SURFACE AREA & VOLUME

4
CHAPTER

FORMULAE

1. If λ, b and h denote respectively the length, breadth and height of a cuboid, then -

(i) total surface area of the cuboid = $2(\lambda b+bh+\lambda h)$ square units.

(ii) Volume of the cuboid

= Area of the base × height = λbh cubic units.

(iii) Diagonal of the cuboid or longest rod

$= \sqrt{\lambda^2 + b^2 + h^2}$ units.

(iv) Area of four walls of a room

$= 2(\lambda + b)h$ sq. units.

2. If the length of each edge of a cube is 'a' units, then-

(i) Total surface area of the cube = $6a^2$ sq. units.

(ii) Volume of the cube = a^3 cubic units

(iii) Diagonal of the cube = $\sqrt{3}$ a units.

3. If r and h denote respectively the radius of the base and height of a right circular cylinder, then -

(i) Area of each end = πr^2

(ii) Curved surface area = $2\pi rh$

= (circumference) height

(iii) Total surface area = $2\pi r(h + r)$ sq. units.

(iv) Volume = $\pi r^2 h$ = Area of the base × height

4. If R and r (R > r) denote respectively the external and internal radii of a hollow right circular cylinder, then -

(i) Area of each end = $\pi(R^2 - r^2)$

(ii) Curved surface area of hollow cylinder $= 2\pi(R + r)\,h$

(iii) Total surface area = $2\pi(R + r)(R + h - r)$

(iv) Volume of material = $\pi h(R^2 - r^2)$

5. If r, h and λ denote respectively the radius of base, height and slant height of a right circular cone, then-

(i) $\lambda^2 = r^2 + h^2$

(ii) Curved surface area = $\pi r\lambda$

(iii) Total surface area = $\pi r^2 + \pi r\lambda$

(iv) Volume = $\frac{1}{3}\pi r^2 h$

6. For a sphere of radius r, we have

(i) Surface area = $4\pi r^2$

(ii) Volume = $\frac{4}{3}\pi r^3$

7. If h is the height, λ the slant height and r_1 and r_2 the radii of the circular bases of a frustum of a cone then -

(i) Volume of the frustum = $\frac{\pi}{3}(r_1^2 + r_1r_2 + r_2^2)\,h$

(ii) Lateral surface area = $\pi(r_1 + r_2)\lambda$

(iii) Total surface area = $\pi\{(r_1 + r_2)\lambda + r_1^2 + r_2^2\}$

(iv) Slant height of the frustum = $\sqrt{h^2 + (r_1 - r_2)^2}$

(v) Height of the cone of which the frustum is a part = $\frac{hr_1}{r_1 - r_2}$

(vi) Slant height of the cone of which the frustum is a part = $\frac{\lambda r_1}{r_1 - r_2}$

(vii) Volume of the frustum

$= \frac{h}{3}\left\{A_1 + A_2 + \sqrt{A_1A_2}\right\}$, where A_1 and A_2 denote the areas of circular bases of the frustum.

❖ EXAMPLES ❖

Ex.1 A circus tent is in the shape of a cylinder, upto a height of 8 m, surmounted by a cone of the same radius 28 m. If the total height of the tent is 13 m, find:

(i) total inner curved surface area of the tent.

(ii) cost of painting its inner surface at the rate of ₹ 3.50 per m^2.

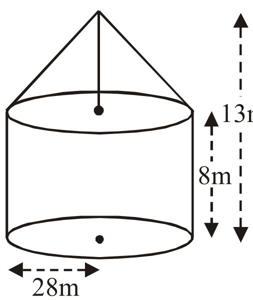
Sol. According to the given statement, the rough sketch of the circus tent will be as shown:

(i) For the cylindrical portion :

$r = 28$ and $h = 8$ m

$\therefore$ Curved surface area $= 2\pi rh$

$= 2 \times \frac{22}{7} \times 28 \times 8 \text{ m}^2 = 1408 \text{ m}^2$



For conical portion :

$r = 28$ m and $h = 13$ m $-$ 8 m $= 5$ m

$\therefore$ $\lambda^2 = h^2 + r^2 \Rightarrow \lambda^2 = 5^2 + 28^2 = 809$

$\Rightarrow$ $\lambda = \sqrt{809}$ m $= 28.4$ m

$\therefore$ Curved surface area $= \pi r\lambda$

$= \frac{22}{7} \times 28 \times 28.4 \text{ m}^2 = 2499.2 \text{ m}^2$

$\therefore$ Total inner curved surface area of the tent.

= C.S.A. of cylindrical portion + C.S.A. of the conical portion

$1408 \text{ m}^2 + 2499.2 \text{ m}^2 = 3907.2 \text{ m}^2$

(ii) Cost of painting the inner surface

$= 3907.2 \times$ ₹ 3.50

$=$ ₹ 13675.20

Ex.2 A cylinder and a cone have same base area. But the volume of cylinder is twice the volume of cone. Find the ratio between their heights.

Sol. Since, the base areas of the cylinder and the cone are the same.

$\Rightarrow$ their radius are equal (same).

Let the radius of their base be r and their heights be h_1 and h_2 respectively.

Clearly, volume of the cylinder $= \pi r^2 h_1$

and, volume of the cone $= \frac{1}{3}\pi r^2 h_2$

Given :

Volume of cylinder = 2 × volume of cone

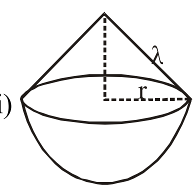
$\Rightarrow$ $\pi r^2 h_1 = 2 \times \frac{1}{3}\pi r^2 h_2$

$\Rightarrow$ $h_1 = \frac{2}{3}h_2 \Rightarrow \frac{h_1}{h_2} = \frac{2}{3}$

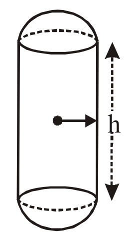
i.e., $h_1 : h_2 = 2 : 3$

Ex.3 Find the formula for the total surface area of each figure given bellow :

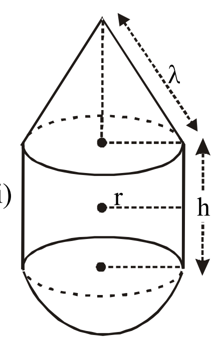
(i)



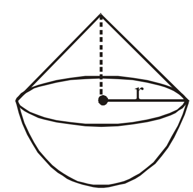
(ii)



(iii)



(iv)



Sol. (i) Required surface area

= C.S.A. of the hemisphere + C.S.A. of the cone

$= 2\pi r^2 + \pi r\lambda = \pi r\,(2r + \lambda)$

(ii) Required surface area

= 2 × C.S.A. of a hemisphere + C.S.A. of the cylinder

$= 2 \times 2\pi r^2 + 2\pi rh = 2\pi r\,(2r + h)$

(iii) Required surface area

= C.S.A. of the hemisphere

+ C.S.A. of the cylinder + C.S.A. of the cone

$= 2\pi r^2 + 2\pi rh + \pi r\lambda = \pi r\,(2r + 2h + \lambda)$

(iv) If slant height of the given cone be λ

$= \lambda^2 = h^2 + r^2 \quad \Rightarrow \quad \lambda = \sqrt{h^2 + r^2}$

And, required surface area

$= 2\pi r^2 + \pi r\lambda = \pi r\,(2r + \lambda)$

$= \pi r\left(2r + \sqrt{h^2 + r^2}\right)$

Ex.4 The radius of a sphere increases by 25%. Find the percentage increase in its surface area.

Sol. Let the original radius be r.

$\Rightarrow$ Original surface area of the sphere $= 4\pi r^2$

Increase radius = r + 25% of r

$= r + \frac{25}{100}r = \frac{5r}{4}$

$\Rightarrow$ Increased surface area

$= 4\pi\left(\frac{5r}{4}\right)^2 = \frac{25\pi r^2}{4}$

Increased in surface area

$= \frac{25\pi r^2}{4} - 4\pi r^2 = \frac{25\pi r^2 - 16\pi r^2}{4} = \frac{9\pi r^2}{4}$

and, percentage increase in surface area

$= \frac{\text{Increase in area}}{\text{Original aera}} \times 100\%$

$= \frac{\frac{9\pi r^2}{4}}{4\pi r^2} \times 100\% = \frac{9}{16} \times 100\%$

= 56.25%

Alternative Method :

Let original radius = 100

$\Rightarrow$ Original C.S.A. $= \pi(100)^2 = 10000\pi$

Increased radius = 100 + 25% of 100 = 125

$\Rightarrow$ Increased C.S.A. $= \pi(125)^2 = 15625\pi$

Increase in C.S.A. $= 15625\pi - 10000\pi$

$= 5625\pi$

$\therefore$ Percentage increase in C.S.A.

$= \frac{\text{Increase in C.S.A.}}{\text{Original C.S.A}} \times 100\%$

$= \frac{5625\pi}{1000\pi} \times 100\% = 56.25\%$

Conversely, if diameter decreases by 20%, the radius also decreases by 20%.

Ex.5 Three solid spheres of radii 1 cm, 6 cm and 8 cm are melted and recasted into a single sphere. Find the radius of the sphere obtained.

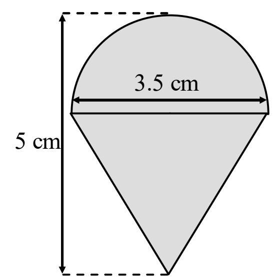
Sol. Let radius of the sphere obtained = R cm.

$\therefore \quad \frac{4}{3} \times \pi R^3 = \frac{4}{3}\pi(1)^3 + \frac{4}{3}\pi(6)^3 + \frac{4}{3}\pi(8)^3.$

$R^3 = 1 + 216 + 512 = 729$

$\therefore \quad R = (9^3)^{1/3} = 9$ cm **Ans.**

Ex.6 In given figure the top is shaped like a cone surmounted by a hemisphere. The entire top is 5 cm in height and the diameter of the top is 3.5 cm. Find the area it has to colour. (Take $\pi = \frac{22}{7}$)



Sol. TSA of the top = CSA of hemisphere + CSA of cone

Now, the curved surface of the hemisphere $= \frac{1}{2}(4\pi r^2) = 2\pi r^2 = \left(2 \times \frac{22}{7} \times \frac{3.5}{2} \times \frac{3.5}{2}\right)\text{cm}^2$

Also, the height of the cone

= height of the top – height (radius) of the hemispherical part

$= \left(5 - \frac{3.5}{2}\right)\text{cm} = 3.25$ cm

So, the slant height of the cone (*l*)

$= \sqrt{r^2 + h^2} = \sqrt{\left(\frac{3.5}{2}\right)^2 + (3.25)^2}$ cm

= 3.7 cm (approx.)

Therefore, CSA of cone = $\pi r l$

$$= \left(\frac{22}{7} \times \frac{3.5}{2} \times 3.7\right) \text{cm}^2$$

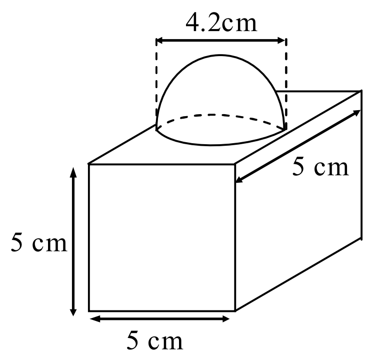
This gives the surface area of the top as

$$\left(2 \times \frac{22}{7} \times \frac{3.5}{2} \times \frac{3.5}{2}\right) \text{cm}^2 + \left(\frac{22}{7} \times \frac{3.5}{2} \times 3.7\right) \text{cm}^2$$

$$= \frac{22}{7} \times \frac{3.5}{2} (3.5 + 3.7) \text{cm}^2$$

$$= \frac{11}{2} \times (3.5 + 3.7) \text{ cm}^2 = 39.6 \text{ cm}^2 \text{ (approx)}$$

Ex.7 The decorative block shown in figure is made of two solids — a cube and a hemisphere. The base of the block is a cube with edge 5 cm, and the hemisphere fixed on the top has a diameter of 4.2 cm. Find the total surface area of the block. (Take $\pi = \frac{22}{7}$)



Sol. The total surface area of the cube $= 6 \times (\text{edge})^2 = 6 \times 5 \times 5 \text{ cm}^2 = 150 \text{ cm}^2$. Note that the part of the cube where the hemisphere is attached is not included in the surface area.

So, the surface area of the block

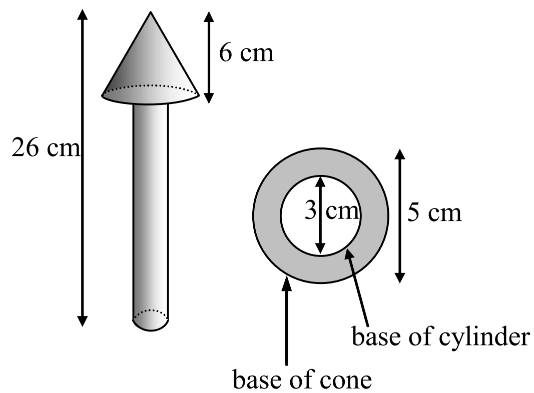
= TSA of cube – base area of hemisphere + CSA of hemisphere

$= 150 - \pi r^2 + 2\pi r^2 = (150 + \pi r^2) \text{ cm}^2$

$= 150 \text{ cm}^2 + \left(\frac{22}{7} \times \frac{4.2}{2} \times \frac{4.2}{2}\right) \text{cm}^2$

$= (150 + 13.86) \text{ cm}^2 = 163.86 \text{ cm}^2$

Ex.8 A wooden toy rocket is in the shape of a cone mounted on a cylinder, as shown in figure. The height of the entire rocket is 26 cm, while the height of the conical part is 6 cm. The base of the conical portion has a diameter of 5 cm, while the best diameter of the cylindrical portion is 3 cm. If the conical portion is to be painted orange and the cylindrical portion yellow, find the area of the rocket painted with each of these colours. (Take $\pi = 3.14$)



Sol. Denote radius of cone by r, slant height of cone by λ, height of cone by h, radius of cylinder by r' and height of cylinder by h'. Then r = 2.5 cm, h = 6 cm, r' = 1.5 cm, h' = 26 – 6 = 20 cm and

$$l = \sqrt{r^2 + h^2} = \sqrt{2.5^2 + 6^2} \text{ cm} = 6.5 \text{ cm}$$

Here, the conical portion has its circular base resting on the base of the cylinder, but the base of the cone is larger than the base of the cylinder. So a part of the base of the cone (a ring) is to be painted.

So, the area to be painted orange

= CSA of the cone + base area of the cone – base area of the cylinder

$= \pi r l + \pi r^2 - \pi (r')^2$

$= \pi[(2.5 \times 6.5) + (2.5)^2 - (1.5)^2] \text{ cm}^2$

$= \pi[20.25] \text{ cm}^2 = 3.14 \times 20.25 \text{ cm}^2$

$= 63.585 \text{ cm}^2$

Now, the area to be painted yellow

= CSA of the cylinder + area of one base of the cylinder

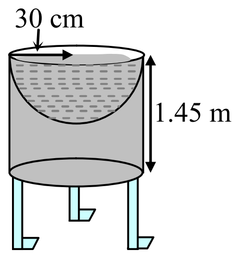
$= 2\pi r'h' + \pi(r')^2$

$= \pi r' (2h' + r')$

$= (3.14 \times 1.5)(2 \times 20 + 1.5) \text{ cm}^2 = 4.71 \times 41.5 \text{ cm}^2$

$= 195.465 \text{ cm}^2$

Ex.9 A bird bath for garden in the shape of a cylinder with a hemispherical depression at one end (see figure). The height of the cylinder is 1.45 m and its radius is 30 cm. Find the total surface area of the bird-bath. (Take $\pi = \frac{22}{7}$)



Sol. Let h be height of the cylinder and r the common radius of the cylinder and hemisphere. Then, the total surface area of the bird-bath

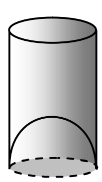
= CSA of cylinder + CSA of hemisphere

= $2\pi rh + 2\pi r^2 = 2\pi r^2 = 2\pi r(h + r)$

= $2 \times \frac{22}{7} \times 30\ (145 + 30)\ cm^2$

= $33000\ cm^2 = 3.3\ m^2$

Ex.10 A juice seller was serving his customers using glasses as shown in figure. The inner diameter of the cylindrical glass was 5 cm, but the bottom of the glass had a hemispherical raised portion which reduced the capacity of the glass. If the height of a glass was 10 cm. Find the apparent capacity of the glass and its actual capacity. (Use $\pi = 3.14$)



Sol. Since the inner diameter of the glass = 5 cm and height = 10 cm, the apparent capacity of the glass = $\pi r^2 h$

= $3.14 \times 2.5 \times 2.5 \times 10\ cm^3 = 196.25\ cm^3$

But the actual capacity of the glass is less by the volume of the hemisphere at the base of the glass.

i.e. it is less by $\frac{2}{3}\pi r^3$

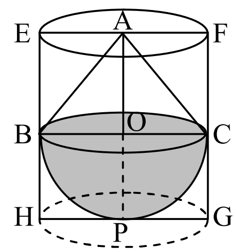
= $\frac{2}{3} \times 3.14 \times 2.5 \times 2.5 \times 2.5\ cm^3 = 32.71\ cm^3$

So, the actual capacity of the glass = apparent capacity of glass – volume of the hemisphere

= $(196.25 - 32.71)\ cm^3$

= $163.54\ cm^2$

Ex.11 A solid toy is in the form of a hemisphere surmounted by a right circular cone. The height of the cone is 2 cm and the diameter of the base is 4 cm. Determine the volume of the toy. If a right circular cylinder circumscribes the toy, find the difference of the volume of the cylinder and the toy. (Take $\pi = 3.14$)



Sol. Let BPC be the hemisphere and ABC be the cone standing on the base of the hemisphere (see figure). The radius BO of the hemisphere (as well as of the cone) = $\frac{1}{2} \times 4$ cm = 2 cm

So, volume of the toy = $\frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h$

$= \left[\frac{2}{3} \times 3.14 \times (2)^3 + \frac{1}{3} \times 3.14 \times (2)^2 \times 2\right] cm^3$

= $25.12\ cm^3$

Now, let the right circular cylinder EFGH circumscribe the given solid. The radius of the base of the right circular cylinder

= HP = BO = 2 cm, and its height is

EH = AO + OP = (2 + 2) cm = 4 cm

So, the volume required

= Volume of the right circular cylinder – volume of the toy

= $(3.14 \times 2^2 \times 4 - 25.12)\ cm^3$

= $25.12\ cm^3$

= $25.12\ cm^3$

Hence, the required difference of the two volumes = $25.12\ cm^3$.

Ex.12 A cone of height 24 cm and radius of base 6 cm is made up of modeling clay. A child reshapes it in the form of a sphere. Find the radius of the sphere.

Sol. Volume of cone = $\frac{1}{3} \times \pi \times 6 \times 6 \times 24$ cm^3

If r is the radius of the sphere, then its volume is $\frac{4}{3}\pi r^3$.

Since the volume of clay in the form of the cone and the sphere remains the same, we have.

$$\frac{4}{3} \times \pi \times r^3 = \frac{1}{3} \times \pi \times 6 \times 6 \times 24$$

$$r^3 = 3 \times 3 \times 24 = 3^3 \times 2^3$$

$$r = 3 \times 2 = 6$$

Therefore, the radius of the sphere is 6 cm.

Ex.13 Selvi's house has an overhead tank in the shape of a cylinder. This is filled by pumping water from a sump (an underground tank) which is in the shape of a cuboid. The sump has dimensions 1.57 m × 1.44 m × 95 cm. The overhead tank has its radius 60 cm and height 95 cm. Find the height of the water left in the sump after the overhead tank has been completely filled with water from the sump which had been full. Compare the capacity of the tank with that of the sump. (Use $\pi = 3.14$)

Sol. The volume of water in the overhead tank equals the volume of the water removed from the shump.

Now the volume of water in the overhead tank (cylinder) = $\pi r^2 h$

$= 3.14 \times 0.6 \times 0.6 \times 0.95$ m^3

The volume of water in the sump when full

$= l \times b \times h = 1.57 \times 1.44 \times 0.95$ m^3

The volume of water left in the sump after filling the tank

$= (1.57\times1.44\times0.95)–(3.14 \times 0.6 \times 0.6 \times 0.95)]$ m^3

$= (1.57 \times 0.6 \times 0.6 \times 0.95 \times 2)$ m^3

So, the height of the water left in the sump

$$= \frac{\text{volume of water left in the sump}}{\lambda \times b}$$

$$= \frac{1.57\times0.6\times0.6\times0.95\times2}{1.57\times1.44} \text{ m}$$

= 0.475 m = 47.5 cm

Also, $\frac{\text{Capacity of tank}}{\text{Capacity of sump}}$

$$= \frac{3.14\times0.6\times0.6\times0.95}{1.57\times1.44\times0.95} = \frac{1}{2}$$

Therefore, the capacity of the tank is half the capacity of the sump.

Ex.14 A copper rod of diameter 1 cm and length 8 cm is drawn into a wire of length 18 m of uniform thickness. Find the thickness of the wire.

Sol. The volume of the rod = $\pi \times \left(\frac{1}{2}\right)^2 \times 8$ cm^3

$= 2\pi$ cm^3.

The length of the new wire of the same volume = 18 m = 1800 cm

If r is the radius (in cm) of cross section of the wire, its volume = $\pi \times r^2 \times 1800$ cm^3

Therefore, $\pi \times r^2 \times 1800 = 2\pi$

i.e. $r^2 = \frac{1}{900}$

i.e. $r = \frac{1}{30}$

So, The diameter of the cross section i.e. the thickness of the wire is $\frac{1}{15}$ cm, i.e. 0.67 mm (approx.)

EXERCISE # 1

Q.1 A cube of 9 cm edge is immersed completely in a rectangular vessel containing water. If the dimensions of the base are 15 cm and 12 cm. Find the rise in water level in the vessel.

Q.2 Three cubes whose edges measure 3 cm, 4 cm and 5 cm respectively to form a single cube. Find its edge. Also, find the surface area of the new cube.

Q.3 Water flows in a tank 150 m × 100 m at the base, through a pipe whose crosssection is 2 dm by 1.5 dm at the speed of 15 km per hour. In what time, will the water be 3 metres deep.

Q.4 A right circular cone is 3.6 cm high and radius of its base is 1.6 cm. It is melted and recast into a right circular cone with radius of its base as 1.2 cm. Find its height.

Q.5 A solid cube of side 7 cm is melted to make a cone of height 5 cm, find the radius of the base of the cone.

Q.6 A toy is in the shape of a right circular cylinder with a hemisphere on one end and a cone on the other. The height and radius of the cylindrical part are 13 cm and 5 cm respectively. The radii of the hemispherical and conical parts are the same as that of the cylindrical part. Calculate the surface area of the toy if height of the conical part is 12 cm.

Q.7 A solid sphere of radius 3 cm is melted and then cast into small spherical balls each of diameter 0.6 cm. Find the number of balls thus obtained.

Q.8 Three solid spheres of iron whose diameters are 2 cm, 12 cm and 16 cm, respectively, are melted into a single solid sphere. Find the radius of the solid sphere.

Q.9 How many spherical bullets can be made out of a solid cube of lead whose edge measures 44 cm, each bullet being 4 cm in diameter.

Q.10 A sphere of diameter 6 cm is dropped in a right circular cylindrical vessel partly filled with water. The diameter of the cylindrical vessel is 12 cm. If the sphere is completely submerged in water, by how much will the level of water rise in the cylindrical vessel ?

Q.11 A spherical canon ball, 28 cm in diameter is melted and cast into a right circular conical mould, the base of which is 35 cm in diameter. Find the height of the cone, correct to one placed of decimal.

Q.12 Length of a class-room is two times its height and its breadth is $1\frac{1}{2}$ times its height. The cost of white-washing the walls at the rate of ₹ 1.60 per m^2 is ₹ 179.20. Find the cost of tiling the floor at the rate of ₹ 6.75 per m^2.

Q.13 A room is half as long again as it is broad. The cost of carpeting the room at ₹ 3.25 per m^2 is ₹ 175.50 and the cost of papering the walls at ₹ 1.40 per m^2 is ₹ 240.80. If 1 door and 2 windows occupy 8 m^2, find the dimensions of the room.

Q.14 A rectangular tank is 225 m by 162 m at the base. With what speed must water flow into it through an aperture 60 cm by 45 cm that the level may be raised 20 cm in 5 hours?

Q.15 An agricultural field is in the form of a rectangle of length 20 m and width 14 m. A pit 6 m long, 3 m wide and 2.5 m deep is dug in a corner of the field and the earth taken out of the pit is spread uniformly over the remaining area of the field. Find the extent to which the level of the field has been raised.

Q.16 A rectangular sheet of paper 44 cm × 18 cm is rolled along its length and a cylinder is formed. Find the radius of the cylinder.

Q.17 A wooden toy is in the form of a cone surmounted on a hemisphere. The diameter of the base of the cone is 6 cm and its height is 4 cm. Find the cost of painting the toy at the rate of ₹ 5 pr 1000 cm^2.

Q.18 A cylindrical container of radius 6 cm and height 15 cm is filled with ice-cream. The whole ice-cream has to be distributed to 10 children in equal cones with hemispherical tops. If the height of the conical portion is four times the radius of its base, find the radius of the ice-cream cone.

Q.19 A solid wooden toy is in the shape of a right circular cone mounted on a hemisphere. If the radius of the hemisphere is 4.2 cm and the total height of the toy is 10.2 cm, find the volume of the wooden toy.

Q.20 A vessel is in the form of a hemispherical bowl mounted by a hollow cylinder. The diameter of the sphere is 14 cm and the total height of the vessel is 13 cm. Find its capacity. (Take $\pi = 22/7$).

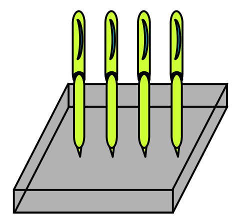
Q.21 A solid is in the form of a cylinder with hemispherical ends. The total height of the solid is 19 cm and the diameter of the cylinder is 7 cm. Find the volume and total surface area of the solid. (use $\pi = 22/7$).

Q.22 Find what length of canvas 2m in width is required to make a conical tent 20 m in diameter and 42m in slant height allowing 10% for folds and stitching. Also find the cost of canvas at the rate of ₹ 60 per metre.

Q.23 From a cube of edge 14 cm, a cone of maximum size is carved out. Find the volume of the cone and of the remaining material.

Q.24 A cone of maximum volume is carved out of a block of wood of size 20 cm × 10 cm × 10 cm. Find the volume of the cone carved out correct to one decimal place.
Take $\pi = 3.1416$.

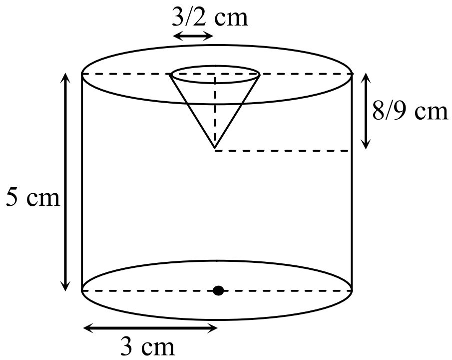
Q.25 A pen stand made of wood is in the shape of a cuboid with four conical depressions to hold pens. The dimensions of the cuboid are 15 cm by 10 cm by 3.5 cm. The radius of each of the depression is 0.5 cm and the depth is 1.4 cm. Find the volume of the wood in the entire stand.



Q.26 From a solid cylinder whose height is 2.4 cm and diameter 1.4 cm, a conical cavity of same height and same diameter is hollowed out. Find the total surface area of the remaining solid to the nearest cm^2.

Q.27 From a solid cylinder whose height is 8 cm and radius is 6 cm, a conical cavity of height 8 cm and of base radius 6 cm, is hollowed out. Find the volume of the remaining solid correct to 4 significant figures. ($\pi = 3.1416$). Also find the total surface area of the remaining solid.

Q.28 A metallic cylinder has radius 3 cm and height 5 cm. It is made of a metal A. To reduce its weight, a conical hole is drilled in the cylinder as shown and it is completely filled with a lighter metal B. The conical hole has a radius of $\frac{3}{2}$ cm and its depth is $\frac{8}{9}$ cm. Calculate the ratio of the volume of the metal A to the volume of the metal B in the solid.



Q.29 An open cylinder vessel of internal diameter 7 cm and height 8 cm stands on a horizontal table. Inside this is placed a solid metallic right circular cone, the diameter of whose base is $\frac{7}{2}$ cm and height 8 cm. Find the volume of water required to fill the vessel.

Q.30 A hemispherical tank full of water is emptied by a pipe at the rate of $3\frac{4}{7}$ litres per second. How much time will it take to empty half the tank, if it is 3 m in diameter ?

Q.31 The volume of a cone is the same as that of the cylinder whose height is 9 cm and diameter 40 cm. Find the radius of the base of the cone if its height is 108 cm.

Q.32 The entire surface of solid cone of base radius 3 cm and height 4 cm is equal to the entire surface of a solid right circular cylinder of diameter 4 cm. Find the ratio of (i) their curved surface; (ii) their volumes.

Q.33 A girl fills a cylindrical bucket 32 cm in height and 18 cm in radius with sand. She empties the bucket on the ground and makes a conical heap of the sand. If the height of the conical heap is 24 cm, find

(i) The radius and

(ii) The slant height of the heap.

Leave your answer in square root form.

Q.34 A hollow metallic cylindrical tube has an internal radius of 3 cm and height 21 cm. The thickness of the metal of the tube is $\frac{1}{2}$ cm. The tube is melted and cast into a right circular cone of height 7 cm. Find the radius of the cone correct to one decimal place.

Q.35 A right circular cone of height 20 cm and base diameter 30 cm is cast into smaller cones of equal sizes with base radius 10 cm and height 9 cm. Find how many cones are made.

Answer Key

1. 4.05 cm	**2.** 6 m, 216 cm^2	**3.** 100 Hours	**4.** 6.4 cm	**5.** 8.09 cm
6.770 cm^2	**7.** 1000	**8.** 9 cm	**9.**2541	**10.** 1 cm
11. 35.84 cm	**12.** ₹. 324	**13.** λ = 9 m, b = 6cm, h = 6 cm		
14. 5400 m/h	**15.** $\frac{1}{21}$kg, $\frac{1}{9}$kg	**16.** 7 cm	**17.** 51 paise	**18.** 3 cm
19. 266.11 cm^3	**20.** 1642.66 cm3	**21.** 641.66 cm^3, 418 cm^2		
22. 726 m, ₹.43560	**23.** $718\frac{2}{3}cm^3, 2025\frac{1}{3}cm^3$		**24.** 523.6 cm^3	**25.** 523.53 cm^3
26. 18 cm^2	**27.** 603.2 cm^3, 603.2 cm^2		**28.** 133 : 2	**29.** $282\frac{1}{3}cm^3$
30. 16.5 minutes	**31.** 10 cm		**32.** (i) 15 : 16 (ii) 3 : 4	
33. (i) 36 cm (ii) $4\sqrt{117}$ cm			**34.** 5.4 cm	**35.** 5.

EXERCISE # 2

Q.1 A sphere of diameter 6 cm is dropped in a right circular cylindrical vessel partly filled with water. The diameter of the cylindrical vessel is 12 cm. If the sphere is completely submerged in water, by how much will the level of water rise in the cylindrical vessel ?

Q.2 The largest sphere is carved out of a cube of side 7 cm. Find the volume of the sphere.

Q.3 A vessel, in the form of a hemispherical bowl, is full of water. Its contents are emptied in a right circular cylindrical. The internal radii of the bowl and the cylinder are 3.5 cm and 7 cm respectively. Find the height to which water will rise in the cylinder.

Q.4 An iron pillar has some part in the form of a right circular cylindrical and the remaining in the form of a right circular cone. The radius of the base of each of the cone and cylinder is 8 cm. The cylindrical part is 240 cm high and the conical part is 36 cm high. Find the weight of the pillar if one cu cm of iron weights 7.8 grams.

Q.5 A solid wooden toy is in the shape of a right circular cone mounted on a hemisphere. If the radius of the hemisphere is 4.2 cm and the total height of the toy is 10.2 cm, find the volume of the wooden toy.

Q.6 A hemispherical bowl of internal diameter 36 cm contains a liquid. This liquid is to be filled in cylindrical bottles of radius 3 cm and height 6 cm. How many bottles are required to empty the bowl?

Q.7 Marbles of diameter 1.4 cm are dropped into a cylindrical beaker of diameter 7 cm, containing some water. Find the number of marbles that should be dropped into the beaker so that the level rises by 5.6 cm.

Q.8 The curved surface area of the right circular cone is 12320 cm^2. If the radius of the base is 56 cm, find its height.

Q.9 The circumference of the edge of a hemispherical bowl is 12 cm. Find the capacity of the bowl.

Q.10 The volume of a vessel in the form of a right circular cylindrical is 448π cm^3 and its height is 7 cm. Find the radius of its base.

Q.11 A building is in the form of a cylinder surmounted by a hemispherical walled dome and contains $41\frac{19}{21}$ m^3 of air. If the internal diameter of the building is equal to its total height above the floor, find the height of the building.

Q.12 The volume of a right circular cylinder of height 7 cm is 567π cm^3. Find its curved surface area.

Q.13 If the radius of the base of a right circular cylinder is halved, keeping the height same, find the ratio of the volume of the reduced to that of the original cylinder.

Q.14 Two right circular cones X and Y are made, X having three times the radius of Y and Y having half the volume of X. Calculate the ratio of heights of X and Y.

Q.15 How many metres of cloth 5m wide will be required to make a conical tent, the radius of whose base is 7m and whose height is 24 cm.

Q.16 The radii of the internal and external surface of a hollow spherical shell are 3 cm and 5 cm respectively. If it is melted and recast into a solid cylinder of height $2\frac{2}{3}$ cm, find the diameter and the curved surface area of the cylinder.

Q.17 A toy is in the form of a cone mounted on a hemisphere of radius 3.5 cm. The total height of the toy is 15.5 cm. Find the total surface area and the volume of the toy.

Q.18 The radii of the ends of the frustum of a right circular cone are 5 metres and 8 metres and its lateral height is 5m Find the lateral surface area and the volume of the frustum. Take $\pi = 3.142$.

Q.19 A hemispherical bowl of internal diameter 30 cm is full of some liquid. This liquid is to be filled into cylindrical shaped bottles each of diameter 5 cm and height 6 cm. Find the number of bottle necessary to empty the bowl.

Q.20 If the radii of the circular ends of a bucket, 45 cm high, are 28 cm and 7 cm, find the capacity and the total surface area of the bucket.

Q.21 A hollow cone is cut by a plane parallel to the base and upper portion is removed. If the curved surface area of the remainder is $\frac{8}{9}$ of the curved surface area of the whole cone, find the ratio of the line segments into which the altitude of the cone is divided by the plane.

Q.22 The radii of the circular ends of a solid frustum of a cone are 33 cm and 27 cm and its slant height is 10 cm. Find its total surface area.

Q.23 The rain water from a roof 22 m × 20 m drains into a cylindrical vessel having diameter of base 2m and height 3.5m. If the vessel is just full, find the rainfall in cm.

Q.24 A bucket made up of a metal sheet is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper ends as 8 cm and 20 cm respectively. Find the cost of the bucket if the cost of the metal sheet used is ₹ 15 per 100 cm^2. (Use $\pi = 3.14$)

Q.25 Water is flowing at the rate of 15 km per hour through a pipe of diameter 14 cm into a rectangular tank which is 50 m long and 44 m wide. Find the time in which the level of water in the tank will rise by 21 cm.

Q.26 A bucket made up of a metal sheet is in the form of a frustum of a cone. Its depth is 24 cm and the diameters of the top and the bottom are 30 cm and 10 cm respectively. Find the cost of milk which can completely fill the bucket at the rate of ₹. 20 per litre and the cost of the metal sheet used, if it costs ₹ 10 per 100 cm^2. (Use $\pi = 3.14$).

Q.27 A bucket is in the form of a frustum of a cone with a capacity of 12308.8 cm^3 of water. The radii of the top and bottom circular ends are 20 cm and 12 cm respectively. Find the height of the bucket and the area of the metal used in making it (Use $\pi = 3.14$).

Q.28 A sphere of diameter 12 cm, is dropped into a right circular cylindrical vessel, partly filled with water. If the sphere is completely submerged in water, The water level in the cylindrical vessel rises by $3\frac{5}{9}$ cm. Find the diameter of the cylindrical vessel.

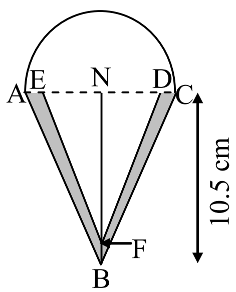
Q.29 Find the number of coins 1.5 cm in diameter and 0.2 cm thick, to be melted to form a right circular cylinder of height 10 cm and diameter 4.5 cm.

Q.30 Water flows out through a circular pipe whose internal radius is 1 cm, at the rate of 80 cm / sec into an empty cylindrical tank, the radius of whose base is 40 cm. By how much will the level of water rise in the tank in half an hour ?

Q.31 The adjoining figure shows the cross–section of an ice–cream cone consisting of a cone surmounted by a hemisphere. The radius of the hemisphere is 3.5 cm and the height of the cone is 10.5 cm. The outer shell ABCDEF is shaded and is not filled with ice cream. AE = DC = 0.5 cm, AB || EF and BC || FD. Calculate

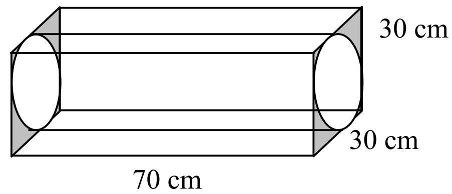
(i) The volume of the ice-cream in the cone (the unshaded portion including the hemisphere) in cm^3;

(ii) The volume of the outer shell (the shaded portion) in cm^3. Give your answer correct to the nearest cm^3.

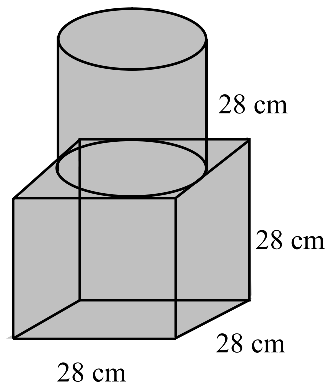


Q.32 (a) The figure (i) given below, shows a cuboidal block of wood through which a circular cylindrical hole of the biggest size is drilled. Find the volume of the wood left in the block.

(b) The figure (ii) given below, shows a solid trophy made of shinning glass. If one cubic centimeter of glass costs ₹ 0.75, find the cost of the glass for making the trophy.



(i)



(ii)

Answer Key

1. 1 cm	**2.** $179\frac{2}{3}$ cm^3	**3.** $\frac{7}{12}$ cm	**4.** 395.37 kg.
5. 266.112 cm^3	**6.** 72	**7.** 150	**8.** 42 cm
9. 19404 cm^3	**10.** 8 cm	**11.** 4 m	**12.** 393 cm^2
13. 1 : 4	**14.** 2 : 9	**15.** 110 m	**16.** 14 cm, $\frac{352}{3}$ cm^2
17. 214.5 cm^2, 243.83 cm^3	**18.** 204.23 m^2, 540.42 m^3	**19.** 60	**20.** 48510 cm^3, 5616.6 cm^2
21. 1 : 2	**22.** 7599.43 cm^2	**23.** 2.5 cm	**24.** ₹ 293.90
25. 2 hr.	**26.** ₹ 163 approx, ₹ 171 approx		**27.** 15 cm, 2160.32 cm^2
28. 9 cm	**29.** 450	**30.** 90 cm	**31.** (i) 175 cm^3 (ii) 50 cm^3
32. (a) 13500 cm^3 (b) ₹ 29400.			