

Topics : Center of Mass, Rigid Body Dynamics, String Wave, Fluid, Electromagnetic Induction, Rigid Body Dynamics

Type of Questions

Single choice Objective ('-1' negative marking) Q.1 to Q.3

(3 marks 3 min.)

M.M., Min.

[9, 9]

Subjective Questions ('-1' negative marking) Q.4

(4 marks 5 min.)

[4, 5]

Comprehension ('-1' negative marking) Q.5 to Q.7

(3 marks 3 min.)

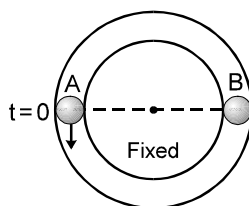
[9, 9]

Match the Following (no negative marking) (2 × 4) Q.8

(8 marks 10 min.)

[8, 10]

1. Particle 'A' moves with speed 10 m/s in a frictionless circular fixed horizontal pipe of radius 5 m and strikes with 'B' of double mass that of A. Coefficient of restitution is $1/2$ and particle 'A' starts its journey at $t = 0$. The time at which second collision occurs is :



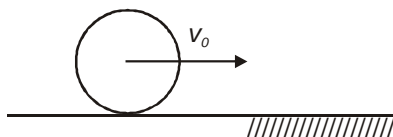
(A) $\frac{\pi}{2}$ s

(B) $\frac{2\pi}{3}$ s

(C) $\frac{5\pi}{2}$ s

(D) 4π s

2. A solid cylinder is sliding on a smooth horizontal surface with velocity v_0 without rotation. It enters on the rough surface. After that it has travelled some distance, select the correct statement:



- (A) Friction force increases its translational kinetic energy.
(B) Friction force increases its rotational kinetic energy.
(C) Friction force increases its total mechanical energy.
(D) Friction force increases its angular momentum about an axis passing through point of contact of the cylinder and the surface.

3. Equation of a standing wave is generally expressed as $y = 2A \sin \omega t \cos kx$. In the equation, quantity ω/k represents

- (A) the transverse speed of the particles of the string.
(B) the speed of either of the component waves.
(C) the speed of the standing wave.
(D) a quantity that is independent of the properties of the string.

4. A block of density 2000 kg/m^3 and mass 10 kg is suspended by a spring of stiffness 100 N/m. The other end of the spring is attached to a fixed support. The block is completely submerged in a liquid of density 1000 kg/m^3 . If the block is in equilibrium position.

- (A) the elongation of the spring is 1 cm.
(B) the magnitude of buoyant force acting on the block is 50 N.
(C) the spring potential energy is 12.5 J.
(D) magnitude of spring force on the block is greater than the weight of the block.

COMPREHENSION

In a certain region, there are non-uniform electrical potential (V_e) as well as gravitational potential (V_g). The electrical potential varies only with x as shown in figure (i), and the gravitational potential varies only with y as shown in figure (ii).

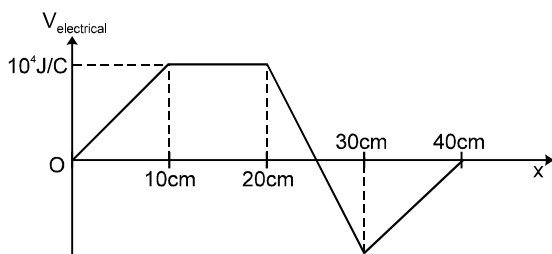


Figure (i)

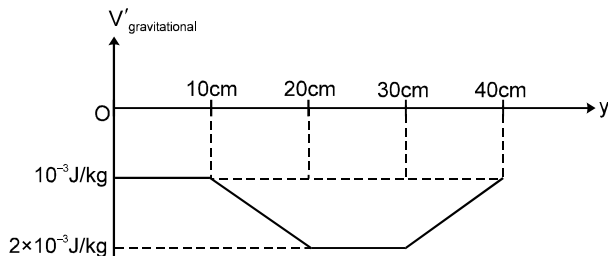
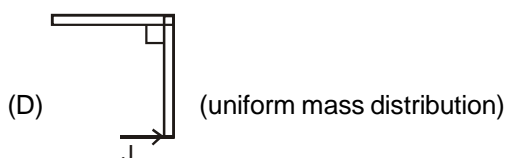
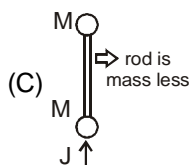
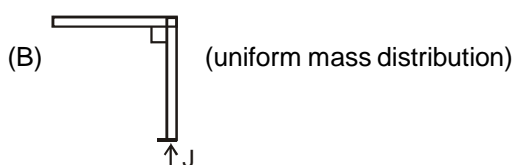
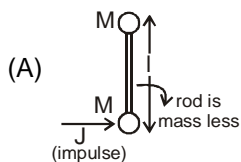


Figure (ii)

Consider a particle of mass 200 kg and charge $20 \mu\text{C}$ in this field.

5. If the particle is released from point (5 cm, 15 cm), it will try to move toward
 - (A) +x direction and +y direction
 - (B) +x direction and -y direction
 - (C) -x direction, +y direction
 - (D) -x direction, -y direction
6. What will be acceleration of the particle at point (25, 35)
 - (A) $(2\hat{i} - \hat{j}) \times 10^{-2} \text{ m/sec}^2$
 - (B) $(2\hat{i} + \hat{j}) \times 10^{-2} \text{ m/sec}^2$
 - (C) $(-2\hat{i} + \hat{j}) \times 10^{-2} \text{ m/sec}^2$
 - (D) $(3\hat{i} - 2\hat{j}) \times 10^{-2} \text{ m/sec}^2$
7. Minimum work required to bring the particle from (5, 15) to (25, 35) is :
 - (A) 0.2 J
 - (B) 0.1 J
 - (C) -0.2 J
 - (D) -0.1 J
8. Match the columns : (All the rigid bodies lie on a smooth horizontal plane. No object is hinged. J is impulse of an impulsive force. Initially system is at rest.)

Column-I



Column-II

(p) Translation occurs

(q) Rotation occurs

(r) Angular momentum (about centre of mass) increases

(s) Linear momentum increases

(t) Angular momentum will be conserved about more than one points in space.

Answers Key

1. (C) 2. (B) 3. (B) 4. (B)(C)
 5. (C) 6. (A)
 7. (D) 8. (A) p, q, r, s,t; (B) p, q, r, s,t;
 (C) p, s,t; (D) p, q, r, s,t

Hints & Solutions

1. For first collision

$$v = 10 \text{ m/s.}$$

$$t_1 = \frac{\pi(5)}{10}$$

$$= \pi/2 \text{ sec.}$$

velocity of sep = e. velocity of app.

$$v_2 - v_1 = \frac{1}{2}(10)$$

$$v_2 - v_1 = 5 \text{ m/s}$$

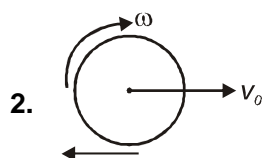
for second collision

$$\therefore t_2 = \frac{2\pi(5)}{5} = 2\pi$$

$$\therefore \text{total time } t = t_1 + t_2$$

$$= \pi/2 + 2\pi$$

$$t = 2.5\pi$$

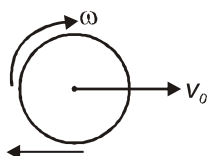


The friction force will reduce v_0 , hence translational K.E.

The friction force will increase ω

There is no torque about the line of contact, angular momentum will remain constant

The frictional force will decrease the mechanical energy.



3. Equation of the component waves are :
 $y = A \sin(\omega t - kx)$ and $y = A \sin(\omega t + kx)$
 where; $\omega t - kx = \text{constant}$ or $\omega t + kx = \text{constant}$
 Differentiating w.r.t. 't' ;

$$\omega - k \frac{dx}{dt} = 0 \quad \text{and} \quad \omega + k \frac{dx}{dt} = 0$$

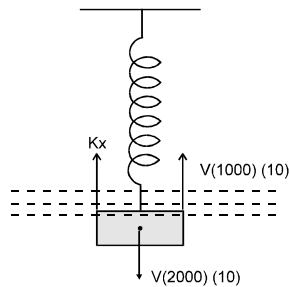
$$\Rightarrow v = \frac{dx}{dt} = \frac{\omega}{k} \quad \text{and} \quad v = -\frac{\omega}{k}$$

i.e.; the speed of component waves is $\left(\frac{\omega}{k}\right)$.

Hence (B)

4. $Kx = V(2000)(10) - V(1000)(10)$

$$= \frac{10}{2000} [1000 \times 10]$$



$$Kx = 50 \text{ N} \quad \dots (b)$$

$$U_{\text{stored}} = \frac{1}{2} \times (100) \left(\frac{50}{100}\right)^2 = \frac{1}{2} \times \frac{2500}{100}$$

$$= 12.5 \text{ J}$$

5. At $x = 5 \text{ cm}$, $\frac{\partial V_e}{\partial x} = \text{slope of figure 1}$

$$\text{at } x = 5 = +ve$$

$$\text{So } F_x = -q \frac{\partial V_e}{\partial x} = -ve$$

$$\text{at } y = 15 \text{ cm}, \frac{\partial V_g}{\partial y} = \text{slope of figure 2}$$

$$\text{at } y = 15 = -ve$$

$$\text{So } F_y = -m \frac{\partial V_g}{\partial y} = +ve$$

So particle will try to move towards $-x$ direction and $+y$ direction.

6. at (25, 35),

$$F_x|_{x=25} = -q \left. \frac{\partial V_e}{\partial x} \right|_{x=25}$$

$$= - \left(- \frac{2 \times 10^4}{0.1} \right) \times (20 \times 10^{-6}) = 4 \text{ N}$$

$$F_y|_{y=35} = -m \left. \frac{\partial V_g}{\partial y} \right|_{y=35}$$

$$= - (200) \left(\frac{10^{-3}}{0.10} \right) = - 2 \text{ N}$$

$$F_{\text{net}} = 4 \hat{i} - 2 \hat{j} = (200) \text{ a}$$

$$\frac{4}{200} \hat{i} - \frac{2}{200} \hat{j} = \text{a}$$

$$\text{a} = (2 \hat{i} - \hat{j}) \times 10^{-2} \text{ m/sec}^2$$

$$\begin{aligned} 7. \quad W_{(5, 15) \rightarrow (25, 35)} &= U_{(25, 35)} - U_{(5, 15)} \\ &= (0 + (200) (-1.5 \times 10^{-3})) - [(20 \times 10^{-6}) \\ &\quad (1/2 \times 10^4) + (200) (-1.5 / 10^{-3})] \\ &= - 0.1 \text{ J} \end{aligned}$$

8. (A) p, q, r, s,t; (B) p, q, r, s,t; (C) p, s,t;
(D) p, q, r, s,t

(p, s) since there is net impulse, translations motion will occurs for all cases.

(r,q) only in C, impulse is passing through centre of mass. Hence rotation will occur and angular momentum will increase in all cases except (C).

(t) About all the points on the line of action of the impulse, torque is zero. Hence angular momentum will conserve for many points.