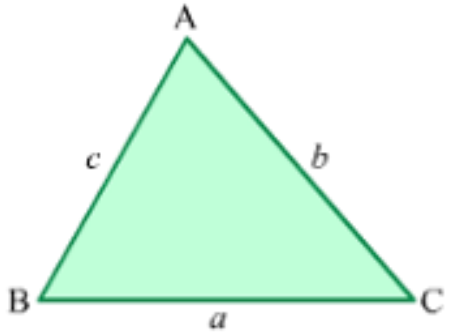
Area and Perimeter of Plane Figure

- **Perimeter** is the length of the boundary of a closed figure.
- The perimeter of a polygon is the sum of the lengths of all its sides.

  In case of a triangle ABC, with sides of lengths a, b and c units:



  Perimeter of ABC = AB + BC + AC = $a + b + c$

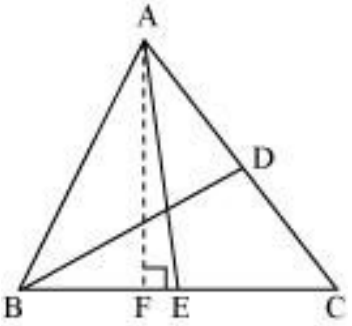
- The **semi-perimeter** of a triangle is half the perimeter of the triangle.

  The semi-perimeter (s) of a triangle with sides a, b and c is $\frac{a+b+c}{2}$.

- The semi-perimeter of a triangle is used for calculating its area when the length of altitude is not known.

- Area of a triangle:
  - Area of a triangle = $\frac{1}{2} \times \text{Base} \times \text{Altitude}$
  - All the congruent triangles are equal in area, but the triangles having equal areas may or may not be congruent.

Example: ΔABC is isosceles with AC = BC = 6 cm. AE and BD are the medians and AF = 4 cm. What is the area of ΔABD?



Solution: In ΔABE and ΔBAD, we have

BE = AD $\left[AC = BC \Rightarrow \frac{1}{2}AC = \frac{1}{2}BC\right]$]

∠ABE = ∠BAD [Angles opposite to equal sides]

AB = AB [Common]

⇒ ΔABE ≅ ΔBAD [By SAS congruency criterion]

Area (ΔABE) = Area (ΔBAD)

Now, Area ΔABE $= \frac{1}{2} \times \text{Base} \times \text{Altitude}$

$= \frac{1}{2} \times BE \times AF$

$= \frac{1}{2} \times \left(\frac{6\,\text{cm}}{2}\right) \times 4\,\text{cm}$

$= 6\ \text{cm}^2$

⇒ Area ΔABD = 6 cm^2

- **Area of triangle using Heron's formula:**

When all the three sides of a triangle are given, its area can be calculated using Heron's formula, which is given by:

Area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

Here, *s* is the semi-perimeter of the triangle and is given by, $s = \frac{a+b+c}{2}$

Example:
Find the area of a triangle whose sides are 9 cm, 28 cm and 35 cm.
Solution:
Let *a* = 9 cm, *b* = 28 cm and *c* = 35 cm

Semi-perimeter, $s = \frac{a+b+c}{2} = \frac{9+28+35}{2}\,\text{cm} = 36\ \text{cm}$

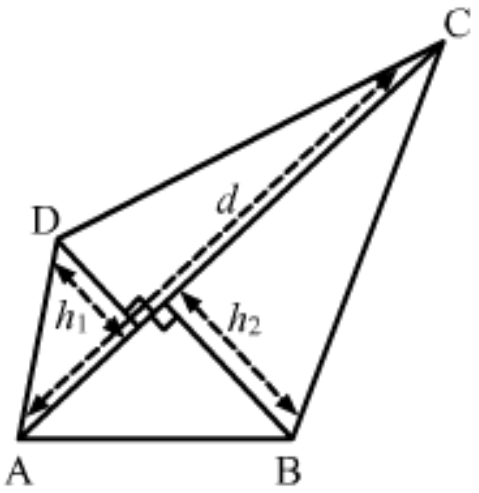
Area of triangle = $\sqrt{36(36-9)(36-28)(36-35)}\ \text{cm}^2$

$= \sqrt{36 \times 27 \times 8 \times 1}\ \text{cm}^2$

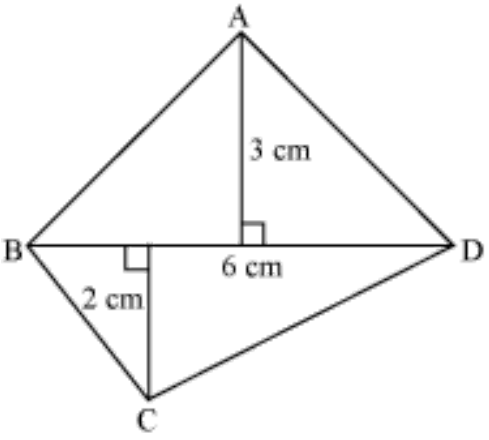
$= 36\sqrt{6}\ \text{cm}^2$

- Area of quadrilateral ABCD = Area of ΔABC + Area of ΔACD

- $= 12\times d\times h1 + 12\times d\times h2 = 12\times d\times h1+h2$



Example: Find the area of the quadrilateral ABCD.



Solution:
Area of the quadrilateral ABCD = Area of ΔABD + Area of ΔBCD
Area of triangle =12 × base × corresponding height
Area of ΔABD $= \frac{1}{2} \times 6\text{cm} \times 3\text{cm} = 9\text{cm}^2$
Area of ΔBCD $= \frac{1}{2} \times 6\text{cm} \times 2\text{cm} = 6\text{cm}^2$
∴ Area of quadrilateral ABCD = 9 cm^2 + 6 cm^2 = 15 cm^2

- Area of rhombus $= \frac{1}{2}$ (Product of its diagonals)

- Area of a rectangle is given by the formula:

Area of a rectangle = length × breadth
Example: How much carpet is required to cover a rectangular floor of length

25 m and breadth 18 m?

Solution: Area of the carpet required = Area of rectangular floor

$= 25 \text{ m} \times 18 \text{ m} = 450 \text{ m}^2$

- Area of a square is given by the formula:

  Area of a square = side × side

  Example: What is the area of a square park of side 10 m 20 cm?

  Solution: Length of park = 10 m 20 cm = 10.2 m

  Area of park = $10.2 \text{ m} \times 10.2 \text{ m} = 104.04 \text{ m}^2$

- Perimeter of a rectangle = 2 (length + breadth)

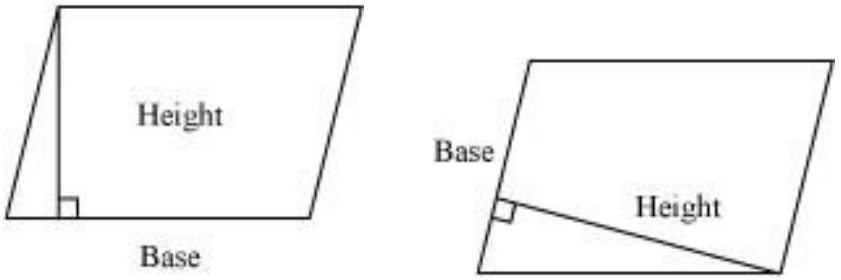
  Example:

  What is the perimeter of a rectangular field whose length and breadth are 15 m and 8 m respectively?

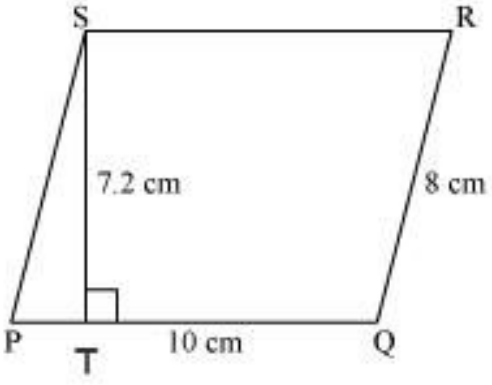
  Solution:

  Perimeter of rectangular field = $2 (15 \text{ m} + 8 \text{ m}) = (2 \times 23) \text{ m} = 46 \text{ m}$

- Area of a parallelogram:
  - The perpendicular dropped on a side from its opposite vertex is known as the height and the side is known as the base.
  - Area of a parallelogram = Base × Height



Example:
Find the height of the parallelogram PQRS corresponding to the base RQ.



Solution:
Let the height corresponding to the base RQ be x cm.
Area of the parallelogram PQRS = PQ × ST
$= 10 \text{ cm} \times 7.2 \text{ cm}$
$= 72 \text{ cm}^2$
Area of the parallelogram = RQ × x
$= 8 \text{ cm} \times x \text{ cm}$
$= 8x \text{ cm}^2$
$\therefore 8x = 72$
$\Rightarrow x = 9$
Thus, the height of the parallelogram corresponding to the base RQ is 9 cm.

- Area and perimeter of various shapes:

Shape	Area	Perimeter
1. Rectangle with adjacent sides a and b	$a \times b$	$2(a + b)$
2. Square with side a	a^2	$4a$

3. Circle with radius r	πr^2	$2\pi r$
4. Triangle with base b and its corresponding height h	$\frac{1}{2} \times b \times h$	Sum of the three sides Sum of the four sides
5. Parallelogram with base b and its corresponding height h	$b \times h$	

Area of trapezium = $\frac{1}{2}$ (Sum of the lengths of the parallel sides) × (Perpendicular distance between them)

- The distance around a circular region is known as its circumference.
- The circumference of a circle = π × Diameter = 2π × Radius

  The value of pi (π) is $\frac{22}{7}$ or 3.14.

- Area of a circle = π × (Radius)^2

  Example: What is the area of a circle whose circumference is 44 cm? $\left(\pi = \frac{22}{7}\right)$

  Solution:
  Circumference = $2\pi r$ = 44 cm
  $\Rightarrow 2 \times \frac{22}{7} \times r = 44\,\text{cm}$
  $\Rightarrow r = 44 \times \frac{7}{22 \times 2} = 7\,\text{cm}$
  ∴ Area of the circle = $\pi r^2 = \frac{22}{7} \times 7 \times 7 = 154\,\text{cm}^2$