Short Notes for Industrial Engineering

Forecasting

- **Simple Moving Average-**

$$F_n = \frac{1}{N}(D_n + D_{n-1} + D_{n-2} + D_{n-3} + \cdots)$$

- **Moving Weight Average-**

$$\text{Weight Moving Average} = \frac{\sum_{t=1}^{n} W_i D_i}{\sum_{i=1}^{n} W_i}$$

- **Single (Simple) Exponential Smoothing-**

$$F_t = F_{t-1} + \alpha(D_{t-1} - F_{t-1})$$
$$or\ F_t = (1-\alpha)F_{t-1} + \alpha D_{t-1}$$

if previous forecasting is not given

$$F_t = \alpha D_t + \alpha(1-\alpha)D_{t-1} + \alpha(1-\alpha)^2 D_{t-2} \dots\dots$$

Where F_t= Smoothed average forecast for period t
F_{t-1}=Previous period forecast
α=Smoothing constant

- **Linear Regression-**

$$Y = a + bX$$
$$\sum y = na + b\sum x$$
$$\sum xy = n\sum x + b\sum x^2$$

- **Forecasting Error-**

$$e_t = (D_t - F_t)$$

- **Bias-**

$$Bias = \frac{1}{N}\sum_{t=1}^{n}(D_t - F_t)$$

- **Mean Absolute Deviation-**

$$MAD = \frac{1}{N}\sum_{t=1}^{n}|D_t - F_t|$$

- **Mean Square Error-**

$$MSE = \frac{1}{N}\sum_{t=1}^{n}(D_t - F_t)^2$$

- **Mean Absolute Percentage Error-**

$$MAPE = \frac{1}{N}\sum_{t=1}^{n}\frac{|D_t - F_t|}{D_t} \times 100$$

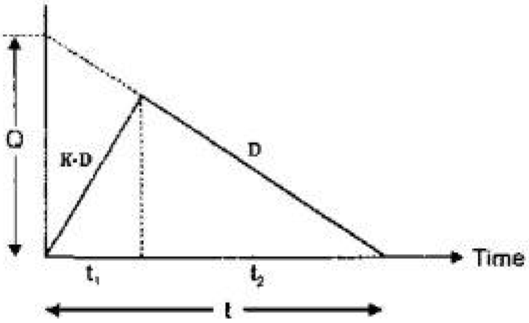
Inventory

If D= demand/year, C_o=Order cost, C_c= Carrying cost, P= Purchase price/unit
Q^*=Economic Order Quantity, K= Production Rate and C_S =Shortage Cost/unit/period

- **Case-1** Purchase Model With Instantaneous Replenishment and Without Shortage-
  1. $EOQ\ Q^* = \sqrt{\frac{2DC_o}{C_c}}$ at $EOQ\ Invertory\ cost = Order\ cost$
  2. $No. of\ order = \frac{D}{Q^*}$
  3. $Time\ Taken\ Per\ Order = \frac{Q^*}{D}$
  4. $Total\ Cost = Unit\ cost + Invertory\ cost + Order\ cost$

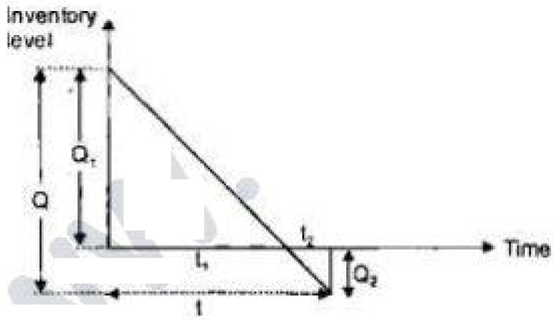
$$= (D \times P) + \left(\frac{Q}{2} \times C_c\right) + \left(\frac{D}{Q} \times C_o\right) = (D \times P) + \sqrt{2DC_cC_o}$$

- **Case-2** Manufacturing Model Without Shortage-



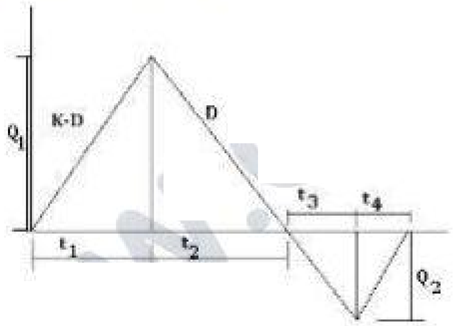
1. $EOQ\ Q^* = \sqrt{\frac{2C_oD}{C_c\left(1-\frac{D}{K}\right)}}$
2. $t_1 = \frac{Q^*}{K}$
3. $t_2 = \frac{Q^*\left(1-\frac{D}{K}\right)}{D}$
4. Total optimum cost $= \sqrt{2DC_cC_o\left(1-\frac{D}{K}\right)}$

- **Case 3** Purchase Model With Shortage-



1. $Q = EOQ = \sqrt{\frac{2DC_o}{C_c}\left(\frac{C_s+C_c}{C_s}\right)}$
2. $Q_1 = \sqrt{\frac{2DC_o}{C_c}\left(\frac{C_s}{C_s+C_c}\right)}$, $Q_2 = Q - Q_1$
3. $t = \frac{Q}{D}$, $t_1 = \frac{Q_1}{D}$ *and* $t_2 = \frac{Q_2}{D}$
4. Total optimum cost $= \sqrt{2DC_cC_o\left(\frac{C_s}{C_s+C_c}\right)}$

- **Case 4** Manufacturing Model With Shortfall



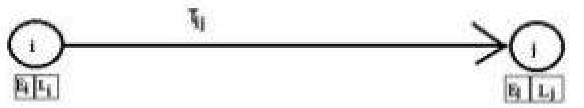
1. $Q = EOQ = \sqrt{\frac{2DC_o}{C_c\times\left(1-\frac{D}{K}\right)}\left(\frac{C_s+C_c}{C_s}\right)}$
2. $Q_1 = \sqrt{\frac{2DC_o}{C_c}\left(\frac{C_s}{C_s+C_c}\right)\times\left(1-\frac{D}{K}\right)}$

3. $Q_1 = \left(1 - \frac{D}{K}\right) Q - Q_2$
4. $t = \frac{Q}{D}, \; t_1 = \frac{Q_1}{K-D}$ and $t_2 = \frac{Q_1}{D}$
1. $t_3 = \frac{Q_3}{D}$ and $t_4 = \frac{Q_2}{K-D}$

- Lead Time Demand + Safety Stock = Reorder Point

PERT and CPM

- EFT = EST + activity time
- LFT = LST + Duration of activity



- **Total Float-** $$(TF_o) = L_j - (E_i + t_{ij}) = LF_{ij} - EF_{ij}$$ $$= (L_j - t_{ij}) = E_i$$ $$= LS_{ij} - ES_{ij}$$
- **Free Float-** FF_o= **(E_j-E_i)-T_{ij}**
- **Independent Float** $IF_o = (E_j - L_j) - t_{ij}$ -
  $= FF_j -$ (Slack of event i)

Example-

1. Total float = $L_2 - (E_1 + t_{12}) = 57 - (20 + 19) = 18$
2. Free float = $E_2 - E_1 - t_{12} = 0$
3. Independent float = $E_2 - (L_1 + t_{12}) = -18$

- **PERT Expected time-** $t_e = \frac{t_0 + 4t_m + t_p}{6}$
  1. t_0 = Optimistic time i.e., shortest possible time to complete the activity if all goes well.
  2. t_p = Pessimistic time i.e., longest time that an activity could take if everything goes wrong.
  3. t_m = Most likely time i.e., normal time of an activity would take.
- **Standard deviation-** $$(\sigma) = \frac{t_p - t_0}{6}$$
- **Variance -** $$(\sigma^2) = \left(\frac{t_p - t_0}{6}\right)^2$$
- **Crashing-** $$\text{Cost Slope (CS)} = \frac{\text{Crash cost} - \text{Normal cost}}{\text{Normal time} - \text{Crash time}}$$
- **Standard Normal Variation** $Z = \frac{T_s - T_e}{\sigma_e}$ **(SNV)-**

Linear Programming

Simplex Method Case 1. Maximization Problem

$$Max\ Z = 3x1 + 5x2$$

$$s/t\quad 3x1 + 2x2 \le 18 \quad -(I)$$

$$x1 \le 4 \quad -(II)$$

$$x2 \le 6 \quad -(III)$$

$$x1, x2 \ge 0$$

Standard Form:

Max Z = $3x_1 + 5x_2 + 0w_1 + 0w_2 + 0w_3$

$3x_1 + 2x_2 + w_1 + 0w_2 + 0w_3 = 18$

$x_1 + 0x_2 + 0w_1 + w_2 + 0w_3 = 4$

$0x_1 + x_2 + 0w_1 + 0w_2 + w_3 = 6$

To prepare initial Table:

Table - I

	c_j	3	5	0	0	0	
c_i	x_i	x_1	x_2	w_1	w_2	w_3	b_i
0	w_1	3	2	1	0	0	18
0	w_2	1	0	0	1	0	4
0	w_3	0	1	0	0	1	6
	I_j	-3	-5	0	0	0	Z=0

- $I_j = (Z_j - c_j) = (\Sigma a_{ij}.c_i) - c_j$

Interpretation of Simplex Table

Table - I

	c_j	3	5	0	0	0		
c_i	x_i	x_1	x_2	w_1	w_2	w_3	b_i	
0	w_1	3	2	1	0	0	18	(18/2=9)
0	w_2	1	0	0	1	0	4	(4/0=infinity)
0	w_3	0	1	0	0	1	6	(6/1=6)
	I_j	-3	-5	0	0	0	Z=0	

- Key Column → Min I_j [Most Negative]
- Key Row → Min positive ratio.

How to get next table ?

- Leaving variable : w3
- Entering variable : x2
- Key no. = 1
- For old key row : New No.= Old No./key No.
- For other rows:

$$New\ No. = Old\ No. - \frac{(Corresponding\ Key\ Row\ No.).(Corresponding\ Key\ Column\ No.)}{Key\ No.}$$

- 18 → 18 - (6*2)/1 = 6
- I(w3) = 0 → 0 - [1*(-5)]/1 = 5

Table - II

	c_j	3	5	0	0	0		
c_i	x_i	x_1	x_2	w_1	w_2	w_3	b_i	Ratio
0	w_1	3	0	1	0	-2	6	(6/3=2)
0	w_2	1	0	0	1	0	4	(4/1=4)
5	x_2	0	1	0	0	1	6	(6/0=infinity)
	I_j	-3	0	0	0	5	Z=30	

- Key Column → Min I_j
- Key Row → Min positive ratio

Table - III

	c_j	3	5	0	0	0		
c_i	x_i	x_1	x_2	w_1	w_2	w_3	b_i	Ratio
3	x_1	1	0	1/3	0	-2/3	2	
0	w_2	0	0	-1/3	1	2/3	2	
5	x_2	0	1	0	0	1	6	
	I_j	0	0	0	0	3	Z=36	

- **This is the final Table**

The Optimal Solution is x1 = 2, x2 = 6

giving Z = 36

Type of Solutions : Basic, Feasible/Infeasible, Optimal/ Non-Optimal, Unique/Alternative Optimal, Bounded/Unbounded, Degenerate/Non-Degenerate

- **Analysis of Solution**
  1. This is a Basic solution, as values of basic variables are Positive
  2. This is a feasible solution, as values of basic variables, not containing Artificial Variable, are Positive and all constraints are satisfied
  3. This Feasible solution is an Optimal, as all values in Index Row are positive.
  4. If there is an Artificial Variable, as Basic variable in final table, it is called as Infeasible solution
  5. This solution is unique Optimal, as the number of zeroes are equal to number of basic variables in Index Row in final Table.
  6. If the number of zeroes are more than number of basic variables in Index Row in final Table, it is a case of more than one optimal solutions.
  7. This is a Bounded Solution, as the values of all Basic variables in final table, are finite positive.
  8. This is a Non-degenerate Solution, as value of none of the basic variables is Zero , in final table.
  9. If value of at least one of the basic variables is Zero in Index Row in final Table, it is a Degenerate Solution.

- **Duality With Example**

  1. Case-1

  $Max.Z = x_1 - x_2 + 3x_3$

  s/t $x_1 + x_2 + x_3 \leq 10$

  $2x_1 - x_2 - x_3 \leq 2$

  $2x_1 - 2x_2 - 3x_3 \leq 6$

  $x_1, x_2, x_3 \geq 0$

  Dual of this would be

  $Min\ Z = 10y_1 + 2y_2 + 6y_3$

  s/t $y_1 + 2y_2 + 2y_3 \geq 1$

  $y_1 - y_2 - 2y_3 \geq -1$

  $y_1 - y_2 - 3y_3 \geq 3$

  $y_1, y_3, y_3 \geq 0$,

2. Case -2

$$\text{Min } Z = 20x_1 + 23x_2$$
$$\text{s/t } -4x_1 - x_2 \le -8$$
$$5x_1 - 3x_2 \le -4$$
$$x_1, x_2 \ge 0$$

Dual of this would be

$$\text{Max } Z = 8y_1 + 4y_2$$
$$\text{S/t } 4y_1 - 5y_2 \le 20$$
$$y_1 + 3y_2 \le 23$$
$$y_1, y_2 \ge 0,$$

3. Case-3

$$\text{Max.} Z_p = x_1 + 2x_2 - 3x_3$$
$$\text{s/t } 2x_1 + x_2 + x_3 \le 10$$

$3x_1 - x_2 + 2x_3 \ge 110$(this need to be converted in less than form)

$$x_1 + 2x_2 - x_3 = 4$$
$$x_1, x_3 \ge 0, \; x_2 \text{ unrestricted}$$

Dual of this would be

$$\text{Min. } Z_d = 10y_1 - 110y_2 + 4y_3$$
$$\text{s/t } 2y_1 - 3y_2 + y_3 \ge 1$$
$$y_1 + y_2 + 2y_3 = 2$$
$$y_1 - 2y_2 - y_3 \ge -3$$
$$y_1, y_2 \ge 0, \; y_3 \text{ unrestricted}$$

Assignment

- **Steps of Solving Assignment Problem-**
  1. Identify the minimum element in each row and **subtract** it from every element of that row.
  2. Identify the minimum element in each column and subtract it from every element of that column
  3. Make the assignments for the reduced matrix obtained from **steps 1 and 2** in the following way:
     - I. For each row or column with a single zero value cell that has not be assigned or eliminated, box ☐ that zero value as an assigned cell.
     - II. For every zero that becomes assigned, cross out (X) all other zeros in the same row and the same column.
     - III. If for a row and a column, there are two or more zeros and one cannot be chosen by inspection, choose the cell arbitrarily for assignment.
     - IV. The above process may be continued until every zero cell is either assigned ☐ or crossed (X)
  4. An optimal assignment is found, if the number of assigned cells equals the number of rows (and columns). In case you have chosen a zero cell arbitrarily, there may be alternate optimal solutions. If no optimal solution is found, go to step 5.
  5. Draw the minimum number of vertical and horizontal lines necessary to cover all the zeros in the reduced matrix obtained from step 3 by adopting the following procedure:
     - I. Mark all the rows that do not have assignments.
     - II. Mark all the columns (not already marked) which have zeros in the marked rows.
     - III. Mark all the rows (not already marked) that have assignments in marked columns.
     - IV. Repeat steps 5 (ii) and (iii) until no more rows or columns can be marked.
     - V. Draw straight lines through all unmarked rows and marked columns.

1. Select the smallest element (i.e., 1) from all the uncovered elements. Subtract this smallest element from all the uncovered elements and add it to the elements, which lie at the intersection of two lines. Thus, we obtain another reduced matrix for fresh assignment.

Note- For maximization problem the matrix is converted into minimization problem by **subtracting** each element by the element having maximum value in the matrix.

Queuing Theory

Arrival rate is λ and mean service rate is denoted by μ.

- **Kendall's notation:-** (a/b/c) : (d/e)
  - a = Probability law for the arrival time
  - b = Probability law according to which the customers are being served.
  - c = number of channels
  - d = capacity of the system
  - e = queue discipline.
- **Traffic Intensity(ρ)** = Mean arrival rate/ Mean service rate = λ / μ
- **Formulas of Queuing Theory**
  1. Expected number of customers in the system (Ls) = $\lambda / (\mu - \lambda)$
  2. Expected number of customers in the queue (Lq) = $\lambda^2 / \mu (\mu - \lambda)$
  3. Expected waiting time for a customer in the queue(Wq) = $\lambda / \mu (\mu - \lambda)$
  4. Expected waiting time for a customer in the system (Ws) = $1 / (\mu - \lambda)$
  5. Probability that the queue is non-empty P(n>1) =$(\lambda / \mu)^2$
  6. Probability that the number of customers ,n in the system exceeds a given number, P(n>=k) = =(λ / μ)k
  7. Expected length of non-empty queue = $\mu / (\mu - \lambda)$.

Sequencing and Scheduling

Sequencing

1. **Johnson's Problem**

   1. **2 machines and n jobs**

      Step 1: Find the minimum among various ti_1 and ti_2.

      Step 2a : If the minimum processing time requires machine 1, place the associated job in the first available position in sequence. Go to Step 3.

      Step 2b : If the minimum processing time requires machine 2, place the associated job in the last available position in sequence. Go to Step 3.

      Step 3: Remove the assigned job from consideration and return to Step 1 until all positions in sequence are filled.

   2. **3 machines and n jobs**

      two conditions of this approach

      - The smallest processing time on machine A is greater than or equal to the greatest processing time on machine B, i.e., Min. $(A_i) \geq$ Max. (B_i)
      - The smallest processing time on machine C is greater than or equal to the greatest processing time on machine B, i.e., Max. $(B_i) \leq$ Min. (C_i)

If **either or both** of the above conditions are satisfied, then we replace the three machines by two fictitious machines G & H with corresponding processing times given by

$G_i = A_i + B_i$

$H_i = B_i + C_i$

Where G_i & H_i are the processing times for ith job on machine G and H respectively

- **Time Study-**

  Normal time = Observed time x Rating factor

  Standard time = Normal time + allowances