Long Answer Questions

Q. 1. Draw a schematic arrangement of Geiger-Marsden experiment for studying a-particle scattering by a thin foil of gold. Describe briefly, by drawing trajectories of the scattered a-particles. How this study can be used to estimate the size of the nucleus?
[CBSE Delhi 2010]

OR

Describe Geiger-Marsden experiment. What are its observations and conclusions?

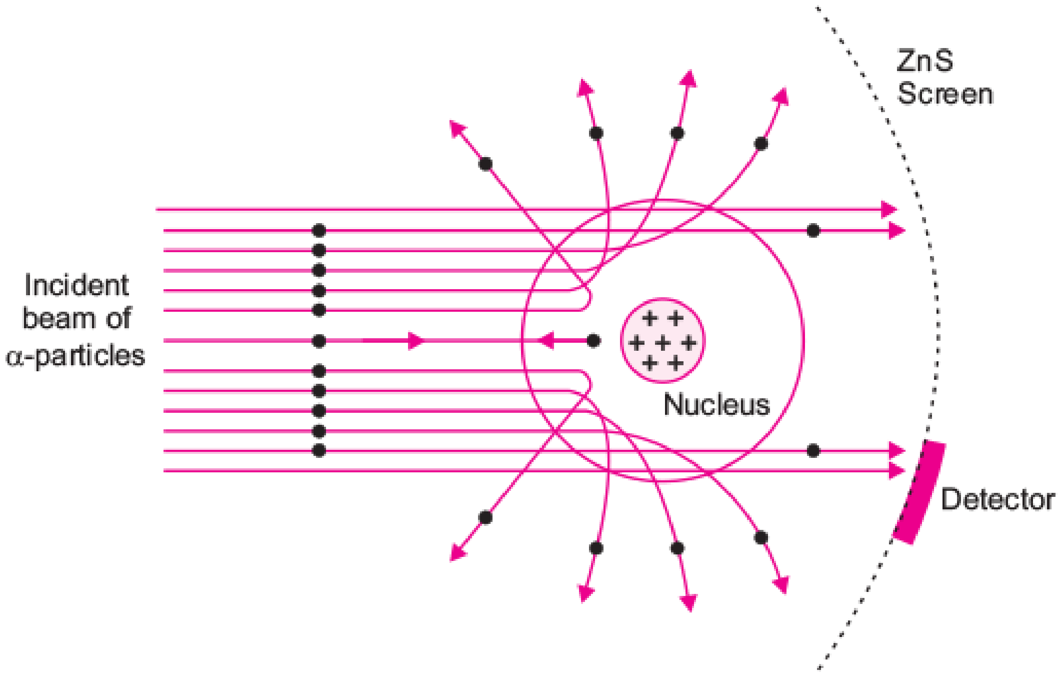
Ans. At the suggestion of Rutherford, in 1911, H. Geiger, and E. Marsden performed an important experiment called Geiger-Marsden experiment (or Rutherford's scattering experiment). It consists of

1. **Source of** a**-particles:** The radioactive source polonium emits high energetic alpha (a-) particles. Therefore, polonium is used as a source of a-particles. This source is placed in an enclosure containing a hole and a few slits A_1, A_2, ..., etc., placed in front of the hole. This arrangement provides a fine beam of a-particles.
2. **Thin gold foil:** It is a gold foil* of thickness nearly 10^{-6} m, a-particles are scattered by this foil. The foil taken is thin to avoid multiple scattering of a-particles, *i.e.*, to ensure that a-particle be deflected by a single collision with a gold atom.
3. **Scintillation counter:** By this the number of a-particles scattered in a given direction may be counted. The entire apparatus is placed in a vacuum chamber to prevent any energy loss of a-particles due to their collisions with air molecules.

Method: When a-particle beam falls on gold foil, the a-particles are scattered due to collision with gold atoms. This scattering takes place in all possible directions. The number of a-particles scattered in any direction is counted by scintillation counter.

Observations and Conclusions

i. Most of a-particles pass through the gold foil undeflected. This implies that *"most part of the atom is hollow."*
ii. a-particles are scattered through all angles. Some a-particles (nearly 1 in 2000), suffer scattering through angles more than 90°, while a still smaller number (nearly 1 in 8000) retrace their path. This implies that when fast moving positively charged a-particles come near gold-atom, then a few of them experience such a strong repulsive force that they turn back. On this basis Rutherford concluded that whole of positive charge of atom is concentrated in a small central core, called the nucleus.



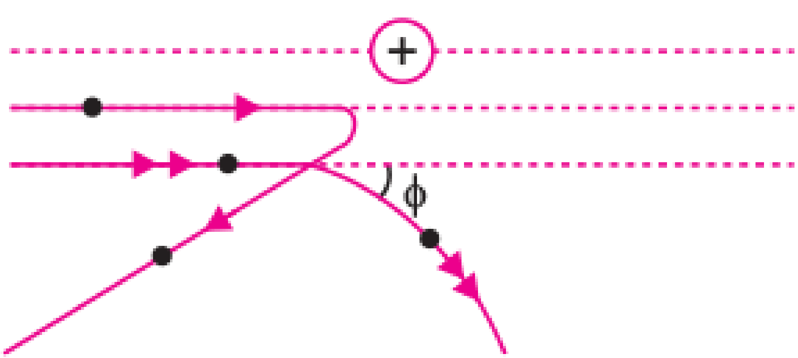
The distance of closest approach of a-particle gives the estimate of nuclear size. If *Ze* is charge of nucleus, E_k–kinetic energy of a particle, 2*e*–charge on a-particle, the size of nucleus r_0 is given by

$$r_0 \text{ is given by } E_k = \frac{1}{4\pi\varepsilon_0}\frac{(Ze)(2e)}{r_0} \quad \Rightarrow \quad r_0 = \frac{1}{4\pi\varepsilon_0}\frac{2\,Ze^2}{E_k}$$

Calculations show that the size of nucleus is of the order of 10^{-14} m, while size of atom is of the order of 10^{-10}m; therefore the size of nucleus is about

$$\frac{10^{-14}}{10^{-10}} = \frac{1}{10{,}000} \text{ times the size of atom.}$$

(iii) The negative charges (electrons) do not influence the scattering process. This implies that nearly whole mass of atom is concentrated in nucleus.



Q. 2. Using the postulates of Bohr's model of hydrogen atom, obtain an expression for the frequency of radiation emitted when atom make a transition from the higher energy state with quantum number ni to the lower energy state with quantum number n_f ($n_f < n_i$). [CBSE (AI) 2013, (F) 2012, 2011]

OR

Using Bohr's postulates, obtain the expression for the total energy of the electron in the stationary states of the hydrogen atom. Hence draw the energy level diagram showing how the line spectra corresponding to Balmer series occur due to transition between energy levels. [CBSE Delhi 2013, Guwahati 2015]

OR

Using Rutherford model of the atom, derive the expression for the total energy of the electron in hydrogen atom. What is the significance of total negative energy possessed by the electron? [CBSE (AI) 2014]

Ans. Suppose m be the mass of an electron and v be its speed in nth orbit of radius r. The centripetal force for revolution is produced by electrostatic attraction between electron and nucleus.

$$\frac{\mathrm{mv}^2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{(Ze)(e)}{r^2} \qquad [\text{from Rutherford model}] \ldots(i)$$

$$\text{or,}\ \mathrm{mv}^2 = \frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r}$$

$$\text{So, Kinetic energy}\ [K] = \frac{1}{2}\mathrm{mv}^2$$

$$K = \frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{2r}$$

$$\text{Potential energy} = \frac{1}{4\pi\varepsilon_0}\frac{(Ze)(-e)}{r} = -\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r}$$

$$\text{Total energy,}\ E = KE + PE$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{2r} + \left(-\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r}\right) = -\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r}$$

For nth orbit, E can be written as E_n

so, $E_n = -\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{2r_n}$ ...(ii)

Negative sign indicates that the electron remains bound with the nucleus (or electron-nucleus form an attractive system)

From Bohr's postulate for quantization of angular momentum.

$$\mathrm{mvr} = \frac{nh}{2\pi} \quad \Rightarrow \quad v = \frac{\mathrm{nh}}{2\pi\ \mathrm{mr}}$$

Substituting this value of v in equation (i), we get

$$\frac{m}{r}\left[\frac{nh}{2\pi\,\mathrm{mr}}\right]^2 = \frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r^2} \quad \text{or} \quad r = \frac{\varepsilon_0 h^2 n^2}{\pi m\ \mathrm{Ze}^2}$$

or, $r_n = \frac{\varepsilon_0 h^2 n^2}{\pi m\ \mathrm{Ze}^2}$ ...(iii)

For Bohr's radius, $n = 1$, *i.e.*, for K shell $r_B = \frac{\varepsilon_0 h^2}{\pi\,\mathrm{Zme}^2}$

Substituting value of r_n in equation (ii), we get

$$E_n = -\frac{1}{4\pi\varepsilon_0}\frac{\mathrm{Ze}^2}{2\left(\frac{\varepsilon_0 h^2 n^2}{\pi\ \mathrm{mZ}\ e^2}\right)} = -\frac{mZ^2e^4}{8\varepsilon_0 h^2 n^2}$$

or, $E_n = -\frac{Z^2\,\mathrm{Rhc}}{n^2}$, where $R = \frac{\mathrm{me}^4}{8\varepsilon_0^2\ \mathrm{ch}^3}$

R is called Rydberg constant.

For hydrogen atom $Z=1$, $E_n = \frac{-\mathrm{Rhc}}{n^2}$

If n_i and n_f are the quantum numbers of initial and final states and E_i & E_f are energies of electron in H-atom in initial and final state, we have

$$E_i = \frac{-Rhc}{n_i^2} \text{ and } E_f = \frac{-Rch}{n_f^2}$$

If ν is the frequency of emitted radiation, we get

$$\nu = \frac{E_i - E_f}{h}$$

$$\nu = \frac{-Rc}{n_i^2} - \left(\frac{-Rc}{n_f^2}\right) \Rightarrow \nu = Rc\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right]$$

For Balmer series $n_f = 2$, while $n_i = 3, 4, 5, \ldots\infty$.

Q. 3. Derive the expression for the magnetic field at the site of a point nucleus in a hydrogen atom due to the circular motion of the electron. Assume that the atom is in its ground state and give the answer in terms of fundamental constants. [CBSE Sample Paper 2016]

Ans. To keep the electron in its orbit, the centripetal force on the electron must be equal to the electrostatic force of attraction. Therefore,

$$\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2} \qquad \ldots(i) \quad (\text{For H atom, } Z = 1)$$

From Bohr's quantisation condition

$$mvr = \frac{nh}{2\pi} \Rightarrow v = \frac{nh}{2\pi\ mr}$$

For K shell, $n=1$

$$v = \frac{nh}{2\pi\ mr} \qquad \ldots(ii)$$

From (i) and (ii), we have

$$\frac{m}{r}\left(\frac{h}{2\pi\, mr}\right)^2 = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}$$

$$\frac{m}{r}\frac{h^2}{4\pi^2 m^2 r^2} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2} \qquad \Rightarrow \qquad \pi\,\mathrm{rme}^2 = \varepsilon_0 h^2$$

$$r = \frac{\varepsilon_0 h^2}{\pi\,\mathrm{me}^2} \qquad \ldots(iii)$$

From (ii) and (iii), we have

$$v = \frac{h\times\pi\,\mathrm{me}^2}{2\pi m\varepsilon_0 h^2} = \frac{e^2}{2\varepsilon_0 h}$$

Magnetic field at the centre of a circular loop $B = \frac{\mu_0 I}{2r}$

$$I = \frac{\mathrm{Ch\,arg}\,e}{\mathrm{Time}} \text{ and } \mathrm{Time} = \frac{2\pi r}{v}$$

$$\therefore \qquad I = \frac{\mathrm{ev}}{2\pi r}$$

So, $$B = \frac{\mu_0\,\mathrm{ev}}{2r\times 2\pi r} = \frac{\mu_0\,\mathrm{ev}}{4\pi r^2} \qquad \ldots.(iv)$$

From (ii), (iii) (iv), we have

$$B = \frac{\mu_0 e.e^2\pi^2 m^2 e^4}{2\varepsilon_0 h\times 4\pi\times\varepsilon_0^2 h^4} \qquad \Rightarrow \qquad B = \frac{\mu_0 e^7\pi m^2}{8{\varepsilon_0}^3 h^5}$$