7. Linear Equations

Equations of Straight Lines

When working with straight lines, there are several ways to arrive at an equation which represents the line.

General Form of Equation of a Line.

The "General Form" of the equation of a straight line is:

$Ax + By + C = 0$

FORMS OF EQUATIONS OF STRAIGHT LINE

- Lines Parallel to Axes : Equation of straight line parallel to x-axis at a distance 'a' is $y = a$ and equation of straight line parallel to y-axis at a distance 'b' is $x = b$.
- Point-Slope Form : The equation of the straight line passing through the point (x_1, y_1) and having slope m is : $y - y_1 = m(x - x_1)$.
- Two-Point Form : The equation of the straight line passing through two point P (x_1, y_1) and Q (x_2, y_2) is :

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} \quad x - x_1$$

or

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

Remember:	**Slope** is found by using the formula: $m = \frac{y_2 - y_1}{x_2 - x_1}$	**Slope** is also expressed as rise/run.

Equation Forms of Straight Lines

Slope Intercept Form	Point Slope Form
Use this form when you know the **slope** and the y-**intercept** (where the line crosses the y-axis).	Use this form when you know a **point** on the line and the **slope** (or can determine the slope).
$y = mx + b$	$y - y_1 = m(x - x_1)$
m = slope b = y-intercept (where line crosses the y-axis.)	m = slope (x_1, y_1) = any point on the line

Horizontal Lines	Vertical Lines
$y = 3$ (or any number) **Lines that are horizontal have a slope of zero.** Horizontal lines have "run", but no "rise". The rise/run formula for slope always yields zero since the **rise = 0.** Since the slope is zero, we have $y = mx + b$ $y = 0 \cdot x + 3$ $y = 3$ This equation also describes what is happening to the y-coordinates on the line. In this case the y-coordinates are always 3.	$x = -2$ (or any number) **Lines that are vertical have no slope (it does not exist).** Vertical lines have "rise", but no "run". The rise/run formula for slope always has a zero denominator and is undefined. The equations for these lines describe what is happening to the x-coordinates. In this example, the x-coordinates are always equal to -2.

Examples:

Examples using Slope-Intercept Form:	Examples using Point-Slope Form:
1. Find the slope and y-intercept for the equation $2y = -6x + 8$. First solve for "y =": $y = -3x + 4$ Remember the form: $y = mx + b$ **Answer:** the slope (m) is -3 the y-intercept (b) is 4	3. Given that the slope of a line is -3 and the line passes through the point (-2,4), write the equation of the line. The slope: $m = -3$ The point $(x_1, y_1) = (-2,4)$ Remember the form: $y - y_1 = m(x - x_1)$ Substitute: $y - 4 = -3(x - (-2))$ ANS. $y - 4 = -3(x + 2)$ If asked to express the answer in "y =" form: $y - 4 = -3x - 6$ $y = -3x - 2$
2. Find the equation of the line whose slope is 4 and the coordinates of the y-intercept are (0,2). In this problem $m = 4$ and $b = 2$. Remember the form: $y = mx + b$ and that b is where the line crosses the y-axis. Substitute: $\mathbf{y = 4x + 2}$	4. Find the slope of the line that passes through the points (-3,5) and (-5,-8). First, find the slope: $m = \frac{y_2 - y_1}{x_2 - x_1}$ $m = \frac{5-(-8)}{-3-(-5)} = \frac{13}{2} = 6.5$ Use either point: (-3,5) Remember the form: $y - y_1 = m(x - x_1)$ Substitute: $y - 5 = 6.5(x - (-3))$ $y - 5 = 6.5(x + 3)$ Ans.

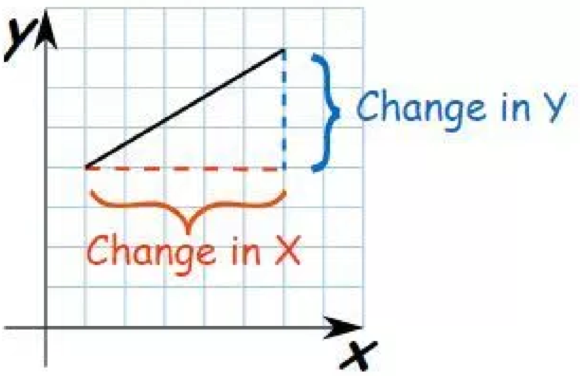
Point-Slope Equation of a Line

Equations of straight line in different forms

(1) Slope form:

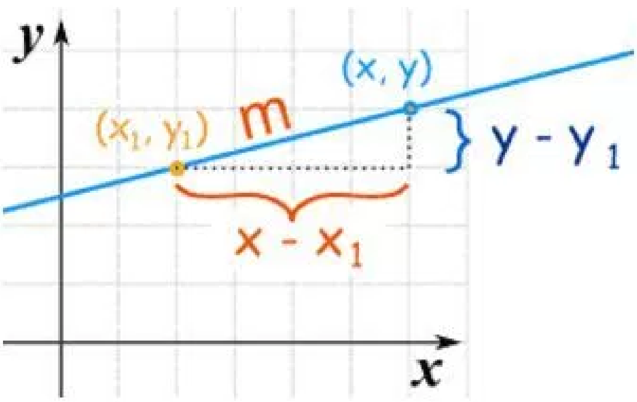
Equation of a line through the origin and having slope m is **y = mx**.

$$m = \frac{\text{Change in Y}}{\text{Change in X}}$$



(2) One point form or Point slope form:

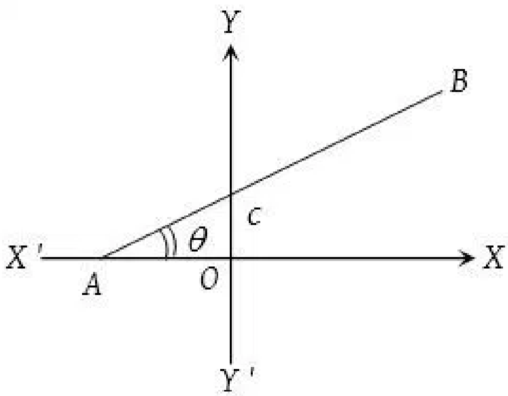
Equation of a line through the point (x_1, y_1) and having slope m is $\mathbf{y - y_1 = m(x - x_1)}$.



$$\text{Slope } m = \frac{\text{change in y}}{\text{change in x}} = \frac{y - y_1}{x - x_1}$$

(3) Slope intercept form:

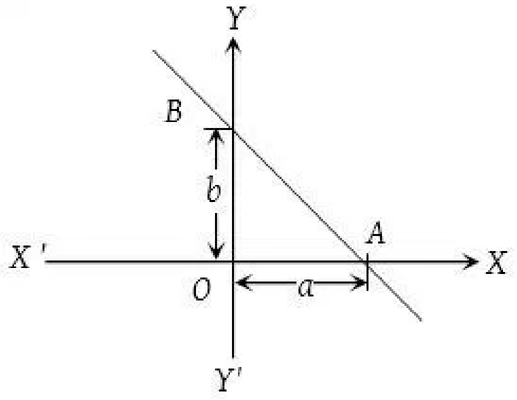
Equation of a line (non-vertical) with slope m and cutting off an intercept c on the y-axis is **y = mx + c**.



The equation of a line with slope m and the x-intercept d is $y = m(x - d)$.

(4) Intercept form:

If a straight line cuts x-axis at A and the y-axis at B then OA and OB are known as the intercepts of the line on x-axis and y-axis respectively.



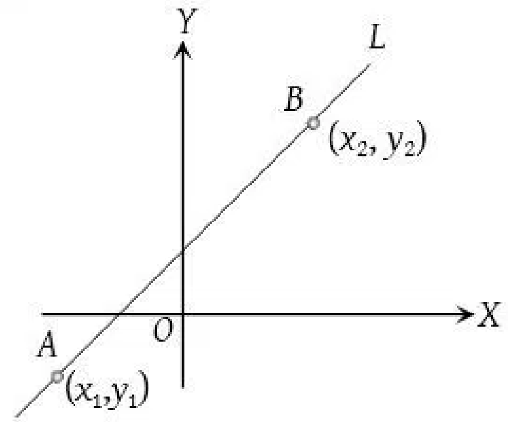
Then, equation of a straight line cutting off intercepts a and b on x–axis and y–axis respectively is $\frac{x}{a} + \frac{y}{b} = 1$.

If given line is parallel to X axis, then X-intercept is undefined.

If given line is parallel to Y axis, then Y-intercept is undefined.

(5) Two point form:

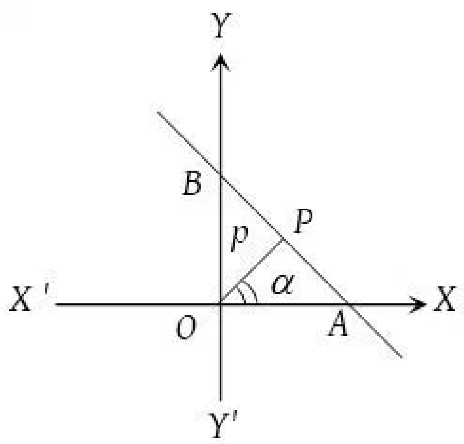
Equation of the line through the points A(x_1, y_1) and B(x_2, y_2) is, $(y - y_1) = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$.



In the determinant form it is gives as $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$ is the equation of line.

(6) Normal or perpendicular form:

The equation of the straight line upon which the length of the perpendicular from the origin is p and this perpendicular makes an angle α with x-axis is x cos α + y sin α = p.

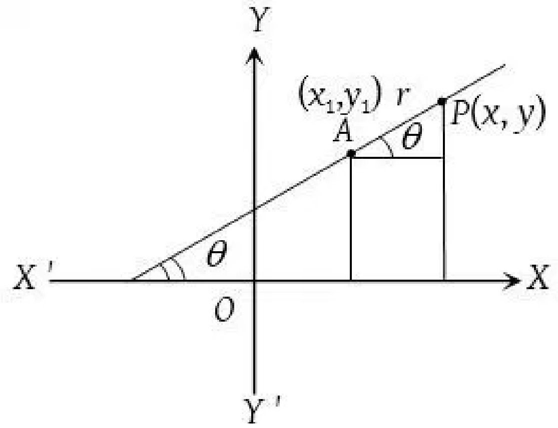


(7) Symmetrical or parametric or distance form of the line:

Equation of a line passing through (x_1, y_1) and making an angle θ with the positive direction of x-axis is ,

$$\frac{x - x_1}{\cos\theta} = \frac{y - y_1}{\sin\theta} = \pm r$$

where r is the distance between the point P(x, y) and $A(x_1, y_1)$.



The co-ordinates of any point on this line may be taken as $(x_1 \pm r\cos\theta, y_1 \pm r\sin\theta)$, known as parametric co-ordinates. ‘r’ is called the parameter.

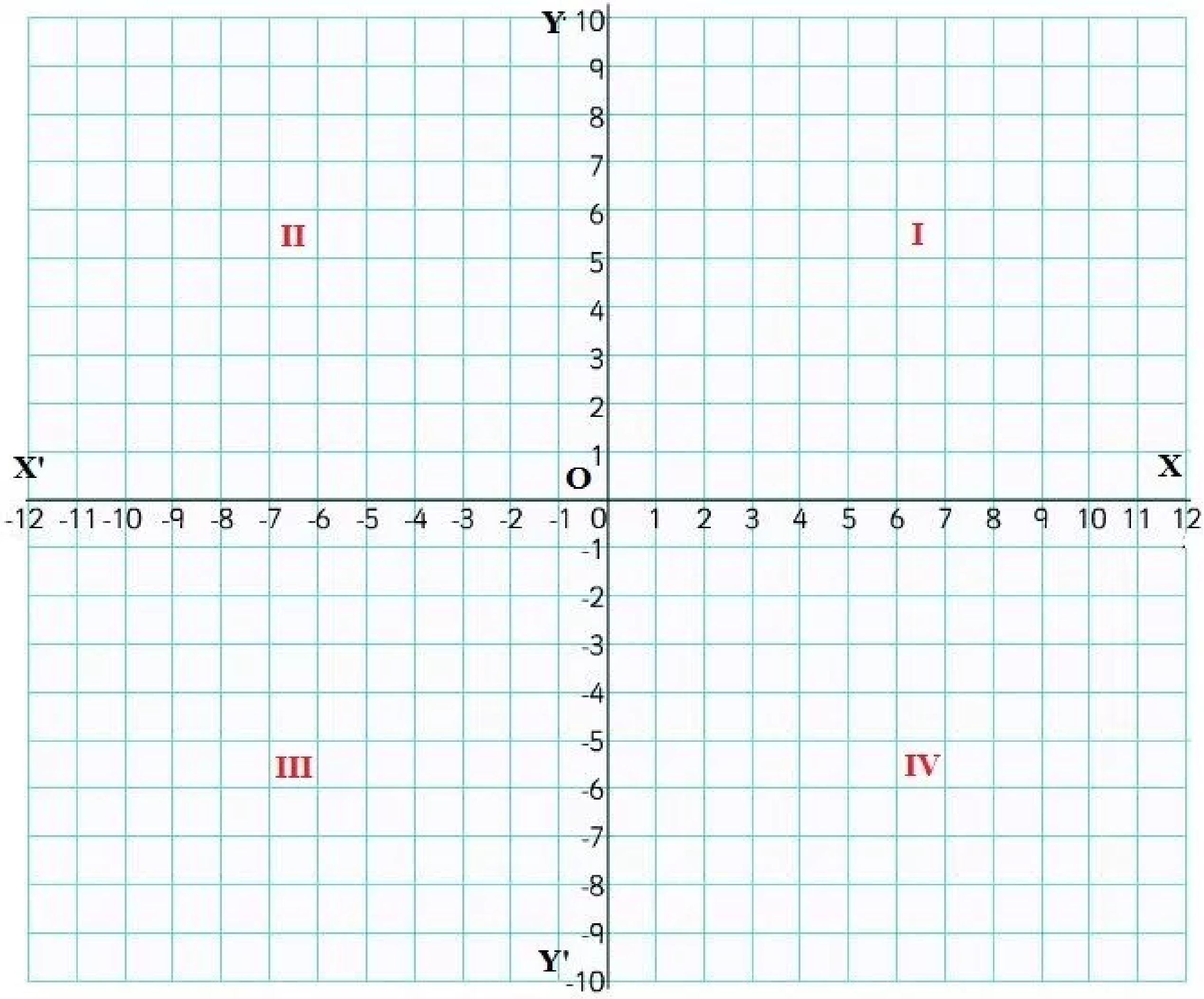
Cartesian Coordinates

Rectangular Or Cartesian coordinates of a point:

The French Mathematician and Philosopher Rene Descartes first published his book La Geometric in 1637 in which he used algebra in the study of geometry. This he did by representing points in a plane by ordered pairs of real numbers, called Cartesian coordinates (named after Rene Descartes). In this section, we shall see how points in a plane are represented by the ordered pair of real numbers.

Cartesian coordinates axes:

Let X’OX and Y’OY be two mutually perpendicular lines through a point O in the plane of a graph paper as shown in Fig. 11.1. The line X’OX is called the r-axis or axis of x the line Y’OY is known as the y-axis or axis of y, and the two lines X’OX and Y’OY taken together are called the coordinate axes or the axes of coordinates. The point O is called the origin.

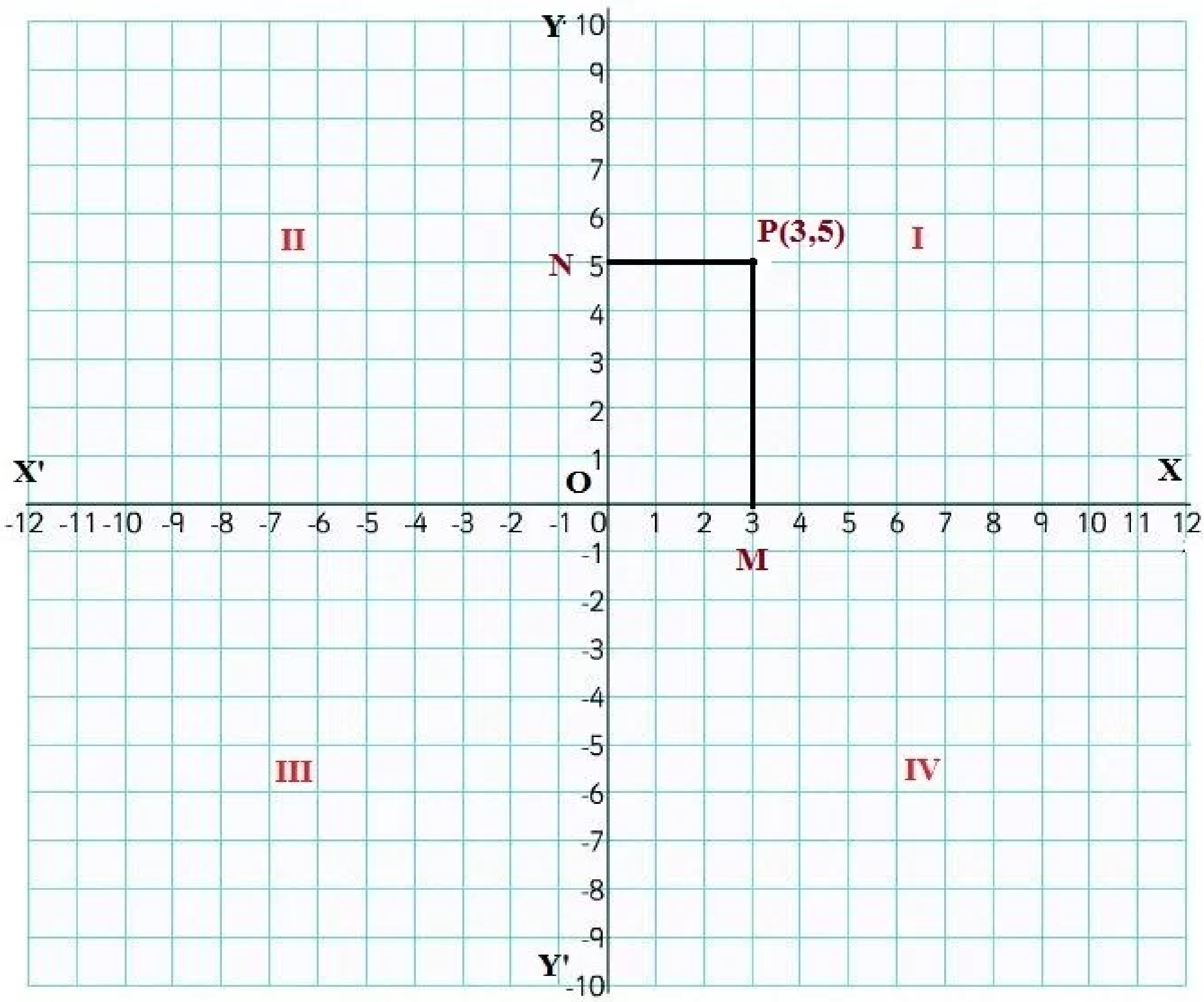


Quadrants:

In Fig. 11.1, the coordinate axes X'OX and Y'OY divide the plane of the graph paper into four regions XOY, X'OY, X'OY' and Y'OX. These four regions are called the quadrants. The regions XOY, X'OY, X'OY' and Y'OX are known as the first, the second, the third and the fourth quadrants respectively.

Cartesian coordinates of a point:

Let X'OX and Y'OY be the coordinate axes and let P be any point on the Plane of the paper. Draw PM ⊥ X'OX and PN ⊥ Y'OY.
The length of the line segment OM is called the x-coordinate or abscissa of point P and the length of the directed line segment ON is called the y-coordinate or ordinate of point P.
If OM = 3 units and ON = 5 units, then the x-coordinate or abscissa of point P is 3 and the y-coordinate or ordinate of P is 5 and we say that the coordinates of P are (3, 5). Note that (3,5) is an ordered pair in which the positions of 3 and 5 cannot he interchanged.



Thus, for a given point P, the abscissa and ordinate are the distances of the point P from y-axis and x-axis respectively.
If we take a point on x-axis, then clearly the distance of this point from x-axis is 0 and therefore the ordinate of this point is 0.
Thus, the ordinate or y-coordinate of every point on x-axis is 0 and the coordinates of a point on x-axis are of the form (x, 0).
Similarly, if we take a point on y-axis then its distance from y-axis is 0 and therefore, the x-coordinate or abscissa of the point is 0.
Thus, the abscissa or coordinate of every point on y-axis is zero and the coordinates of a point on y-axis are of the form (0, y)
The coordinate of the origin are taken as (0, 0).

Convention of signs:

Let X'OX and Y'OY be the coordinate axes. As discussed earlier that regioons XOY, X'OY, X'OY' and Y'OX are known as the first, the second, the third and the fourth quadrants respectively. The ray OX is taken as positive x-axis, OX' as negative x-axis,OY as positive y-axis and OY'as negative y-axis.
This means that any distance measured along OX will be taken as positive and the distance moved along OX' will be negative. Similarly, the distance moved along or parallel to OY will taken as positive and the distance along OY' will be negative.
In view of the above sign convention, we find that:
In I quadrant: $x > 0, y > 0$
In II quadrant: $x < 0, y > 0$
In III quadrant: $x < 0, y < 0$
In IV quadrant: $x > 0, y < 0$.

Plotting of points:

In order to plot the points in a plane, we may u the following algorithm:

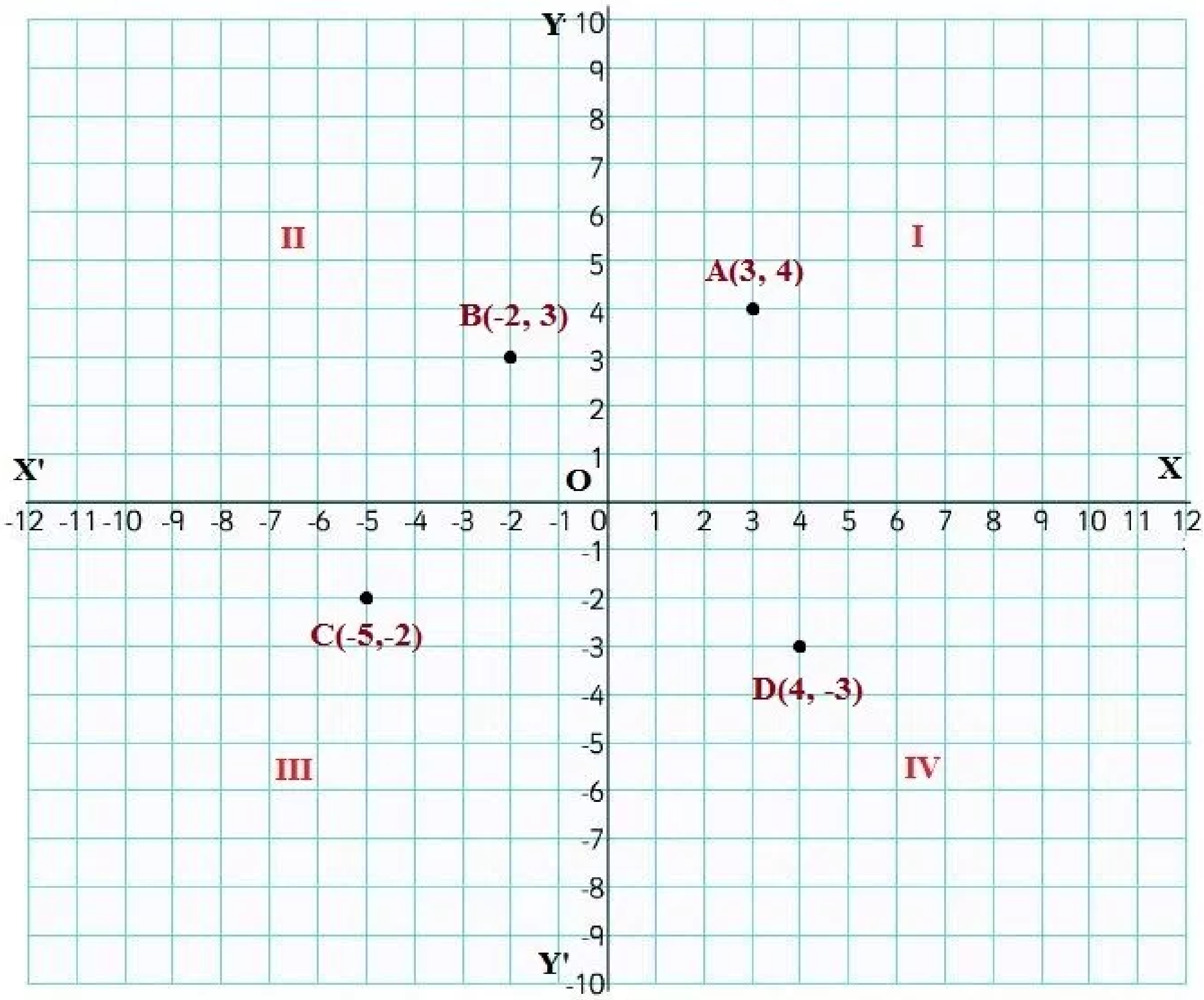
Algorithm:

STEP I: Draw two mutually perpendicular lines on the graph paper, One horizontal and other vertical.
STEP II: Mark their intersection point as O(origin). The horizontal line as X'OX and the vertical line as Y'OY. The line X'OX is the x-axis and the line Y'OY as the y-axis.
STEP III: Choose a suitable scale on x-axis and y-axis and mark the points on both the axes.
STEP IV: Obtain the coordinates of the point which is to be plotted. Let the point be P (a, b). To plot this point, start from the origin and move |a| units along OX or OX' according as 'a'is positive or negative. Suppose we arrive at point M. From point M move vertically upward or down ward through |b| units according as b is positive or negative. The point where we arrive finally is the required point P (a, b).
The following examples will illustrate the above procedure.

Example: Plot the following points on a graph paper:

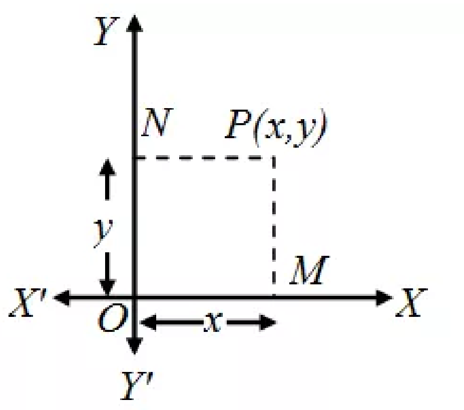
(1) (3,4) 01) (- 2, 3) (iii) (- 5, – 2) (iv) (4, – 3)

Solution: X'OX and Y'OY be the coordinate axes. Then the given four points may be plotted as given below:



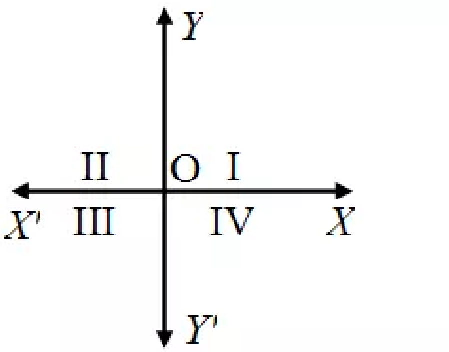
What Is The Cartesian Coordinate System

In Cartesian co-ordinates the position of a point P is determined by knowing the distances from two perpendicular lines passing through the fixed point. Let O be the fixed point called the origin and XOX' and YOY', the two perpendicular lines through O, called Cartesian or Rectangular co-ordinates axes.



Draw PM and PN perpendiculars on OX and OY respectively. OM (or NP) and ON (or MP) are called the x-coordinate (or abscissa) and y-coordinate (or ordinate) of the point P respectively.

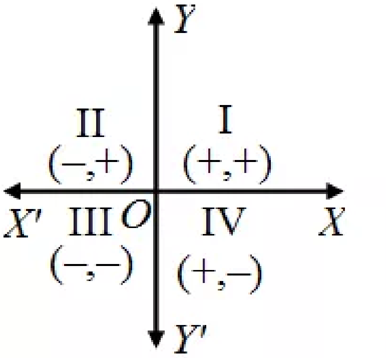
1. **Axes of Co-ordinates**
   In the figure OX and OY are called as x-axis and y-axis respectively and both together are known as axes of co-ordinates.
2. **Origin**
   It is point O of intersection of the axes of co-ordinates.
3. **Co-ordinates of the Origin**
   It has zero distance from both the axes so that its abscissa and ordinate are both zero. Therefore, the coordinates of origin are (0, 0).
4. **Abscissa**
   The distance of the point P from y-axis is called its abscissa. In the figure OM = PN is the Abscissa.
5. **Ordinate**
   The distance of the point P from x-axis is called its ordinate. ON = PM is the ordinate in the figure.
6. **Quadrant**
   The axes divide the plane into four parts. These four parts are called quadrants. So, the plane consists of axes and quadrants. The plane is called the cartesian plane or the coordinate plane or the xy-plane. These axes are called the co-ordinate axes.
   A quadrant is 1/4 part of a plane divided by co-ordinate axes.

   (i) XOY is called the first quadrant
   (ii) YOX’ the second.
   (iii) X’OY’ the third.
   (iv) Y’OX the fourth
   as marked in the figure.

RULES OF SIGNS OF CO-ORDINATES

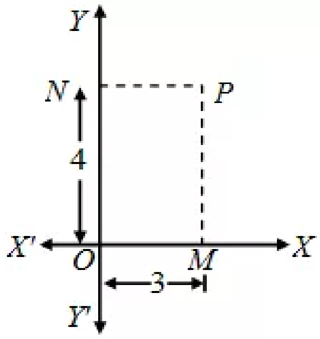
1. In the first quadrant, both co-ordiantes i.e., abscissa and ordinate of a point are positive.
2. In the second quadrant, for a point, abscissa is negative and ordinate is positive.
3. In the third quadrant, for a point, both abscissa and ordinate are negative.
4. In the fourth quadrant, for a point, the abscissa is positive and the ordinate is negative.



Quadrant	x-co-ordinate	y-cordinate	Point
First quadrant	+	+	(+,+)
Second quadrant	–	+	(–,+)
Third quadrant	–	–	(–,–)
Fourth quadrant	+	–	(+,–)

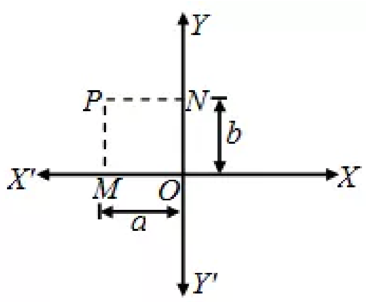
Cartesian Coordinate System Example Problems With Solutions

Example 1: From the adjoining figure find



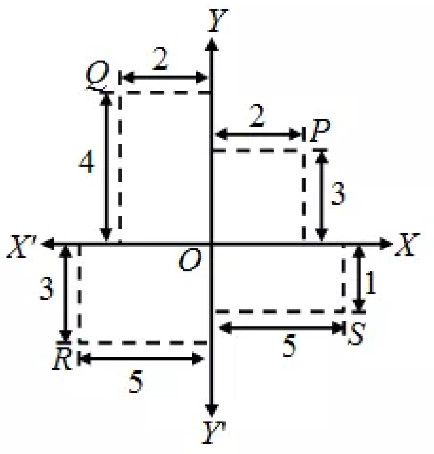
(i) Abscissa
(ii) Ordinate
(iii) Co-ordinates of a point P
Solution: (i) Abscissa = PN = OM = 3 units
(ii) Ordinate = PM = ON = 4 units
(iii) Co-ordinates of the point P = (Abscissa, ordinate) = (3, 4)

Example 2: Determine



(i) Abscissa (ii) ordinate (iii) Co-ordinates of point P given in the following figure.
Solution: (i) Abscissa of the point P = – NP = –OM = – a
(ii) Ordinate of the point P = MP = ON = b
(iii) Co-ordinates of point P = (abscissa, ordinate)
= (–a, b)

Example 3: Write down the (i) abscissa (ii) ordinate (iii) Co-ordinates of P, Q, R and S as given in the figure.



Solution: **Point P**
Abscissa of P = 2; Ordinate of P = 3
Co-ordinates of P = (2, 3)

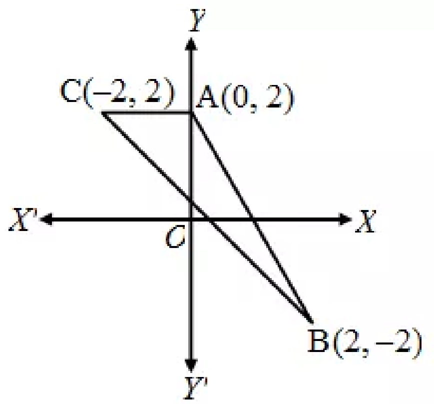
Point Q
Abscissa of Q = – 2; Ordinate of Q = 4
Co-ordinate of Q = (–2, 4)

Point R
Abscissa of R = – 5; Ordinate of R = – 3
Co-ordinates of R = (–5, –3)

Point S
Abscissa of S = 5; Ordinate of S = – 1
Co-ordinates of S = (5, – 1)

Example 4: Draw a triangle ABC where vertices A, B and C are (0, 2), (2, – 2), and (–2, 2) respectively.

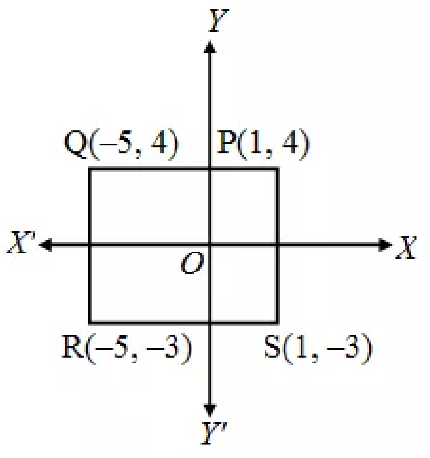
Solution: Plot the point A by taking its abscissa O and ordinate = 2.
Similarly, plot points B and C taking abscissa 2 and –2 and ordinates – 2 and 2 respectively. Join A, B and C. This is the required triangle.



Example 5: Draw a rectangle PQRS in which vertices P, Q, R and S are (1, 4), (–5, 4), (–5, –3) and (1, – 3) respectively.

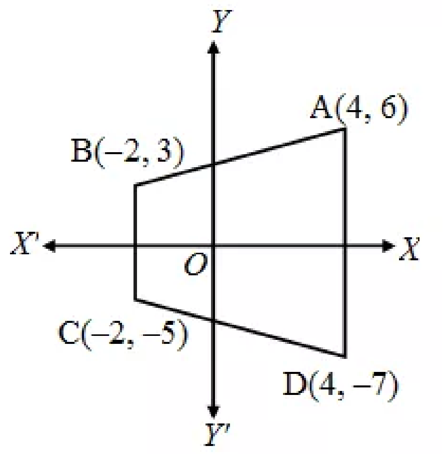
Solution: Plot the point P by taking its abscissa 1 and ordinate – 4.
Similarly, plot the points Q, R and S taking abscissa as –5, –5 and 1 and ordinates as 4, – 3 and –3 respectively.

Join the points PQR and S. PQRS is the required rectangle.



Example 6: Draw a trapezium ABCD in which vertices A, B, C and D are (4, 6), (−2, 3), (−2, −5) and (4, −7) respectively.

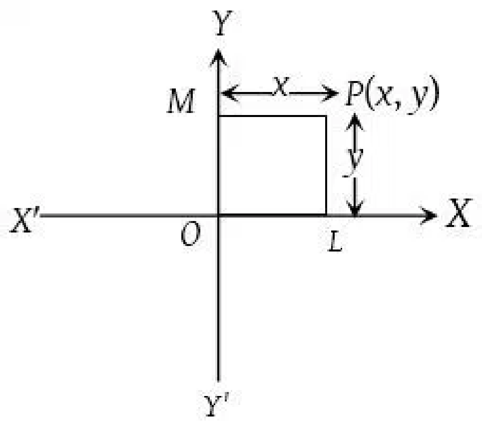
Solution: Plot the point A taking its abscissa as 4 and ordinate as 6.
Similarly plot the point B, C and D taking abscissa as – 2, −2 and 4 and ordinates as 3, – 5, and −7 respectively. Join A, B, C and D ABCD is the required trapezium.



Polar and Cartesian Coordinates

Cartesian co-ordinates of a point

This is the most popular co-ordinate system.



Axis of x: The line XOX′ is called axis of x.

Axis of y: The line YOY' is called axis of y.

Co-ordinate axes: x axis and y axis together are called axis of co-ordinates or axes of reference.

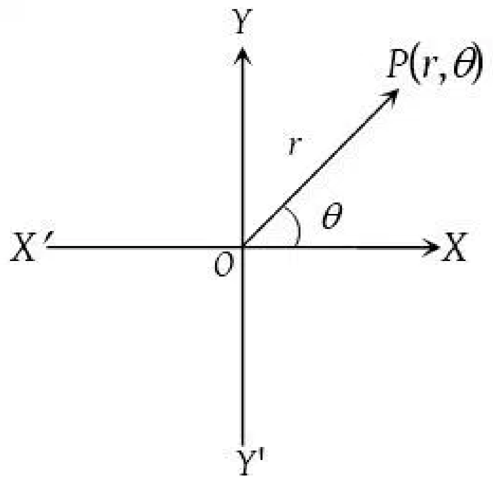
Origin: The point 'O' is called the origin of co-ordinates or the origin.

Let OL = x and OM = y which are respectively called the abscissa (or x-coordinate) and the ordinate (or y-coordinate). The co-ordinate of P are (x, y).
Here, co-ordinates of the origin is (0, 0). The y co-ordinates of every point on x-axis is zero.
The x co-ordinates of every point on y-axis is zero.

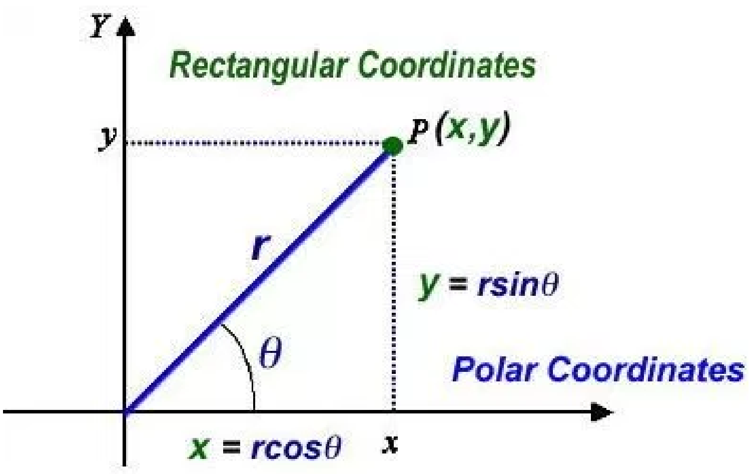
Oblique axes: If both the axes are not perpendicular then they are called as oblique axes.

Polar co-ordinates

Let OX be any fixed line which is usually called the initial line and O be a fixed point on it. If distance of any point P from the O is 'r' and $\angle XOP = \theta$, then (r, θ) are called the polar co-ordinates of a point P.



To convert from Polar Coordinates (r,θ) to Cartesian Coordinates (x,y):



If (x, y) are the cartesian co-ordinates of a point P, then $x = r\cos\theta$; $y = r\sin\theta$; and

$$r = \sqrt{x^2 + y^2}\ ;\ \theta = \tan^{-1}\left(\frac{y}{x}\right)$$