**CBSE Class 11 Mathematics**
**Sample Papers 01 (2020-21)**

**Maximum Marks: 80**
**Time Allowed: 3 hours**

**General Instructions:**

i. This question paper contains two parts A and B. Each part is compulsory. Part A carries 24 marks and Part B carries 56 marks
ii. Part-A has Objective Type Questions and Part -B has Descriptive Type Questions
iii. Both Part A and Part B have choices.

**Part – A:**

i. It consists of two sections- I and II.
ii. Section I comprises of 16 very short answer type questions.
iii. Section II contains 2 case studies. Each case study comprises of 5 case-based MCQs. An examinee is to attempt any 4 out of 5 MCQs.

**Part – B:**

i. It consists of three sections- III, IV and V.
ii. Section III comprises of 10 questions of 2 marks each.
iii. Section IV comprises of 7 questions of 3 marks each.
iv. Section V comprises of 3 questions of 5 marks each.
v. Internal choice is provided in 3 questions of Section –III, 2 questions of SectionIV and 3 questions of Section-V. You have to attempt only one of the alternatives in all such questions.

## Part - A Section - I

1. Let A, B and C be three sets. If $A \subset B$ and $B \subset C$, is it true that $A \subset C$? If not give an example.

OR

Let A = {a, b} and B = {a , b, c}. Is $\therefore A \subset B$, What is $A \cup B = \{a, b, c\} = B$

2. In which octant does the given point (-3 , -1, 4) lie.
3. Prove that cos 50° cos 10° - sin 50° sin 10° $= \frac{1}{2}$

OR

Express as the product of sines and cosines: sin 15x - sin x.

4. Express the complex number sin $50^0$ + i cos $50^0$ in the polar form.
5. In how many ways can 5 letters be posted in 4 letter boxes?

OR

In how many ways can the letters of the word PARALLEL be arranged so that all L's do not come together?

6. Find the arithmetic mean between: 14 and -6
7. Find the slope of the line passing through the points ( 4, -3) and ( 6, -3).

OR

Write the equation of the line through the points (1, –1) and (3, 5)

8. Find the equation of a circle with centre (-3, -2) and radius 6.
9. Given that N = {1, 2, 3, ..., 100}, then
   Write the subset B of N, whose element are represented by x + 2, where x ∈ N.

OR

If A and B are two sets such that n(A) = 17, n(B) = 23 and n(A ∪ B) = 38, find number of elements in exactly one of A and B.

10. The letters of word 'SOCIETY' are placed at random in a row. What is the probability that three vowels come together?
11. Find the distance between (2, 3, 5) and (4, 3, 1) pairs of points.
12. How many different words can be formed from the letters of the word 'GANESHPURI'? In how many of these words: the vowels are always together?
13. Determine whether the functions are even or odd or neither: g(x) = $3x^2 + 1$.
14. Prove that: $\sin^2 (n + 1) A - \sin^2 nA = \sin (2n + 1) A \sin A$.
15. Write down the solution set of the inequation x < 6, when the replacement W.

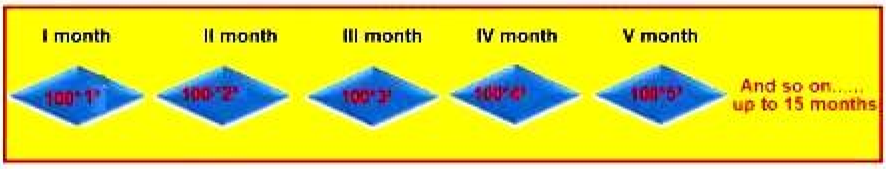
16. State whether $\{x \in Z : x < 5\}$ is infinite or finite set.

## Section - II

17. **Read the Case study given below and attempt any 4 sub parts:**

Ratan wants to open an RD for the marriage of his daughter, He visited the branch of SBI at sector 3, Gurgaon.

There he made an agreement with the bank.




According to this agreement, he would deposit Rs $100 \times n^2$ every month ( here n =1 to 15).

Other terms and conditions are as follows:

I. He has to pay a minimum of six instalments.
II. If he continues the deposit up to 15 months then the bank will pay 20% extra as a bonus.
III. If he breaks the deposit after 6 months then the bank will pay 10% extra as a bonus
IV. If he breaks the deposit after 10 months then the bank will pay 15% extra as a bonus.
V. No other interest will be paid by the bank.

**Answer the following questions:**

i. How much amount would be accumulated after 15 months?
   a. Rs 14,40,000
   b. Rs 11,02,500
   c. Rs 10,00,000
   d. Rs 15,00,000
ii. How much total amount would Ratan get after 15 months?
   a. Rs 14,40,000
   b. Rs 17,28,000
   c. Rs 13,23,000
   d. Rs 15,00,000
iii. How much total amount would Ratan get if he breaks the deposit after 10 months?
   a. Rs 3,45,875
   b. Rs 3,50,000
   c. Rs 3,23,000

d. Rs 3,47,875

iv. How much total amount would Ratan get if he breaks the deposit after 6 months?

a. Rs 65,875

b. Rs 50,000

c. Rs 50,715

d. Rs 60,000

v. How much amount did Ratan pay in the 10th month?

a. Rs 1,00,000

b. Rs 729,000

c. Rs 50,715

d. Rs 60,000

18. **Read the Case study given below and attempt any 4 subparts:**

One evening, four friends decided to play a card game Rummy. Rummy is a card game that is played with decks of cards. To win the rummy game a player must make a valid declaration by picking and discarding cards from the two piles given. One pile is a closed deck, where a player is unable to see the card that he is picking, while the other is an open deck that is formed by the cards discarded by the players. To win at a rummy card game, the players have to group cards invalid sequences and sets.

In rummy, the cards rank low to high starting with Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, and King. Ace, Jack, Queen, and King each have 10 points. The remaining cards have a value equal to their face value. For example, 5 cards will have 5 points, and so on.

Four cards are drawn from a pack of 52 playing cards, then:

i. How many different ways can this is done

a. $\frac{52!}{4!48!}$

b. $\frac{48!}{4!52!}$

c. $\frac{52!4!}{48!}$

d. $\frac{48!4!}{52!}$

ii. exactly one card of each suit

a. 13 ways.

b. $(13)^4$ ways

c. $(13)^2$ ways.

d. $13_{C_1}$

iii. all cards of the same suit

a. 2060 ways.

b. 2800 ways.

c. 2860 ways.

d. 2000 ways.

iv. all club cards

a. 751 ways.

b. 175 ways.

c. 517 ways.

d. 715 ways.

v. The value of P(n, n – 1) is

a. n

b. 2n

c. n!

d. 2n!

vi. If If $^nP_r = 3024$ and $^nC_r = 126$ then find n and r.

a. 9, 4

b. 10, 3

c. 12, 4

d. 11, 4

**Part - B Section - III**

19. If X and Y are two sets such that X has 40 elements $X \cup Y$ has 60 elements and $X \cap Y$ has 10 elements, how many elements does Y have?

20. Let X = {2, 3, 4, 5} and Y = {7, 9, 11, 13, 15, 17}. Define a relation f from X to Y by: f = {(x, y) : $x \in X, y \in Y$ and y = 2x + 3}.

i. Write f in roster form.

ii. Find dom (f) and range (f).

iii. Show that f is a function from X to Y.

OR

if A = {1, 3, 5} and B = {2, 3}, then show that $A \times B \neq B \times A$.

21. Show that the sum $(1 + i^2 + i^4 + \ldots + i^{2n})$ is 0 when n is odd and 1 when n is even.

22. Solve: $ix^2 + 4x - 5i = 0$.

23. If $\frac{(1+i)^2}{2-i} = x + iy$, then find the value of x + y.

OR

Simplify and express $(-2+\sqrt{-3})^{-1}$ in the form (a + ib).

24. On her vacations Veena visits four cities (A, B, C and D) in a random order. What is the probability of A either first or second?

25. Differentiate $x^4 \tan x$.

26. Events E and F are such that P (not E or not F) = 0.25 state whether E and F are mutually exclusive.

27. An analysis of monthly wages paid to the workers of two firms A and B belonging to the same industry gives the following results:

| | **Firm A** | **Firm B** |
|---|---|---|
| Number of workers | 1000 | 1200 |
| Average monthly wages | ₹ 2800 | ₹ 2800 |
| Variance of distribution of wages | 100 | 169 |

In which firm, A or B is there greater variability in individual wages?

28. Write the value of the expression $\frac{1-4\sin 10^\circ \sin 70^\circ}{2\sin 10^\circ}$

OR

Prove that $\tan 6^\circ \tan 42^\circ \tan 66^\circ \tan 78^\circ = 1$.

**Section - IV**

29. Let $x_1, x_2, x_3, \ldots, x_n$ be n values of a variable X, and let $X_j = a + hu_i$, i = 1, 2, . . . n, where $u_1, u_2, \ldots, u_n$ are the values of variable U. Then, prove that Var (X) = $h^2$ Var (U), $h \neq 0$.

30. Let $f = \left\{\left(x, \frac{x^2}{1+x^2}\right) : x \in R\right\}$ be a function from R into R. Determine the range of f.

31. If the $n^{th}$ term of the A.P. 9, 7, 5, ... is same as the $n^{th}$ term of the A.P. 15, 12, 9, ... find n.

OR

Insert three geometric means between $\frac{1}{3}$ and 432.

32. Find the equation of the parabola with vertex at the origin and focus F(0, 5).

33. How many four digit different numbers, greater than 5000 can be formed with the digits 1, 2, 5, 9, 0 when repetition of digits is not allowed?

34. Point R(h, k) divides a line segment between the axis in the ratio 1 : 2. Find equation of the line.

OR

Find equation of the line through the point (0, 2) making an angle $\frac{2\pi}{3}$ with the positive x-axis. Also, find the equation of line parallel to it an crossing the y-axis at a distance of 2 units below the origin.

35. In a group of 400 people in USA, 250 can speak Spanish and 200 can speak English. How many people can speak both Spanish and English?

**Section - V**

36. Differentiate If y = $\sqrt{\frac{\sec x - \tan x}{\sec x + \tan x}}$ show that $\frac{dy}{dx} =$ (sec x tan x + sec x)

OR

Differentiate the function by first principle: $e^{\sqrt{ax+b}}$.

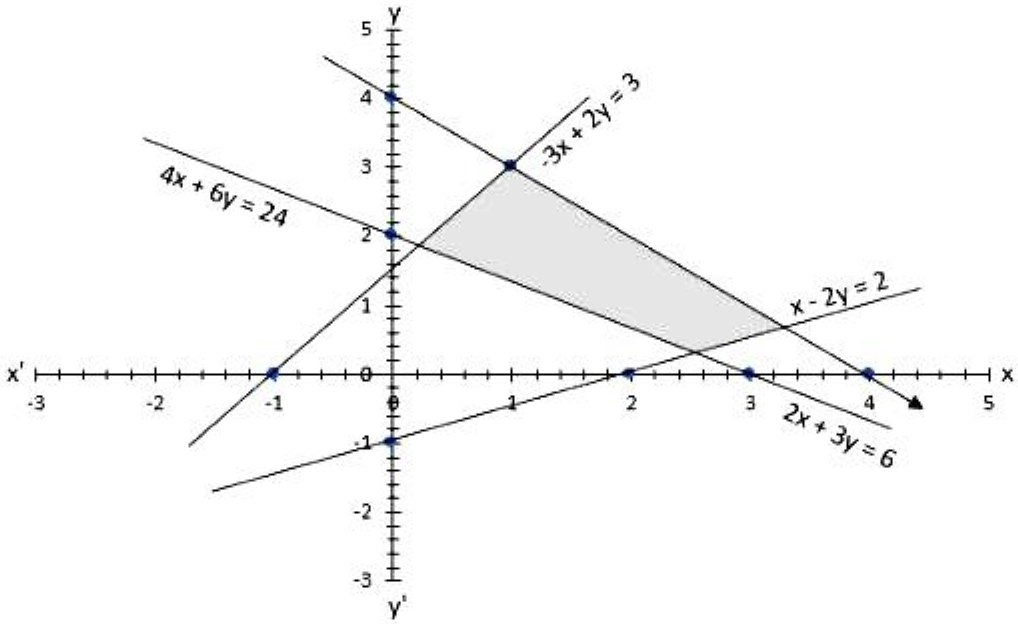
37. Find the domain and range of the real function f(x) defined by

$$f(x) = \begin{Bmatrix} 1 - x_t & x < 0 \\ 1, & x = 0 \\ x - 1, & x > 0 \end{Bmatrix}$$ and draw its graph.

OR

If f (x) = $x^2$ find $\frac{f(1.1) - f(1)}{(1.1 - 1)}$.

38. Find the linear inequations for which the shaded area in the figure is the solution set. Draw the diagram of the solution set of the linear inequations:




OR

Check whether the half plane $3x + 4y \geq 6$ contains the origin. Also, shade the half plane not containing the origin.

**CBSE Class 11 Mathematics**

**Sample Papers 01 (2020-21)**

## Solution

### Part - A Section - I

1. Given, A ⊂ B and B ⊂ C

   Thus, it is true A ⊂ C

   Let a be any arbitrary element of set A, so a ∈ A

   As A ⊂ B, so a ∈ B

   Also B ⊂ C, so a ∈ C

   So as A ⊂ B and B ⊂ C, all the elements of set A is contained in set C and hence we can say A ⊂ C

   OR

   Here A = {a, b} and B = {a, b, c} All elements of set A are present in set B

   $\therefore A \subset B$, Now $A \cup B = \{a, b, c\} = B$

2. Point (-3, -1, 4) lies in octant III

3. We know that $\cos\theta\cos\phi - \sin\theta\sin\phi = \cos(\theta + \phi)$

   Put $\theta = 50^\circ$ and $\phi = 10^\circ$ in the this identity, we get

   cos 50° cos 10° - sin 50° sin 10° = cos (50° + 10°) = cos 60° $= \frac{1}{2}$.

   OR

   Let y = sin 5x - sin x, then

   y = 2 sin $\left(\frac{5x-x}{2}\right)$ cos $\left(\frac{5x+x}{2}\right)$ {∵ sin A - sin B = 2 sin $\left(\frac{A-B}{2}\right)$ cos $\left(\frac{A+B}{2}\right)$}

   = 2 sin 2x cos 3x

4. Suppose, $z = sin50^o + i\ cos50^o$

   $= sin(90^o - 40^o) + i\ cos(90^o - 40^o)$

   = cos $40^o$ + i sin $40^o$

5. Each letter has 4 possible letter boxes option.

   So the number of ways in which 5 letters can be posted in 4 letter boxes = 4 × 4 × 4 × 4 × 4 = $4^5$ (Each 4 for each letter.)

OR

To find: number of words where L do not come together

Let the three L's be treated as a single letter say Z

Number of words with L not the together = Total number of words - Words with L's together

The new word is PARAEZ

Total number of words = $\frac{8!}{2!3!}$ = 3360

Words with L together = $\frac{6!}{2!}$ = 360

Words with L, not together = 3360 - 360 = 3000

There are 3000 words where L do not come together.

6. Thus , Arithmetic mean between 14 and -6

   = $\frac{14+(-6)}{2} = \frac{8}{2}$ = 4

7. Let C(4, - 3) and D(6, - 3) be the given points. Then, we have

   slope of CD $= \frac{-3-(-3)}{6-4} = \frac{-3+3}{2} = \frac{0}{2} = 0$

OR

We know that,

$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

Here $x_1 = 1, y_1 = -1, x_2 = 3$ and $y_2 = 5$.

$y - (-1) = \frac{5-(-1)}{3-1}(x - 1)$

or $-3x + y + 4 = 0$.

which is the required equation.

8. The general equation of a circle with centre (h, k) and Radius r is given by: $(x - h)^2 + (y - k)^2 = r^2$

   Putting value of the centre(h,k) = (-3 , -2) and radius(r = 6) of the circle in the above equation, we get

   $\Rightarrow (x + 3))^2 + (y + 2))^2 = 6^2$

   $\Rightarrow (x + 3)^2 + (y + 2)^2 = 36$

   It is required equation of circle.

9. Here,B = $\{y \mid y = x + 2, x \in N\}$ then

   If,

$1 \in N$, y = 1 + 2 = 3

$2 \in N$, y = 2 + 2 = 4,

and so on. Therefore, B = {3, 4, 5, 6, ... , 100}

OR

Therefore,

Number of elements in exactly one of A and B = n(A) + n(B) - 2n (A $\cap$ B) = 17 + 23 - 2 $\times$ 2 = 36.

10. There are 7 letters in the word

$\therefore$ Total events = 7!

Now there are 3 vowels in the word 'SOCIETY' when we put three vowels altogether and consider as one letter. Then favorable events = 5! $\times$ 3!

Thus required probability $= \frac{5! \times 3!}{7!} = \frac{1}{7}$

11. Let A(2, 3, 5) and B(4, 3, 1) be two points. Then

$AB = \sqrt{(4-2)^2 + (3-3)^2 + (1-5)^2}$ [using distance formula]

$= \sqrt{4+0+16} = \sqrt{20} = 2\sqrt{5}$ units

12. If we fix the first letter as P and the last letter as I, the remaining 8 letters can be arranged in 8! ways to form the words.

$\therefore$The Number of words that start with P and end with I = 8! = 40320

13. The given function is g(x) = $3x^2 + 1$

$\therefore$ g(-x) = $3(-x^2) + 2 = 3x^2 + 1$ = g(x) for all x $\in$ R.

So, g(x) is an even function.

14. LHS = $\sin^2(n + 1)A - \sin^2 nA$

By using the formula $\sin^2 X - \sin^2 Y = \sin(X + Y) \sin(X - Y)$

and taking X = (n + 1)A and Y = nA

= sin[(n + 1 + n)A] sin[(n + 1 - n)A] we get

= sin[(n + 1)A + nA] sin[(n + 1)A - nA]

= sin(2n + 1)A sin A

= RHS

Hence proved.

15. Solution set = $\{x \in W : x < 6\}$= {0, 1, 2, 3, 4, 5}

16. Infinite, $\because \{x \in Z : x < 5\}$ = {....,-3, -2, -1, 0, 1, 2, 3, 4} which is infinite.

**Section - II**

17. i. (a) Rs. 14,40,000
    ii. (b) Rs. 17,28,000
    iii. (d) Rs. 3,47,875
    iv. (c) Rs.50,715
    v. (a) Rs. 1,00,000

18. i. (a) $\frac{52!}{4!48!}$
    ii. (b) $(13)^4$ ways
    iii. (c) 2860 ways.
    iv. (c) n!
    v. (a) 9, 4

**Part - B Section - III**

19. Here n(X) = 40, $n(X \cup Y) = 60$ and $n(X \cap Y) = 10$

    We know that $n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$

    60 = 40 + n(Y) - 10

    $\therefore n(Y) = 60 - 30 = 30.$

20. Here we have, X = {2, 3, 4, 5} and Y = {7, 9, 11, 13, 15, 17}. We need to define a relation f from X to Y by: f = {(x, y) : x $\in$ X, y $\in$ Y and y = 2x + 3}.

    Here X = {2 ,3, 4, 5} and y = 2x + 3

    Now, x = 2 $\Rightarrow$ y = (2 $\times$ 2 + 3) = 7,

    x = 3 $\Rightarrow$ y = (2 $\times$ 3 + 3) = 9,

    x = 4 $\Rightarrow$ y = (2 $\times$ 4 + 3) = 11,

    x = 5 $\Rightarrow$ y = (2 $\times$ 5 + 3) = 13.

    i. $\therefore$ f = {(2, 7), (3, 9), (4, 11), (5, 13)}.

    ii. Clearly, dom (f) = {2, 3, 4, 5} and range (f) = {7, 9, 11, 13} $\subset$ Y.

    iii. It is clear that no two distinct ordered pairs in f have the same first coordinate.

    $\therefore$ f is a function from X to Y

OR

Given, = $\{1, 3, 5\}$ and B = $\{2, 3\}$.

Now, A $\times$ B = {1,3, 5} $\times$ {2, 3}

$\therefore$ A $\times$ B = $\{(1,2),(1,3),(3,2),(3,3),(5,2),(5,3)\}$ ...(i) and B $\times$ A = {2, 3} $\times$ {1, 3, 5}

$\therefore$B $\times$ A = {$(2,1),(2,3),(2,5),(3,1),(3,3),(3,5)$} ....(ii)

From eq. (i) and (ii), we get

A $\times$ B $\neq$ B $\times$ A

**Hence proved.**

21. Let S = 1 + $i^2$ + $i^4$ + ... + $i^{2n}$

This is a GP having (n +1) terms with a = 1 and r = $i^2$ = -1.

$\therefore \quad S=\frac{a(1-r^{n+1})}{(1-r)}=\frac{1\times\left\{1-(i^2)^{n+1}\right\}}{(1-i^2)}$

$S=\frac{\left\{1-(-1)^{n+1}\right\}}{1-(-1)}=\frac{\left\{1-(-1)^{n+1}\right\}}{2}$

$S=\begin{cases}\frac{1}{2}(1-1)=0,\text{ when } n \text{ is odd}\\ \frac{1}{2}(1+1)=1,\text{ when } n \text{ is even}\end{cases}$

22. We have, $ix^2 + 4x - 5i = 0$ ...(i)

On comparing Eq. (i) with $ax^2 + bx + c = 0$, we get

a = i, b = 4 and c = - 5i

$\because x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$\therefore x=\frac{-4\pm\sqrt{(4)^2-4\times i(-5i)}}{2\times i}$

$=\frac{-4\pm\sqrt{16+20i^2}}{2i}$

$=\frac{-4\pm\sqrt{16-20}}{2i}$ [$\because i^2 = -1$]

$=\frac{-4\pm\sqrt{-4}}{2i}=\frac{-4\pm 2i}{2i}$

$=\frac{4i^2\pm 2i}{2i}$ = 2i $\pm$ 1 [$\because -1 = i^2$]

$\therefore$ x = 2i + 1 and x = 2i - 1

Hence, the roots of the given equation are 2i + 1 and 2i - 1.

23. Given that $\frac{(1+i)^2}{2-i}$ = x + iy $\Rightarrow \frac{1+i^2+2i}{2-i}$ = x + iy

$\Rightarrow \frac{1-1+2i}{2-i}$ = x + iy $\Rightarrow \frac{2i}{2-i}$ = x + iy

$\Rightarrow \frac{2i(2+i)}{(2-i)(2+i)}$ = x + iy $\Rightarrow \frac{4i+2i^2}{4-i^2}$ = x + iy [multyply & divide by 2 + i]

$\Rightarrow \frac{4i-2}{4+1}$ = x + iy [$\because i^2 = -1$]

$\Rightarrow \frac{-2+4i}{5}$ = x + iy $\Rightarrow \frac{-2}{5}+\frac{4}{5}$ i = x + iy

Compare the real part and imaginary parts, we get

$x = \frac{-2}{5}$ and $y = \frac{4}{5}$

Hence, $x + y = \frac{-2}{5} + \frac{4}{5} = \frac{2}{5}$.

OR

Given: $(-2 + \sqrt{-3})^{-1}$

We can re-write the above equation as

$= \frac{1}{-2+\sqrt{-3}}$

$= \frac{1}{-2+\sqrt{3i^2}} [\because i^2 = -1]$

$= \frac{1}{-2+i\sqrt{3}}$

Now, rationalizing

$= \frac{1}{-2+i\sqrt{3}} \times \frac{-2-i\sqrt{3}}{-2-i\sqrt{3}}$

$= \frac{-2-i\sqrt{3}}{(-2+i\sqrt{3})(-2-i\sqrt{3})}$

$= \frac{-2-i\sqrt{3}}{(-2)^2-(i\sqrt{3})^2}$ $[(a + b)(a - b) = (a^2 - b^2)]$

$= \frac{-2-i\sqrt{3}}{4-(3i^2)}$

$= \frac{-2-i\sqrt{3}}{4-3(-1)} [\because i^2 = -1]$

$= \frac{-2-i\sqrt{3}}{4+3}$

$= \frac{-2-i\sqrt{3}}{7}$

$= -\frac{2+i\sqrt{3}}{7}$

$= -\frac{2}{7} - \frac{\sqrt{3}}{7}i$

24. The number of arrangements (orders) in which Veena can visit four cities A, B, C, or D is 4! i.e. 24.

Therefore, n (S) = 24.

Sample space for the experiment is given by

S = {ABCD, ABDC, ACBD, ACDB, ADBC, ADCB

BACD, BADC, BDAC, BDCA, BCAD, BCDA

CABD, CADB, CBDA, CBAD, CDAB, CDBA

DABC, DACB, DBCA, DBAC, DCAB, DCBA}

Let G be the event 'Veena visits A either first or second'.

Here G = {ABCD, ABDC, ADBC, ACDB, ACBD, ADCB

BACD, BADC, CABD, CADB, DABC, DACB}

n(G) = 12

Therefore, P(G) = $\frac{n(G)}{n(S)} = \frac{12}{24} = \frac{1}{2}$

25. To find: Differentiation of $x^4$ tan x

Formula used: (i) (uv)' = u'v + uv' (Using Leibnitz or product rule)

(ii) $\frac{dx^n}{dx} = nx^{n-1}$

(iii) $\frac{d\tan x}{dx} = \sec^2 x$

Let u = $x^4$ and v = tan x

$u' = \frac{du}{dx} = \frac{dx^4}{dx} = 4x^3$

$v' = \frac{dv}{dx} = \frac{d\tan x}{dx} = \sec^2 x$

Put the above obtained values in the formula:-

(uv)' = u'v + uv'

$(x^4 \tan x)' = 4x^3 \times \tan x + x^4 \times \sec^2 x$

$= 4x^3 \tan x + x^4 \sec^2 x$

$= x^3 (4\tan x + x\sec^2 x)$.

26. Given, P (not E or not F) = 0.25

$\Rightarrow P\left(\overline{E} \cup \overline{F}\right) = 0.25$

$\Rightarrow P\left(\overline{E \cap F}\right) = 0.25 [\because (\overline{A} \cup \overline{B}) = (\overline{A \cap B}) \text{ by De Morgan's law}]$

$\Rightarrow 1 - P(E \cap F) = 0.25$

$\therefore P(E \cap F) = 1 - 0.25 = 0.75 \neq \phi$

Hence, E and F are not mutually exclusive events.

27. We observe that the average monthly wages in both the firms is same i.e. Rs. 2800.

Therefore, the firm with a greater variance will have more variability.

Now, Variance of A = 100, Variance of B = 169

Thus, firm B has greater variability in individual wages.

28. Let y = $\frac{1-4\sin 10^\circ \sin 70^\circ}{2\sin 10^\circ}$, then

y = $\frac{1-4\sin 10^\circ \sin 70^\circ}{2\sin 10^\circ}$

= $\frac{1-2[2\sin 10^\circ \sin 70^\circ]}{2\sin 10^\circ}$

= $\frac{1-2[\cos(10^\circ-70^\circ)-\cos(10^\circ+70^\circ)]}{2\sin 10^\circ}$

= $\frac{1-2[\cos(-60^\circ)-\cos 80^\circ]}{2\sin 10^\circ}$

= $\frac{1-2[\cos 60^\circ-\cos 80^\circ]}{2\sin 10^\circ}$

$= \frac{1-2\left[\frac{1}{2} - \cos(90^\circ - 10^\circ)\right]}{2\sin 10^\circ}$

$= \frac{1-2\times\frac{1}{2}+2\cos(90^\circ-10^\circ)}{2\sin 10^\circ}$

$= \frac{2\sin 10^\circ}{2\sin 10^\circ}$

= 1

OR

To prove: tan 6° tan 42° tan 66° tan 78° = 1

Now L.H.S = 6° tan 42° tan 66° tan 78°

$= \frac{\tan 6^\circ \tan 42^\circ \tan 66^\circ \tan 78^\circ}{\tan 54^\circ \tan 18^\circ} \times \tan 54^\circ \tan 18^\circ$

$= \frac{(\tan 6^\circ \tan 54^\circ \tan 66^\circ)(\tan 18^\circ \tan 42^\circ \tan 72^\circ)}{\tan 54^\circ \tan 18^\circ}$

$= \frac{[\tan 3(6^\circ)][\tan 3(18^\circ)]}{\tan 54^\circ \tan 18^\circ} = \frac{\tan 18^\circ \tan 54^\circ}{\tan 54^\circ \tan 18^\circ}$

= 1 = RHS

∴ LHS = RHS

Hence proved.

## Section - IV

29. From given information We have,

$x_i = a + h\, u_i$, i = 1, 2, ... , n

$\Rightarrow \sum_{i=1}^{n} x_i = \sum_{i=1}^{n} (a + h\, u_i)$

$\Rightarrow \sum_{i=1}^{n} x_i = na + h \sum_{i=1}^{n} u_i$

$\Rightarrow \frac{1}{n} \sum_{i=1}^{n} x_i = a + h \left(\frac{1}{n} \sum_{i=1}^{n} u_i\right)$

$\Rightarrow \bar{X} = a + h\bar{u}$ [ $\left[\because \bar{X} = \frac{1}{n} \sum_{i=1}^{n} x_i \text{ and } \bar{U} = \frac{1}{n} \sum_{i=1}^{n} u_i\right]$

$\therefore x_i - \bar{X} = (a + h\, u_i) - (a + h\,\bar{U})$, i = 1, 2, ..., n

$\Rightarrow x_i - \bar{X} = h\,(u_i - \bar{U})$, i = 1, 2, ..., n

$\Rightarrow (x_i - \bar{X})^2 = h^2\,(u_i - \bar{U})^2$, i = 1, 2, ..., n

$\Rightarrow \sum_{i=1}^{n} (x_i - \bar{X})^2 = h^2 \sum_{i=1}^{n} (u_i - \bar{U})^2$

$\Rightarrow \frac{1}{n}\Sigma\left(x_i - \bar{X}\right)^2 = h^2 \left\{\frac{1}{n} \sum_{i=1}^{n} \left(u_i - \bar{U}\right)^2\right\}$ [Dividing both sides by n]

$\Rightarrow$ Var (X) = $h^2$ Var (U)

Hence proved.

30. Here $f(x) = \frac{x^2}{1+x^2}$

Put $y = \frac{x^2}{1+x^2} \Rightarrow y + yx^2 = x^2 \Rightarrow x^2(1 - y) = y$

$\Rightarrow x^2 = \frac{y}{1-y} \Rightarrow x = \pm\sqrt{\frac{y}{1-y}}$

$\frac{y}{1-y} \geq 0$

$\Rightarrow \frac{y}{y-1} \leq 0$

$\Rightarrow 0 \leq y < 1$

$\Rightarrow y \in [0, 1)$

$\therefore$ Range of f(x) = [0, 1)

31. According to the question , we can write ,

Considering 9, 7, 5 ...

a = 9, d = (7 - 9) = -2

$n^{th}$ term = 9 + (n - 1)(-2)[$a_n$ = a + (n - 1) d]

= 9 - 2n + 2

= 11 - 2n ...(i)

Considering 15, 12, 9, ...

a = 15, d = (12 - 15) = -3

$n^{th}$ term = 15 + (n - 1)(-3)[$a_n$ = a + (n - 1)d]

= 15 - 3n + 3

= 18 - 3n ...(ii)

Equating (i) and (ii), we get:

11 - 2n = 18 - 3n

n = 7

Thus, 7th terms of both the A.P. are the same.

OR

Given: the numbers $\frac{1}{3}$ and 432.

By using Formula, r = $\left(\frac{b}{a}\right)^{\frac{1}{n+1}}$ where n is the number of geometric mean.

Suppose $G_1$, $G_2$ and $G_3$ be the three geometric mean

Then r = $\left(\frac{b}{a}\right)^{\frac{1}{n+1}}$

$\Rightarrow r = \left(\frac{432}{\frac{1}{3}}\right)^{\frac{1}{3+1}}$

$\Rightarrow r = \left(\frac{432\times 2}{1}\right)^{\frac{1}{4}}$ $\Rightarrow r = 6$

$G_1 = ar = \frac{1}{3} \times 6 = 2$

$G_2 = ar^2 = \frac{1}{3} \times 6^2 = 12$

$G_3 = ar^3 = \frac{1}{3} \times 6^3 = \frac{1}{3} \times 216 = 72$

Therefore, three geometric mean between $\frac{1}{3}$ and 432 are 2, 12 and 72.

32. given that parabola with vertex at the origin and focus F(0, 5).

Given focus F(0, 5) is of the form F(0, a)

For Vertex A(0, 0) and Focus F(0, a), equation of parabola is

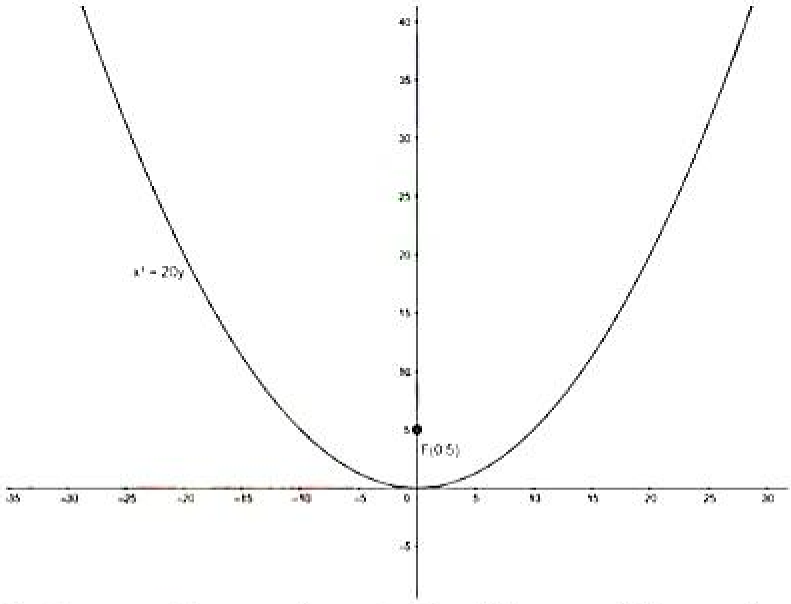
$x^2 = 4ay$

Here, a = 5

Therefore, equation of parabola,

$\Rightarrow x^2 = 4(5)$

$\Rightarrow x^2 = 20y$




33. In the question, we have to find the possible number of 4 digit numbers greater than 5000 formed by the numbers 0, 1, 2, 5, 9 when repetition of digits is not allowed.

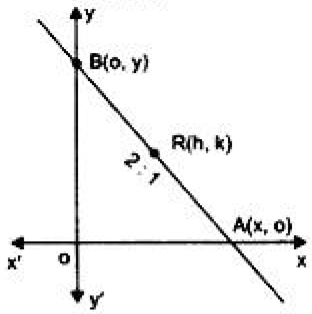
We will use the concept of multiplication because there are four sub-jobs dependent on each other because a number appearing on any one place will not appear in any other place.

For the first place from left we have two choices 5 and 9 because only then our number will be greater than 5000 , for the second place we have 4 choices because out of two one is assigned to the first position from left and total choice of numbers are 5, so 5 - 1 = 4, the number of choices will decrease by one as we keep on going right side.

The number of ways in which we can form six-digit numbers with the help of given numbers is $2 \times 4 \times 3 \times 2 = 2 \times 4! = 2 \times 24 = 48$

34. Let A(x, 0) and B (0, y) be two points where the line intersect x and y axis respectively and R(h, k) is a point divides AB in the ratio 1: 2.

Then $\frac{2x+0}{2+1} = h$ and $\frac{0+y}{2+1} = k$




$\Rightarrow x = \frac{3}{2}h$ and y = 3 k

Now equation of required line is

$\frac{x}{\frac{3}{2}h} + \frac{y}{3k} = 1 \Rightarrow \frac{2x}{3h} + \frac{y}{3k} = 1$

$\Rightarrow$ 2kx + hy = 3 kh

OR

Here $m = \tan\frac{2\pi}{3}$ = tan 120° = tan (90 + 30) = -cot 30° = $-\sqrt{3}$

Equation of the line passing through point (0, 2) having slope $-\sqrt{3}$ is

$y - 2 = -\sqrt{3}(x - 0) \Rightarrow \sqrt{3}x + y - 2 = 0$

Now the line parallel to this line has slope $-\sqrt{3}$.

Here c= -2

Putting these values in y = mx + c, we have

$y = -\sqrt{3}x - 2 \Rightarrow -\sqrt{3}x - y - 2 = 0$

35. Let S be the set of people who speak Spanish, and E be the set of people who speak English

$\therefore n(S \cup E) = 400, n(S) = 250, n(E) = 200$

$n(S \cap E) = ?$

We know that:

$n(S \cup E) = n(S) + n(E) - n(S \cap E)$

$\therefore 400 = 250 + 200 - n(S \cap E)$

$\Rightarrow 400 = 450 - n(S \cap E)$

$\Rightarrow n(S \cap E) = 450 - 400$

$\therefore n(S \cap E) = 50$

Thus, 50 people can speak both Spanish and English.

**Section - V**

36. We have to show that $\frac{dy}{dx} = (\sec x \tan x + \sec x)$

where, it is given that

$y = \sqrt{\frac{\sec x - \tan x}{\sec x + \tan x}}$

$y = \sqrt{\frac{\frac{1}{\cos x} \frac{\sin x}{1}}{\cos x} + \frac{\sin x}{\cos x}} = \sqrt{\frac{1-\sin x}{1+\sin x}}$

$u = 1 - \sin x, v = 1 + \sin x, x = \frac{1-\sin x}{1+\sin x}$

if $z = \frac{u}{v}$

$\frac{dz}{dx} = \frac{v \times \frac{du}{dx} - u \times \frac{dv}{dx}}{v^2}$

$= \frac{(1+\sin x)\times(-\cos x)-(1-\sin x)\times(\cos x)}{(1+\sin x)^2}$

$= \frac{-\cos x - \sin x \cos x - \cos x + \sin x \cos x}{(1+\sin x)^2}$

$= \frac{-2\cos x}{(1+\sin x)^2}$

According to the chain rule of differentiation

$\frac{dy}{dx} = \frac{dy}{dz} \times \frac{dz}{dx}$

$= \left[-\frac{\cos x}{1} \times \left(\frac{1-\sin x}{1}\right)^{-\frac{1}{2}}\right] \times \left[\frac{1}{(1+\sin x)^{2-\frac{1}{2}}}\right]$

$= \left[\cos x \times (1+\sin x)^{-\frac{1}{2}}\right] \times (1-\sin x)^{-\frac{3}{2}} \times \left(\frac{1+\sin x}{1+\sin x}\right)^{\frac{3}{2}}$

Multiplying and dividing by $(1+\sin x)^{\frac{3}{2}}$

$= \left[\cos x \times (1+\sin x)^{\frac{2}{2}-\frac{1}{2}}\right] \times (1-\sin x)^{-\frac{2}{2}} \times \left(\frac{1}{1+\sin x}\right)^{\frac{3}{2}}$

$= \left[\cos x \times (1+\sin x)^{\frac{2}{2}-\frac{1}{2}}\right] \times (1-\sin x)^{-\frac{2}{2}} \times (1+\sin x)^{-\frac{2}{2}}$

$= \left[\cos x \times (1+\sin x)^1\right] \times (1-\sin^2 x)^{-\frac{3}{2}}$

$= \left[\cos x \times (1+\sin x)^1\right] \times (\cos^2 x)^{-\frac{3}{2}}$

$= \left[\cos x \times (1+\sin x)^1\right] \times (\cos x)^{-3}$

$= \left[(1+\sin x)^1\right] \times (\cos x)^{-3+1}$
$= \frac{1+\sin x}{\cos^2 x}$
$= \frac{1}{\cos^2 x} \times \frac{1+\sin x}{\cos^2 x}$
$= \sec x \left(\frac{1}{\cos x} + \frac{\sin x}{\cos x}\right)$
= sec x (sec x + tan x)

Hence proved

OR

We need to find derivative of f(x) = $e^{\sqrt{ax+b}}$ by first principle.

Derivative of a function f(x) is given by -

f'(x) = $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ [where h is a very small positive number]

∴ derivative of f(x) = $e^{\sqrt{ax+b}}$ is given as -

$$\mathrm{f}'(\mathrm{x}) = \lim_{\mathrm{h}\to 0} \frac{e^{\sqrt{a(x+h)+b}} - e^{\sqrt{ax+b}}}{h}$$

$$\Rightarrow \mathrm{f}(\mathrm{x}) = \lim_{\mathrm{h}\to 0} \frac{e^{\sqrt{ax+ah+b}} - e^{\sqrt{ax+b}}}{h}$$

Taking $e^{\sqrt{ax+b}}$common, we have –

$$\Rightarrow \mathrm{f}(\mathrm{x}) = \lim_{h\to 0} \frac{e^{\sqrt{ax+b}}\left(e^{\sqrt{ax+ah+b} - \sqrt{ax+b}} - 1\right)}{h}$$

Using algebra of limits –

$$\Rightarrow f'(x) = \lim_{h\to 0} e^{\sqrt{ax+b}} \times \lim_{h\to 0} \frac{\left(e^{\sqrt{ax+ah+b} - \sqrt{ax+b} - 1}\right)}{h}$$

$$\Rightarrow f'(x) = e^{\sqrt{ax+b}} \lim_{h\to 0} \frac{\left(e^{\sqrt{2x+ah+b}} - \sqrt{ax+b} - 1\right)}{h}$$

As $\lim_{h\to 0} \frac{\left(e^{\sqrt{2x+ah+b}} - \sqrt{ax+b} - 1\right)}{h}$ takes 0/0 form.

we need to take some steps to find its value.

As h $\to 0 \Rightarrow (\sqrt{ax+ah+b} - \sqrt{ax+b}) \to 0$

∴ Multiplying numerator and denominator by $\sqrt{ax+ah+b} - \sqrt{ax+b}$

∴ f'(x) = $e^{\sqrt{ax+b}} \lim_{h\to 0} \frac{e^{\sqrt{2x+ah+b} - \sqrt{ax+b}} - 1}{\sqrt{ax+ah+b} - \sqrt{ax+b}} \times \frac{\sqrt{ax+ah+b} - \sqrt{ax+b}}{h}$

Again using algebra of limits, we have –

⇒ f'(x) = $e^{\sqrt{ax+b}} \lim_{h\to 0} \frac{e^{\sqrt{ax+ah+b} - \sqrt{ax+b} - 1}}{\sqrt{ax+ah+b} - \sqrt{ax+b}} x \lim_{h\to 0} \frac{\sqrt{ax+ah+b} - \sqrt{ax+b}}{h}$

Use the formula: $\lim_{x\to 0}\frac{e^x-1}{x}$ = $\log_e e$ = 1

$\Rightarrow$ f'(x) = $e^{\sqrt{ax+b}}\lim_{h\to 0}\frac{\sqrt{ax+ah+b}-\sqrt{ax+b}}{h}$

Again we get an indeterminate form, so multiplying and dividing $\sqrt{ax+ah+b}+\sqrt{ax+b}$ to get rid of indeterminate form.

$\therefore$ f'(x) = $e^{\sqrt{ax+b}}\lim_{h\to 0}\frac{\sqrt{ax+ah+b}-\sqrt{ax+b}}{h}\times\frac{\sqrt{ax+ah+b}+\sqrt{ax+b}}{\sqrt{ax+ah+b}+\sqrt{ax+b}}$

Using $a^2 - b^2$ = (a + b)(a – b), we have –

$\Rightarrow$ f'(x) = $e^{\sqrt{ax+b}}\lim_{h\to 0}\frac{(\sqrt{ax+ah+b})^2-(\sqrt{ax+b})^2}{h(\sqrt{ax+ah+b}+\sqrt{ax+b})}$

Using algebra of limits we have –

$\Rightarrow$ f'(x) = $e^{\sqrt{ax+b}}\lim_{h\to 0}\frac{ax+ah+b-ax-b}{h}\times\lim_{h\to 0}\frac{1}{\sqrt{2x+ah+b}+\sqrt{ax+b}}$

$\Rightarrow$ f'(x) = $e^{\sqrt{ax+b}}\lim_{h\to 0}\frac{ah}{h}\times\frac{1}{\sqrt{ax+a(0)+b}+\sqrt{ax+b}}$

$\Rightarrow$ f'(x) = $e^{\sqrt{ax+b}}\lim_{h\to 0}a\times\frac{1}{2\sqrt{ax+b}}$

$\therefore$ f'(x) = $\frac{ae^{\sqrt{ax+b}}}{2\sqrt{ax+b}}$

Therefore, Derivative of f(x) = $e^{\sqrt{ax+b}}$ = $\frac{ae^{\sqrt{ax+b}}}{2\sqrt{ax+b}}$

37. Consider, $f(x)=\begin{cases}1-x, & x<0\\ 1, & x=0\\ x-1, & x>0\end{cases}$

For $x>0, x-1>-1$ and for $x<0, -x>0$

$\Rightarrow 1-x>1$

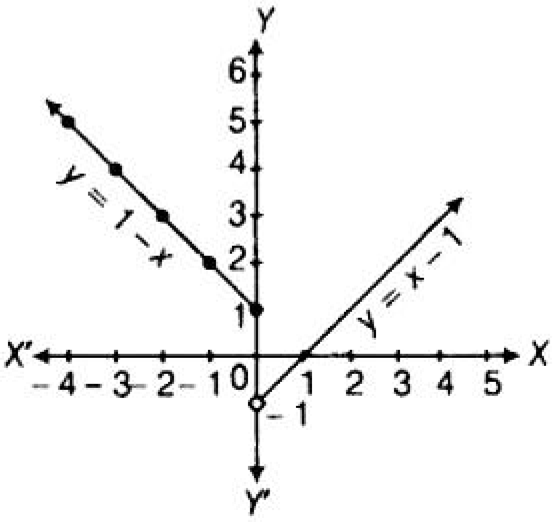
Hence, for x $\in$ R, f( x ) > -1

Domain = Set of real numbers;

Range = Set of real numbers > -1.

Some points on graph are

| x | -4 | -3 | -2 | -1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| y | 5 | 4 | 3 | 2 | 1 | 0 | 1 | 2 | 3 | 4 | 5 |




OR

Here f (x) = $x^2$

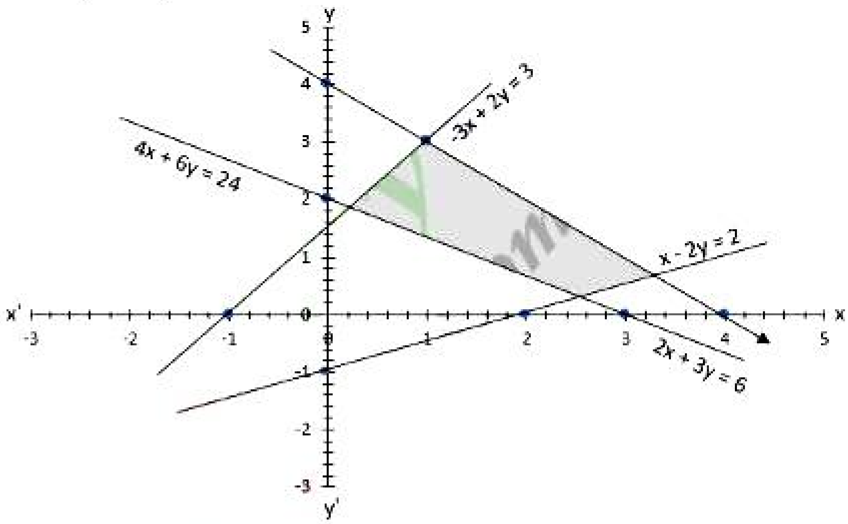
At x = 1.1

f(1.1) = $(1.1)^2$ = 1.21

f (1) = $(1)^2$ = 1

$\therefore \frac{f(1.1)-f(1)}{(1.1-1)} = \frac{1.21-1}{0.1} = \frac{0.21}{0.1} = \mathbf{2.1}$

38.




Consider the line, 2x + 3y = 6, we observe that the shaded region and the origin are on the opposite sides of the line 2x + 3y = 6 and (0,0) does not satisfy the inequation, 2x + 3y $\geq$ 6. So, the first inequation is 2x + 3y $\geq$ 6.

Consider the line, 4x + 6y = 24, we observe that the shaded region and the origin are on the same side of the line 4x + 6y = 24 and (0,0) satisfies the linear inequation 4x + 6y $\leq$ 24. So, the second inequation is 4x + 6y $\leq$ 24.

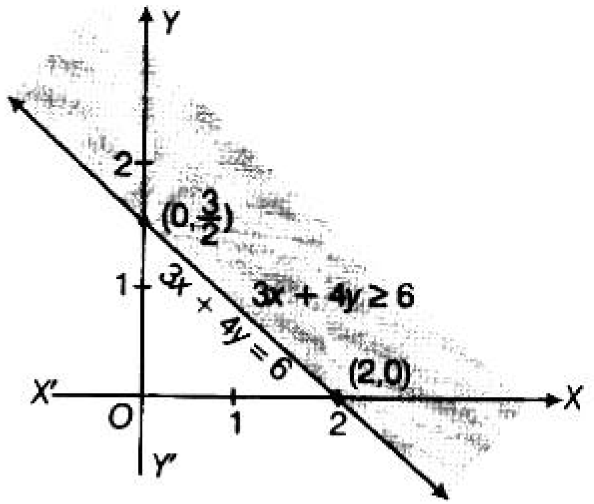
Consider the line, $-3x + 2y = 3$, we observe that the shaded region and the origin are on the same side of the line $-3x + 2y = 3$ and $(0,0)$ satisfies the linear equation $-3x + 2y \leq 3$.
So, the third inequation is $-3x + 2y \leq 3$.
Finally, consider the line, $x - 2y = 2$, we observe that the shaded region and the origin are on the same side of the line $x - 2y = 2$ and $(0,0)$ satisfies the linear inequation, $x - 2y \leq 2$.
So, the fourth inequation is $x - 2y \leq 2$.
We also notice that the shaded region is above the x-axis and is on the right side of the y-axis. So, we must have $x \geq 0$ and $y \geq 0$.
Thus, the linear inequations corresponding to the given solution set are:
$2x + 3y \geq 6, 4x + 6y \leq 24, -3x + 2y \leq 3, x - 2y \leq 2, x \geq 0, y \geq 0$

OR

The given half plane is $3x + 4y \geq 6$ ... (i)
On putting $x = 0$ and $y = 0$ in Eq. (i), we get
$0 + 0 \geq 6$
$\Rightarrow 0 \geq 6$, which is false.
$\therefore$ The given half plane does not contain the origin.
Now, in equation form given inequality can be written as
$3x + 4y = 6$ ... (ii)
On putting $x = 0$ in Eq. (ii), we get
$4y = 6 \Rightarrow y = \frac{3}{2}$
So, the line $3x + 4y = 6$ meet Y - axis at $\left(0, \frac{3}{2}\right)$
On putting $y = 0$ in Eq. (ii), we get
$3x + 0 = 6$
$\Rightarrow x = 2$
Thus, the line $3x + 4y = 6$ meet X - axis at $x = 2$ i.e., at the point $(2, 0)$

| | | |
|---|---|---|
| x | 0 | 2 |
| y | $\frac{3}{2}$ | 0 |

Since, given inequality has the sign $\geq$ , so join the points $\left(0, \frac{3}{2}\right)$ and $(2, 0)$ with a dark line.




Now, the origin is not contained by the given half plane, so we shade that half plane which does not contain origin i.e., the region above the line $3x + 4y = 6$. Hence, shaded region give the solution set.