7. Quadratic Equations

Exercise 7.1

1 A. Question

Which of the following is a quadratic polynomial?

(i) $2 - \frac{1}{3}x^2$ (ii) $x + \frac{1}{\sqrt{x}}$ (iii) $x + \frac{1}{x}$ (iv) $x^2 + 3\sqrt{x} + 2$

Answer

(i) On solving the equations,

$$2 - \frac{1}{3}x^2$$

Re-writing in the format of $ax^2 + bx + c = 0$

$$\left(-\frac{1}{3}\right)x^2 + x(0) + 2 = 0$$

$\because a = -\frac{1}{3}$ b = 0 & c = 2

So, following the ideal pattern of a quadratic polynomial $2 - \frac{1}{3}x^2$ is a quadratic polynomial.

(ii) On solving the equations,

$$x + \frac{1}{\sqrt{x}}$$

$\because$ it can't be re-written in the format of $ax^2 + bx + c = 0$

So, following the ideal pattern of a quadratic polynomial, $x + \frac{1}{\sqrt{x}}$ is not a quadratic polynomial.

(iii) On solving the equations,

$$x + \frac{1}{x}$$

∵ it can't be re-written in the format of $ax^2 + bx + c = 0$

So, following the ideal pattern of a quadratic polynomial, $x+\frac{1}{x}$ is not a quadratic polynomial.

(iv) On solving the equations,

$x^2 + 3\sqrt{x} + 2$

∵ it can't be re-written in the format of $ax^2 + bx + c = 0$

So, following the ideal pattern of a quadratic polynomial, $x^2 + 3\sqrt{x} + 2$ is not a quadratic polynomial.

1 B. Question

Which of the following is a quadratic polynomial?

(i) $2x^2+1$ (ii) $x^2 + \frac{1}{\sqrt{x}}$ (iii) $\sqrt{x^2+1} + \frac{1}{\sqrt{x}}$ (iv) $3\sqrt{x^2+1} + x$

Answer

(i) On solving the equations,

$2x^2 + 1 = 0$

Re-writing in the format of $ax^2 + bx + c = 0$

$(2)x^2 + (0)x + 1 = 0$

∵ $a = 2$ $b = 0$ & $c = 1$.

So, following the ideal pattern of a quadratic polynomial $2x^2 + 1$ is a quadratic polynomial.

(ii) On solving the equations,

$$x^2 + \frac{1}{\sqrt{x}}$$

∵ it can't be re-written in the format of $ax^2 + bx + c = 0$

So, following the ideal pattern of a quadratic polynomial, $x^2+\frac{1}{\sqrt{x}}$ is not a quadratic polynomial.

(iii) On solving the equations,

$$\sqrt{x^2 + 1} + \frac{1}{\sqrt{x}}$$

∵ it can't be re-written in the format of $ax^2 + bx + c = 0$

So, following the ideal pattern of a quadratic polynomial, $\sqrt{x^2 + 1} + \frac{1}{\sqrt{x}}$ is not a quadratic polynomial.

(iv) On solving the equations,

$3\sqrt{(x^2 + 1)} + x = 0$

∵ it can't be re-written in the format of $ax^2 + bx + c = 0$

So, following the ideal pattern of a quadratic polynomial, $3\sqrt{(x^2 + 1)} + x = 0$ is not a quadratic polynomial.

2. Question

Which of the following is a polynomial?

(i) $2x + \frac{1}{3x^2}$ (ii) $\frac{\sqrt{3}}{2} + x^2$ (iii) $y^2 + y^{-3}$ (iv) $3\sqrt{x} + 7$

Answer

(i) On solving the equations,

$$2x + \frac{1}{3x^2} = 0$$

$$2x + 3x^{-2} = 0$$

∵ After simplifying the equation, one of the term has a negative (-2) exponent.

So, following the ideal pattern of a polynomial, $2x + \frac{1}{3x^2} = 0$ is not a polynomial.

(ii) On solving the equations,

$$\frac{\sqrt{3}}{2} + x^2 = 0$$

∵ After simplifying the equation, as it has a positive (2) exponent.

So, following the ideal pattern of a polynomial, $\frac{\sqrt{3}}{2} + x^2 = 0$ is a polynomial.

(iii) On solving the equations,

$$y^2 + \frac{1}{y^3} = 0$$

$$y^2 + y^{-3} = 0$$

∵ After simplifying the equation, as the one of the term has a negative (-3) exponent.

So, following the ideal pattern of a polynomial, $y^2 + \frac{1}{y^3} = 0$ is not a polynomial.

(iv) On solving the equations,

$$3\sqrt{x} + 7 = 0$$

$$3x^{\frac{1}{2}} + 7 = 0$$

∵ After simplifying the equation, as the expression has a degree of $\frac{1}{2}$.

So, following the ideal pattern of a polynomial, $3x^{\frac{1}{2}} + 7 = 0$ is not a polynomial.

3. Question

Fill in the blanks:

(i) $x^2 + x + 3$ is a .... polynomial.

(ii) $ax^n + bx + c$ is a quadratic polynomial if n = ...

(iii) The value of the quadratic polynomial $x^2 — 5x + 4$ for x = — 1 is ......

(iv) The degree of the polynomial $2x^2 + 4x — x^3$is ....

(v) A real number a will be called the zero of the quadratic polynomial $ax^2 + bx + c$ if .... is equal to zero.

Answer

(i) Quadratic, because it is in the form of $ax^2 + bx^2 + c = 0$

(ii) n = 2, and also a≠0, as it will make the polynomial 0.

(iii) Putting the value of x = -1, in $x^2 — 5x + 4$

$(-1)^2 - 5(-1) + 4$

$1 + 5 + 4$

10

∴ The value is 10.

(iv) ∵ The degree is the highest power of the term in the expression, so it is 3.

(v) ∵ The zeroes of a polynomial are α & β.

∴ to be zero, $\alpha x^2 + b\alpha + c = 0$ & $\beta x^2 + b\beta + c = 0$

4 A. Question

Find the zeroes of the quadratic polynomial $9 - x^2$.

Answer

$-x^2 + 9 = 0$

$-x^2 = -9$

$x^2 = 9$

∴ $x = \pm 3$

∴ The zeroes of the given polynomial are 3 & -3.

4 B. Question

Find the zeroes of the quadratic polynomial $4x^2 - 1$.

Answer

$4x^2 - 1 = 0$

$4x^2 = 1$

$x^2 = \frac{1}{4}$

∴ $x = \pm\frac{1}{2}$

∴ The zeroes of the given polynomial are $\frac{1}{2}$ & $-\frac{1}{2}$.

4 C. Question

Which of the following are the zeroes of the quadratic polynomial $9 - 4x^2$?

(a) 4 (b) 9

(c) $\frac{3}{2}$ (d) $\frac{2}{3}$

Answer

$9 - 4x^2 = 0$

$4x^2 = 9$

$x^2 = \frac{9}{4}$

$\therefore x = \pm\frac{3}{2}$

∴ The zeroes of the given polynomial are $\frac{3}{2}$ & $-\frac{3}{2}$ and the option (c) is correct.

4 D. Question

Find the zeroes of the polynomial $4 - \frac{1}{2}x^2$

(a) 2 (b) $2\sqrt{2}$ (c) 0 (d) 4

Answer

$4 - \frac{1}{2}x^2 = 0$

$\frac{1}{2}x^2 = 4$

$x^2 = 8$

$\therefore x = \pm 2\sqrt{2}$

∴ The zeroes of the given polynomial $2\sqrt{2}$ and the option (b) is correct.

5 A. Question

Is — 2 a zero of the quadratic polynomial $3x^2 + x - 10$?

Answer

Putting the value of -2, in the given polynomial,

$3(-2)^2 + (-2) - 10$

$3(4) - 2 - 10$

12 – 12

0

∵ the value comes out to be 0.

∴ -2 is one of the zeroes and, yes $3x^2 + x — 10$ is a quadratic polynomial.

5 B Type. Question

Is — 1 a zero of the quadratic polynomial $x^2 + 2x — 3$?

Answer

Putting the value of -1, in the given polynomial,

$(-1)^2 + 2(-1) - 3$

1 – 2 – 3

3 – 3

0

∵ the value comes out to be 0.

∴ -1 is one of the zeroes of the given polynomial.

6 A. Question

Which of the following is a polynomial? Find its degree and the zeroes.

$2 - \frac{1}{2} x^2$

Answer

∵ The highest power is 2, so the degree is also 2.

Equating the expression with 0,

$$2 - \frac{1}{2}x^2 = 0$$

$$2 = \frac{1}{2}x^2$$

$x^2 = 4$

∴ $x = \pm 2$

Yes, the above expression is a polynomial, as it has no negative powers in any of the terms and its zeroes are 2 & -2.

6 B. Question

Which of the following is a polynomial? Find its degree and the zeroes.

$x + \frac{1}{\sqrt{x}}$

Answer

∵ the power of a term is in negative $(-\frac{1}{2})$.

∴ The above given expression is not a polynomial.

7 A. Question

Which of the following is a polynomial '? Find its zeroes.

(i) $x^2 + \sqrt{x} + 2$ (ii) $x + \frac{1}{x}$ (iii) $4 - \frac{1}{4}x^2$

Answer

In the above expressions, only the thirdone has the positive power unlike others.

∴ It is the only polynomial.

Equating the expression with 0,

$4 - \frac{1}{4}x^2 = 0$

$4 = \frac{1}{4}x^2$

$x^2 = 16$

∴ $x = \pm 4$

The zeroes of the polynomial $4 - \frac{1}{4}x^2$ are 4 & -4.

7 B. Question

Which of the following expressions is a polynomial? Find the degree and zeroes of the polynomial.

(i) $\frac{x}{2} + \frac{2}{x}$ (ii) x^2 +2x

Answer

In the above expressions, only the secondone has a positive power, unlike others.

∴ It is the only polynomial.

Equating the expression with 0,

$x^2 + 2x = 0$

x (x+2) = 0

∴ x = 0 or x + 2 = 0

X = 0 Or x = -2

The zeroes of the polynomial x^2 +2x are 2 & -2, having a degree of 2, being the highest power of the terms in the same expression.

7 C. Question

Which among the expressions 1- $\frac{1}{16}z^2$ and $z^2 + z + 1$ is a polynomial in z? Find its zeroes and degree.

Answer

∵ The highest power is 2, so the degree is also 2, in both the expressions.

Equating the expression with 0,

$$1 - \frac{1}{16}z^2 = 0$$

$$1 = \frac{1}{16}z^2$$

x^2 =16

∴ x = ±4

Yes, the above expression $(1 - \frac{1}{16}z^2)$ is a polynomial, and its zeroes are 4 & -4.

Equating the expression with 0,

$z^2 + z + 1 = 0$

Using Sreedharacharya formula, $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$ax^2+bx+c = 0$

$x = \frac{(-(1)) \pm \sqrt{(1)^2 - 4(1)(1)}}{2(1)}$

$x = \frac{-1 \pm \sqrt{1-4}}{2}$

$x = \frac{-1 \pm \sqrt{-3}}{2}$

∵ it does not have real values.

∴ The zeroes of $z^2 + z + 1$ are complex numbers, though it is a polynomial having the degree 2.

8. Question

Find the zeroes of the quadratic polynomial $x^2 — 6x + 8$.

Answer

Equating the expression with 0,

$x^2 — 6x + 8 = 0$

On factorising it further,

$x^2 - 4x - 2x + 8 = 0$

$x(x - 4) – 2(x - 4) = 0$

$(x - 4)(x - 2) = 0$

∴ $x = 4$ or $x = 2$

∴ The zeroes of $x^2 — 6x + 8$ are 4 & 2.

9 A. Question

Find the zeroes of the quadratic polynomial:

$2x^2 + x — 1$

Answer

Equating the expression with 0,

$2x^2 + x - 1 = 0$

On factorising it further,

$2x^2 - x + 2x - 1 = 0$

$x(2x - 1) + 1(2x - 1) = 0$

$(2x - 1)(x + 1) = 0$

$\therefore x = \frac{1}{2}$ or $x = -1$

$\therefore$ The zeroes of $2x^2 + x — 1$ are $\frac{1}{2}$ and -1.

9 B. Question

Find the zeroes of the quadratic polynomial:

$2x^2 — 5x + 2$

Answer

Equating the expression with 0,

$2x^2 - 5x + 2 = 0$

On factorising it further,

$2x^2 - 4x - x + 2 = 0$

$2x(x - 2) - 1(x - 2) = 0$

$(2x - 1)(x - 2) = 0$

$\therefore x = \frac{1}{2}$ or $x = 2$

$\therefore$ The zeroes of $2x^2 — 5x + 2$ are $\frac{1}{2}$ and 2.

9 C. Question

Find the zeroes of the quadratic polynomial:

$5x^2 - 4x — 1$

Answer

Equating the expression with 0,

$5x^2 - 4x - 1 = 0$

On factorising it further,

$5x^2 - 5x + x - 1 = 0$

$5x(x - 1) + 1(x - 1) = 0$

$(5x + 1)(x - 1) = 0$

$\therefore x = -\frac{1}{5}$ or $x = 1$

$\therefore$ The zeroes of $5x^2 - 4x — 1$ are $-\frac{1}{5}$ and 1.

9 D. Question

Find the zeroes of the quadratic polynomial:

$x^2 — 2x + 3$

Answer

Equating the expression with 0,

$x^2 — 2x + 3 = 0$

Using Sreedharacharya formula, $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$ax^2 + bx + c = 0$

$x = \frac{(-(-2)) \pm \sqrt{(-2)^2 - 4(1)(3)}}{2(1)}$

$x = \frac{2 \pm \sqrt{4 - 12}}{2}$

$x = \frac{2 \pm \sqrt{-8}}{2}$

$\because$ it does not have real values.

$\therefore$ The zeroes of $x^2 — 2x + 3$ are complex numbers.

9 E. Question

Find the zeroes of the quadratic polynomial:

$3x^2 — 10x + 3$

Answer

Equating the expression with 0,

$3x^2 - 10x + 3 = 0$

On factorising it further,

$3x^2 - 9x - x + 3 = 0$

$3x(x - 3) - 1(x - 3) = 0$

$(3x - 1)(x - 3) = 0$

$\therefore x = \frac{1}{3}$ or $x = 3$

$\therefore$ The zeroes of $3x^2 — 10x + 3$ are $\frac{1}{3}$ and 3.

9 F. Question

Find the zeroes of the quadratic polynomial:

$3x^2 + 5x + 2$

Answer

Equating the expression with 0,

$3x^2 + 5x + 2 = 0$

On factorising it further,

$3x^2 + 3x + 2x + 2 = 0$

$3x(x + 1) + 2(x + 1) = 0$

$(3x + 2)(x + 1) = 0$

$\therefore x = -\frac{2}{3}$ or $x = -1$

$\therefore$ The zeroes of $3x^2 + 5x + 2$ are $-\frac{2}{3}$ and -1.

9 G. Question

Find the zeroes of the quadratic polynomial:

$4x^2 — x — 5$

Answer

Equating the expression with 0,

$4x^2 — x — 5 = 0$

On factorising it further,

$4x^2 + 4x - 5x - 5 = 0$

$4x(x + 1) -5(x + 1)=0$

$(4x - 5) (x + 1) = 0$

$\therefore x = \frac{5}{4}$ or $x = -1$

$\therefore$ The zeroes of $4x^2 — x — 5$ are $\frac{5}{4}$ and -1.

Exercise 7.2

1 A. Question

Check whether the following are quadratic equations:

$(x - 2) (x + 1) = (x - 1) (x + 3)$

Answer

Given; $(x - 2) (x + 1) = (x - 1) (x + 3)$

$\Rightarrow x^2 + x - 2x - 2 = x^2 + 3x - x - 3$

$\Rightarrow x^2 - x - 2 - x^2 - 2x + 3 = 0$

$\Rightarrow -3x + 1 = 0$

$\because$ The highest power of x in the equation is 1;

$\therefore$ It is not a quadratic equation.

1 B. Question

Check whether the following are quadratic equations:

$(x - 2)^2 + 1 = 2x - 3$

Answer

Given; $(x - 2)^2 + 1 = 2x - 3$

$\Rightarrow x^2 - 2x + 4 + 1 = 2x - 3$

$\Rightarrow x^2 - 2x + 5 - 2x + 3 = 0$

$\Rightarrow x^2 - 4x + 8 = 0$

∵ The highest power of x in the equation is 2;

∴ It is a quadratic equation.

1 C. Question

Check whether the following are quadratic equations:

$x (x + 1) + 8 = (x + 2) (x - 2)$

Answer

Given; $x (x + 1) + 8 = (x + 2) (x - 2)$

$\Rightarrow x^2 + x + 8 = x^2 - 2^2$

$\Rightarrow x^2 + x + 8 - x^2 + 4 = 0$

$\Rightarrow x + 12 = 0$

∵ The highest power of x in the equation is 1;

∴ It is not a quadratic equation.

1 D. Question

Check whether the following are quadratic equations:

$(x - 3) (2x + 1) = x (x + 5)$

Answer

Given; $(x - 3) (2x + 1) = x (x + 2)$

$\Rightarrow 2x^2 + x - 6x - 3 = x^2 + 5x$

$\Rightarrow 2x^2 + x - 6x - 3 - x^2 - 5x = 0$

$\Rightarrow x^2 - 10x - 3 = 0$

∵ The highest power of x in the equation is 2;

∴ It is a quadratic equation.

1 E. Question

Check whether the following are quadratic equations:

$x (2x + 3) = x^2 + 1$

Answer

Given; $x (2x + 3) = x^2 + 1$

$\Rightarrow 2x^2 + 3x = x^2 + 1$

$\Rightarrow 2x^2 + 3x - x^2 - 1 = 0$

$\Rightarrow x^2 + 3x - 1 = 0$

$\because$ The highest power of x in the equation is 2;

$\therefore$ It is a quadratic equation.

1 F. Question

Check whether the following are quadratic equations:

$x^2 + 3x + 1 = (x - 2)^2$

Answer

Given; $x^2 + 3x + 1 = (x - 2)^2$

$\Rightarrow x^2 + 3x + 1 = x^2 - 4x + 4$

$\Rightarrow x^2 + 3x + 1 - x^2 - 4x - 4 = 0$

$\Rightarrow -x - 3 = 0$

$\because$ The highest power of x in the equation is 1;

$\therefore$ It is not a quadratic equation.

1 G. Question

Check whether the following are quadratic equations:

$(x + 1)(x - 1) = (x + 2)(x + 3)$

Answer

Given; $(x + 1)(x - 1) = (x + 2)(x + 3)$

$\Rightarrow x^2 - 1^2 = x^2 + 5x + 6$

$\Rightarrow x^2 - 1 - x^2 - 5x - 6 = 0$

$\Rightarrow -5x - 7 = 0$

$\because$ The highest power of x in the equation is 1;

$\therefore$ It is not a quadratic equation.

1 H. Question

Check whether the following are quadratic equations:

$(x - 1)^2 = (x + 1)^2$

Answer

Given; $(x - 1)^2 = (x + 1)^2$

$\Rightarrow x^2 - x + 1^2 = x^2 + x + 1^2$

$\Rightarrow x^2 - x + 1 - x^2 - x - 1 = 0$

$\Rightarrow -2x = 0$

$\because$ The highest power of x in the equation is 1;

$\therefore$ It is not a quadratic equation.

2 A. Question

Check whether the following are quadratic equations:

$(x + 2)^3 = x^3 - 4$

Answer

Given; $(x + 2)^3 = x^3 - 4$

$\Rightarrow x^3 + 6x^2 + 12x + 2^3 = x^3 - 4$

$\Rightarrow x^3 + 6x^2 + 12x + 2^3 - x^3 + 4 = 0$

$\Rightarrow 6x^2 + 12x + 12 = 0$

$\because$ The highest power of x in the equation is 2;

$\therefore$ It is a quadratic equation.

2 B. Question

Check whether the following are quadratic equations:

$$x - \frac{1}{x} = 8$$

Answer

Given; $x - \frac{1}{x} = 8$

$\Rightarrow x^2 - 1 = 8x$

$\Rightarrow x^2 - 8x - 1 = 0$

$\because$ The highest power of x in the equation is 2;

$\therefore$ It is a quadratic equation.

2 C. Question

Check whether the following are quadratic equations :

$$2x^2 - 3\sqrt{x} + 5 = 0$$

Answer

Given; $2x^2 - 3\sqrt{x} + 5 = 0$

$\Rightarrow 2x^2 + 5 = 3\sqrt{x}$

$\Rightarrow \left(2x^2 + 5\right)^2 = \left(3\sqrt{x}\right)^2$

$\Rightarrow 4x^4 + 20x^2 + 25 = 9x$

$\Rightarrow 4x^4 + 20x^2 - 9x + 25 = 0$

$\because$ The highest power of x in the equation is 4;

$\therefore$ It is not a quadratic equation.

2 D. Question

Check whether the following are quadratic equations :

$$x^2 + \frac{1}{x} = 5$$

Answer

Given; $x^2 + \frac{1}{x} = 5$

$\Rightarrow x^3 + 1 = 5x$

$\Rightarrow x^3 - 5x + 1 = 0$

$\because$ The highest power of x in the equation is 3;

$\therefore$ It is not a quadratic equation.

2 E. Question

Check whether the following are quadratic equations :

$$x^2 - \frac{1}{x^2} = 8$$

Answer

Given; $x^2 - \frac{1}{x^2} = 8$

⇒ $x^4 - 1 = 8x^2$

⇒ $x^4 - 8x^2 - 1 = 0$

∵ The highest power of x in the equation is 4;

∴ It is not a quadratic equation.

3 A. Question

Represent the following situations mathematically:

John and Jivanti together have 45 marbles. Both of them lost 5 marbles each and the product of the number of marbles they now have is 124. We would like to find out how many marbles they had to start with.

Answer

Let 'x' be the number of marbles John had.

∴ Jivanti will have 45 – x marbles.

[∵ the total number of marbles is 45]

As both of them lost 5 marbles each; marbles they now have will be x – 5 and 40 – x respectively.

⇒ Product of the number of marbles = (x – 5) (40 – x)

∴ $40x - x^2 - 200 + 5x = 124$ [∵ the product is 124]

⇒ $x^2 - 45x + 324 = 0$

3 B. Question

Represent the following situations mathematically:

A shopkeeper buys a number of books for Rs. 80. If he had bought four more books for the same amount, the book would have cost Re. 1 less.

Answer

Let ‘x’ be the number of books bought by the shopkeeper.

∴ Cost of one book = Rs. 80 ÷ x.

[∵ the total cost of books is Rs. 80]

∴ Cost of one book when he buys 4 more books for same rate = Rs. 80 ÷ (x + 4).

When he buys four more books for the same amount; it will cost Re. 1 per book less than the previous.

∴ 80 ÷ (x + 4) = (80 ÷ x) − 1

$$\Rightarrow \frac{80}{x+4} = \frac{80}{x} - 1$$

⇒ 80x = (x + 4) (80 − x)

⇒ $80x = 80x - x^2 + 320 - 4x$

⇒ $x^2 + 4x + 320 = 0$

3 C. Question

Represent the following situations mathematically:

A cottage industry produces a certain number of toys in a day. The cost of production of each toy (in rupees) was found to be 55 minus the number of toys produced in a day. On a particular day, the total cost of production was Rs. 750. We would like to find out the number of toys produced on that day.

Answer

Let ‘x’ be the number of toys produced on that day.

∴ Cost of production of each toy that day= Rs. 55 − x.

[∵ The cost of production of each toy (in rupees) is 55 minus the number of toys produced in a day]

∴ Cost of production of x toys = Rs. x (55 − x)

[∵ On a particular day, the total cost of production was Rs. 750.]

∴ x (55 − x) = 750

⇒ $x^2 - 55x + 750 = 0$

3 D. Question

Represent the following situations mathematically:

The sum of the squares of two positive integers is 117. If the square of the smaller number equals four times the larger number, we need to find the integers.

Answer

Let ‘x’ and ‘y’ be the smaller and larger integer respectively.

∴ $x^2 + y^2 = 117$.

[∵ The sum of the squares of two positive integers is 117]

∴ $x^2 = 4y$

[∵ the square of the smaller number equals four times the larger number.]

∴ $y^2 + 4y - 117 = 0$

4 A. Question

Represent the following situations in the form of the quadratic equation.

Divide 16 into two parts such that twice of the square of larger part exceeds the square of the smaller part by 164.

Answer

Let ‘x’ be the one part and the other part will be 16 – x.

[∵ 16 is being divided]

∵ Twice of the square of larger part exceeds the square of the smaller part by 164.

∴ $2x^2 = (16 - x)^2 + 164$

∴ $2x^2 = 256 - 32x + x^2 + 164$

⇒ $x^2 + 32x - 420 = 0$

4 B. Question

Represent the following situations in the form of the quadratic equation.

One year ago, a man was eight times as old as his son. Now, his age is equal to the square of his son's age.

Answer

Let ‘y’ and ‘x’ be the present age of the man and son.

∵ One year ago the man was eight times as old as his son.

$\therefore$ y − 1 = 8 (x − 1)

$\because$ Now, his age is equal to the square of his son's age.

$\therefore y = x^2$

$\therefore x^2 - 1 = 8(x - 1)$

$\Rightarrow x^2 - 8x + 7 = 0$

4 C. Question

Represent the following situations in the form of the quadratic equation.

A train travels a distance of 300 km at a constant speed. If the speed of the train 'is increased by 5 km an hour. The journey would have taken two hours less.

Answer

Let ‘x’ be the speed of the train in km per hour.

$\therefore$ Time taken to cover 300 km = $\frac{300}{x}$

[$\because$ Time $= \frac{\text{Distance}}{\text{Speed}}$.]

$\therefore$ Time taken when speed is increased by 5 = $\frac{300}{x+5}$

$\because$ The journey would have taken two hours less when speed is decreased.

$$\Rightarrow \text{Difference in time} = \frac{300}{x} - \frac{300}{(x+5)} = 2$$

4 D. Question

Represent the following situations in the form of the quadratic equation.

The hypotenuse of a right-angled triangle is 6 metres more than twice of the shortest side. The third side is two metres less than the hypotenuse.

Answer

Let ‘x’ be the length of the shortest side.

$\because$ The hypotenuse of a right–angled triangle is 6 metres more than twice of the shortest side.

∴ length of hypotenuse = 2x + 6

∵ The third side is two metres less than the hypotenuse.

∴ length of third side = hypotenuse – 4 = 2x + 4

By applying Pythagoras theorem; Hypotenuse square is equal to sum of the squares of other two sides.

$\Rightarrow (2x + 6)^2 = x^2 + (2x + 4)^2$

4 E. Question

Represent the following situations in the form of the quadratic equation.

A piece of cloth costs Rs. 200. If the piece was 5 metre longer and each metre of cloth costs Rs. 2 less, the cost of the piece would have remained unchanged.

Answer

Let ‘x’ be the length of cloth.

∵ Cost of x metre is Rs. 200.

∴ Cost per metre = Rs. 200 ÷ x

∴ Cost per metre when total size is x + 5 = Rs. 200 ÷ (x + 5)

∵ Cost of 5 metre longer cloth is Rs. 2 less for each metre.

$$\Rightarrow \frac{200}{x} - \frac{200}{x+5} = 2$$

Exercise 7.3

1. Question

Determine whether $x = \frac{3}{2}$ and $x = -\frac{4}{3}$ are the solutions of the equation $6x^2 - x - 12 = 0$ or not.

Answer

Put both the values of x in the equation.

When $x = \frac{3}{2}$

$$6\left(\frac{3}{2}\right)^2 - \frac{3}{2} - 12 = 0$$

$$6 \times \frac{9}{4} - \frac{3}{2} - 12 = 0$$

$$\frac{54 - 48 - 6}{4}$$

$$\frac{54 - 54}{4}$$

= 0

When $x = -\frac{4}{3}$

$$6\left(-\frac{4}{3}\right)^2 - \frac{4}{3} - 12 = 0$$

$$6\left(\frac{16}{9}\right) + \frac{4}{3} - 12 = 0$$

$$\frac{96 + 12 - 108}{9}$$

$$\frac{108 - 108}{9}$$

= 0

R.H.S = L.H.S

Therefore, $x = \frac{3}{2}$ and $x = -\frac{4}{3}$ are the solutions of the given equation.

2. Question

Determine whether (i) x = 1, (ii) x = 3 are the solutions of the equation $x^2 - 5x + 4 = 0$ or not.

Answer

Put both the values of x in the equation.

When x = 1

$1^2 - 5(1) + 4$

1 – 5 + 4

= 0

Therefore, it is the solution to the equation.

When $x = 3$

$3^2 - 5(3) + 4 = 0$

$9 - 15 + 4$

$= -2$

Therefore, it is not the solution to the equation.

3. Question

Determine whether $x = \sqrt{3}$ and $x = -2\sqrt{3}$ are solutions of the equation $x^2 - 3\sqrt{3}x + 6 = 0$

Answer

Put both the values of x in the equation.

When $x = \sqrt{3}$

$(\sqrt{3})^2 - 3\sqrt{3}(\sqrt{3}) + 6 = 0$

$3 - (3)3 + 6$

$3 - 9 + 6$

$9 - 9$

$= 0$

Therefore, it is the solution to the equation.

When $x = -2\sqrt{3}$

$(-2\sqrt{3})^2 - 3\sqrt{3}(-2\sqrt{3}) + 6 = 0$

$12 + 18 + 6$

$= 36$

Therefore, it is not the solution to the equation.

4. Question

For $2x^2 - 5x - 3 = 0$, determine which of the following are solutions?

(i) $x = 3$ (ii) $x = -2$

(iii) $x = -\frac{1}{2}$ (iv) $x = -\frac{1}{3}$

Answer

The two possible solutions are x = 3 and $= -\frac{1}{2}$.

Since this question is given in standard form, meaning that it follows the form: $ax^2 + by + c = 0$, we can use the quadratic formula to solve for x:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(2)(-3)}}{2 \times 2}$$

$$x = \frac{5 \pm \sqrt{25 + 24}}{4}$$

$$x = \frac{5 \pm 7}{4}$$

$$x = \frac{5 + 7}{4} \quad x = \frac{5 - 7}{4}$$

$$x = \frac{12}{4} \quad x = \frac{-2}{4}$$

x = 3 $x = -\frac{1}{2}$

That value of x is correct as well!

Therefore, the two possible solutions are:

x=3

x=−0.50

5. Question

Determine whether (i) x= √2, (ii) x = −2√2 are the solutions of the equation $x^2 + \sqrt{2}\,x - 4 = 0$ or not.

Answer

Put x = √2

$(\sqrt{2})^2 + \sqrt{2}(\sqrt{2}) - 4 = 0$

$2 + 2 - 4$

$4 - 4$

$= 0$

Therefore, it is the solution to the equation.

When $x = -2\sqrt{2}$

$(-2\sqrt{2})^2 + \sqrt{2}(-2\sqrt{2}) - 4 = 0$

$8 - 4 - 4$

$= 0$

Therefore, it is the solution to the equation.

6. Question

Show that $x = — 3$ is a solution of $x^2 + 6x + 9 = 0$.

Answer

Put $x = -3$ in the equation.

$(-3)^2 + 6(-3) + 9$

$9 - 18 + 9$

$=0$

Hence it is a solution

7. Question

Show that $x = — 3$ is a solution of $2x^2 + 5x -3 = 0$.

Answer

Put $x = -3$ in the equation.

$2(-3)^2 + 5(-3) -3 = 0$

$18 - 15 -3$

$18 - 18$

$= 0$

Therefore, $x = -3$ is the solution of the equation.

8. Question

Show that x = — 2 is a solution of $3x^2 + 13x + 14 = 0$.

Answer

The given quadratic equation is $3x^2 + 13x + 14 = 0$

Putting x = – 2,

L.H.S.

$3.(-2)^2 + 13.(-2) + 14$

3 x 4 – 26 + 14

12 – 26 + 14

26 – 26

= 0

Hence, x = – 2 is a solution of 3x + 13x + 14 = 0

9. Question

For what value of k, $x = \frac{2}{3}$ is the solution of the equation

$kx^2 - x - 2 = 0$.

Answer

$x = \frac{2}{3}$

$kx^2 - x - 2 = 0$

$k\left(\frac{2}{3}\right)^2 - \frac{2}{3} - 2$

$\frac{4k}{9} - \frac{2}{3} - 2$

$\frac{4k - 6 - 18}{9}$

4k – 24 = 0

4k = 24

k = 6

10. Question

For what value of k, $x = -\frac{1}{2}$ is a solution of the equation $3x^2 + 2kx — 3 = 0$

Answer

Put the value of x in the equation.

$3x^2 + 2kx — 3 = 0$

$$3\left(-\frac{1}{2}\right)^2 + 2k\left(-\frac{1}{2}\right) - 3 = 0$$

$$\frac{3}{4} - \frac{2k}{2} - 3 = 0$$

$$\frac{3 - 4k - 12}{4} = 0$$

$-9 - 4k = 0$

$-4k = 9$

$$k = -\frac{9}{4}$$

11. Question

For what values of a and b, $x = \frac{3}{4}$ and x = — 2 are solutions of the equation $ax^2 + bx — 6 = 0$.

Answer

Put x = 3/4

$ax^2 + bx — 6 = 0$

$$a\left(\frac{3}{4}\right)^2 + b\left(\frac{3}{4}\right) — 6 = 0$$

$$\frac{9a}{16} + \frac{3b}{4} - 5 = 0$$

$$\frac{9a + 12b - 96}{16} = 0$$

$9a + 12b - 96 = 0$ divide by 3

$3a + 4b - 32 = 0$

$3a + 4b = 32$ (1)

Put $x = -2$

$ax^2 + bx - 6 = 0$

$a(-2)^2 + b(-2) - 6 = 0$

$4a - 2b - 6 = 0$

$4a - 2b = 6$ (2)

Eliminate (1) and (2)

$3a + 4b = 32$

$4a - 2b = 6 \times 2$

$$\begin{array}{r} 3a + 4b = 32 \\ \underline{8a - 4b = 12} \\ \underline{11a = 44} \end{array}$$

$a = 4$

Put $a = 4$ in equation (1).

$3a + 4b = 32$

$3(4) + 4b = 32$

$12 + 4b = 32$

$4b = 32 - 12$

$4b = 20$

$b = 5$

12. Question

For what value of k, $x = a$ is a solution of the equation

$x^2 - (a + b)x + k = 0.$

Answer

$x^2 - (a + b)x + k = 0$

Put x = a

$a^2 - (a + b)\,a + k$

$a^2 - a^2 + ab + k = 0$

ab + k = 0

k = –ab

13. Question

Determine the value of k, a and b in each of the following quadratic equation, for which the given value of x is the root of the given quadratic equation:

(i) $kx^2 — 5x + 6 = 0$; x = 2

(ii) $6x^2 + kx — \sqrt{6} = 0$; $x = -\frac{\sqrt{3}}{2}$

(iii) $ax^2 — 13x + b = 0$; x =2 and x = —2 find a, b

(iv) $ax^2 + bx - 10 = 0$; $x = -\frac{2}{5}$ and $x = \frac{5}{3}$

Answer

(i) $kx^2 - 5x + 6 = 0$

Put x = 2

(ii) $k^2 - 5(2) + 6 = 0$

4k – 10 + 6 = 0

4k = 4

k = 1

(iii) $6x^2 + kx — \sqrt{6} = 0$

Put x = – 2/√3

$$6\left(-\frac{\sqrt{3}}{2}\right)^2 + k\left(-\frac{\sqrt{3}}{2}\right) - \sqrt{6} = 0$$

$$\frac{18}{4} - \frac{\sqrt{3}k}{2} - \sqrt{6} = 0$$

$$\frac{18-2\sqrt{3}k-\ 4\sqrt{6}}{4}=0$$

18 – 2√3 k – 4√6 = 0

18 – 4√6 = 2√3k

$$\frac{18-\ 4\sqrt{6}}{2\sqrt{3}}=k$$

$$\frac{(18-\ 4\sqrt{6})2\sqrt{3}}{2\sqrt{3}\times\ 2\sqrt{3}}=k$$

$$\frac{18\sqrt{3}-\ 12\sqrt{2}}{12}=k$$

3√3 – 2√2 = k

(iv) $ax^2 + bx - 10 = 0$

Put $x=-\frac{\sqrt{3}}{2}$

$$a\left(-\frac{2}{5}\right)^2+b\left(-\frac{2}{5}\right)-10=0$$

$$\frac{4a}{25}-\frac{2b}{5}-10=0$$

$$\frac{4a-10b-250}{25}=0$$

4a – 10b – 250 = 0

4a – 10b = 250 (1)

Put x = 3/5

$ax^2 + bx - 10 = 0$

$$a\left(\frac{5}{3}\right)^2+b\left(\frac{5}{3}\right)-10=0$$

$$\frac{25a}{9}+\frac{5b}{3}-10=0$$

$$\frac{25a+15b-90}{9}=0$$

$25a + 15b - 90 = 0$ (divide by 5)

$5a + 3b = 18$ (2)

Eliminate (1) and (2)

$4a - 10b = 250 \times 5$

$5a + 3b = 18 \times 4$

$20a - 50b = 1250$

$\underline{-20a - 12b = -72}$

$\underline{\quad -62b = 1178}$

$b = -19$

Put $b = -19$ in (1)

$4a - 10b = 250$

$4a - 10(-19) = 250$

$4a + 190 = 250$

$4a = 60$

$a = 15$

14 A. Question

Find the roots of the following quadratic equations by factorisation:

$2x^2 — 5x + 3 = 0$

Answer

$2x - 2x - 3x + 3 = 0$

$2x (x - 1) - 3(x - 1) = 0$

$(2x - 3) (x - 1) = 0$

$2x - 3 = 0$

$x = \frac{3}{2}$

$x - 1 = 0$

$x = 1$

Therefore, the roots of the equation are $\frac{3}{2}$, 1.

14 B. Question

Find the roots of the following quadratic equations by factorisation:

$3x^2 — 2\sqrt{6}x + 2 = 0$

Answer

$3x^2 - \sqrt{6}\,x - \sqrt{6}\,x + 2 = 0$

$3x^2 - \sqrt{2}\sqrt{3}x - \sqrt{2}\sqrt{3}x + 2 = 0$

$\sqrt{3}x(\sqrt{3}x - \sqrt{2}) - \sqrt{2}\,(\sqrt{3}x - \sqrt{2}) = 0$

$\sqrt{3}x - \sqrt{2} = 0$ $\sqrt{3}x - \sqrt{2} = 0$

$x = \frac{\sqrt{2}}{\sqrt{3}}$ $x = \frac{\sqrt{2}}{\sqrt{3}}$ Therefore, the roots of the equation are $\frac{\sqrt{2}}{\sqrt{3}}, \frac{\sqrt{2}}{\sqrt{3}}$.

14 C. Question

Find the roots of the following quadratic equations by factorisation:

$3x^2 — 14x — 5 = 0$

Answer

$3x^2 — 15x + x — 5 = 0$

$3x\,(x - 5) + (x - 5) = 0$

$(3x + 1)\,(x - 5) = 0$

$3x + 1 = 0$ $x - 5 = 0$

$x = -\frac{1}{3}$ $x=5$

Therefore, the roots of the equation are $-\frac{1}{3}, 5$.

14 D. Question

Find the roots of the following quadratic equations by factorisation:

$$\sqrt{3}x^2 + 10x + 7\sqrt{3} = 0$$

Answer

Now, to find the roots by factorisation, we need to factorise 10 such that the sum is 10 and the product is $7\sqrt{3} \times \sqrt{3} = 21$

We can do that by 7 and 3.

So,

$\sqrt{3}x^2 + 10x + 7\sqrt{3} = 0$

$\sqrt{3}x^2 + 3x + 7x + 7\sqrt{3} = 0$

$\sqrt{3}x(x + \sqrt{3}) + 7(x + \sqrt{3}) = 0$

$(\sqrt{3}x + 7)(x + \sqrt{3}) = 0$

$(\sqrt{3}x + 7) = 0$

$\sqrt{3}x = -7$

$$x = -\frac{7}{\sqrt{3}}$$

Or

$x + \sqrt{3} = 0$

$x = -\sqrt{3}$

Hence, the solutions of the given quadratic equations are $-\sqrt{3}$ and $-\frac{7}{\sqrt{3}}$.

14 E. Question

Find the roots of the following quadratic equations by factorisation:

$\sqrt{7}\, y^2 - 6y - 13\sqrt{7} = 0$

Answer

$\sqrt{7}\, y^2 - 13y + 7y - 13\sqrt{7} = 0$

$y(\sqrt{7}\, y - 13) + \sqrt{7}(\sqrt{7}\, y - 13) = 0$

$(\sqrt{7}\, y - 13)(y + \sqrt{7}) = 0$

$\sqrt{7}\, y - 13 = 0 \; y + \sqrt{7} = 0$

$y = \frac{13}{\sqrt{7}} \; y = -\sqrt{7}$

Rationalise

$$y = \frac{13}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}}$$

$$y = \frac{13\sqrt{7}}{7}$$

Therefore, the roots of the equation are $\frac{13\sqrt{7}}{7}, -\sqrt{7}$.

14 F. Question

Find the roots of the following quadratic equations by factorisation:

$4x^2 - 4a^2x + a^4 - b^4 = 0$

Answer

$4x^2 - \{2(a^2 + b^2) + 2(a^2 - b^2)\}x + (a^2 + b^2)(a^2 - b^2) = 0$

$4x^2 - 2(a^2 + b^2)x + 2(a^2 - b^2)x + (a^2 + b^2)(a^2 - b^2) = 0$

$2x\{2x - (a^2 + b^2)\} - (a^2 - b^2)\{2x - (a^2 + b^2)\} = 0$

$\{2x - (a^2 - b^2)\}\{2x - (a^2 + b^2)\} = 0$

$2x - (a^2 - b^2) = 0 \; 2x - (a^2 + b^2) = 0$

$2x = (a^2 - b^2) \; 2x = (a^2 + b^2)$

$$x = \frac{a^2 - b^2}{2} \; x = \frac{a^2 + b^2}{2}$$

Therefore, the roots of the equation are $\frac{a^2 - b^2}{2}, \frac{a^2 + b^2}{2}$.

15 A. Question

$a^2b^2x^2 + b^2x - a^2x - 1 = 0, a \neq 0, b \neq 0$

Answer

$b^2x\{a^2x + 1\} - 1\{a^2x + 1\} = 0$

$(b^2x - 1)(a^2x + 1) = 0$

$b^2x - 1 = 0 \; a^2x + 1 = 0$

$$x = \frac{1}{b^2} \; x = \frac{-1}{a^2}$$

Therefore, the roots of the equation are $\frac{1}{b^2}, \frac{-1}{a^2}$.

15 B. Question

$36x^2 - 12ax + (a^2 - b^2) = 0$

Answer

$(6x)^2 - 2(6x)a + a^2 — b^2 = 0$

Using Identity:

$(x - y)^2 = x^2 + y^2 - 2xy$

Here, $(6x - a)^2 = (6x)^2 - 2(6x)a + a^2$

$(6x - a)^2 - b^2 = 0$

Using identity:

$x^2 - y^2 = (a + b)(a - b)$

$(6x - a + b)(6x - a - b) = 0$

$6x = a - b$ $6x = a + b$

$$x = \frac{a - b}{6} \quad x = \frac{a + b}{6}$$

Therefore, the roots of the equation are $\frac{a-b}{6}, \frac{a+b}{6}$

15 C. Question

$10ax^2 — 6x + 15ax - 9 = 0, a \neq 0$

Answer

$2x(5ax - 3) + 3(5ax - 3) = 0$

$2x+3 = 0$ $5ax - 3 = 0$

$$x = -\frac{3}{2} \quad x = \frac{3}{50}$$

Therefore, roots of the equation are $\frac{3}{50}, -\frac{3}{2}$.

15 D. Question

$12abx^2 — (9a^2 — 8b^2)x — 6ab = 0$

Answer

$12abx^2 — 9a^2x — 8b^2x — 6ab = 0$

$3ax(4bx - 3a) + 2b(4bx - 3a) = 0$

$3ax + 2b = 0$ $4bx - 3a = 0$

$$x = -\frac{2b}{3a}\ x = \frac{3a}{4b}$$

Therefore, roots of the equation are $-\frac{2b}{3a}, \frac{3a}{4b}$.

15 E. Question

$4x^2 — 2 (a^2 + b^2) x + a^2b^2 = 0$

Answer

$4x^2 — 2a^2x+ 2b^2x + a^2b^2 = 0$

$2x (2x – a^2) – b^2 (2x – a^2) = 0$

$2x – b^2 = 0$ $2x – a^2 = 0$

$$x = \frac{b^2}{2}\ x = \frac{a^2}{2}$$

16 A. Question

Find the roots of the following quadratic equations, if they exist by the method of completing the square:

$5x^2 – 6x – 2 = 0$

Answer

$5x^2 – 6x – 2 = 0$

Dividing by 5

$$x^2 - \frac{6x}{5} - \frac{2}{5} = 0$$

We know

$(a – b)^2 = a^2 – 2ab + b^2$

Here, a=x and $-2ab = -\frac{6x}{5}$

$-2xb = -\frac{6x}{5}$ ($\because$ a =x)

$-2b = -\frac{6}{5}$

$$b = -\frac{6}{5 \times (-2)}$$

$$b = \frac{3}{5}$$

∴ Equation becomes

$$x^2 - \frac{6x}{5} - \frac{2}{5} = 0$$

Add and subtract $\left(\frac{3}{5}\right)^2$

$$x^2 - \frac{6x}{5} - \frac{2}{5} + \left(\frac{3}{5}\right)^2 - \left(\frac{3}{5}\right)^2 = 0$$

$$x^2 - \frac{6x}{5} + \left(\frac{3}{5}\right)^2 - \frac{2}{5} - \left(\frac{3}{5}\right)^2 = 0$$

$$\left(x - \frac{3}{5}\right)^2 = \left(\frac{3}{5}\right)^2 + \frac{2}{5}$$

$$\left(x - \frac{3}{5}\right)^2 = \frac{9}{25} + \frac{2}{5}$$

$$\left(x - \frac{3}{5}\right)^2 = \frac{9 + 2(5)}{25}$$

$$\left(x - \frac{3}{5}\right)^2 = \frac{9 + 10}{25}$$

$$\left(x - \frac{3}{5}\right)^2 = \frac{19}{25}$$

$$\left(x - \frac{3}{5}\right)^2 = \frac{(\sqrt{19})^2}{(5)^2}$$

Canceling squares both sides

$$\left(x - \frac{3}{5}\right) = \pm \frac{\sqrt{19}}{5}$$

Solving

$$\left(x - \frac{3}{5}\right) = \frac{\sqrt{19}}{5} \quad \left(x - \frac{3}{5}\right) = -\frac{\sqrt{19}}{5}$$

$$x = \frac{\sqrt{19}}{5} + \frac{3}{5} \quad x = \frac{-\sqrt{19}}{5} + \frac{3}{5}$$

$$x = \frac{\sqrt{19}+3}{5}\ x = \frac{-\sqrt{19}+3}{5}$$

So, $x = \frac{\sqrt{19}+3}{5}$ and $x = \frac{-\sqrt{19}+3}{5}$ are the roots of the equation.

16 B. Question

Find the roots of the following quadratic equations, if they exist by the method of completing the square:

$2x^2 – 5x + 3 = 0$

Answer

$2x^2 – 5x + 3 = 0$

Dividing by 2

$$x^2 - \frac{5x}{2} + \frac{3}{2} = 0$$

$$x^2 - \frac{5x}{2} = -\frac{3}{2}$$

Add a coefficient of $\left(\frac{x}{2}\right)^2$ to both sides

$$x^2 - \frac{5x}{2} + \left(\frac{5}{4}\right)^2 = -\frac{3}{2} + \left(\frac{5}{4}\right)^2$$

$$\left(x - \frac{5}{4}\right)^2 = -\frac{3}{2} + \frac{25}{16}$$

$$\left(x - \frac{5}{4}\right)^2 = \frac{-24+25}{16}$$

$$\left(x - \frac{5}{4}\right)^2 = \frac{1}{16}$$

$$x - \frac{5}{4} = \sqrt{\frac{1}{16}}$$

$$x = \frac{1}{4} + \frac{5}{4}\ x = -\frac{1}{4} + \frac{5}{4}$$

$$x = \frac{6}{4} = \frac{3}{2}\ x = \frac{4}{4} = 1$$

16 C. Question

Find the roots of the following quadratic equations, if they exist by the method of completing the square:

$9x^2 - 15x + 6 = 0$

Answer

Dividing by 9

$$x^2 - \frac{15x}{9} + \frac{6}{9} = 0$$

$$x^2 - \frac{15x}{9} = -\frac{2}{3}$$

Add a coefficient of $\left(\frac{x}{2}\right)^2$ to both sides

$$x^2 - \frac{15x}{9} + \left(\frac{15}{18}\right)^2 = -\frac{2}{3} + \left(\frac{15}{18}\right)^2$$

$$\left(x - \frac{15}{18}\right)^2 = -\frac{2}{3} + \frac{25}{36}$$

$$\left(x - \frac{15}{18}\right)^2 = \frac{25 - 24}{36}$$

$$\left(x - \frac{15}{18}\right)^2 = \frac{1}{36}$$

$$x - \frac{15}{18} = \frac{\sqrt{1}}{\sqrt{36}}$$

$$x = \frac{1}{6} + \frac{5}{6}\ x = -\frac{1}{6} + \frac{5}{6}$$

$$x = \frac{6}{6} = 1\ x = \frac{4}{6} = \frac{2}{3}$$

16 D. Question

Find the roots of the following quadratic equations, if they exist by the method of completing the square:

$x^2 - 9x + 18 = 0$

Answer

$x^2 - 9x = -18$

Add the coefficient of $\left(\frac{x}{2}\right)^2$ to both sides

$$x^2 - 9x + \left(\frac{9}{2}\right)^2 = -18 + \left(\frac{9}{2}\right)^2$$

$$\left(x - \frac{9}{2}\right)^2 = -18 + \frac{81}{4}$$

$$\left(x - \frac{9}{2}\right)^2 = \frac{-72 + 81}{4}$$

$$\left(x - \frac{9}{2}\right)^2 = \frac{9}{4}$$

$$x - \frac{9}{2} = \frac{\sqrt{9}}{\sqrt{4}}$$

$$x - \frac{9}{2} = \frac{\pm 3}{2}$$

$$x = -\frac{3}{2} + \frac{9}{2}\ x = \frac{3}{2} + \frac{9}{2}$$

$$x = \frac{6}{2} = 3\ x = \frac{12}{2} = 6$$

16 E. Question

Find the roots of the following quadratic equations, if they exist by the method of completing the square:

$2x^2 + x + 4 = 0$

Answer

$$x^2 + \frac{x}{2} + 2 = 0$$

$$x^2 + \frac{x}{2} = -2$$

Add the coefficient of $\left(\frac{x}{2}\right)^2$ to both sides

$$x^2 + \frac{x}{2} + \left(\frac{1}{4}\right)^2 = -2 + \left(\frac{1}{4}\right)^2$$

$$\left(x+\frac{1}{4}\right)^2 = -2+\frac{1}{16}$$

$$\left(x+\frac{1}{4}\right)^2 = \frac{-32+1}{16}$$

$$\left(x+\frac{1}{4}\right)^2 = \frac{-31}{16}$$

$$x+\frac{1}{4} = \sqrt{-\frac{31}{16}}$$

Since root cannot be negative

Therefore, no real roots exist.

17 A. Question

Find the roots of each of the following quadratic equations if they exist by the method of completing the squares:

$2x^2 - 5x + 3 = 0$

Answer

$x^2 - \frac{5x}{2} + \frac{3}{2} = 0$

$$x^2 - \frac{5x}{2} = -\frac{3}{2}$$

Add the coefficient of $\left(\frac{x}{2}\right)^2$ to both sides

$$x^2 - \frac{5x}{2} + \left(\frac{5}{4}\right)^2 = -\frac{3}{2} + \left(\frac{5}{4}\right)^2$$

$$\left(x-\frac{5}{4}\right)^2 = -\frac{3}{2}+\frac{25}{16}$$

$$\left(x-\frac{5}{4}\right)^2 = \frac{-24+25}{16}$$

$$\left(x-\frac{5}{4}\right)^2 = \frac{1}{16}$$

$$x-\frac{5}{4} = \frac{\sqrt{1}}{\sqrt{16}}$$

$$x = \frac{1+5}{4} \quad x = \frac{-1+5}{4}$$

$$x = \frac{6}{4} = \frac{3}{2} \quad x = \frac{4}{4} = 1$$

17 B. Question

Find the roots of each of the following quadratic equations if they exist by the method of completing the squares:

$x^2 - 6x + 4 = 0$

Answer

$x^2 - 6x = -4$

Add the coefficient of $\left(\frac{x}{2}\right)^2$ to both sides

$x^2 - 6x + (3)^2 = -4 + (3)^2$

$(x - 3)^2 = -4 + 9$

$x - 3 = \sqrt{5}$

$x = \pm\sqrt{5} + 3$

17 C. Question

Find the roots of each of the following quadratic equations if they exist by the method of completing the squares:

$\sqrt{5}x^2 + 9x + 4\sqrt{5} = 0$

Answer

Divide by $\sqrt{5}$

$$x^2 + \frac{9x}{\sqrt{5}} = -4$$

Add the coefficient of $\left(\frac{x}{2}\right)^2$ to both sides

$$x^2 + \frac{9x}{\sqrt{5}} + \left(\frac{9}{2\sqrt{5}}\right)^2 = -4 + \left(\frac{9}{2\sqrt{5}}\right)^2$$

$$\left(x + \frac{9}{2\sqrt{5}}\right)^2 = -4 + \frac{81}{20}$$

$$\left(x+\frac{9}{2\sqrt{5}}\right)^2=\frac{-80+81}{20}$$

$$x+\frac{9}{2\sqrt{5}}=\frac{\sqrt{1}}{\sqrt{20}}$$

$$x=\frac{1}{2\sqrt{5}}-\frac{9}{2\sqrt{5}}\ x=\frac{-1}{2\sqrt{5}}-\frac{9}{2\sqrt{5}}$$

$$x=-\frac{8}{2\sqrt{5}}=-\frac{4}{\sqrt{5}}\ x=-\frac{10}{2\sqrt{5}}=-\frac{5}{\sqrt{5}}$$

$$x=-\frac{5}{\sqrt{5}}\times\frac{\sqrt{5}}{\sqrt{5}}=-\frac{5\sqrt{5}}{5}=-\sqrt{5}$$

17 D. Question

Find the roots of each of the following quadratic equations if they exist by the method of completing the squares:

$2x^2 + \sqrt{15}\,x + \sqrt{2} = 0$

Answer

Divide by 2

$$x^2+\frac{\sqrt{15}x}{2}=-\frac{\sqrt{2}}{2}$$

Add the coefficient of $\left(\frac{x}{2}\right)^2$ to both sides

$$x^2+\frac{\sqrt{15}x}{2}+\left(\frac{\sqrt{15}}{4}\right)^2=-\frac{\sqrt{2}}{2}+\left(\frac{\sqrt{15}}{4}\right)^2$$

$$\left(x+\frac{\sqrt{15}}{4}\right)^2=-\frac{\sqrt{2}}{2}+\frac{15}{16}$$

$$\left(x+\frac{\sqrt{15}}{4}\right)^2=\frac{-8\sqrt{2}+15}{16}$$

Since root cannot be negative

Therefore, it has no real roots.

17 E. Question

Find the roots of each of the following quadratic equations if they exist by the method of completing the squares:

$x^2 + x + 3 = 0$

Answer

$x^2 + x = -3$

Add the coefficient of $\left(\frac{x}{2}\right)^2$ to both sides

$$x^2 + x + \left(\frac{1}{2}\right)^2 = -3 + \left(\frac{1}{2}\right)^2$$

$$\left(x + \frac{1}{2}\right)^2 = -3 + \frac{1}{4}$$

$$\left(x + \frac{1}{2}\right)^2 = \frac{-12 + 1}{4}$$

$$\left(x + \frac{1}{2}\right)^2 = \frac{-11}{4}$$

$$x + \frac{1}{2} = \sqrt{-\frac{11}{4}}$$

Since root cannot be negative

Therefore, it has no real roots.

18 A. Question

Solve the following equations by the method of completion of a square.

$5x^2 - 24x - 5 = 0$

Answer

Dividing by 5

$$x^2 - \frac{24x}{5} - \frac{5}{5} = 0$$

$$x^2 - \frac{24x}{5} = 1$$

Add the coefficient of $\left(\frac{x}{2}\right)^2$ to both sides

$$x^2 - \frac{24x}{5} + \left(\frac{24}{10}\right)^2 = 1 + \left(\frac{24}{10}\right)^2$$

$$\left(x - \frac{24}{10}\right)^2 = 1 + 5.76$$

$$x - \frac{24}{10} = \sqrt{6.76}$$

x = 2.6 + 2.4 x = −2.6 + 2.4

x = 5 x = – 0.2

18 B. Question

Solve the following equations by the method of completion of a square.

$7x^2 - 13x - 2 = 0$

Answer

Divide by 7

$$x^2 - \frac{13x}{7} = \frac{2}{7}$$

Add the coefficient of $\left(\frac{x}{2}\right)^2$ to both sides

$$x^2 - \frac{13x}{7} + \left(\frac{13}{14}\right)^2 = \frac{2}{7} + \left(\frac{13}{14}\right)^2$$

$$\left(x - \frac{13}{14}\right)^2 = \frac{2}{7} + \frac{169}{196}$$

$$\left(x - \frac{13}{14}\right)^2 = \frac{56 + 169}{196}$$

$$\left(x - \frac{13}{14}\right)^2 = \frac{225}{196}$$

$$x - \frac{13}{14} = \frac{\sqrt{225}}{\sqrt{196}}$$

$$x - \frac{13}{14} = \frac{15}{14}$$

$$x = \frac{15 + 13}{14} \quad x = \frac{-15 + 13}{14}$$

$$x = \frac{28}{14} = 2\ x = -\frac{2}{14} = -\frac{1}{7}$$

18 C. Question

Solve the following equations by the method of completion of a square.

$15x^2 + 53x + 42 = 0$

Answer

Divide by 15

$$x^2 + \frac{53x}{15} = -\frac{42}{15}$$

Add the coefficient of $\left(\frac{x}{2}\right)^2$ to both sides

$$x^2 + \frac{53x}{15} + \left(\frac{53}{30}\right)^2 = -\frac{42}{15} + \left(\frac{53}{30}\right)^2$$

$$\left(x + \frac{53}{30}\right)^2 = -\frac{42}{15} + \frac{2809}{900}$$

$$\left(x + \frac{53}{30}\right)^2 = \frac{-2520 + 2809}{900}$$

$$x + \frac{53}{30} = \frac{\sqrt{289}}{\sqrt{900}}$$

$$x = \frac{17}{30} - \frac{53}{30}\ x = -\frac{17}{30} - \frac{53}{30}$$

$$x = -\frac{6}{5}\ x = -\frac{7}{3}$$

18 D. Question

Solve the following equations by the method of completion of a square.

$7x^2 + 2x - 5 = 0$

Answer

Divide by 7

$$x^2 + \frac{2x}{7} = \frac{5}{7}$$

Add the coefficient of $\left(\frac{x}{2}\right)^2$ to both sides

$$x^2+\frac{2x}{7}+\left(\frac{2}{14}\right)^2=\frac{5}{7}+\left(\frac{2}{14}\right)^2$$

$$\left(x+\frac{2}{14}\right)^2=\frac{5}{7}+\frac{4}{196}$$

$$\left(x+\frac{2}{14}\right)^2=\frac{140+4}{196}$$

$$\left(x+\frac{2}{14}\right)^2=\frac{144}{196}$$

$$x+\frac{2}{14}=\frac{\sqrt{144}}{\sqrt{196}}$$

$$x+\frac{2}{14}=\frac{12}{14}$$

$$x=\frac{12-2}{14}\ x=\frac{-12-2}{14}$$

$$x=\frac{10}{14}=\frac{5}{7}\ x=-\frac{14}{14}=-1$$

Exercise 7.4

1 A. Question

Write the discriminate of each of the following quadratic equation:

$x^2 + 4x + 3 =0$

Answer

d = b^2 – 4ac

d = $(4)^2$ – 4 (1) (3)

d = 16 – 12

d = 4

1 B. Question

Write the discriminate of each of the following quadratic equation:

$4x^2 + 5x + 7 = 0$

Answer

$d = b^2 - 4ac$

$d = (5)^2 - 4\ (4)\ (7)$

$d = 25 - 112$

$d = -87$

1 C. Question

Write the discriminate of each of the following quadratic equation:

$2x^2 + 4x + 5 = 0$

Answer

$d = b^2 - 4ac$

$d = (4)^2 - 4\ (2)\ (5)$

$d = 16 - 40$

$d = -24$

1 D. Question

Write the discriminate of each of the following quadratic equation:

$3x^2 + 5x + 6 = 0$

Answer

$d = b^2 - 4ac$

$d = (5)^2 - 4\ (3)\ (6)$

$d = 25 - 72$

$d = -47$

1 E. Question

Write the discriminate of each of the following quadratic equation:

$\sqrt{3}\ x^2 — 2\sqrt{2} - 2\sqrt{3} = 0$

Answer

$d = b^2 - 4ac$

$d = (-2\sqrt{2})^2 - 4\ (\sqrt{3})\ (-2\sqrt{3})$

$d = 8 + 24$

$d = 32$

2 A. Question

Examine whether the following quadratic equations have real roots or not:

$7x^2 + 8x — 1 = 0$

Answer

TO CHECK REAL ROOTS, ‘d’ SHOULD BE GREATER THAN 0.

$d = b^2 - 4ac$

$d = (8)^2 2 - 4\ (7)\ (-1)$

$d = 64 - 28$

$d = 92$

Yes, roots are real.

2 B. Question

Examine whether the following quadratic equations have real roots or not:

$2x^2 + 3x + 4 = 0$

Answer

TO CHECK REAL ROOTS, ‘d' SHOULD BE GREATER THAN 0.

$d = b^2 - 4ac$

$d = (3)^2 - 4(2)(4)$

$d = 9 - 32$

$d = - 23$

No, roots aren’t real.

2 C. Question

Examine whether the following quadratic equations have real roots or not:

$x^2 — 12x — 16 = 0$

Answer

TO CHECK REAL ROOTS, ‘d' SHOULD BE GREATER THAN 0.

$d = b^2 - 4ac$

$d = (-12)^2 - 4 (1) (-16)$

$d = 144 - 64$

$d = 208$

Yes, roots are real .

2 D. Question

Examine whether the following quadratic equations have real roots or not:

$x^2 + x - 1 = 0$

Answer

TO CHECK REAL ROOTS, ‘d' SHOULD BE GREATER THAN 0.

$d = b^2 - 4ac$

$d = (1)^2 - 4 (1) (-1)$

$d = 1 + 4$

$d = 5$

Yes, roots are real.

2 E. Question

Examine whether the following quadratic equations have real roots or not:

$x^2 — 10x + 2 = 0$

Answer

TO CHECK REAL ROOTS, ‘d' SHOULD BE GREATER THAN 0.

$d = b^2 - 4ac$

$d = (-10)^2 - 4 (1) (2)$

$d = 100 - 8$

$d = 92$

Yes, roots are real.

3 A. Question

Find whether the following quadratic equations have a repeated root :

$9x^2 — 12x + 4 = 0$

Answer

REPEATED ROOTS MEAN d = 0.

$d = b^2 - 4ac$

$d = (-12)^2 - 4(9)(4)$

$d = 144 - 144$

$d = 0$

Yes, roots are repeated.

3 B. Question

Find whether the following quadratic equations have a repeated root :

$y^2 — 6y + 6 = 0$

Answer

REPEATED ROOTS MEAN d = 0.

$d = b^2 - 4ac$

$d = (-6)^2 - 4(1)(6)$

$d = 36 - 24$

$d = 12$

∴ roots are not repeated.

3 C. Question

Find whether the following quadratic equations have a repeated root :

$9x^2 + 4x + 6 = 0$

Answer

REPEATED ROOTS MEAN d = 0.

$d = b^2 - 4ac$

$d = (4)^2 - 4\,(9)\,(6)$

$d = 16 - 216$

$d = -200$

∴ roots are not repeated.

3 D. Question

Find whether the following quadratic equations have a repeated root :

$16y^2 — 40y + 25 = 0$

Answer

REPEATED ROOTS MEAN d = 0 .

$d = b^2 - 4ac$

$d = (-40)^2 - 4\ (16)\ (25)$

$d = 1600 - 1600$

$d = 0$

∴ roots are repeated.

3 E. Question

Find whether the following quadratic equations have a repeated root :

$x^2 + 6x + 9 = 0$

Answer

REPEATED ROOTS MEAN d = 0 .

$d = b^2 - 4ac$

$d = (6)^2 - 4(1)(9)$

$d = 36 - 36$

$d = 0$

∴ roots are repeated.

4 A. Question

Comment upon the nature of roots of the following equations:

$4x^2 + 7x + 2 = 0$

Answer

$d = b^2 - 4ac$

$d = (7)^2 - 4\ (4)\ (2)$

$d = 49 - 32$

$d = 17$

Since, $d>0$, roots are unique and real.

4 B. Question

Comment upon the nature of roots of the following equations:

$x^2 + 10x + 39 = 0$

Answer

$d = b^2 - 4ac$

$d = (10)^2 - 4\ (1)\ (39)$

$d = 100 - 156$

$d = -56$

Since, $d < 0$, no real roots exists.

5 A. Question

Without solving, determine whether the following equations have real roots or not. If yes, find them:

$2x^2 — 4x + 3 = 0$

Answer

$d = b^2 - 4ac$

$d = (-4)^2 - 4\ (2)\ (3)$

$d = 16 - 24$

$d = -8$

Since, $d < 0$, no real roots exist for the given equation.

5 B. Question

Without solving, determine whether the following equations have real roots or not. If yes, find them:

$$y^2 - \frac{2}{3}y + \frac{1}{9} = 0$$

Answer

d = b^2 – 4ac

$$d = \left(\frac{-2}{3}\right)^2 - 4(1)\left(\frac{1}{9}\right)$$

$$d = \frac{4}{9} - \frac{4}{9}$$

d = 0

Since, d = 0, roots are real and equal for the given equation.

$$x = \frac{-b \pm \sqrt{d}}{2a}$$

$$x = \frac{-\left(-\frac{2}{3}\right) \pm \sqrt{0}}{2 \times 1}$$

$$x = \frac{2}{3} \times \frac{1}{2}$$

$$x = \frac{1}{3}$$

6 A. Question

Without finding the roots, comment upon the nature of roots of each of the following quadratic equations:

$2x^2$— 6x + 3 = 0

Answer

d = b^2 – 4ac

d = $(-6)^2$ – 4 (2) (3)

d = 36 – 24

d = 12

∴ Roots are real and unique.

6 B. Question

Without finding the roots, comment upon the nature of roots of each of the following quadratic equations:

$2x^2 — 5x — 3 = 0$

Answer

$d = b^2 - 4ac$

$d = (-5)^2 - 4\ (2)\ (-3)$

$d = 25 + 24$

$d = 49$

∴ Roots are real and unique.

7 A. Question

Find the value of k for which the quadratic equation

$4x^2 — 2\ (k + 1)\ x + (k + 4) = 0$ has equal roots.

Answer

Since roots are equal

∴ d=0 ….(1)

$4x^2 — 2\ (k + 1)\ x + (k + 4) = 0$

$d = b^2 - 4ac$

$d = (-2\ (k + 1)^2 - 4\ (4)\ (k + 4)$

$d = (-\ 2k - 2)^2 - 16k - 64$

$d = 4k^2 + 4 + 8k - 16k - 64$

$(\because\ (a - b)^2 = a^2 + b^2 - 2ab)$

$d = 4k^2 - 8k - 60$

From (1), d = 0

∴ Equation will be:

$4k^2 - 8k - 60 = 0$

Dividing by 4

$k^2 - 2k - 15 = 0$

$4k^2 - 5k + 3k - 15 = 0$

$k(k - 5) + 3(k - 5) = 0$

$(k - 5)(k + 3) = 0$

$K - 5 = 0$ $k + 3 = 0$

$K = 5$ $k = -3$

7 B. Question

Find the value of k, so that the quadratic equation

$(k + 1)x^2 - 2(k - 1)x + 1 = 0$ has equal roots.

Answer

Since roots are equal

$\therefore d=0$ ….(1)

$(k + 1)x^2 - 2(k - 1)x + 1 = 0$

$d = b^2 - 4ac$

$d = (-2(k-1))^2 - 4(k+1)(1)$

$d = (-2k+2)^2 - 4k - 4$

$d=4k^2 + 4 - 8k - 4k - 4$

$(\because (a + b)^2 = a^2 + b^2 + 2ab)$

$d = 4k^2 - 12k$

From (1), $d = 0$

$\therefore$ Equation will be:

$0 = 4k^2 - 12k$

$4k^2 = 12k$

$$k^2 = \frac{12}{4}k$$

$k^2 = 3k$

$k^2 - 3k = 0$

$k(k - 3) = 0$

k = 0 or k – 3 = 0

k = 3

∴ Values of k are 0, 3.

8 A. Question

For what values of k, does the following quadratic equation has equal roots.

$9x^2 + 8kx + 16 = 0$

Answer

Since roots are equal

∴ d=0 (1)

$9x^2 + 8kx + 16 = 0$

$d = b^2 - 4ac$

$d = (8k)^2 - 4 (9) (16)$

From (1), d = 0

∴ Equation will be:

$0 = 64k^2 - 576$

$$k^2 = \frac{576}{64}$$

$k^2 = 9$

$k = \pm\sqrt{9}$

k = 3, –3

∴ values of k are –3, 3 .

8 B. Question

$(k + 4)x^2 + (k + 1)x + 1 = 0$

Answer

Since roots are equal

∴ d=0 (1)

$(k + 4)x^2 + (k + 1)x + 1 = 0$

$d = b^2 - 4ac$

$d = (k - 1)^2 - 4\ (k + 4)\ (1)$

$d = (-2k + 2)^2 - 4k - 4$

$d = k^2 + 1 + 2k - 4k - 16$

From (1), d = 0

∴ Equation will be:

$0 = k^2 + 1 + 2k - 4k - 16$

$k^2 - 2k - 15 = 0$

$k^2 - 5k + 3k - 15 = 0$

$k(k - 5) + 3\ (k - 5) = 0$

$(k - 5)\ (k + 3) = 0$

$K - 5 = 0\ k + 3 = 0$

$k = 5\ k = -3$

∴ Values of k are –3, 5.

8 C. Question

$k^2x^2 — 2(2k — 1)x + 4 = 0$

Answer

Since roots are equal

∴ d=0 (1)

$k^2x^2 — 2(2k — 1)x + 4 = 0$

$d = b^2 - 4ac$

$d = (-2(k-1))^2 - 4\ (k^2)\ (4)$

$d = (-2k+2)^2 - 4k - 4$

$d = (-4k+2)^2 - 16k^2$

$(\because (a - b)^2 = a^2 + b^2 - 2ab)$

$d = 16k^2 - 16k + 4 - 16k^2$

d = −16k + 4

From (1), d = 0

∴ Equation will be:

0 = −16k + 4

16k = 4

$$k = \frac{4}{16}$$

$$k = \frac{1}{4}$$

∴ Values of k are is $\frac{1}{4}$.

9. Question

If the roots of the equation $(a — b)x^2 + (b — c)\,x + (c — a) = 0$ are equal, prove that 2a = b + c.

Answer

Since roots are equal

∴ d=0 (1)

$(a — b)x^2 + (b — c)\,x + (c — a) = 0$

$d = b^2 - 4ac$

$d = (b-c)^2 - 4\,(a-b)\,(c-a)$

$d = b^2 + c^2 - 2bc - 4\,[a\,(c - a) - b\,(c - a)]$

$d = b^2 + c^2 - 2bc - 4\,[ac - a^2 - bc + ba]$

From (1), d = 0

∴ Equation will be:

$0 = b^2 + c^2 - 2bc - 4ac + 4a^2 + 4bc - 4ba$

$b^2 + c^2 - (2a)^2\ 2bc + 2c\,(-2a) + 2(-2a)b = 0$

$(b + c - 2a)^2 = 0$

$(b + c - 2a) = 0$

$b + c = 2a$

Hence proved.

10. Question

If — 5 is a root of the quadratic equation $2x^2 + 2px — 15 = 0$ and the quadratic equation $p(x^2 + x) + k = 0$ has equal roots, find the value of k.

Answer

$2x^2 + 2px — 15 = 0$

Put $x = -5$

$2(-5)^2 + 2p(-5) — 15 = 0$

$50 - 10p - 15 = 0$

$35 = 10p$

$p = 3.5$

Equation $p(x^2 + x) + k = 0$ has equal roots i.e. $d = 0$

$p(x^2 + x) + k = 0$

$p = 3.5$

$3.5(x^2 + x) + k = 0$

$3.5x^2 + 3.5x + k = 0$

$d = b^2 - 4ac$

$d = (3.5)^2 - 4(3.5)(k)$

$d = 12.25 - 14k$

Putting $d = 0$

∴ Equation will be:

$0 = 12.25 - 14k$

$14k = 12.25$

$$k = \frac{12.25}{14}$$

$k = 0.875$

11 A. Question

Find the values of k, for which the given equation has real roots:

$2x^2 — 10x + k = 0$

Answer

Roots are equal

∴ d = 0

$d = b^2 – 4ac$

$d = (–10)^2 – 4 (2) (k)$

d = 100 – 8k

Put d = 0

0 = 100 – 8k

8k = 100

$$k = \frac{100}{8}$$

$$k = \frac{25}{2}$$

11 B. Question

Find the values of k, for which the given equation has real roots:

$kx^2 — 6x — 2 = 0$

Answer

Roots are equal

∴ d = 0

$d = b^2 – 4ac$

$d = (–6)^2 – 4 (–2) (k)$

d = 36 + 8k

Put d = 0

0 = 36 + 8k

$-8k = 36$

$$k = \frac{-36}{8}$$

$$k = \frac{-9}{2}$$

11 C. Question

Find the values of k, for which the given equation has real roots:

$kx^2 + 4x + 1 = 0$

Answer

Roots are equal

$\therefore$ d = 0

$d = b^2 - 4ac$

$d = (4)^2 - 4\ (1)\ (k)$

$d = 16 - 4k$

Put d = 0

$0 = 16 - 4k$

$4k = 16$

$k = 4$

11 D. Question

Find the values of k, for which the given equation has real roots:

$kx^2 - 2\sqrt{5}x + 4 = 0$

Answer

Roots are equal

$\therefore$ d = 0

$d = b^2 - 4ac$

$d = (-2\sqrt{5})^2 - 4\ (k)\ (4)$

$d = 20 - 16k$

Put d = 0

$0 = 20 - 16k$

$16k = 20$

$$k = \frac{20}{16}$$

$$k = \frac{5}{4}$$

11 E. Question

Find the values of k, for which the given equation has real roots:

$x^2 + k(4x + k — 1) + 2 = 0$

Answer

Roots are equal

$\therefore d = 0$

$d = b^2 - 4ac$

$d = (4k)^2 - 4 (k^2 - k + 2) (1)$

$d = 16k^2 - 4k^2 + 4k - 8$

Put $d = 0$

$0 = 16k^2 - 4k^2 + 4k - 8$

$12k^2 + 4k^2 + 4k - 8 = 0$ (divide by 4)

$3k^2 + k - 2 = 0$

$3k^2 + 3k - 2k - 2 = 0$

$3k (k + 1) - 2k (k + 1) = 0$

$(3k-2k)(k+1) = 0$

$(3k-2k)=0$ or $(k+1) = 0$

$k = 0$ $k = -1$

12. Question

Prove that the equation $x^2(a^2 + b^2) + 2x(ac + bd) + (c^2 + d^2)= 0$ has no real root, if $ad \neq bc$.

Answer

$x^2(a^2 + b^2) + 2x(ac + bd) + (c^2 + d^2)= 0$

$d = b^2 - 4ac$

$d = (2ac + 2bd)^2 - 4 (a^2 + b^2) (c^2 + d^2)$

$d = 4a^2c^2 + 4b^2d^2 + 8abcd - 4 [a^2 (c^2 + d^2) + b^2 (c^2+d^2)]$

$d = 4a^2c^2 + 4b^2d^2 + 8abcd - 4 [a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2]$

$d = 4a^2c^2 + 4b^2d^2 + 8abcd - 4a^2c^2 - 4a^2d^2 - 4b^2c^2 - 4b^2 d^2$

$d = 8abcd - 4a^2d^2 - 4b^2c^2$

$d = 8abcd - 4(a^2d^2 + b^2 c^2)$

$d = -4 (a^2 d^2 + b^2c^2 - 2abcd)$

$d = -4 [(ad + bc)^2]$

For $ad \neq bc$

$d= -4 \times$ [value of $(ad + bc)^2$]

$\therefore$ d is always negative

So, $d < 0$

The given equation has no real roots.

Exercise 7.5

1. Question

Divide 12 into two parts such that their product is 32.

Answer

Let the first number be ‘X’, so the other number will be ’(12–X)’.

$\because X (12 - X) = 32$

$12X - X^2 = 32$

$X^2 - 12X + 32 = 0$

On factorising further,

$X^2 - 4X - 8X + 32 = 0$

$X(X - 4) - 8(X - 4) = 0$

(X – 4)(X – 8) = 0

So, X = 4 or 8

∴ The numbers are 4 and 8.

2. Question

Two numbers differ by 3 and their product is 504. Find the numbers.

Answer

Let the first number be 'X', so the other number will be '(X+3)'.

∵ X(X + 3) = 504

$X^2 + 3X - 504 = 0$

On applying Sreedhracharya formula

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{-3 \pm \sqrt{9 - 4(1)(-504)}}{2(1)}$$

$$\frac{-3 \pm \sqrt{9 + 2016}}{2(1)}$$

$$\frac{-3 \pm \sqrt{2025}}{2}$$

$$\frac{-3 \pm 45}{2}$$

X = –24 or 21.

∴ if the first number is – 24 , then the other number is – 21.

∴ if the first number is 21 , then the other number is 24.

∴ The numbers are not (21,24) & (– 21, – 24).

3. Question

Find two consecutive positive integers, the sum of whose squares is 365.

Answer

Let the first number be 'X', so the other number will be '(X+1)'.

$\because X^2+(X+1)^2 = 365$

$X^2 + X^2 + 1 + 2X - 365 = 0$

$2X^2 + 2X - 364 = 0$

$X^2 + X - 182 = 0$

On applying Sreedhracharya formula

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{-1 \pm \sqrt{1 - 4(1)(-182)}}{2(1)}$$

$$\frac{-1 \pm \sqrt{1 + 728}}{2(1)}$$

$$\frac{-1 \pm \sqrt{729}}{2}$$

$$\frac{-1 \pm 27}{2}$$

$\therefore X = -14$ or $X = 13$.

$\therefore$ The numbers are 13 & 14.

4. Question

The difference of two numbers is 4. If the difference of their reciprocals is $\frac{4}{21}$, find the two numbers.

Answer

Let the first number be 'X', so the other number will be '(X+4)'.

$$\because \frac{1}{X} - \frac{1}{X+4} = \frac{4}{21}$$

$$\frac{X + 4 - X}{X^2 + 4X} = \frac{4}{21}$$

On simplifying further,

$X^2 + 4X - 21 = 0$

On applying Sreedhracharya formula

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{-4 \pm \sqrt{16 - 4(1)(-21)}}{2(1)}$$

$$\frac{-4 \pm \sqrt{16 + 84}}{2(1)}$$

$$\frac{-4 \pm \sqrt{100}}{2}$$

$$\frac{-4 \pm 10}{2}$$

∴ X = –7 or X = 3.

∴ The numbers are –7 & –3 or 3 & 7.

5. Question

The sum of two numbers is 18 and the sum of their reciprocals is $\frac{1}{4}$. Find the numbers.

Answer

Let the first number be ‘X’, so the other number will be ’(18–X)’.

$$\because \frac{1}{X} + \frac{1}{18 - X} = \frac{1}{4}$$

$$\frac{18 - X + X}{-X^2 + 18X} = \frac{1}{4}$$

On simplifying further,

$-X^2 + 18X - 72 = 0$

$X^2 - 18X + 72 = 0$

On applying Sreedhracharya formula

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{-(-18) \pm \sqrt{(-18)^2 - 4(1)(72)}}{2(1)}$$

$$\frac{18 \pm \sqrt{324 - 288}}{2(1)}$$

$$\frac{18 \pm \sqrt{36}}{2}$$

$$\frac{18 \pm 6}{2}$$

X = 6 or X = 12

∴ The numbers are 6 & 12.

6. Question

The sum of the squares of three consecutive positive integers is 50. Find the integers.

Answer

Let the first number be ‘X’, so the other numbers will be ’(X+1)’ & ‘(X+2).

$\because X^2 + (X + 1)^2 + (X + 2)^2 = 50$

$X^2 + X^2 + 1 + 2X + X^2 + 4 + 4X = 50$

On simplifying further,

$3X^2 + 6X - 45 = 0$

$X^2 + 2X - 15 = 0$

On applying Sreedhracharya formula

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{-2 \pm \sqrt{4 - 4(1)(-15)}}{2(1)}$$

$$\frac{-2 \pm \sqrt{4 + 60}}{2(1)}$$

$$\frac{-2 \pm \sqrt{64}}{2}$$

$$\frac{-2 \pm 8}{2}$$

X = – 5 or 3

∴ X = 3 (Only Positive values)

∴ X + 1 = 4

∴ X +2 = 5

∴ The numbers are 3, 4 & 5.

7. Question

Find three consecutive positive integers such that the sum of the square of the first and the product of the other two is 154.

Answer

Let the first number be ‘X’, so the other numbers will be ’(X+1)’ & ‘(X+2).

∵ $X^2 + (X + 1)(X + 2) = 154$

$X^2 + X^2 + 2X + X + 2 = 154$

On simplifying further,

$2X^2 + 3X - 152 = 0$

On applying Sreedhracharya formula

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{-3 \pm \sqrt{3^2 - 4(2)(-152)}}{2(2)}$$

$$\frac{-3 \pm \sqrt{9 + 1216}}{4}$$

$$\frac{-3 \pm \sqrt{1225}}{4}$$

$$\frac{-3 \pm 35}{4}$$

X = – 9.5 or 8

∴ X = 8 (Only whole values)

∴ X +1 = 9

∴ X + 2 = 10

∴ The numbers are 8, 9 & 10.

8. Question

A two–digit number is such that the product of its digits is 14. If 45 is added to the number, the digits interchange their places. Find the number.

Answer

Let the units digit be 'Y', so the tens digit will be X.

∴ the number is 10X + Y

∵ X . Y =14 ------ (i)

$$10X + Y + 45 = 10Y + X$$

On simplifying further,

9X – 9Y + 45 = 0

X – Y + 5 = 0

Putting the value of $X = \frac{14}{Y}$ from equation ------ (i)

$$\frac{14}{Y} - Y + 5 = 0$$

$14 - Y^2 + 5Y = 0$

$Y^2 - 5Y - 14 = 0$

On applying Sreedhracharya formula

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(-14)}}{2(1)}$$

$$\frac{5 \pm \sqrt{25 + 56}}{2}$$

$$\frac{5 \pm \sqrt{81}}{2}$$

$$\frac{5 \pm 9}{2}$$

Y = – 2 or 7

∴ Y = 7 (Only Positive values)

$\therefore\ X = \frac{14}{7}$

$\therefore$ X = 2

$\therefore$ The number is 27 {2 (10) + 7}.

9. Question

The difference of squares of two natural numbers is 45. The square of the smaller number is four times the larger number. Find the numbers.

Answer

Let the larger number be 'X', so the smaller number will be 'Y'.

$\therefore X^2 - Y^2 = 45$ ----- (i)

$\because 4X = Y^2$ ------ (ii)

On simplifying further,

Putting the value of $Y^2 = 4X$ from equation ------ (ii)

$X^2 - 4X = 45$

$X^2 - 4X - 45 = 0$

On applying Sreedhracharya formula

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-45)}}{2(1)}$$

$$\frac{4 \pm \sqrt{16 + 180}}{2}$$

$$\frac{4 \pm \sqrt{196}}{2}$$

$$\frac{4 \pm 14}{2}$$

X = – 5 or 9

$\therefore$ X = 9 (Only natural number, as given in the question)

$\therefore Y = \sqrt{(4*9)}$

∴ Y = 6

∴ The numbers are 9 & 6.

10. Question

The difference of two numbers is 5 and the difference of their reciprocals is $\frac{1}{10}$ Find the numbers.

Answer

Let the first number be ‘X’, so the other number will be ’(X+5)’.

$$\because \frac{1}{X} - \frac{1}{X+5} = \frac{1}{10}$$

$$\frac{X+5-X}{X^2+5X} = \frac{1}{10}$$

On simplifying further,

$X^2 + 5X - 50 = 0$

On applying Sreedhracharya formula

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{-5 \pm \sqrt{5^2 - 4(1)(-50)}}{2(1)}$$

$$\frac{-5 \pm \sqrt{25 + 200}}{2(1)}$$

$$\frac{-5 \pm \sqrt{225}}{2}$$

$$\frac{-5 \pm 15}{2}$$

∴ X = –10 or X = 5.

Then other numbers will be (X+5) = {–10+5} & {5+5} i.e., –5 or 10.

∴ The numbers are –10 & –5 or 5 & 10.

11. Question

The sum of a number and its reciprocal is $\frac{10}{3}$. Find the number.

Answer

Let the number be 'X', so the reciprocal will be '$(\frac{1}{X})$'.

$$\because \frac{1}{X} + X = \frac{10}{3}$$

$$\frac{1 + X^2}{X} = \frac{10}{3}$$

On simplifying further,

$3 + 3X^2 = 10X$

$3X^2 - 10X + 3 = 0$

On applying Sreedhracharya formula

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{-(-10) \pm \sqrt{(-10)^2 - 4(3)(3)}}{2(3)}$$

$$\frac{10 \pm \sqrt{100 - 36}}{6}$$

$$\frac{10 \pm \sqrt{64}}{6}$$

$$\frac{10 \pm 8}{6}$$

$\therefore$ X = 3 or $X = \frac{1}{3}$.

Then other numbers will be $\frac{1}{X} = \frac{1}{3}$ & 3.

$\therefore$ The numbers are 3 or $\frac{1}{3}$.

12. Question

Divide 12 into two parts such that the sum of their squares is 74.

Answer

Let the first number be ‘X’, so the other number will be ’(12–X)’.

∵ $X^2 + (12 - X)^2 = 74$

$X^2 + 144 + X^2 - 24X = 74$

On simplifying further,

$2X^2 - 24X + 70 = 0$

$X^2 - 12X + 35 = 0$

On applying Sreedhracharya formula

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{-(-12) \pm \sqrt{(-12)^2 - 4(1)(35)}}{2(1)}$$

$$\frac{12 \pm \sqrt{144 - 140}}{2(1)}$$

$$\frac{12 \pm \sqrt{4}}{2}$$

$$\frac{12 \pm 2}{2}$$

∴ X = 5 or X = 7.

Then other numbers will be (12–X) = {12–5} & {12–7} i.e., 7 or 5.

∴ The number 12 is divided into two parts namely 5 & 7.

13. Question

The sum of the squares of two consecutive natural numbers is 421. Find the numbers.

Answer

Let the first number be ‘X’, so the other number will be ’(X+1)’.

∵ $X^2 + (X + 1)^2 = 421$

$X^2 + X^2 + 1 + 2X = 421$

On simplifying further,

$2X^2 + 2X - 420 = 0$

$X^2 + X - 210 = 0$

On applying Sreedhracharya formula

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{-(1) \pm \sqrt{(1)^2 - 4(1)(-210)}}{2(1)}$$

$$\frac{-1 \pm \sqrt{1 + 840}}{2(1)}$$

$$\frac{-1 \pm \sqrt{841}}{2}$$

$$\frac{-1 \pm 29}{2}$$

∴ X = –15 or X = 14.

∴ X = 14 (Only natural number, as given in the question)

∴ Then other numbers will be 14 & 15.

14. Question

A two–digit number is such that the product of its digits is 18. When 63 is subtracted from the number, the digits interchange their places. Find the number.

Answer

Let the units digit be 'Y', so the tens digit will be X.

∴ the number is 10X + Y

∵ X . Y =18 –––––– (i)

10X+Y–63=10Y+X

On simplifying further,

9X – 9Y – 63 = 0

X – Y – 7 = 0

Putting the value of $X = \frac{18}{Y}$ from equation –––––– (i)

$$\frac{18}{Y} - Y - 7 = 0$$

$18 - Y^2 - 7Y = 0$

$Y^2 + 7Y - 18 = 0$

On applying Sreedhracharya formula

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{-7 \pm \sqrt{(7)^2 - 4(1)(-18)}}{2(1)}$$

$$\frac{-7 \pm \sqrt{49 + 72}}{2}$$

$$\frac{-7 \pm \sqrt{121}}{2}$$

$$\frac{-7 \pm 11}{2}$$

$Y = -9$ or 2

$\therefore Y = 2$ (Only Positive values)

$$\therefore\ X = \frac{18}{2}$$

$\therefore X = 9$

$\therefore$ The number is 92 {9(10)+2}.

15. Question

A two–digit number is 5 times the sum of its digits and is also equalto5 more than twice the product of its digits. Find the number.

Answer

Let the units digit be 'Y', so the tens digit will be X.

$\therefore$ the number is $10X + Y$

$\because 10X + Y = 5(X + Y)$

On simplifying further,

$10X - 5X - 5Y + Y = 0$

$5X - 4Y = 0$ ------ (i)

$\because$ 10X + Y = 2XY + 5

10X– 2XY + Y – 5 = 0 ------ (ii)

Putting the value of $X = \frac{4Y}{5}$ from equation ------ (i)

$$8Y - \frac{8Y^2}{5} + Y - 5 = 0$$

$40Y - 8Y^2 + 5Y - 25 = 0$

$8Y^2 - 45Y + 25 = 0$

On applying Sreedhracharya formula

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{-(-45) \pm \sqrt{(-45)^2 - 4(8)(25)}}{2(8)}$$

$$\frac{45 \pm \sqrt{2025 - 800}}{16}$$

$$\frac{45 \pm \sqrt{1225}}{16}$$

$$\frac{45 \pm 35}{16}$$

$Y = \frac{5}{8}$ or 5

$\therefore$ Y = 5

$\therefore X = \frac{4*5}{5}$ from equation (i) $X = \frac{4Y}{5}$

$\therefore$ X = 4

$\therefore$ The number is 45 {4(10)+5}.

16. Question

The denominator of a fraction is one more than twice the numerator. If the sum of the fraction and its reciprocal is $2\frac{19}{15}$, find the fraction.

Answer

Let the numerator be 'X', so the denominator will be '(2X+1)'.

∴ the fraction is $\frac{X}{2X+1}$

$$\because \frac{X}{2X+1} + \frac{2X+1}{X} = \frac{58}{21}$$

On simplifying further,

$$\frac{X^2 + 4X^2 + 4X + 1}{2X^2 + X} = \frac{58}{21}$$

$21(5X^2 + 4X + 1) = 58\,(2X^2 + X)$

$105X^2 + 84X + 21 = 116X^2 + 58X$

$11X^2 - 26X - 21 = 0$------ (i)

On applying Sreedhracharya formula

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{-(-26) \pm \sqrt{(-26)^2 - 4(11)(-21)}}{2(11)}$$

$$\frac{26 \pm \sqrt{676 + 924}}{22}$$

$$\frac{26 \pm \sqrt{1600}}{22}$$

$$\frac{26 \pm 40}{22}$$

$X= -\frac{7}{11}$ or 3

∴ X = 3 (Only positive values)

∴ Numerator is = 3 & denominator is (2X+1) = 7

∴ The fraction is $\frac{3}{7}$.

17. Question

The numerator of a fraction is one more than its denominator. If its reciprocal is subtracted from it, the difference is $\frac{11}{30}$. Find the fraction.

Answer

Given: Numerator of a fraction is one more than its denominator.

To Find: The fraction

Assumption: Let the denominator be x.

Numerator = x + 1

Therefore, the fraction $= \frac{x+1}{x}$

From the second case, we get,

$$\frac{x+1}{x} - \frac{x}{x+1} = \frac{11}{30}$$

Taking L.C.M we get,

$$\frac{(x+1)^2 - x^2}{x(x+1)} = \frac{11}{30}$$

$$\frac{x^2 + 1 + 2x - x^2}{x(x+1)} = \frac{11}{30}$$

$$\frac{2x+1}{x^2+x} = \frac{11}{30}$$

Cross-multiplying we get,

$30(2x + 1) = 11(x^2 + x)$

$60x + 30 = 11x^2 + 11x$

$11x^2 + 11x - 60x - 30 = 0$

$11x^2 - 49x - 30 = 0$

Now, we need to factorise such that, on multiplication we get 330 and on substraction we get 49.

Therefore, 55 and 6 can be the factors.

So, equation becomes,

$11x^2 - (55x - 6x) - 30 = 0$

$11x^2 - 55x + 6x - 30 = 0$

$11x(x - 5) + 6(x - 5) = 0$

$(11x + 6)(x - 5) = 0$

So, 11x + 5 = 0 or x – 5 = 0

$$x = -\frac{5}{11} \text{ or } x = 5$$

$$x + 1 = -\frac{5}{11} + 1 = \frac{6}{11} \text{ or } x + 1 = 5 + 1 = 6$$

So the possible fractions are:

$$\frac{x+1}{x} = \frac{\frac{6}{11}}{-\frac{5}{11}} = -\frac{6}{5}$$

Or

$$\frac{x+1}{x} = \frac{6}{5}$$

18. Question

The numerator of a fraction is one more than its denominator. If its reciprocal is added to it the sum is $\frac{61}{30}$. Find the fraction.

Answer

Let the denominator be 'X', so the numerator will be '(X+1)'.

∴ the fraction is $\frac{XX}{XX+1}$

$$\because \frac{X}{X+1} + \frac{X+1}{X} = \frac{61}{30}$$

On simplifying further,

$$\frac{X^2 + X^2 + 2X + 1}{X^2 + X} = \frac{61}{30}$$

$30(2X^2 + 2X + 1) = 61\ (X^2 + X)$

$60X^2 + 60X + 30 = 61X^2 + 61X$

$X^2 + X - 30 = 0$------ (i)

On applying Sreedhracharya formula

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{-1 \pm \sqrt{(1)^2 - 4(1)(-30)}}{2(1)}$$

$$\frac{-1 \pm \sqrt{1 + 120}}{2}$$

$$\frac{-1 \pm \sqrt{121}}{2}$$

$$\frac{-1 \pm 11}{2}$$

X= – 6 or 5

∴ X = 5 (Only positive values)

∴ Denominator is = 5 & numerator is (X+1) = 6

∴ The fraction is $\frac{6}{5}$.

19. Question

The numerator of a fraction is 3 more than its denominator. If its reciprocal is subtracted from it, the difference is $\frac{33}{28}$. Find the fraction.

Answer

Let the denominator be 'X', so the numerator will be '(X+3)'.

∴ the fraction is $\frac{X+3}{X}$

$$\because \frac{X+3}{X} - \frac{X}{X+3} = \frac{33}{28}$$

On simplifying further,

$$\frac{X^2 + 6X + 9 - X^2}{X^2 + 3X} = \frac{33}{28}$$

$28(6X + 9) = 33(X^2 + 3X)$

$168X + 252 = 33X^2 + 99X$

$33X^2 – 69X – 252 = 0$

$11X^2 – 13X – 84 = 0$ ------ (i)

On applying Sreedhracharya formula

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{-(-13) \pm \sqrt{(-13)^2 - 4(11)(-84)}}{2(11)}$$

$$\frac{13 \pm \sqrt{169 + 3696}}{22}$$

$$\frac{13 \pm \sqrt{3865}}{22}$$

$\therefore$ it does not have any real values.

20. Question

The denominator of a fraction exceeds its numerator by 3. If one is added to both numerator and denominator, the difference between the new and the original fractions 1 becomes $\frac{1}{24}$. Find the original fraction.

Answer

Let the denominator be '(X+3)', so the numerator will be 'X'.

$\therefore$ the original fraction is $\frac{X}{X+3}$

$\therefore$ the new fraction is $\frac{X+1}{X+4}$

$$\because \frac{X+1}{X+4} - \frac{X}{X+3} = \frac{1}{24}$$

On simplifying further,

$$\frac{X^2 + 3X + X + 3 - X^2 - 4X}{X^2 + 3X + 4X + 12} = \frac{1}{24}$$

$24(3) = X^2 + 7X + 12$

$X^2 + 7X - 60 = 0$ ------ (i)

On applying Sreedhracharya formula

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{-(7) \pm \sqrt{(7)^2 - 4(1)(-60)}}{2(1)}$$

$$\frac{-7 \pm \sqrt{49 + 240}}{2}$$

$$\frac{-7 \pm \sqrt{289}}{2}$$

$$\frac{-7 \pm 17}{2}$$

∴ X = –12 or 5.

∴ the numerator is X = 5 and denominator (X+3) will be 8 and the fraction will be $\frac{5}{8}$, as taking (–12) will form a fraction i.e., $\frac{4}{3}$ (not satisfying the conditions) .

21. Question

The denominator of a fraction exceeds its numerator by 3. If 3 is added to both numerator and denominator, the difference between the new and the original fraction is $\frac{9}{88}$. Find the original fraction.

Answer

Let the denominator be '(X+3)', so the numerator will be 'X'.

∴ the original fraction is $\frac{X}{X+3}$

∴ the new fraction is $\frac{X+3}{X+6}$

$$\because \frac{X+3}{X+6} - \frac{X}{X+3} = \frac{9}{88}$$

On simplifying further,

$$\frac{X^2 + 6X + 9 - X^2 - 6X}{X^2 + 3X + 6X + 18} = \frac{9}{88}$$

$88(9) = 9(X^2 + 9X + 18)$

$X^2 + 9X - 70 = 0$ ------ (i)

On applying Sreedhracharya formula

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{-(9) \pm \sqrt{(9)^2 - 4(1)(-70)}}{2(1)}$$

$$\frac{-9 \pm \sqrt{81 + 280}}{2}$$

$$\frac{-9 \pm \sqrt{361}}{2}$$

$$\frac{-9 \pm 19}{2}$$

∴ X = –14 or 5.

∴ the numerator is X = 5 and denominator (X+3) will be 8 and the fraction will be $\frac{5}{8}$, as taking (–14) will form a fraction i.e., $\frac{14}{11}$ (not satisfying the conditions) .

22. Question

The numerator of a fraction is 3 less than denominator. If 2 is added to both 29 numerator as well as denominator, then sum of the new and original fraction is $\frac{19}{15}$. Find the fraction.

Answer

Let the denominator be '(X+3)', so the numerator will be 'X'.

∴ the original fraction is $\frac{X}{X+3}$

∴ the new fraction is $\frac{X+2}{X+5}$

$$\because \frac{X+2}{X+5} + \frac{X}{X+3} = \frac{19}{15}$$

On simplifying further,

$$\frac{X^2 + 5X + 6 + X^2 + 5X}{X^2 + 3X + 5X + 15} = \frac{19}{15}$$

$15(2X^2 + 10X + 6) = 19(X^2 + 8X + 15)$

$30X^2 + 150X + 90 = 19X^2 + 152X + 285$

$11X^2 – 2X – 195 = 0$ ------- (i)

On applying Sreedhracharya formula

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{-(-2) \pm \sqrt{(-2)^2 - 4(11)(-195)}}{2(11)}$$

$$\frac{2 \pm \sqrt{4 + 8580}}{22}$$

$$\frac{2 \pm \sqrt{8584}}{22}$$

∴ it does not have real values.

23. Question

The numerator of a fraction is 2 less than the denominator. If 1 is added to both numerator and denominator the sum of the new and original fraction is $\frac{19}{15}$. Find the original fraction.

Answer

Given: Numerator of a fraction is 2 less than the denominator

To find: The fraction

Assumption: Let the denominator be x

Numerator = x – 2

Therefore, the fraction $= \frac{x-2}{x}$

If one is added to the numerator and denominator, fraction becomes $= \frac{x-2+1}{x+1} = \frac{x-1}{x+1}$

Sum of the fractions $= \frac{x-2}{x} + \frac{x-1}{x+1}$

Sum of the fractions $= \frac{19}{15}$

Therefore,

$$\frac{x-2}{x} + \frac{x-1}{x+1} = \frac{19}{15}$$

Taking L.C.M we get,

$$\frac{(x-2)(x+1)+x(x-1)}{x(x+1)}=\frac{19}{15}$$

$$\frac{x^2+x-2x-2+x^2-x}{x^2+x}=\frac{19}{15}$$

$$\frac{2x^2-2x-2}{x^2+x}=\frac{19}{15}$$

Cross-multiplying we get,

$30x^2 - 30x - 30 = 19x^2 + 19x$

$30x^2 - 19x^2 - 30x - 19x - 30 = 0$

$11x^2 - 49x - 30 = 0$

Now we need to factorise such that, on multiplication we get 330 and on substraction we get 49.

So, equation becomes,

$11x^2 - (55x - 6x) - 30 = 0$

$11x^2 - 55x + 6x - 30 = 0$

$11x(x - 5) + 6(x - 5) = 0$

$(11x + 6)(x - 5) = 0$

So, $11x + 5 = 0$ or $x - 5 = 0$

$$x = -\frac{5}{11}\ or\ x = 5$$

$$x-2 = -\frac{5}{11}-2 = -\frac{27}{11}\ or\ x = 5-2 = 3$$

Putting these values in fraction $=\frac{x-2}{x}$

Hence, the possible fractions are,

$$\frac{x-2}{x}=\frac{-\frac{27}{11}}{-\frac{5}{11}}=\frac{27}{5}$$

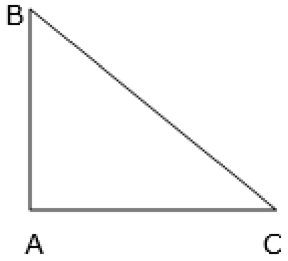
$$\frac{x-2}{x}=\frac{3}{5}$$

24. Question

The hypotenuse of a right–angled triangle is 6 cm more than twice the shortest side. If the third side is 2 cm less than the hypotenuse, find the sides of the triangle.

Answer

Let the shortest side(AC) be ‘(X)’cms, so the hypotenuse (BC) will be ‘(2X+6)’ cms, as demonstrated in the figure drawn below:



$\therefore$ AB = 2X + 4 cms

$\because$ $(BC)^2 = (AC)^2 + (AB)^2$

$\therefore (2X + 6)^2 = X^2 + (2X + 4)^2$

On simplifying further,

$4X^2 + 24X + 36 = X^2 + 4X^2 + 16X + 16$

Using the identity of $a^2 + b^2 + 2ab = (a + b)^2$

$X^2 - 8X - 20 = 0$ –––––– (i)

On applying Sreedhracharya formula

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(-20)}}{2(1)}$$

$$\frac{8 \pm \sqrt{64 + 80}}{2}$$

$$\frac{8 \pm \sqrt{144}}{2}$$

$$\frac{8 \pm 12}{2}$$

$\therefore$ X = –2 or 10.

∴ the shortest side (AC) is X = 10 cms (Only positive values), hypotenuse (2X+6) i.e., (BC) is 26 cms and the other side (AB) is (2X+4) i.e., 24 cms .

25. Question

The sum of the areas of two squares is 640 m^2. If the difference in their perimeters be 64 m, find the sides of the two squares.

Answer

Let the sides of the squares be '(X)'cms and '(Y) cms.

$\therefore X^2 + Y^2 = 640$ -----(i)

$\because$ Area = $(\text{Side})^2$

$\because$Perimeter = 4 (Side)

$\therefore 4X - 4Y = 64$

On simplifying further,

$X - Y = 16$ ------(ii)

Squaring the above mentioned equation, i.e., equation (ii)

$X^2 + Y^2 - 2XY = 256$

Using the identity of $a^2 + b^2 - 2ab = (a - b)^2$

Putting the value of equation (i) in equation (ii)

$\therefore 640 - 2XY = 256$

$2XY = 384$

$XY = 192$

Putting the value of $X = Y + 16$ in equation (i)

$(Y+16)^2 + Y^2 = 640$

$Y^2 + 256 + 32Y + Y^2 = 640$

$2Y^2 + 32Y - 384 = 0$

$Y^2 + 16Y - 192 = 0$

On applying Sreedhracharya formula

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{-(16) \pm \sqrt{(16)^2 - 4(1)(-192)}}{2(1)}$$

$$\frac{-16 \pm \sqrt{256 + 768}}{2}$$

$$\frac{-16 \pm \sqrt{1024}}{2}$$

$$\frac{-16 \pm 32}{2}$$

∴ X = – 12 or 24.

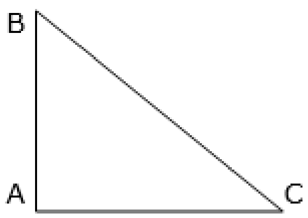
∴ the side is X = 24 cms (Only positive values), other square's side is Y=X–16 i.e., 8 cms.

26. Question

The hypotenuse of a right triangle is 3√5cm. If the smaller side is tripled and the longer side doubled, new hypotenuse will be 9√5 cm. How long are the sides of the triangle?

Answer

Let the shortest side(AC) be '(X)'cms, and the longer side (AB) be '(Y)' cms, as demonstrated in the figure drawn below:



∴ BC = 3√10 cms

∵ $(BC)^2 = (AC)^2 + (AB)^2$

∴ $(3\sqrt{10})^2 = X^2 + (Y)^2$

On simplifying further,

$(Y)^2 = 90 - X^2$

$X^2 + Y^2 - 90 = 0$ ------ (i)

As per the question,

New smaller side = '(3X)' cms

New longer side = '(2Y)' cms

$\therefore$ BC = $9\sqrt{5}$ cms

$\because$ $(BC)^2 = (AC)^2 + (AB)^2$

$\therefore (9\sqrt{5})^2 = (3X)^2 + (2Y)^2$

On simplifying further,

$4Y^2 + 9X^2 = 405$

$4X^2 + 9Y^2 - 405 = 0$ ------ (ii)

Putting the value of X^2, from equation (i) in equation (ii)

$X^2 = 90 - Y^2$

On simplifying further,

$(360-4Y)^2 + 9Y^2 - 405 = 0$

$5Y^2 = 45$

Y = ± 3 cms(Only positive values),

$\therefore$ X = $\sqrt{(90-9)}$.

$\therefore$ X = 9 cms

$\therefore$ the shortest side (AC) is X = 9 cms hypotenuse i.e., (BC) is $3\sqrt{10}$ cms and the other side (AB) is 3 cms .

27. Question

A teacher on attempting to arrange the students for mass drill in the form of a solid square found that 24 students were left. When he increased the size of the square by one student, he found that lie was short of 25 students. Find the number of students.

Answer

Ok now we know that area of square = side^2

So no. of students in the line if side was x (assumption)

No. of students = x^2+24 (as there were 24 exrltra students)

No. Of students after increasing 1 student in square = $(x+1)^2$–25 (as there were 25 less students)

So $x^2 + 24 = (x + 1)^2 - 25$

$x^2 + 24 = x^2 + 1 + 2x - 25$

$x^2 - x^2 + 24 + 25 - 1 = 2x$

$48 = 2x$

So $x = 24$

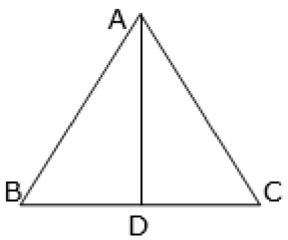
So no. Of students = $x^2 + 24 = 24^2 + 24 = 576 + 24 = 600$

$\therefore$ the no. of students = 600.

28. Question

The area of a triangle is 30 sq cm. Find the base if the altitude exceeds the base by 7 cm.

Answer



Let the length of base = P cm.

As base exceeds the base by 7cm ,

then , length of altitude = (P + 7)cm

now, area of triangle = $\frac{1}{2} \times \text{altitude} \times \text{base}$

given, area of triangle = 30 cm^2

so, $30\ cm^2 = \frac{1}{2} \times (P + 7) \times P$

$\Rightarrow 30 \times 2 = P^2 + 7P$

$\Rightarrow 60 = P^2 + 7P$

$\Rightarrow P^2 + 7P - 60 = 0$

$\Rightarrow P^2 + 12P - 5P - 60 = 0$

$\Rightarrow P(P + 12) - 5(P + 12) = 0$

$\Rightarrow (P + 12)(P - 5) = 0$

$\Rightarrow P = 5 , -12$

but length can't be negative so, $P \neq -12$

hence, P = 5 cm e.g., base = 5cm

29. Question

Is it possible to design a rectangular mango grove whose length is twice its breadth, and area is 800 m^2? If so, find its length and breadth.

Answer

Let breadth be X cmlength = 2X cmsarea=8002X (X)=800X(X)=400

X^2 = 400X =20Yes it is possible to design a rectangular mangrove having breadth =20 cm and length=40 cm.

30. Question

I want to design a rectangular park whose breadth is 3 m less than its length. Its area is to be 4 square metres more than the area of a park that has already been made in the shape of an isosceles triangle with its base as breadth of the rectangular park and altitude 12 m. Is it possible to have such a rectangular park? If so, find its length and breadth.

Answer

Let the length be l

then breadth= $l - 3$

area of rectangle= $l(l - 3) = l^2 - 3l$

area of triangle= $\frac{1}{2}(l - 3)(12) = 6l - 18$

given

area of rectangle is 4sq mt more than triangle

so

area of rectangle – 4= area of triangle

$l^2 - 3l - 4 = 6l - 18$

$l^2 - 9l + 14 = 0$

$l^2 - 7l - 2l + 14 = 0$

$l(l - 7) - 2(l - 7) = 0$

$(l - 7)(l - 2) = 0$

Yes, it is possible, to design a rectangular park.

L = 7 and 2

L = 2 is neglected as when length is 2 the breadth will be negative which is not possible...

so length= 7m

breadth= $l - 3$ =7–3 = 4m

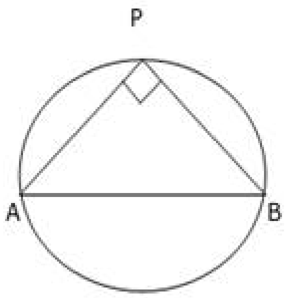
Yes it is possible to design a rectangular park having length & breadth 7 mts & 4 mts respectively.

31. Question

A pole has to be erected at a point on the boundary of a circular park of diameter 13 meters in such a way that the difference of its distances from two diametrically opposite fixed gates A and B on the boundary is 7 metres. Is it possible to do so? If yes, at what distances from the two gates should the pole be erected?

Answer

Let P be the position of the pole and A and B be the opposite fixed gates.



PA = ‘a’ mts

PB = ‘b’ mts

PA – PB = 7 m

⇒ a – b = 7

⇒ a = 7 + b ....(1)

In Δ PAB,

$AB^2 = AP^2 + BP^2$

⇒ $(17) = (a)^2 + (b)^2$

⇒ $a^2 + b^2 = 289$

⇒ Putting the value of a = 7 + b in the above,

⇒ $(7 + b)^2 + b^2 = 289$

⇒ $49 + 14b + 2b^2 = 289$

⇒ $2b^2 + 14b + 49 - 289 = 0$

⇒ $2b^2 + 14b - 240 = 0$

Dividing the above by 2, we get.

⇒ $b^2 + 7b - 120 = 0$

⇒ $b^2 + 15b - 8b - 120 = 0$

⇒ $b(b + 15) - 8(b + 15) = 0$

⇒ $(b - 8)(b + 15) = 0$

⇒ $b = 8$ or $b = -15$

Since this value cannot be negative, so b = 8 is the correct value.

Yes it is possible to erect a pole.

Putting b = 8 in (1), we get.

a = 7 + 8

a = 15 m

Hence PA = 15 m and PB = 8 m

So, the distance from the gate A to pole is 15 m and from gate B to the pole is 8 m.

32. Question

Is the following situation possible? If so, determine their present ages. The sum of the ages of a mother and her daughter is 20 years. Four years ago, the product of their ages in years was 48.

Answer

Let the mother age be ‘X’ years, then her daughter’s age will be ‘(20–X)’ years.

As per the question,

$(X - 4)(20 - X - 4) = 48$

$(X - 4)(16 - X) = 48$

$16X - 64 - X^2 + 4X = 48$

$-X^2 + 20X - 112 = 0$

$X^2 - 20X + 112 = 0$

On applying Sreedhracharya formula

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{-(-20) \pm \sqrt{(-20)^2 - 4(1)(112)}}{2(1)}$$

$$\frac{20 \pm \sqrt{400 + 448}}{2}$$

$$\frac{20 \pm \sqrt{848}}{2}$$

∵ it does not have real values, then it is not possible for the above situation to happen.

33. Question

A train covers a distance of 90 km at a uniform speed. Had the speed been 15 kmph more, it would have taken 30 minutes less for the journey. Find the original speed of the train.

Answer

Let the speed of the train and the time taken to cover the same be 'X' km/hr and 'Y' hrs respectively.

∵ Distance = Speed * Time

∴ As per the question,

$XY = 90$ ------ (i)

$(X + 15)(Y - 0.5) = 90$ ----- (ii)

∵ LHS = RHS

∴ Equating the LHS of both equations, and simplifying it further

$XY = XY - 0.5X + 15Y - 7.5$

$0.5X - 15Y + 7.5 = 0$ ------- (iii)

Multiplying the above equation by 10,

5X – 150Y + 75 = 0

Simplyfying it further,

X – 30Y + 15 = 0

Putting the value of Y, in equation (iii)

$$X - 30\left(\frac{90}{X}\right) + 15 = 0$$

$X^2 + 15X - 2700 = 0$

On applying Sreedhracharya formula

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{-(15) \pm \sqrt{(15)^2 - 4(1)(2700)}}{2(1)}$$

$$\frac{-15 \pm \sqrt{225 + 10800}}{2}$$

$$\frac{-15 \pm \sqrt{11025}}{2}$$

$$\frac{-15 \pm 105}{2}$$

X = – 60 or X = 45

∴ the original speed of train is 45 km/hr as speed can't be negative.

34. Question

An aeroplane left 30 minutes later than its scheduled time and in order to reach its destination 1500 km away in time, it had to increase its speed by 250 km/hr from its usual speed. Determine its usual speed.

Answer

Let the usual speed be x km /hr.

Actual speed = (x + 250) km/hr.

Time taken at actual speed = $\left(\frac{1500}{x}\right)$ hr.

Difference between the two times taken = $\frac{1}{2}$ hr.

Speed at that time = (x + 250) km/hr

$$\frac{\text{Distance}}{\text{time}} = x + 250$$

$$\frac{1500}{\text{time}} = x + 250$$

$$\frac{1500}{x + 250} = \text{time}$$

Then, According to the question,

$$\frac{1500}{x} - \frac{1500}{x+250} = 30 \times \frac{1}{60}$$

$$1500\left(\frac{1}{x} - \frac{1}{(x+250)}\right) = \frac{1}{2}$$

$$3000 \times [\frac{x + 250 - x}{x^2 + 250}] = 1$$

3000{250} = x^2 + 250

0 = x^2 + 250x – 750000

0 = x^2 + (1000–750)x – 750000

0 = x^2 + 1000x – 750x – 750000

0 = x(x + 1000) – 750(x + 1000)

0 = (x + 1000) (x – 750)

x = –1000 or x = 750

Usual speed = x = 750 km /hr

⇒ x = 750 【 speed cannot be negative】

Hence , the usual speed of the aeroplane was 750 km / hr.

35. Question

The speed of a boat in still water is 15 km/h. It can go 30 km upstream and return downstream to the original point in 4 hours and 30 minutes. Find the speed of the stream.

Answer

Let the speed of the boat be ‘X’ km/ hr, time taken for upstream and downstream be T1 hrs & T2 hrs respectively.

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$T1 = \frac{30}{15+X}$ ------- (i) (Downstream)

$T2 = \frac{30}{15-X}$ ------- (ii) (Upstream)

Adding equation equation (i) & equation (ii),

$$T1 + T2 = \frac{30}{15 + X} + \frac{30}{15 - X}$$

$$\frac{9}{2} = \frac{450 - 30X + 450 + 30X}{225 - X^2}$$

$2025 - 9X^2 = 1800$

$9X^2 = 225$

$X^2 = 25$

$X = \pm 5$

∵ speed can't be negative.

∴ The speed of the stream be 5 Km/hr

36. Question

An express train takes 1 hour less than a passenger train to travel 132 km between Mysore and Bangalore (without taking into consideration the time they stop at intermediate stations). If the average speed of the express train is 11 km/hr more than that of the passenger train, find the average speed of the two trains.

Answer

Let the average speed of the passenger train be 'x' km/hr and the average speed of the express train be (x + 11) km/hr

Distance between Mysore and Bangalore = 132 km

It is given that the time taken by the express train to cover the distance of 132 km is 1 hour less than the passenger train to cover the same distance.

So, time taken by passenger train = $\frac{132}{x}$ hr

The time taken by the express train = $\left\{\frac{132}{x} + 11\right\}$ hr

Now, according to the question

$$\left\{\frac{132}{x+11}\right\} = \frac{132}{x} + 1$$

After taking L.C.M. of $\frac{132}{x} + 1$ and then solving it we get $\frac{132+x}{x}$.

Now,

$$\left\{\frac{132}{x+11}\right\} = \frac{132+x}{x}$$

By cross multiplying, we get

$132x = x^2 + 132x + 11x + 1452$

$x^2 + 11x - 1452 = 0$

$x^2 + 44x - 33x - 1452 = 0$

$x(x + 44) - 33(x + 44) = 0$

$(x + 33)(x + 44) = 0$

$x - 44$ or $x = 33$

As the speed cannot be in negative therefore, x = 33 or the speed of the passenger train = 33 km/hr and the speed of express train is 33 + 11 = 44 km/hr.

37. Question

The sum of the reciprocals of Rehman's age (in years) 3 years ago and 5 years from now is $\frac{1}{3}$. Find his present age.

Answer

Let the present age of Rehman be x years.

So, 3 years age his age was = (x – 3) years

The reciprocal = $\frac{1}{x-3}$

And, after 5 years the age will be = (x + 5) years

The reciprocal = $\frac{1}{x+5}$

So, according to the question

$$\frac{1}{x-3} + \frac{1}{x+5} = \frac{1}{3}$$

Taking L. C. M. of (x – 3) and (x + 5)

$$\frac{(x+5)+x-3}{x^2+2x-15}=\frac{1}{3}$$

$$\frac{(2x+2)}{x^2+2x-15}=\frac{1}{3}$$

$6x + 6 = x^2 + 2x - 15$

$x^2 + 2x - 6x - 15 - 6 = 0$

$x^2 - 4x - 21 = 0$

$x^2 - 7x + 3x - 21 = 0$

$x(x - 7) + 3(x - 7) = 0$

$(x - 7)(x + 3) = 0$

$x = 7$ and $x = -3$

$x = -3$ is not possible because age cannot be negative.

So, $x = 7$

Therefore present age of Rehman is 7 years.

38. Question

The sum of the ages (in years) of a son and his father is 35 and the their ages is 150. Find their ages.

Answer

Let the father and his son's age be 'X' yrs and 'Y' yrs respectively.

$X + Y = 35X \times Y = 150X = \frac{150150}{Y\ \ Y} + Y = 35150 + Y^2 = 35YY^2 - 35Y + 150 = 0(Y-5)\ (Y-30)=0Y=5\therefore$ the son's age (Y) = 5 yrs and father's age (X) = 30 yrs.

39. Question

If a boy's age and his father's age amount together to 24 years. Fourth pan i product of their ages exceeds the boy's age by 9 years. Find how old they are?

Answer

Let the father and his son's age be 'X' yrs and '(24–X)' yrs respectively.

As per the question,

$$\frac{\{(24-X)(X)\}}{4} - 24 + X = 9$$

$24X - X^2 - 96 + 4X = 36$

$X^2 - 28X + 132 = 0$

$X^2 - 22X - 6X + 132 = 0$

$X(X - 22) - 6(X - 22) = 0$

$(X - 6)(X - 22) = 0$

$\therefore X = 22$ & boy'age is 2 years.

$\therefore X = 6$ & boy'age is 18 years. (practically not possible)

$\therefore$ The age of father and his son are 22 years & 2 years respectively.

40. Question

The product of the ages of two sisters is 104. The difference between their ages is 5. Find their ages.

Answer

Let the sister and her sister's age be 'X' yrs and '(X+5)' yrs respectively.

As per the question,

$X(X + 5) = 104$

$X^2 + 5X - 104 = 0$

$X^2 + 13X - 8X + 132 = 0$

$X(X + 13) - 8(X + 13) = 0$

$(X + 13)(X - 8) = 0$

$\therefore X = 8$ or $X = -13$

$\therefore$ the ages of sisters are 8 years & 13 years respectively, as the age can't be negative.

41. Question

Seven years ago Varun's age was five times the square of Swati's age. Three years hence, Swati's age will be two–fifth of Varun's age. Find their present ages.

Answer

Let seven years, age of Swati was x years and age of Varun was $5x^2$ years.

Present age of Swati = (x + 7) years

Present age of Varun = $(5x^2 + 7)$ years

Given, after 3 years swati's age will be $\frac{2}{5}$ th of Varun's age.

Age of Swati after 3 years = (x + 7 + 3) years = (x + 10) years

Age of Varun after 3 years = $(5x^2 + 7 + 3)$ years = $(5x^2 + 10)$ years

Given, age of Swati after 3 years = Two–fifths age of varun after 3 years

$$(x + 10) = \frac{2}{5} \times (5x^2 + 10)$$

$(x + 10) = 2(x^2 + 2)$

$x + 10 = 2x^2 + 4$

$2x^2 - x + 4 - 10 = 0$

$2x^2 - x - 6 = 0$

$2x^2 - 4x + 3x - 6 = 0$

$2x(x - 2) + 3(x - 2) = 0$

x = 2 [Age can't be negative]

Therefore, the present age of Swati = (2 + 7) = 9 year and the present age of Varun = $5(2)^2+7=27$ years.

42 A. Question

In a class test, the sum of Kamal's marks in Mathematics and English is 40. Had he got 3 marks more in Mathematics and 4 marks less in English, the product of his marks would have been 360. Find his marks in two subjects separately.

Answer

Let the marks scored in maths be 'X'.

Marks in English is '(40–X)'.

As, per the question,

If he got 3 marks in maths & 4 marks less in English, Marks in Maths $=X+3$

Marks in English $= 40–X–4 = 36–X$

Product $= 360$

$(36 – X)(X + 3) = 360$

$(36X + 108 – X^2 – 3X) = 360$

$(33X + 108 – X^2) = 360$

$X^2 – 33X + 360 – 108 = 0$

$X^2 – 33X + 252 = 0$

$X^2 – 21X – 12X + 252 = 0$

$X(X – 21) – 12(X – 21) = 0$

$(X– 12)(X – 21) = 0$

$X = 12$ or 21

∴ If marks in Maths = 12 then marks in English = 40 – 12 = 28

If marks in Maths = 21 then marks in English = 40 – 21 = 19

42 B. Question

In a class test, the sum of Gagan marks in Mathematics and English is 45. If he had 1 more mark in Mathematics and 1 less in English, the product of marks would have been 500. Find the original marks obtained by Gagan in Mathematics and English separately.

Answer

Let the marks in maths be ‘X’ and English be ‘Y’.

$X + Y = 45$ equation 1

$X = 45 – Y$

$(X + 1)(Y – 1) = 500$

$(45 – Y – 1)(Y – 1) = 500$

$Y^2 – 43Y + 456 = 0$

By solving this quadratic equation

$Y^2 – 24Y – 19Y + 456 = 0$

$Y(Y - 24) - 19(Y - 24) = 0$

we get two values of Y

$Y1 = 24$

$Y2 = 19$

substitute both this values in equation 1

$X1 + 24 = 45$

$X1 = 45 - 24$

$= 21$

$X2 + 19 = 45$

$X2 = 45 - 19$

$= 26$

∴ the marks in maths is 21, then marks in English is 24 and if the marks in maths is 26, then marks in English is 19.

43. Question

Rs. 6500 were divided equally among a certain number of persons. Had there bees 1 15 more persons, each would have got Rs. 30 less. Find the original number of persons.

Answer

Let x be the no. of person and y is amount taken by each person.

$\frac{6500}{x} = y$ ......(1)

when 15 more person appear then

$\frac{6500}{x+15} = y - 30$ ......(2)

On solving both (1) and (2) equation

subtracting (2) from (1)

$$\frac{6500}{x} - \frac{6500}{x + 15} = 30$$

$$\left(\frac{1}{x} - \frac{1}{x + 15}\right) 6500 = 30$$

$$\frac{x + 15 - x}{x^2 + 15x} = \frac{30}{6500}$$

$3250 = x^2 + 15x$

$x^2 + 15x - 3250 = 0$

On applying Sreedhracharya formula

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{-(15) \pm \sqrt{(15)^2 - 4(1)(-3250)}}{2(1)}$$

$$\frac{-15 \pm \sqrt{225 + 13000}}{2}$$

$$\frac{-15 \pm \sqrt{13225}}{2}$$

$$\frac{-15 \pm 115}{2}$$

X = – 65 or X = 50

∴ The persons are 50 (As positive values are only considered).

44. Question

300 apples are distributed equally among a certain number of students. Had there been 10 more students, each would have received one apple less. Find the number of students.

Answer

Let x be the no. of students and y is the number of apples taken by each person.

$\frac{300}{x} = y$ ------(1)

when 10 more students appear then,

$\frac{300}{x+10} = y - 1$ -----(2)

On solving both (1) and (2) equation

subtracting (2) from (1)

$$\frac{300}{x} - \frac{300}{x+10} = 1$$

$$\left(\frac{1}{x} - \frac{1}{x+10}\right)300 = 1$$

$$\frac{x+10-x}{x^2+10x} = \frac{1}{300}$$

$3000 = x^2 + 10x$

$x^2 + 10x - 3000 = 0$

On applying Sreedhracharya formula

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{-(10) \pm \sqrt{(10)^2 - 4(1)(-3000)}}{2(1)}$$

$$\frac{-10 \pm \sqrt{100 + 12000}}{2}$$

$$\frac{-10 \pm \sqrt{12100}}{2}$$

$$\frac{-10 \pm 110}{2}$$

X = – 60 or X = 50

∴ The number of students are 50. (as only positive values are considered).

45. Question

A shopkeeper buys a number of books for Rs. 1200. If he had bought 10 more books for the same amount, each book would have cost Rs. 20 less. How many books did he buy?

Answer

Let x be the no. of books and y is the cost of each book.

$\frac{1200}{x} = y$ ------(1)

when 10 more students appear then,

$\frac{1200}{x+10} = y - 20$ ----(2)

On solving both (1) and (2) equation

subtracting (2) from (1)

$$\frac{1200}{x} - \frac{1200}{x+10} = 20$$

$$\left(\frac{1}{x} - \frac{1}{x+10}\right)1200 = 20$$

$$\frac{x+10-x}{x^2+10x} = \frac{20}{1200}$$

$600 = x^2 + 10x$

$x^2 + 10x - 600 = 0$

On applying Sreedhracharya formula

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{-(10) \pm \sqrt{(10)^2 - 4(1)(-600)}}{2(1)}$$

$$\frac{-10 \pm \sqrt{100 + 2400}}{2}$$

$$\frac{-10 \pm \sqrt{2500}}{2}$$

$$\frac{-10 \pm 50}{2}$$

X = – 30 or X = 20

∴ The number of books, he purchased are 20. (as only positive values are considered).

46. Question

One–fourth of a herd of camels was seen in the forest. Twice the square root of the herd had gone to mountains and the remaining 15 camels were seen on the bank of the river. Find the total number of camels.

Answer

Let the total number of camels be x.

Number of camels in forest = $\frac{x}{4}$.

Number of camels gone to mountains = $2\sqrt{x}$

Remaining camels on bank of the river = 15

Total camels = $\frac{x}{4} + 2\sqrt{x} + 15$

$$x = \frac{x}{4} + 2\sqrt{x} + 15$$

$4x = x + 8\sqrt{x} + 60$

$3x - 60 = 8\sqrt{x}$

Squaring both the sides,

$9x^2 + 3600 - 360x = 64x$

$9x^2 - 424x + 3600 = 0$

On simplifying further,

$9x^2 - 100x - 324x + 3600 = 0$

$x(9x - 100) - 36(9x - 100) = 0$

$(x - 36)(9x - 100) = 0$

$\therefore x = \frac{100}{9}$ or $X = 36$

$\therefore$ The number of camels is 36. (As whole values are only considered)

47. Question

A party of tourists booked a room in a hotel for Rs. 1200. Three of the members failed to pay. As a result, others had to pay Rs. 20 more (each). How many tourists were there in the party?

Answer

Amount for the booking of hotel = Rs. 1200

Let there be x tourists

When all the tourist paid money, each share = Rs. $\frac{1200}{x}$

When 3 members failed to pat, its Rs. $\frac{1200}{x-3}$

So, according to the question:

$$\frac{1200}{x-3} - \frac{1200}{x} = 20$$

$$\frac{1200x - 1200x + 3600}{x(x-3)} = 20$$

$3600 = 20 \times x(x - 3)$

$3600 = 20x^2 - 60x$

$20x^2 - 60x - 3600 = 0$

On simplifying further,

$x^2 - 3x - 180 = 0$

$x^2 - 15x + 12x - 180 = 0$

$x(x - 15) + 12(x - 15) = 0$

Therefore, $x = -12$ or $x = 15$

∴ The number of camels is 15. (As positive values are only considered)

48. Question

Two pipes running together can fill a cistern in 6 minutes. If one pipe takes 5 minutes more than the other to fill the cistern, find the time in which each pipe would fill the cistern.

Answer

Let the first pipe fill the cistern in 'X' minutes, then the second pipe requires '(X+5)' minutes to fill it.

Applying the concept of Unitary Method,

In one minute, both pipes will fill the part of cistern as below:

$$\frac{1}{X} + \frac{1}{X+5} = \frac{1}{6}$$

$$\frac{X+5+X}{X^2+5X} = \frac{1}{6}$$

$$\frac{(2X+5)}{X^2+5X} = \frac{1}{6}$$

$12X + 30 = X^2 + 5X$

$X^2 - 7X - 30 = 0$

On factorising the same.

$X^2 - 10X + 3X - 30 = 0$

$X(X - 10) + 3(X - 10) = 0$

$(X - 10)(X + 3) = 0$

$\therefore X = -3$ or $X = 10$.

Then the first pipe will fill the cistern in 'X' minutes i.e., 10 minutes and the second pipe will fill the cistern in '(X+5)' minutes i.e., 15 minutes.

49. Question

Two pipes running together can fill a cistern in $\frac{30}{11}$ minute. If one pipe takes 1 minutes more than the other to fill the cistern, find the time in which each pipe would fill the cistern.

Answer

Let the first pipe fill the cistern in 'X' minutes, then the second pipe requires '(X+1)' minutes to fill it.

Applying the concept of Unitary Method,

In one minute, both pipes will fill the part of cistern as below:

$$\frac{1}{X} + \frac{1}{X + 1} = \frac{11}{30}$$

$$\frac{X + 1 + X}{X^2 + X} = \frac{11}{30}$$

$$\frac{(2X + 1)}{X^2 + X} = \frac{11}{30}$$

$60X + 30 = 11X^2 + 11X$

$11X^2 - 49X - 30 = 0$

On applying Sreedhracharya formula

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{-(-49) \pm \sqrt{(-49)^2 - 4(11)(-30)}}{2(11)}$$

$$\frac{49 \pm \sqrt{2401 + 1320}}{22}$$

$$\frac{49 \pm \sqrt{3721}}{22}$$

$$\frac{49 \pm 61}{22}$$

X = $-\frac{6}{11}$ or X = 5

∴ The time required to fill the cistern is 5 & 6 minutes respectively. (as only positive values are considered).

50. Question

Two pipes running together can fill a cistern in $\frac{40}{13}$ minutes. If one pipe takes 3 minutes more than the other to fill the cistern, find the time in which each pipe would fill the cistern.

Answer

Let the first pipe fill the cistern in 'X' minutes, then the second pipe requires '(X+3)' minutes to fill it.

Applying the concept of Unitary Method,

In one minute, both pipes will fill the part of cistern as below:

$$\frac{1}{X} + \frac{1}{X+3} = \frac{13}{40}$$

$$\frac{X+3+X}{X^2+3X} = \frac{13}{40}$$

$$\frac{(2X+3)}{X^2+3X} = \frac{13}{40}$$

$80X + 120 = 13X^2 + 39X$

$13X^2 - 41X - 120 = 0$

On applying Sreedhracharya formula

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{-(-41) \pm \sqrt{(-41)^2 - 4(13)(-120)}}{2(13)}$$

$$\frac{41 \pm \sqrt{1681 + 6240}}{26}$$

$$\frac{41 \pm \sqrt{7921}}{26}$$

$$\frac{41 \pm 89}{26}$$

X = $-\frac{24}{13}$ or X = 5

∴ The time required to fill the cistern is 5 & 8 minutes respectively. (as only positive values are considered).