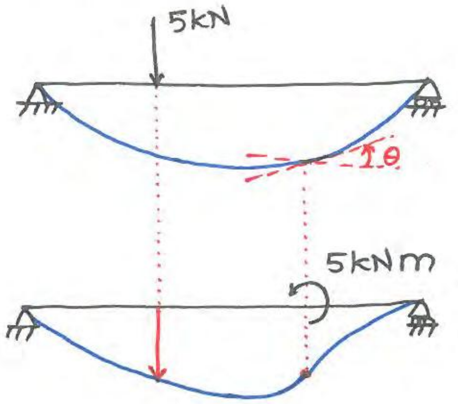
$$P_1\Delta_1 + P_2\Delta_2 = Q_1\delta_1$$

$$30\times 8 + 40\times 5 = W\times 8$$

$$\Rightarrow W = 55\text{ kN}.$$

5.9 Maxwell's Reciprocal Theorem:-

It is the special case of Betti's law where a single force/moment in P-force system and a single force/moment in Q-force system of equal magnitude are present.



from Betti's law:-

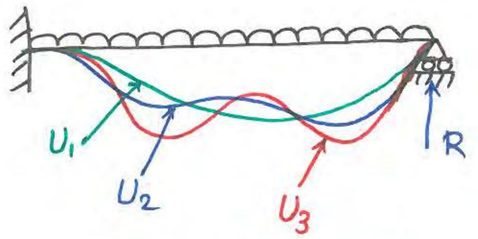
$$P_1\Delta_1 = Q_1\delta_1$$

$$\Rightarrow 5\times\Delta = 5\times\theta$$

$$\Rightarrow \Delta = \theta$$

5.10 Theorem of Least Work / Strain Energy Method.

In any statically indeterminate structure the redundant should be such as total internal energy of a structure is minimum.



U_1 is minimum.

$$\frac{\partial U}{\partial R} = 0$$

$$\Rightarrow \frac{\int \frac{M^2 dx}{2EI}}{\partial R} = 0$$

$$\Rightarrow \int \frac{M\frac{\partial M}{\partial R}dx}{EI} = 0$$

In case of settlement of support:-

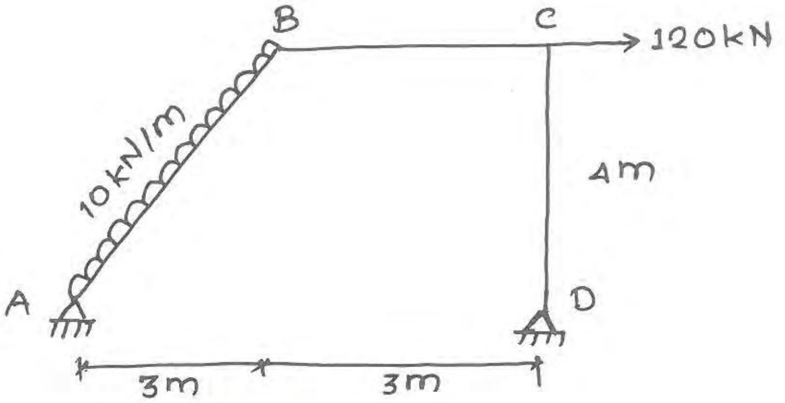
$$\frac{\partial U}{\partial R} = \Delta$$

$$\int \frac{M\frac{\partial M}{\partial R}dx}{EI} = \Delta$$

M= +ve if clockwise /sagging/ comp on ref. face

Δ = +ve if along R.

Ex. Analyze the given frame using strain energy method.
Horizontal settlement of support D is $\frac{10}{EI}$ towards right.

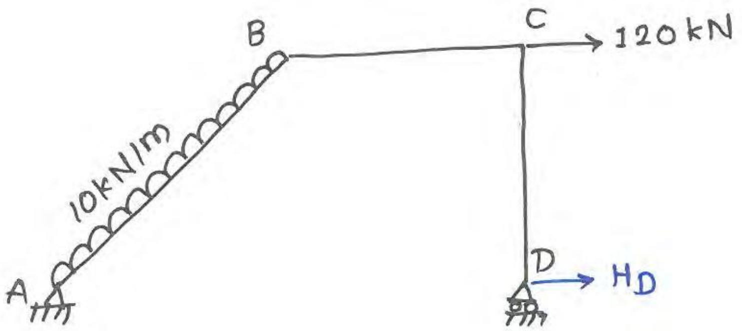


Step I: Calculate DSI.

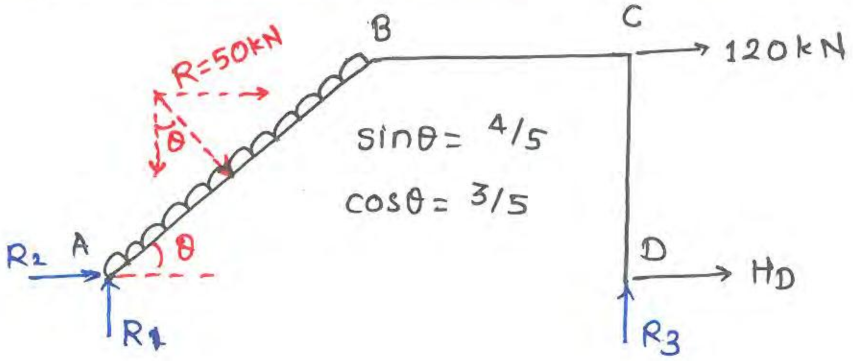
DSI = 1

Step II: Identify redundants.

In this case, redundant must be in the direction of settlement of support to consider the effect of settlement so considering horizontal reaction at D as redundant.



Step III: Calculate other support reactions



$$\Sigma F_x = 0$$

$$\Rightarrow R_2 + H_D + 120 + 50\sin\theta = 0$$

$$\Rightarrow R_2 = -H_D - 160 \quad \cdots (i)$$

$$\Sigma F_y = 0$$

$$\Rightarrow R_1 + R_3 - 50\cos\theta = 0$$

$$\Sigma M_z = 0$$

$$\Rightarrow \Sigma M_A = 0$$

$$\Rightarrow 50 \times \frac{5}{2} + 120\times 4 - R_3 \times 6 \quad \text{---(iii)}$$

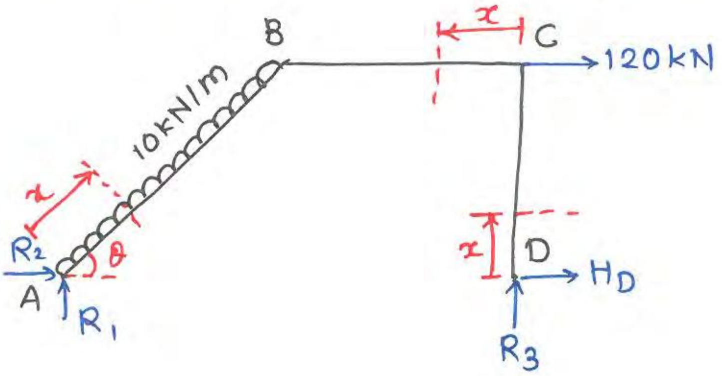
from eq^n (i), (ii) and (iii)

$$R_1 = -70.83\ kN$$

$$R_2 = -H_D - 160$$

$$R_3 = 100.83\ kN$$

Step IV: Make strain energy of structure minimum.



For DC:-

$M = -H_D \cdot x$ (-ve becoz anticlockwise)

$$\frac{\partial M}{\partial H_D} = -x$$

For CB:-

$$M = -R_3 \cdot x - H_D \times 4$$

$$= -100.83x - H_D \times 4$$

$$\frac{\partial M}{\partial H_D} = -4$$

For AB:-

$M = R_1\, x\cos\theta - R_2\, x\sin\theta - \frac{wx^2}{2}$ (+ve if clockwise)

$$= -70.83 \times x \times \frac{3}{5} - (-H_D - 160) \times x \times \frac{4}{5} - \frac{10x^2}{2}$$

$$= 85.51x - 5x^2 + 0.8 H_D x$$

$$\frac{\partial M}{\partial H_D} = 0.8x$$

$$\frac{\partial U}{\partial H_D} = \Delta_D$$

$$\int_0^4 \frac{M\frac{\partial M}{\partial H_D}dx}{EI} + \int_0^3 \frac{M\frac{\partial M}{\partial H_D}dx}{EI} + \int_0^5 \frac{M\frac{\partial M}{\partial H_D}dx}{EI} = \frac{10}{EI}$$ (+ve becoz along H_D)

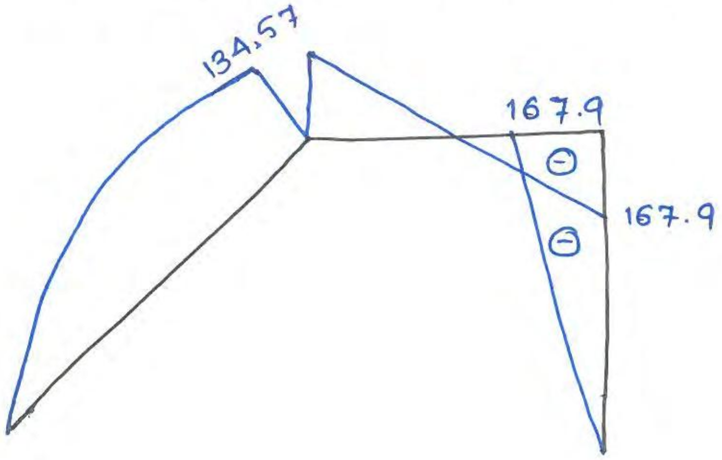
$$\Rightarrow \int_0^4 (-H_D x)(-x)\,dx + \int_0^3 (-100.83x - 4H_D)(-4)\,dx$$

$$+ \int_0^5 (85.51x - 5x^2 + 0.8H_D x)(0.8x)\,dx = 10$$

$$\Rightarrow \quad H_D = -41.98\ kN$$

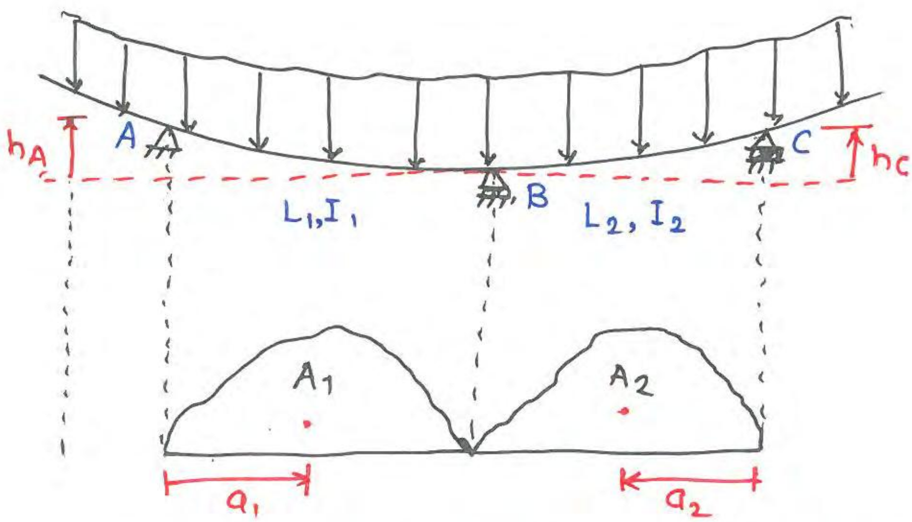
Step V: BMD :-

Draw BMD as discussed in section 1.16.



5.11. Three-Moment Equation.

This method is used to analyse fixed and continuous beams

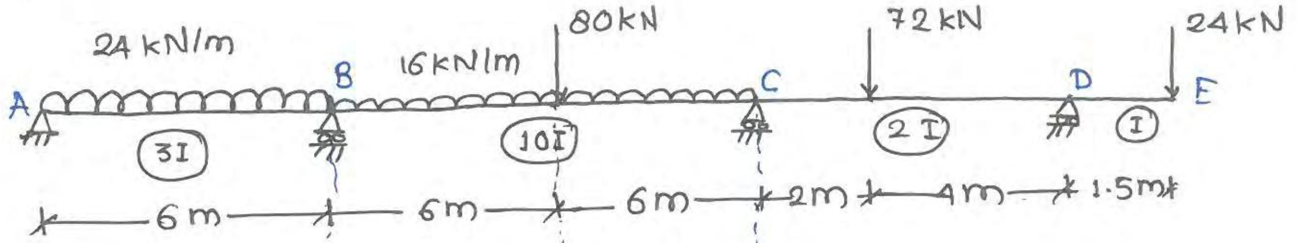


$$M_A\left(\frac{L_1}{I_1}\right) + 2M_B\left(\frac{L_1}{I_1} + \frac{L_2}{I_2}\right) + M_C\left(\frac{L_2}{I_2}\right) = \frac{-6A_1a_1}{L_1I_1} - \frac{6A_2a_2}{L_2I_2} + \frac{6Eh_A}{L_1} + \frac{6Eh_C}{L_2}$$

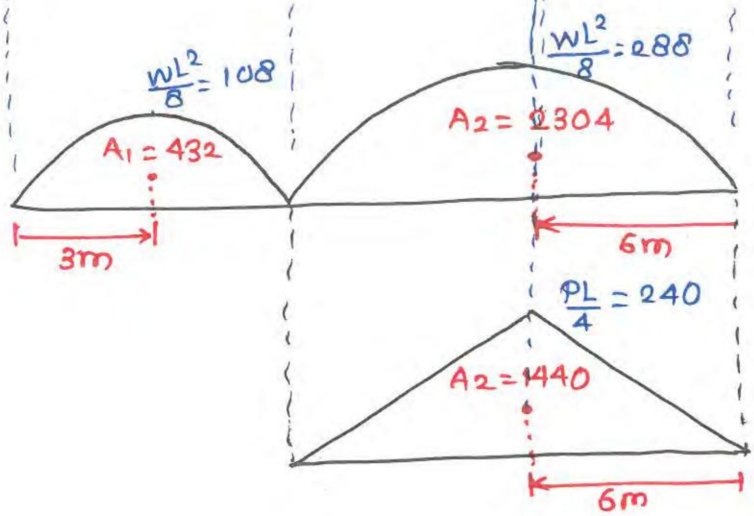
M_A, M_B, M_C = These are BM so +ve if sagging

h_A and h_C = +ve if vertically upward w.r.t. middle support.

Ex.



DSI = 2 so Two equations are required to analyze the structure.



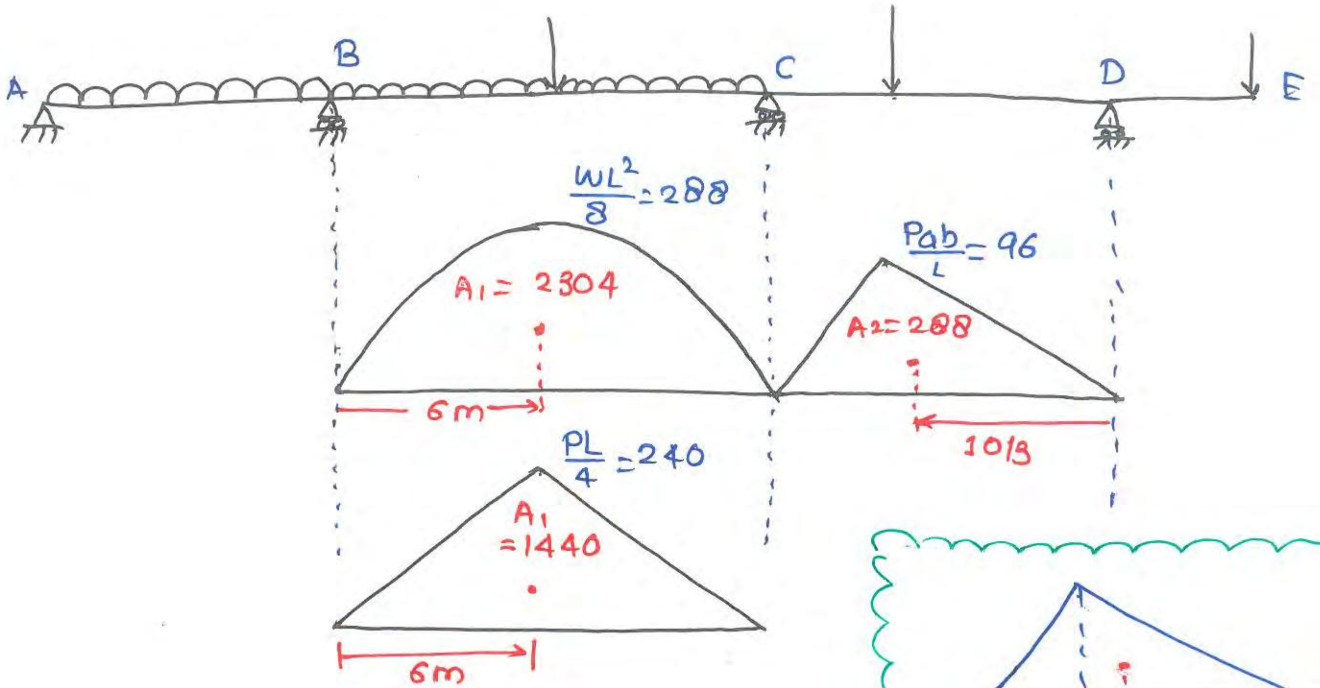
For AB and BC:

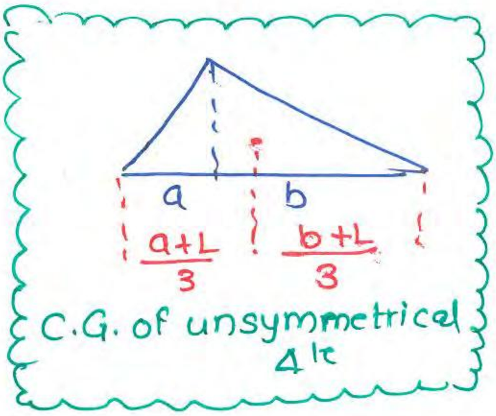
$$M_A\left(\frac{L_1}{I_1}\right) + 2M_B\left(\frac{L_1}{I_1}+\frac{L_2}{I_2}\right) + M_C\left(\frac{L_2}{I_2}\right) = \frac{-6A_1a_1}{L_1I_1} - \frac{6A_2a_2}{L_2I_2}$$

$$\Rightarrow 0\left(\frac{6}{3I}\right) + 2M_B\left(\frac{6}{3I}+\frac{12}{10I}\right) + M_c\left(\frac{12}{10I}\right) = \frac{-6\times432\times3}{6\times3I} - \left(\frac{6\times2304\times6}{12\times10I}+\frac{6\times1440\times6}{12\times10I}\right)$$

$$\Rightarrow 6.4M_B + 1.2M_C = 1555.2 \quad \text{-----}(1)$$

PROBLEM CONT ON NEXT PG.





For BC & CD :-

$$M_B\left(\frac{L_1}{I_1}\right)+2M_C\left(\frac{L_1}{I_1}+\frac{L_2}{I_2}\right)+M_D\left(\frac{L_2}{I_2}\right)=\frac{-6A_1a_1}{L_1I_1}-\frac{6A_2a_2}{L_2I_2}$$

$$\Rightarrow M_B\left(\frac{12}{10I}\right)+2M_c\left(\frac{12}{10I}+\frac{6}{2I}\right)+(-36)\left(\frac{6}{2I}\right)=-\left(\frac{6\times2304\times6}{12\times10I}+\frac{6\times1440\times6}{12\times10I}\right)-\frac{6\times288\times10/3}{6\times2I}$$

$\left\{ \begin{array}{l} M_D = -36\,kNm \\ -ve \text{ becoz hogging} \end{array} \right\}$

$$\Rightarrow 1.2M_B + 8.4M_C = -1495.2 \quad \text{--- (ii)}$$

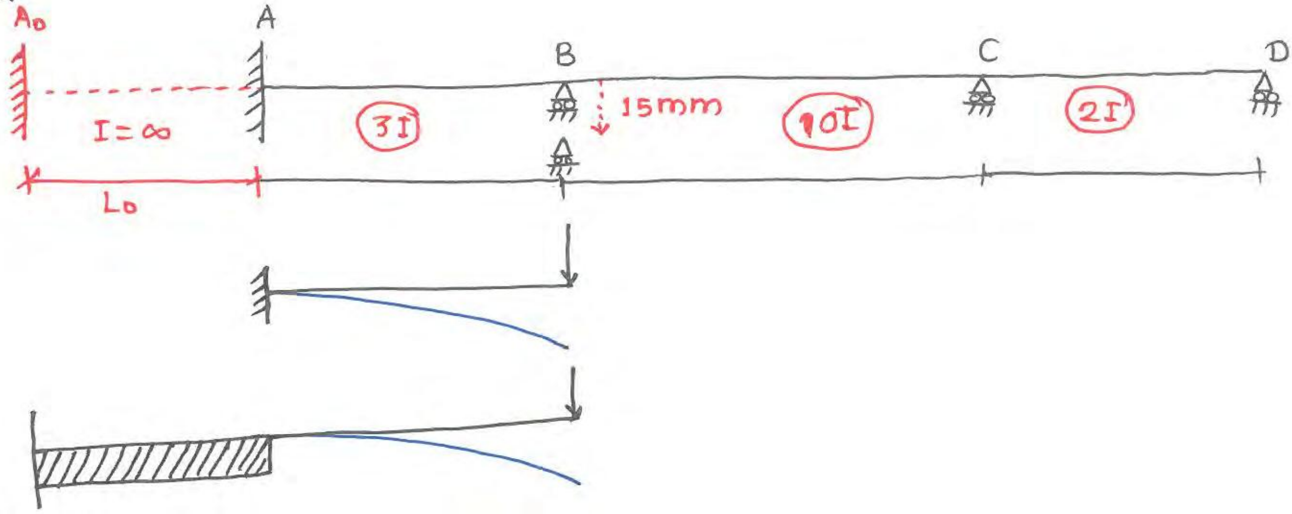
from eq^n (i) and (ii)

$M_B = -215.39$ kNm

$M_C = -147.22$ kNm

-ve sign indicates hogging BM.

Ex.2.



DSI = 3

so three equations are required to analyze the structure.

For A_oA and AB:-

$$M_{A_o}\left(\frac{L_1}{I_1}\right) + 2M_A\left(\frac{L_1}{I_1} + \frac{L_2}{I_2}\right) + M_B\left(\frac{L_2}{I_2}\right) = \frac{6Eh_{A_o}}{L_1} + \frac{6Eh_B}{L_2}$$

$$\Rightarrow M_{A_o}\left(\frac{L_o}{\infty}\right) + 2M_A\left(\frac{L_o}{\infty} + \frac{6}{3I}\right) + M_B\left(\frac{6}{3I}\right) = \frac{6E \times 0}{L_o} + \frac{6E(-0.015)}{6}$$

$$\Rightarrow \quad 2M_A + M_B = -600 \quad \cdots (i)$$

For AB and BC

$$M_A\left(\frac{L_1}{I_1}\right) + 2M_B\left(\frac{L_1}{I_1} + \frac{L_2}{I_2}\right) + M_C\left(\frac{L_2}{I_2}\right) = \frac{6Eh_A}{L_1} + \frac{6Eh_C}{L_2}$$

$$\Rightarrow M_A\left(\frac{6}{3I}\right) + 2M_B\left(\frac{6}{3I} + \frac{12}{10I}\right) + M_C\left(\frac{12}{10I}\right) = \frac{6E(0.015)}{6} + \frac{6E(0.015)}{12}$$

$$\Rightarrow \quad M_A + 3.2M_B + 0.6M_C = 900 \quad \cdots (ii)$$

For BC and CD

$$M_B\left(\frac{L_1}{I_1}\right) + 2M_C\left(\frac{L_1}{I_1} + \frac{L_2}{I_2}\right) + M_D\left(\frac{L_2}{I_2}\right) = \frac{6Eh_B}{L_1} + \frac{6Eh_D}{L_2}$$

$$\Rightarrow M_B\left(\frac{12}{10I}\right) + 2M_C\left(\frac{12}{10I} + \frac{6}{2I}\right) + M_D\left(\frac{6}{2I}\right) = \frac{6E(-0.015)}{12} + \frac{6E \times 0}{6}$$

$$\Rightarrow \quad 0.6M_B + 4.2M_C = -300 \quad \cdots (iii)$$

from equation (i), (ii) and (iii)

$M_A = -537.7 \text{ kNm}$

$M_B = 475.4 \text{ kNm}$

$M_C = -139.34 \text{ kNm}.$