Rational Numbers

- The numbers, which can be written in the form $\frac{p}{q}$, where *p* and *q* are integers and *q* $\neq 0$, are called rational numbers. Rational numbers can be positive as well as negative. Rational numbers include all integers and fractions.

  For example
  $-\frac{2}{7}, \frac{41}{366}, 2 = \frac{2}{1}$, etc.

- Equivalent rational numbers can be obtained by multiplying the numerator and denominator of a rational number by the same non-zero integer.

  Example:

  Write three equivalent rational numbers of $-\frac{2}{5}$.

  Solution:

  $-\frac{2}{5} \times \frac{2}{2} = -\frac{4}{10}$
  $-\frac{2}{5} \times \frac{3}{3} = -\frac{6}{15}$
  $-\frac{2}{5} \times \frac{4}{4} = -\frac{8}{20}$

  $\therefore \frac{4}{10}, -\frac{6}{15}, -\frac{8}{20}$ are three equivalent rational numbers of $-\frac{2}{5}$.

- A rational number is said to be in **standard form** if its numerator and denominator have no common factor other than 1.
- Rational numbers can be reduced to the standard form by dividing their numerator and denominator by their HCF.

  Example:

  Reduce $-\frac{12}{54}$ into standard form.

  Solution:

  HCF (12, 54) = 6

$$-\frac{12}{54} = \frac{-12 \div 6}{54 \div 6} = -\frac{2}{9}$$

- **Rational numbers on number line**

  Rational numbers can be represented on number line in the similar manner like fractions and integers.

  Negative rational numbers are marked to the left of 0 while positive rational numbers are marked to the right of 0.

  Example:

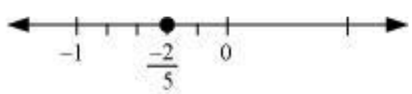
  Represent $-\frac{2}{5}$ on number line.

  Solution:

  The given rational number is negative. Therefore, it will lie to the left of 0.

  The space between –1 and 0 is divided into 5 equal parts. Therefore, each part represents $-\frac{1}{5}$.

  Marking $-\frac{2}{5}$ at 2 units to the left of 0, we obtain the number line as shown below.



- To find rational numbers between any two given rational numbers, firstly we have to make their denominators same and then find the respective rational numbers.

  Example:

  Find some rational numbers between $\frac{1}{6}$ and $\frac{7}{8}$.

  Solution:

  The L.C.M. of 6 and 8 is 24.
  Now, we can write

$$\frac{1}{6} = \frac{1\times4}{6\times4} = \frac{4}{24}$$
$$\frac{7}{8} = \frac{7\times3}{8\times3} = \frac{21}{24}$$

Therefore, some of the rational numbers between $\frac{4}{24}\left(\frac{1}{6}\right)$ and $\frac{21}{24}\left(\frac{7}{8}\right)$ are $\frac{5}{24}, \frac{6}{24}, \frac{7}{24}, \frac{8}{24}, \frac{9}{24}, \frac{10}{24}, \frac{11}{24}, \frac{12}{24}, \frac{13}{24}, \frac{14}{24}, \frac{15}{24}, \frac{16}{24}, \frac{17}{24}, \frac{18}{24}, \frac{19}{24}, \frac{20}{24}$

- Two positive rational numbers can be compared as in fractions.
- Two negative rational numbers can be compared by ignoring their negative signs and then reversing their order.

Example: Compare $-\frac{2}{3}$ and $-\frac{1}{5}$

Solution: To compare $-\frac{2}{3}$ and $-\frac{1}{5}$, we first compare $\frac{2}{3}$ and $\frac{1}{5}$.

HCF (3, 5) = 15

$\therefore \frac{2}{3} = \frac{2\times5}{3\times5} = \frac{10}{15}$ and $\frac{1}{5} = \frac{1\times3}{5\times3} = \frac{3}{15}$

Now, $\frac{10}{15} > \frac{3}{15}$

$\Rightarrow \frac{2}{3} > \frac{1}{5}$

$\Rightarrow -\frac{2}{3} < -\frac{1}{5}$

- Rational numbers between two rational numbers can be found by first converting them to rational numbers with same denominator.

Example: Find 4 rational numbers between $-\frac{2}{3}$ and $-\frac{1}{5}$.

Solution:

$-\frac{2}{3} \times \frac{5}{5} = -\frac{10}{15}$

$= \frac{1}{5} \times \frac{3}{3} = -\frac{3}{15}$

$\therefore \frac{10}{15} < -\frac{9}{15} < -\frac{8}{15} < -\frac{7}{15} < -\frac{6}{15} < -\frac{3}{15}$

Thus, four rational numbers between $-\frac{3}{2}$ and $-\frac{1}{5}$ are $-\frac{9}{15}, -\frac{8}{15}, -\frac{7}{15}$ and $-\frac{6}{15}$

- There are unlimited rational numbers between two rational numbers.

All the operations on rational numbers are performed as in fractions.

Example: Solve

1. $-\frac{3}{4} + \frac{5}{6}$

2. $\frac{2}{7} - \frac{3}{5}$

Solution:

1. $-\frac{3}{4} + \frac{5}{6}$

$$= \frac{-3\times3+5\times2}{12} = \frac{-9+10}{12} = \frac{1}{12}$$

2. $\frac{2}{7} - \frac{3}{5}$

$$= \frac{2\times5-3\times7}{35} = \frac{10-21}{35} = -\frac{11}{35}$$

- When 0 is added to any rational number, say $\frac{p}{q}$, the same rational number is obtained. Therefore, 0 is the additive identity of rational numbers.

$$\frac{p}{q} + 0 = \frac{p}{q} = 0 + \frac{p}{q}$$

- $-\frac{p}{q}$ is the additive inverse of the rational number $\frac{p}{q}$.

Example: $-\frac{4}{7}$ is the additive inverse of the rational number $\frac{4}{7}$.

Example: Solve

1. $\frac{2}{9} \times \left(-\frac{4}{3}\right)$
2. $-\frac{3}{7} \div \frac{11}{21}$

Solution:

1. $\frac{2}{9} \times \left(-\frac{4}{3}\right)$

$$= \frac{2\times(-4)}{9\times3} = -\frac{8}{27}$$

2. $-\frac{3}{7} \div \frac{11}{21}$

$$= -\frac{3}{7} \times \frac{21}{11} = -\frac{9}{11}$$