

14. Differentials, Errors and Approximations

Exercise 14.1

1. Question

If $y = \sin x$ and x changes from $\frac{\pi}{2}$ to $\frac{22}{14}$, what is the approximate change in y ?

Answer

Given $y = \sin x$ and x changes from $\frac{\pi}{2}$ to $\frac{22}{14}$.

Let $x = \frac{\pi}{2}$ so that $x + \Delta x = \frac{22}{14}$

$$\Rightarrow \frac{\pi}{2} + \Delta x = \frac{22}{14}$$

$$\therefore \Delta x = \frac{22}{14} - \frac{\pi}{2}$$

On differentiating y with respect to x , we get

$$\frac{dy}{dx} = \frac{d}{dx}(\sin x)$$

We know $\frac{d}{dx}(\sin x) = \cos x$

$$\therefore \frac{dy}{dx} = \cos x$$

When $x = \frac{\pi}{2}$, we have $\frac{dy}{dx} = \cos\left(\frac{\pi}{2}\right)$.

$$\Rightarrow \left(\frac{dy}{dx}\right)_{x=\frac{\pi}{2}} = 0$$

Recall that if $y = f(x)$ and Δx is a small increment in x , then the corresponding increment in y , $\Delta y = f(x + \Delta x) - f(x)$, is approximately given as

$$\Delta y = \left(\frac{dy}{dx}\right) \Delta x$$

Here, $\frac{dy}{dx} = 0$ and $\Delta x = \frac{22}{14} - \frac{\pi}{2}$

$$\Rightarrow \Delta y = (0) \left(\frac{22}{14} - \frac{\pi}{2}\right)$$

$$\therefore \Delta y = 0$$

Thus, there is approximately no change in y .

2. Question

The radius of a sphere shrinks from 10 to 9.8 cm. Find approximately the decrease in its volume.

Answer

Given the radius of a sphere changes from 10 cm to 9.8 cm.

Let x be the radius of the sphere and Δx be the change in the value of x .

Hence, we have $x = 10$ and $x + \Delta x = 9.8$

$$\Rightarrow 10 + \Delta x = 9.8$$

$$\Rightarrow \Delta x = 9.8 - 10$$

$$\therefore \Delta x = -0.2$$

The volume of a sphere of radius x is given by

$$V = \frac{4}{3}\pi x^3$$

On differentiating V with respect to x , we get

$$\frac{dV}{dx} = \frac{d}{dx} \left(\frac{4}{3}\pi x^3 \right)$$

$$\Rightarrow \frac{dV}{dx} = \frac{4\pi}{3} \frac{d}{dx} (x^3)$$

$$\text{We know } \frac{d}{dx} (x^n) = nx^{n-1}$$

$$\Rightarrow \frac{dV}{dx} = \frac{4\pi}{3} (3x^2)$$

$$\therefore \frac{dV}{dx} = 4\pi x^2$$

$$\text{When } x = 10, \text{ we have } \frac{dV}{dx} = 4\pi(10)^2.$$

$$\Rightarrow \left(\frac{dV}{dx} \right)_{x=10} = 4\pi \times 100$$

$$\Rightarrow \left(\frac{dV}{dx} \right)_{x=10} = 400\pi$$

Recall that if $y = f(x)$ and Δx is a small increment in x , then the corresponding increment in y , $\Delta y = f(x + \Delta x) - f(x)$, is approximately given as

$$\Delta y = \left(\frac{dy}{dx} \right) \Delta x$$

$$\text{Here, } \frac{dV}{dx} = 400\pi \text{ and } \Delta x = -0.2$$

$$\Rightarrow \Delta V = (400\pi)(-0.2)$$

$$\therefore \Delta V = -80\pi$$

Thus, the approximate decrease in the volume of the sphere is $80\pi \text{ cm}^3$.

3. Question

A circular metal plate expands under heating so that its radius increases by $k\%$. Find the approximate increase in the area of the plate, if the radius of the plate before heating is 10 cm.

Answer

Given the radius of a circular plate initially is 10 cm and it increases by $k\%$.

Let x be the radius of the circular plate, and Δx is the change in the value of x .

$$\text{Hence, we have } x = 10 \text{ and } \Delta x = \frac{k}{100} \times 10$$

$$\therefore \Delta x = 0.1k$$

The area of a circular plate of radius x is given by

$$A = \pi x^2$$

On differentiating A with respect to x , we get

$$\frac{dA}{dx} = \frac{d}{dx}(\pi x^2)$$

$$\Rightarrow \frac{dA}{dx} = \pi \frac{d}{dx}(x^2)$$

We know $\frac{d}{dx}(x^n) = nx^{n-1}$

$$\Rightarrow \frac{dA}{dx} = \pi(2x)$$

$$\therefore \frac{dA}{dx} = 2\pi x$$

When $x = 10$, we have $\frac{dA}{dx} = 2\pi(10)$.

$$\Rightarrow \left(\frac{dA}{dx}\right)_{x=10} = 20\pi$$

Recall that if $y = f(x)$ and Δx is a small increment in x , then the corresponding increment in y , $\Delta y = f(x + \Delta x) - f(x)$, is approximately given as

$$\Delta y = \left(\frac{dy}{dx}\right) \Delta x$$

Here, $\frac{dA}{dx} = 20\pi$ and $\Delta x = 0.1k$

$$\Rightarrow \Delta A = (20\pi)(0.1k)$$

$$\therefore \Delta A = 2k\pi$$

Thus, the approximate increase in the area of the circular plate is $2k\pi \text{ cm}^2$.

4. Question

Find the percentage error in calculating the surface area of a cubical box if an error of 1% is made in measuring the lengths of the edges of the cube.

Answer

Given the error in the measurement of the edge of a cubical box is 1%.

Let x be the edge of the cubical box, and Δx is the error in the value of x .

Hence, we have $\Delta x = \frac{1}{100} \times x$

$$\therefore \Delta x = 0.01x$$

The surface area of a cubical box of radius x is given by

$$S = 6x^2$$

On differentiating A with respect to x , we get

$$\frac{dS}{dx} = \frac{d}{dx}(6x^2)$$

$$\Rightarrow \frac{dS}{dx} = 6 \frac{d}{dx}(x^2)$$

We know $\frac{d}{dx}(x^n) = nx^{n-1}$

$$\Rightarrow \frac{dS}{dx} = 6(2x)$$

$$\therefore \frac{dS}{dx} = 12x$$

Recall that if $y = f(x)$ and Δx is a small increment in x , then the corresponding increment in y , $\Delta y = f(x + \Delta x) - f(x)$, is approximately given as

$$\Delta y = \left(\frac{dy}{dx} \right) \Delta x$$

Here, $\frac{dS}{dx} = 12x$ and $\Delta x = 0.01x$

$$\Rightarrow \Delta S = (12x)(0.01x)$$

$$\therefore \Delta S = 0.12x^2$$

The percentage error is,

$$\text{Error} = \frac{0.12x^2}{6x^2} \times 100\%$$

$$\Rightarrow \text{Error} = 0.02 \times 100\%$$

$$\therefore \text{Error} = 2\%$$

Thus, the error in calculating the surface area of the cubical box is 2%.

5. Question

If there is an error of 0.1% in the measurement of the radius of a sphere, find approximately the percentage error in the calculation of the volume of the sphere.

Answer

Given the error in the measurement of the radius of a sphere is 0.1%.

Let x be the radius of the sphere and Δx be the error in the value of x .

$$\text{Hence, we have } \Delta x = \frac{0.1}{100} \times x$$

$$\therefore \Delta x = 0.001x$$

The volume of a sphere of radius x is given by

$$V = \frac{4}{3} \pi x^3$$

On differentiating V with respect to x , we get

$$\frac{dV}{dx} = \frac{d}{dx} \left(\frac{4}{3} \pi x^3 \right)$$

$$\Rightarrow \frac{dV}{dx} = \frac{4\pi}{3} \frac{d}{dx} (x^3)$$

$$\text{We know } \frac{d}{dx} (x^n) = nx^{n-1}$$

$$\Rightarrow \frac{dV}{dx} = \frac{4\pi}{3} (3x^2)$$

$$\therefore \frac{dV}{dx} = 4\pi x^2$$

Recall that if $y = f(x)$ and Δx is a small increment in x , then the corresponding increment in y , $\Delta y = f(x + \Delta x) - f(x)$, is approximately given as

$$\Delta y = \left(\frac{dy}{dx} \right) \Delta x$$

Here, $\frac{dV}{dx} = 4\pi x^2$ and $\Delta x = 0.001x$

$$\Rightarrow \Delta V = (4\pi x^2)(0.001x)$$

$$\therefore \Delta V = 0.004\pi x^3$$

The percentage error is,

$$\text{Error} = \frac{0.004\pi x^3}{\frac{4}{3}\pi x^3} \times 100\%$$

$$\Rightarrow \text{Error} = \frac{0.004 \times 3}{4} \times 100\%$$

$$\Rightarrow \text{Error} = 0.003 \times 100\%$$

$$\therefore \text{Error} = 0.3\%$$

Thus, the error in calculating the volume of the sphere is 0.3%.

6. Question

The pressure p and the volume v of a gas are connected by the relation $pv^{1.4} = \text{const.}$ Find the percentage error in p corresponding to a decrease of $\frac{1}{2}\%$ in v .

Answer

Given $pv^{1.4} = \text{constant}$ and the decrease in v is $\frac{1}{2}\%$.

Hence, we have $\Delta v = -\frac{\frac{1}{2}}{100} \times v$

$$\therefore \Delta v = -0.005v$$

We have $pv^{1.4} = \text{constant}$

Taking log on both sides, we get

$$\log(pv^{1.4}) = \log(\text{constant})$$

$$\Rightarrow \log p + \log v^{1.4} = 0 \quad [\because \log(ab) = \log a + \log b]$$

$$\Rightarrow \log p + 1.4 \log v = 0 \quad [\because \log(a^m) = m \log a]$$

On differentiating both sides with respect to v , we get

$$\frac{d}{dp}(\log p) \times \frac{dp}{dv} + \frac{d}{dv}(1.4 \log v) = 0$$

$$\Rightarrow \frac{d}{dp}(\log p) \times \frac{dp}{dv} + 1.4 \frac{d}{dv}(\log v) = 0$$

We know $\frac{d}{dx}(\log x) = \frac{1}{x}$

$$\Rightarrow \frac{1}{p} \times \frac{dp}{dv} + 1.4 \times \frac{1}{v} = 0$$

$$\Rightarrow \frac{1}{p} \frac{dp}{dv} + \frac{1.4}{v} = 0$$

$$\Rightarrow \frac{1}{p} \frac{dp}{dv} = -\frac{1.4}{v}$$

$$\therefore \frac{dp}{dv} = -\frac{1.4}{v}p$$

Recall that if $y = f(x)$ and Δx is a small increment in x , then the corresponding increment in y , $\Delta y = f(x + \Delta x) - f(x)$, is approximately given as

$$\Delta y = \left(\frac{dy}{dx}\right) \Delta x$$

Here, $\frac{dp}{dv} = -\frac{1.4}{v}p$ and $\Delta v = -0.005v$

$$\Rightarrow \Delta p = \left(-\frac{1.4}{v}p\right)(-0.005v)$$

$$\Rightarrow \Delta p = (-1.4p)(-0.005)$$

$$\therefore \Delta p = 0.007p$$

The percentage error is,

$$\text{Error} = \frac{0.007p}{p} \times 100\%$$

$$\Rightarrow \text{Error} = 0.007 \times 100\%$$

$$\therefore \text{Error} = 0.7\%$$

Thus, the error in p corresponding to the decrease in v is 0.7%.

7. Question

The height of a cone increases by $k\%$, its semi-vertical angle remaining the same. What is the approximate percentage increase in (i) in total surface area, and (ii) in the volume, assuming that k is small.

Answer

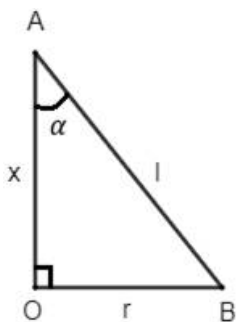
Given the height of a cone increases by $k\%$.

Let x be the height of the cone and Δx be the change in the value of x .

$$\text{Hence, we have } \Delta x = \frac{k}{100} \times x$$

$$\therefore \Delta x = 0.01kx$$

Let us assume the radius, the slant height and the semi-vertical angle of the cone to be r , l and α respectively as shown in the figure below.



From the above figure, using trigonometry, we have

$$\tan \alpha = \frac{OB}{OA}$$

$$\Rightarrow \tan \alpha = \frac{r}{x}$$

$$\therefore r = x \tan(\alpha)$$

We also have

$$\cos \alpha = \frac{OA}{AB}$$

$$\Rightarrow \cos \alpha = \frac{x}{l}$$

$$\Rightarrow l = \frac{x}{\cos \alpha}$$

$$\therefore l = x \sec(\alpha)$$

(i) The total surface area of the cone is given by

$$S = \pi r^2 + \pi r l$$

From above, we have $r = x \tan(\alpha)$ and $l = x \sec(\alpha)$.

$$\Rightarrow S = \pi(x \tan(\alpha))^2 + \pi(x \tan(\alpha))(x \sec(\alpha))$$

$$\Rightarrow S = \pi x^2 \tan^2 \alpha + \pi x^2 \tan(\alpha) \sec(\alpha)$$

$$\Rightarrow S = \pi x^2 \tan(\alpha) [\tan(\alpha) + \sec(\alpha)]$$

On differentiating S with respect to x , we get

$$\frac{dS}{dx} = \frac{d}{dx} [\pi x^2 \tan \alpha (\tan \alpha + \sec \alpha)]$$

$$\Rightarrow \frac{dS}{dx} = \pi \tan \alpha (\tan \alpha + \sec \alpha) \frac{d}{dx} (x^2)$$

We know $\frac{d}{dx} (x^n) = nx^{n-1}$

$$\Rightarrow \frac{dS}{dx} = \pi \tan \alpha (\tan \alpha + \sec \alpha) (2x)$$

$$\therefore \frac{dS}{dx} = 2\pi x \tan \alpha (\tan \alpha + \sec \alpha)$$

Recall that if $y = f(x)$ and Δx is a small increment in x , then the corresponding increment in y , $\Delta y = f(x + \Delta x) - f(x)$, is approximately given as

$$\Delta y = \left(\frac{dy}{dx} \right) \Delta x$$

Here, $\frac{dS}{dx} = 2\pi x \tan \alpha (\tan \alpha + \sec \alpha)$ and $\Delta x = 0.01kx$

$$\Rightarrow \Delta S = (2\pi x \tan(\alpha) [\tan(\alpha) + \sec(\alpha)])(0.01kx)$$

$$\therefore \Delta S = 0.02k\pi x^2 \tan(\alpha) [\tan(\alpha) + \sec(\alpha)]$$

The percentage increase in S is,

$$\text{Increase} = \frac{\Delta S}{S} \times 100\%$$

$$\Rightarrow \text{Increase} = \frac{0.02k\pi x^2 \tan \alpha (\tan \alpha + \sec \alpha)}{\pi x^2 \tan \alpha (\tan \alpha + \sec \alpha)} \times 100\%$$

$$\Rightarrow \text{Increase} = 0.02k \times 100\%$$

$$\therefore \text{Increase} = 2k\%$$

Thus, the approximate increase in the total surface area of the cone is $2k\%$.

(ii) The volume of the cone is given by

$$V = \frac{1}{3} \pi r^2 x$$

From above, we have $r = x \tan(\alpha)$.

$$\Rightarrow V = \frac{1}{3} \pi (x \tan \alpha)^2 x$$

$$\Rightarrow V = \frac{1}{3} \pi (x^2 \tan^2 \alpha) x$$

$$\Rightarrow V = \frac{1}{3} \pi x^3 \tan^2 \alpha$$

On differentiating V with respect to x , we get

$$\frac{dV}{dx} = \frac{d}{dx} \left(\frac{1}{3} \pi x^3 \tan^2 \alpha \right)$$

$$\Rightarrow \frac{dV}{dx} = \frac{1}{3} \pi \tan^2 \alpha \frac{d}{dx} (x^3)$$

We know $\frac{d}{dx} (x^n) = nx^{n-1}$

$$\Rightarrow \frac{dV}{dx} = \frac{1}{3} \pi \tan^2 \alpha (3x^2)$$

$$\therefore \frac{dV}{dx} = \pi x^2 \tan^2 \alpha$$

Recall that if $y = f(x)$ and Δx is a small increment in x , then the corresponding increment in y , $\Delta y = f(x + \Delta x) - f(x)$, is approximately given as

$$\Delta y = \left(\frac{dy}{dx} \right) \Delta x$$

Here, $\frac{dV}{dx} = \pi x^2 \tan^2 \alpha$ and $\Delta x = 0.01kx$

$$\Rightarrow \Delta V = (\pi x^2 \tan^2 \alpha)(0.01kx)$$

$$\therefore \Delta V = 0.01k\pi x^3 \tan^2 \alpha$$

The percentage increase in V is,

$$\text{Increase} = \frac{\Delta V}{V} \times 100\%$$

$$\Rightarrow \text{Increase} = \frac{0.01k\pi x^3 \tan^2 \alpha}{\frac{1}{3} \pi x^3 \tan^2 \alpha} \times 100\%$$

$$\Rightarrow \text{Increase} = \frac{0.01k}{\frac{1}{3}} \times 100\%$$

$$\Rightarrow \text{Increase} = 0.03k \times 100\%$$

$$\therefore \text{Increase} = 3k\%$$

Thus, the approximate increase in the volume of the cone is $3k\%$.

8. Question

Show that the relative error in computing the volume of a sphere, due to an error in measuring the radius, is approximately equal to three times the relative error in the radius.

Answer

Let the error in measuring the radius of a sphere be $k\%$.

Let x be the radius of the sphere and Δx be the error in the value of x .

Hence, we have $\Delta x = \frac{k}{100} \times x$

$$\therefore \Delta x = 0.01kx$$

The volume of a sphere of radius x is given by

$$V = \frac{4}{3} \pi x^3$$

On differentiating V with respect to x , we get

$$\frac{dV}{dx} = \frac{d}{dx} \left(\frac{4}{3} \pi x^3 \right)$$

$$\Rightarrow \frac{dV}{dx} = \frac{4\pi}{3} \frac{d}{dx} (x^3)$$

We know $\frac{d}{dx} (x^n) = nx^{n-1}$

$$\Rightarrow \frac{dV}{dx} = \frac{4\pi}{3} (3x^2)$$

$$\therefore \frac{dV}{dx} = 4\pi x^2$$

Recall that if $y = f(x)$ and Δx is a small increment in x , then the corresponding increment in y , $\Delta y = f(x + \Delta x) - f(x)$, is approximately given as

$$\Delta y = \left(\frac{dy}{dx} \right) \Delta x$$

Here, $\frac{dV}{dx} = 4\pi x^2$ and $\Delta x = 0.01kx$

$$\Rightarrow \Delta V = (4\pi x^2)(0.01kx)$$

$$\therefore \Delta V = 0.04k\pi x^3$$

The percentage error is,

$$\text{Error} = \frac{0.04k\pi x^3}{\frac{4}{3} \pi x^3} \times 100\%$$

$$\Rightarrow \text{Error} = \frac{0.04k \times 3}{4} \times 100\%$$

$$\Rightarrow \text{Error} = 0.03k \times 100\%$$

$$\therefore \text{Error} = 3k\%$$

Thus, the error in measuring the volume of the sphere is approximately three times the error in measuring its radius.

9 A. Question

Using differentials, find the approximate values of the following:

$$\sqrt{25.02}$$

Answer

Let us assume that $f(x) = \sqrt{x}$

Also, let $x = 25$ so that $x + \Delta x = 25.02$

$$\Rightarrow 25 + \Delta x = 25.02$$

$$\therefore \Delta x = 0.02$$

On differentiating $f(x)$ with respect to x , we get

$$\frac{df}{dx} = \frac{d}{dx}(\sqrt{x})$$

$$\Rightarrow \frac{df}{dx} = \frac{d}{dx}\left(x^{\frac{1}{2}}\right)$$

$$\text{We know } \frac{d}{dx}(x^n) = nx^{n-1}$$

$$\Rightarrow \frac{df}{dx} = \frac{1}{2}x^{\frac{1}{2}-1}$$

$$\Rightarrow \frac{df}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$$

$$\therefore \frac{df}{dx} = \frac{1}{2\sqrt{x}}$$

$$\text{When } x = 25, \text{ we have } \frac{df}{dx} = \frac{1}{2\sqrt{25}}$$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=25} = \frac{1}{2 \times 5}$$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=25} = 0.1$$

Recall that if $y = f(x)$ and Δx is a small increment in x , then the corresponding increment in y , $\Delta y = f(x + \Delta x) - f(x)$, is approximately given as

$$\Delta y = \left(\frac{dy}{dx}\right) \Delta x$$

$$\text{Here, } \frac{df}{dx} = 0.1 \text{ and } \Delta x = 0.02$$

$$\Rightarrow \Delta f = (0.1)(0.02)$$

$$\therefore \Delta f = 0.002$$

$$\text{Now, we have } f(25.02) = f(25) + \Delta f$$

$$\Rightarrow f(25.02) = \sqrt{25} + 0.002$$

$$\Rightarrow f(25.02) = 5 + 0.002$$

$$\therefore f(25.02) = 5.002$$

$$\text{Thus, } \sqrt{25.02} \approx 5.002$$

9 B. Question

Using differentials, find the approximate values of the following:

$$(0.009)^{1/3}$$

Answer

$$\text{Let us assume that } f(x) = x^{\frac{1}{3}}$$

$$\text{Also, let } x = 0.008 \text{ so that } x + \Delta x = 0.009$$

$$\Rightarrow 0.008 + \Delta x = 0.009$$

$$\therefore \Delta x = 0.001$$

On differentiating $f(x)$ with respect to x , we get

$$\frac{df}{dx} = \frac{d}{dx} \left(x^{\frac{1}{3}} \right)$$

$$\text{We know } \frac{d}{dx} (x^n) = nx^{n-1}$$

$$\Rightarrow \frac{df}{dx} = \frac{1}{3} x^{\frac{1}{3}-1}$$

$$\Rightarrow \frac{df}{dx} = \frac{1}{3} x^{-\frac{2}{3}}$$

$$\therefore \frac{df}{dx} = \frac{1}{3x^{\frac{2}{3}}}$$

$$\text{When } x = 0.008, \text{ we have } \frac{df}{dx} = \frac{1}{3(0.008)^{\frac{2}{3}}}$$

$$\Rightarrow \left(\frac{df}{dx} \right)_{x=0.008} = \frac{1}{3((0.2)^3)^{\frac{2}{3}}}$$

$$\Rightarrow \left(\frac{df}{dx} \right)_{x=0.008} = \frac{1}{3(0.2)^2}$$

$$\Rightarrow \left(\frac{df}{dx} \right)_{x=0.008} = \frac{1}{3(0.04)}$$

$$\Rightarrow \left(\frac{df}{dx} \right)_{x=0.008} = \frac{1}{0.12}$$

$$\Rightarrow \left(\frac{df}{dx} \right)_{x=0.008} = 8.3333$$

Recall that if $y = f(x)$ and Δx is a small increment in x , then the corresponding increment in y , $\Delta y = f(x + \Delta x) - f(x)$, is approximately given as

$$\Delta y = \left(\frac{dy}{dx} \right) \Delta x$$

$$\text{Here, } \frac{df}{dx} = 8.3333 \text{ and } \Delta x = 0.001$$

$$\Rightarrow \Delta f = (8.3333)(0.001)$$

$$\therefore \Delta f = 0.0083333$$

$$\text{Now, we have } f(0.009) = f(0.008) + \Delta f$$

$$\Rightarrow f(0.009) = (0.008)^{\frac{1}{3}} + 0.0083333$$

$$\Rightarrow f(0.009) = ((0.2)^3)^{\frac{1}{3}} + 0.0083333$$

$$\Rightarrow f(0.009) = 0.2 + 0.0083333$$

$$\therefore f(0.009) = 0.2083333$$

$$\text{Thus, } (0.009)^{1/3} \approx 0.2083333$$

9 C. Question

Using differentials, find the approximate values of the following:

$$(0.007)^{1/3}$$

Answer

Let us assume that $f(x) = x^{\frac{1}{3}}$

Also, let $x = 0.008$ so that $x + \Delta x = 0.007$

$$\Rightarrow 0.008 + \Delta x = 0.007$$

$$\therefore \Delta x = -0.001$$

On differentiating $f(x)$ with respect to x , we get

$$\frac{df}{dx} = \frac{d}{dx} \left(x^{\frac{1}{3}} \right)$$

We know $\frac{d}{dx} (x^n) = nx^{n-1}$

$$\Rightarrow \frac{df}{dx} = \frac{1}{3} x^{\frac{1}{3}-1}$$

$$\Rightarrow \frac{df}{dx} = \frac{1}{3} x^{-\frac{2}{3}}$$

$$\therefore \frac{df}{dx} = \frac{1}{3x^{\frac{2}{3}}}$$

When $x = 0.008$, we have $\frac{df}{dx} = \frac{1}{3(0.008)^{\frac{2}{3}}}$

$$\Rightarrow \left(\frac{df}{dx} \right)_{x=0.008} = \frac{1}{3((0.2)^3)^{\frac{2}{3}}}$$

$$\Rightarrow \left(\frac{df}{dx} \right)_{x=0.008} = \frac{1}{3(0.2)^2}$$

$$\Rightarrow \left(\frac{df}{dx} \right)_{x=0.008} = \frac{1}{3(0.04)}$$

$$\Rightarrow \left(\frac{df}{dx} \right)_{x=0.008} = \frac{1}{0.12}$$

$$\Rightarrow \left(\frac{df}{dx} \right)_{x=0.008} = 8.3333$$

Recall that if $y = f(x)$ and Δx is a small increment in x , then the corresponding increment in y , $\Delta y = f(x + \Delta x) - f(x)$, is approximately given as

$$\Delta y = \left(\frac{dy}{dx} \right) \Delta x$$

Here, $\frac{df}{dx} = 8.3333$ and $\Delta x = 0.001$

$$\Rightarrow \Delta f = (8.3333)(-0.001)$$

$$\therefore \Delta f = -0.0083333$$

Now, we have $f(0.007) = f(0.008) + \Delta f$

$$\Rightarrow f(0.007) = (0.008)^{\frac{1}{3}} - 0.0083333$$

$$\Rightarrow f(0.007) = ((0.2)^3)^{\frac{1}{3}} - 0.0083333$$

$$\Rightarrow f(0.007) = 0.2 - 0.0083333$$

$$\therefore f(0.007) = 0.1916667$$

$$\text{Thus, } (0.007)^{1/3} \approx 0.1916667$$

9 D. Question

Using differentials, find the approximate values of the following:

$$\sqrt{401}$$

Answer

Let us assume that $f(x) = \sqrt{x}$

Also, let $x = 400$ so that $x + \Delta x = 401$

$$\Rightarrow 400 + \Delta x = 401$$

$$\therefore \Delta x = 1$$

On differentiating $f(x)$ with respect to x , we get

$$\frac{df}{dx} = \frac{d}{dx}(\sqrt{x})$$

$$\Rightarrow \frac{df}{dx} = \frac{d}{dx}\left(x^{\frac{1}{2}}\right)$$

We know $\frac{d}{dx}(x^n) = nx^{n-1}$

$$\Rightarrow \frac{df}{dx} = \frac{1}{2}x^{\frac{1}{2}-1}$$

$$\Rightarrow \frac{df}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$$

$$\therefore \frac{df}{dx} = \frac{1}{2\sqrt{x}}$$

When $x = 400$, we have $\frac{df}{dx} = \frac{1}{2\sqrt{400}}$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=400} = \frac{1}{2 \times 20}$$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=400} = \frac{1}{40}$$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=400} = 0.025$$

Recall that if $y = f(x)$ and Δx is a small increment in x , then the corresponding increment in y , $\Delta y = f(x + \Delta x) - f(x)$, is approximately given as

$$\Delta y = \left(\frac{dy}{dx}\right) \Delta x$$

Here, $\frac{df}{dx} = 0.025$ and $\Delta x = 1$

$$\Rightarrow \Delta f = (0.025)(1)$$

$$\therefore \Delta f = 0.025$$

Now, we have $f(401) = f(400) + \Delta f$

$$\Rightarrow f(401) = \sqrt{400} + 0.025$$

$$\Rightarrow f(401) = 20 + 0.025$$

$$\therefore f(401) = 20.025$$

Thus, $\sqrt{401} \approx 20.025$

9 E. Question

Using differentials, find the approximate values of the following:

$$(15)^{1/4}$$

Answer

Let us assume that $f(x) = x^{\frac{1}{4}}$

Also, let $x = 16$ so that $x + \Delta x = 15$

$$\Rightarrow 16 + \Delta x = 15$$

$$\therefore \Delta x = -1$$

On differentiating $f(x)$ with respect to x , we get

$$\frac{df}{dx} = \frac{d}{dx} \left(x^{\frac{1}{4}} \right)$$

We know $\frac{d}{dx} (x^n) = nx^{n-1}$

$$\Rightarrow \frac{df}{dx} = \frac{1}{4} x^{\frac{1}{4}-1}$$

$$\Rightarrow \frac{df}{dx} = \frac{1}{4} x^{-\frac{3}{4}}$$

$$\therefore \frac{df}{dx} = \frac{1}{4x^{\frac{3}{4}}}$$

When $x = 16$, we have $\frac{df}{dx} = \frac{1}{4(16)^{\frac{3}{4}}}$

$$\Rightarrow \left(\frac{df}{dx} \right)_{x=16} = \frac{1}{4(2^4)^{\frac{3}{4}}}$$

$$\Rightarrow \left(\frac{df}{dx} \right)_{x=16} = \frac{1}{4(2^3)}$$

$$\Rightarrow \left(\frac{df}{dx} \right)_{x=16} = \frac{1}{4(8)}$$

$$\Rightarrow \left(\frac{df}{dx} \right)_{x=16} = \frac{1}{32}$$

$$\Rightarrow \left(\frac{df}{dx} \right)_{x=16} = 0.03125$$

Recall that if $y = f(x)$ and Δx is a small increment in x , then the corresponding increment in y , $\Delta y = f(x + \Delta x) - f(x)$, is approximately given as

$$\Delta y = \left(\frac{dy}{dx} \right) \Delta x$$

Here, $\frac{df}{dx} = 0.03125$ and $\Delta x = -1$

$$\Rightarrow \Delta f = (0.03125)(-1)$$

$$\therefore \Delta f = -0.03125$$

Now, we have $f(15) = f(16) + \Delta f$

$$\Rightarrow f(15) = (16)^{\frac{1}{4}} - 0.03125$$

$$\Rightarrow f(15) = (2^4)^{\frac{1}{4}} - 0.03125$$

$$\Rightarrow f(15) = 2 - 0.03125$$

$$\therefore f(15) = 1.96875$$

$$\text{Thus, } (15)^{1/4} \approx 1.96875$$

9 F. Question

Using differentials, find the approximate values of the following:

$$(255)^{1/4}$$

Answer

Let us assume that $f(x) = x^{\frac{1}{4}}$

Also, let $x = 256$ so that $x + \Delta x = 255$

$$\Rightarrow 256 + \Delta x = 255$$

$$\therefore \Delta x = -1$$

On differentiating $f(x)$ with respect to x , we get

$$\frac{df}{dx} = \frac{d}{dx} \left(x^{\frac{1}{4}} \right)$$

We know $\frac{d}{dx} (x^n) = nx^{n-1}$

$$\Rightarrow \frac{df}{dx} = \frac{1}{4} x^{\frac{1}{4}-1}$$

$$\Rightarrow \frac{df}{dx} = \frac{1}{4} x^{-\frac{3}{4}}$$

$$\therefore \frac{df}{dx} = \frac{1}{4x^{\frac{3}{4}}}$$

When $x = 256$, we have $\frac{df}{dx} = \frac{1}{4(256)^{\frac{3}{4}}}$

$$\Rightarrow \left(\frac{df}{dx} \right)_{x=256} = \frac{1}{4(4^4)^{\frac{3}{4}}}$$

$$\Rightarrow \left(\frac{df}{dx} \right)_{x=256} = \frac{1}{4(4^3)}$$

$$\Rightarrow \left(\frac{df}{dx} \right)_{x=256} = \frac{1}{4(64)}$$

$$\Rightarrow \left(\frac{df}{dx} \right)_{x=256} = \frac{1}{256}$$

$$\Rightarrow \left(\frac{df}{dx} \right)_{x=256} = 0.00390625$$

Recall that if $y = f(x)$ and Δx is a small increment in x , then the corresponding increment in y , $\Delta y = f(x + \Delta x) - f(x)$, is approximately given as

$$\Delta y = \left(\frac{dy}{dx} \right) \Delta x$$

Here, $\frac{df}{dx} = 0.00390625$ and $\Delta x = -1$

$$\Rightarrow \Delta f = (0.00390625)(-1)$$

$$\therefore \Delta f = -0.00390625$$

Now, we have $f(255) = f(256) + \Delta f$

$$\Rightarrow f(255) = (256)^{\frac{1}{4}} - 0.00390625$$

$$\Rightarrow f(255) = (4^4)^{\frac{1}{4}} - 0.00390625$$

$$\Rightarrow f(255) = 4 - 0.00390625$$

$$\therefore f(255) = 3.99609375$$

Thus, $(255)^{1/4} \approx 3.99609375$

9 G. Question

Using differentials, find the approximate values of the following:

$$\frac{1}{(2.002)^2}$$

Answer

Let us assume that $f(x) = \frac{1}{x^2}$

Also, let $x = 2$ so that $x + \Delta x = 2.002$

$$\Rightarrow 2 + \Delta x = 2.002$$

$$\therefore \Delta x = 0.002$$

On differentiating $f(x)$ with respect to x , we get

$$\frac{df}{dx} = \frac{d}{dx} \left(\frac{1}{x^2} \right)$$

$$\Rightarrow \frac{df}{dx} = \frac{d}{dx} (x^{-2})$$

We know $\frac{d}{dx} (x^n) = nx^{n-1}$

$$\Rightarrow \frac{df}{dx} = -2x^{-2-1}$$

$$\Rightarrow \frac{df}{dx} = -2x^{-3}$$

$$\therefore \frac{df}{dx} = -\frac{2}{x^3}$$

When $x = 2$, we have $\frac{df}{dx} = -\frac{2}{2^3}$

$$\Rightarrow \left(\frac{df}{dx} \right)_{x=2} = -\frac{2}{8}$$

$$\Rightarrow \left(\frac{df}{dx} \right)_{x=2} = -0.25$$

Recall that if $y = f(x)$ and Δx is a small increment in x , then the corresponding increment in y , $\Delta y = f(x + \Delta x) - f(x)$, is approximately given as

$$\Delta y = \left(\frac{dy}{dx} \right) \Delta x$$

Here, $\frac{df}{dx} = -0.25$ and $\Delta x = 0.002$

$$\Rightarrow \Delta f = (-0.25)(0.002)$$

$$\therefore \Delta f = -0.0005$$

Now, we have $f(2.002) = f(2) + \Delta f$

$$\Rightarrow f(2.002) = \frac{1}{(2)^2} - 0.0005$$

$$\Rightarrow f(2.002) = \frac{1}{4} - 0.0005$$

$$\Rightarrow f(2.002) = 0.25 - 0.0005$$

$$\therefore f(2.002) = 0.2495$$

Thus, $\frac{1}{(2.002)^2} \approx 0.2495$

9 H. Question

Using differentials, find the approximate values of the following:

$\log_e 4.04$, it being given that $\log_{10} 4 = 0.6021$ and $\log_{10} e = 0.4343$

Answer

$\log_e 4.04$, it being given that $\log_{10} 4 = 0.6021$ and $\log_{10} e = 0.4343$

Let us assume that $f(x) = \log_e x$

Also, let $x = 4$ so that $x + \Delta x = 4.04$

$$\Rightarrow 4 + \Delta x = 4.04$$

$$\therefore \Delta x = 0.04$$

On differentiating $f(x)$ with respect to x , we get

$$\frac{df}{dx} = \frac{d}{dx} (\log_e x)$$

$$\text{We know } \frac{d}{dx} (\log_e x) = \frac{1}{x}$$

$$\therefore \frac{df}{dx} = \frac{1}{x}$$

$$\text{When } x = 4, \text{ we have } \frac{df}{dx} = \frac{1}{4}$$

$$\Rightarrow \left(\frac{df}{dx} \right)_{x=4} = 0.25$$

Recall that if $y = f(x)$ and Δx is a small increment in x , then the corresponding increment in y , $\Delta y = f(x + \Delta x) - f(x)$, is approximately given as

$$\Delta y = \left(\frac{dy}{dx} \right) \Delta x$$

Here, $\frac{df}{dx} = 0.25$ and $\Delta x = 0.04$

$$\Rightarrow \Delta f = (0.25)(0.04)$$

$$\therefore \Delta f = 0.01$$

Now, we have $f(4.04) = f(4) + \Delta f$

$$\Rightarrow f(4.04) = \log_e 4 + 0.01$$

$$\Rightarrow f(4.04) = \frac{\log_{10} 4}{\log_{10} e} + 0.01 \left[\because \log_b a = \frac{\log_e a}{\log_e b} \right]$$

$$\Rightarrow f(4.04) = \frac{0.6021}{0.4343} + 0.01$$

$$\Rightarrow f(4.04) = 1.3863689 + 0.01$$

$$\therefore f(4.04) = 1.3963689$$

Thus, $\log_e 4.04 \approx 1.3963689$

9 I. Question

Using differentials, find the approximate values of the following:

$\log_e 10.02$, it being given that $\log_e 10 = 2.3026$

Answer

$\log_e 10.02$, it being given that $\log_e 10 = 2.3026$

Let us assume that $f(x) = \log_e x$

Also, let $x = 10$ so that $x + \Delta x = 10.02$

$$\Rightarrow 10 + \Delta x = 10.02$$

$$\therefore \Delta x = 0.02$$

On differentiating $f(x)$ with respect to x , we get

$$\frac{df}{dx} = \frac{d}{dx}(\log_e x)$$

$$\text{We know } \frac{d}{dx}(\log_e x) = \frac{1}{x}$$

$$\therefore \frac{df}{dx} = \frac{1}{x}$$

$$\text{When } x = 10, \text{ we have } \frac{df}{dx} = \frac{1}{10}$$

$$\Rightarrow \left(\frac{df}{dx} \right)_{x=10} = 0.1$$

Recall that if $y = f(x)$ and Δx is a small increment in x , then the corresponding increment in y , $\Delta y = f(x + \Delta x) - f(x)$, is approximately given as

$$\Delta y = \left(\frac{dy}{dx} \right) \Delta x$$

$$\text{Here, } \frac{df}{dx} = 0.1 \text{ and } \Delta x = 0.02$$

$$\Rightarrow \Delta f = (0.1)(0.02)$$

$$\therefore \Delta f = 0.002$$

Now, we have $f(10.02) = f(10) + \Delta f$

$$\Rightarrow f(10.02) = \log_e 10 + 0.002$$

$$\Rightarrow f(10.02) = 2.3026 + 0.002$$

$$\therefore f(10.02) = 2.3046$$

Thus, $\log_e 10.02 \approx 2.3046$

9 J. Question

Using differentials, find the approximate values of the following:

$\log_{10} 10.1$, it being given that $\log_{10} e = 0.4343$

Answer

$\log_{10} 10.1$, it being given that $\log_{10} e = 0.4343$

Let us assume that $f(x) = \log_{10} x$

Also, let $x = 10$ so that $x + \Delta x = 10.1$

$$\Rightarrow 10 + \Delta x = 10.1$$

$$\therefore \Delta x = 0.1$$

On differentiating $f(x)$ with respect to x , we get

$$\frac{df}{dx} = \frac{d}{dx} (\log_{10} x)$$

$$\Rightarrow \frac{df}{dx} = \frac{d}{dx} \left(\frac{\log_e x}{\log_e 10} \right) \left[\because \log_b a = \frac{\log_c a}{\log_c b} \right]$$

$$\Rightarrow \frac{df}{dx} = \frac{d}{dx} (\log_e x \times \log_{10} e) \left[\because \frac{1}{\log_a b} = \log_b a \right]$$

$$\Rightarrow \frac{df}{dx} = \log_{10} e \times \frac{d}{dx} (\log_e x)$$

$$\Rightarrow \frac{df}{dx} = 0.4343 \frac{d}{dx} (\log_e x)$$

$$\text{We know } \frac{d}{dx} (\log_e x) = \frac{1}{x}$$

$$\Rightarrow \frac{df}{dx} = 0.4343 \times \frac{1}{x}$$

$$\therefore \frac{df}{dx} = \frac{0.4343}{x}$$

$$\text{When } x = 10, \text{ we have } \frac{df}{dx} = \frac{0.4343}{10}$$

$$\Rightarrow \left(\frac{df}{dx} \right)_{x=10} = 0.04343$$

Recall that if $y = f(x)$ and Δx is a small increment in x , then the corresponding increment in y , $\Delta y = f(x + \Delta x) - f(x)$, is approximately given as

$$\Delta y = \left(\frac{dy}{dx} \right) \Delta x$$

$$\text{Here, } \frac{df}{dx} = 0.04343 \text{ and } \Delta x = 0.1$$

$$\Rightarrow \Delta f = (0.04343)(0.1)$$

$$\therefore \Delta f = 0.004343$$

Now, we have $f(10.1) = f(10) + \Delta f$

$$\Rightarrow f(10.1) = \log_{10} 10 + 0.004343$$

$$\Rightarrow f(10.1) = 1 + 0.004343 [\because \log_a a = 1]$$

$$\therefore f(10.1) = 1.004343$$

Thus, $\log_{10} 10.1 \approx 1.004343$

9 K. Question

Using differentials, find the approximate values of the following:

$\cos 61^\circ$, it being given that $\sin 60^\circ = 0.86603$ and $1^\circ = 0.01745$ radian

Answer

$\cos 61^\circ$, it being given that $\sin 60^\circ = 0.86603$ and $1^\circ = 0.01745$ radian

Let us assume that $f(x) = \cos x$

Also, let $x = 60^\circ$ so that $x + \Delta x = 61^\circ$

$$\Rightarrow 60^\circ + \Delta x = 61^\circ$$

$$\therefore \Delta x = 1^\circ = 0.01745 \text{ radian}$$

On differentiating $f(x)$ with respect to x , we get

$$\frac{df}{dx} = \frac{d}{dx}(\cos x)$$

$$\text{We know } \frac{d}{dx}(\cos x) = -\sin x$$

$$\therefore \frac{df}{dx} = -\sin x$$

$$\text{When } x = 60^\circ, \text{ we have } \frac{df}{dx} = -\sin 60^\circ$$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=60^\circ} = -0.86603$$

Recall that if $y = f(x)$ and Δx is a small increment in x , then the corresponding increment in y , $\Delta y = f(x + \Delta x) - f(x)$, is approximately given as

$$\Delta y = \left(\frac{dy}{dx}\right) \Delta x$$

$$\text{Here, } \frac{df}{dx} = -0.86603 \text{ and } \Delta x = 0.01745$$

$$\Rightarrow \Delta f = (-0.86603)(0.01745)$$

$$\therefore \Delta f = -0.0151122$$

Now, we have $f(61^\circ) = f(60^\circ) + \Delta f$

$$\Rightarrow f(61^\circ) = \cos(60^\circ) - 0.0151122$$

$$\Rightarrow f(61^\circ) = 0.5 - 0.0151122$$

$$\therefore f(61^\circ) = 0.4848878$$

Thus, $\cos 61^\circ \approx 0.4848878$

9 L. Question

Using differentials, find the approximate values of the following:

$$\frac{1}{\sqrt{25.1}}$$

Answer

Let us assume that $f(x) = \frac{1}{\sqrt{x}}$

Also, let $x = 25$ so that $x + \Delta x = 25.1$

$$\Rightarrow 25 + \Delta x = 25.1$$

$$\therefore \Delta x = 0.1$$

On differentiating $f(x)$ with respect to x , we get

$$\frac{df}{dx} = \frac{d}{dx} \left(\frac{1}{\sqrt{x}} \right)$$

$$\Rightarrow \frac{df}{dx} = \frac{d}{dx} \left(\frac{1}{x^{\frac{1}{2}}} \right)$$

$$\Rightarrow \frac{df}{dx} = \frac{d}{dx} \left(x^{-\frac{1}{2}} \right)$$

$$\Rightarrow \frac{df}{dx} = -\frac{1}{2} x^{-\frac{1}{2}-1}$$

$$\Rightarrow \frac{df}{dx} = -\frac{1}{2} x^{-\frac{3}{2}}$$

$$\therefore \frac{df}{dx} = -\frac{1}{2x^{\frac{3}{2}}}$$

When $x = 25$, we have $\frac{df}{dx} = -\frac{1}{2(25)^{\frac{3}{2}}}$

$$\Rightarrow \left(\frac{df}{dx} \right)_{x=25} = -\frac{1}{2(5^2)^{\frac{3}{2}}}$$

$$\Rightarrow \left(\frac{df}{dx} \right)_{x=25} = -\frac{1}{2(5^3)}$$

$$\Rightarrow \left(\frac{df}{dx} \right)_{x=25} = -\frac{1}{2(125)}$$

$$\Rightarrow \left(\frac{df}{dx} \right)_{x=25} = -\frac{1}{250}$$

$$\Rightarrow \left(\frac{df}{dx} \right)_{x=25} = -0.004$$

Recall that if $y = f(x)$ and Δx is a small increment in x , then the corresponding increment in y , $\Delta y = f(x + \Delta x) - f(x)$, is approximately given as

$$\Delta y = \left(\frac{dy}{dx} \right) \Delta x$$

Here, $\frac{df}{dx} = -0.004$ and $\Delta x = 0.1$

$$\Rightarrow \Delta f = (-0.004)(0.1)$$

$$\therefore \Delta f = -0.0004$$

Now, we have $f(25.1) = f(25) + \Delta f$

$$\Rightarrow f(25.1) = \frac{1}{\sqrt{25}} - 0.0004$$

$$\Rightarrow f(25.1) = \frac{1}{5} - 0.0004$$

$$\Rightarrow f(25.1) = 0.2 - 0.0004$$

$$\therefore f(15) = 0.1996$$

$$\text{Thus, } \frac{1}{\sqrt{25.1}} \approx 0.1996$$

9 M. Question

Using differentials, find the approximate values of the following:

$$\sin\left(\frac{22}{14}\right)$$

Answer

Let us assume that $f(x) = \sin x$

$$\text{Let } x = \frac{\pi}{2} \text{ so that } x + \Delta x = \frac{22}{14}$$

$$\Rightarrow \frac{\pi}{2} + \Delta x = \frac{22}{14}$$

$$\therefore \Delta x = \frac{22}{14} - \frac{\pi}{2}$$

On differentiating $f(x)$ with respect to x , we get

$$\frac{df}{dx} = \frac{d}{dx}(\sin x)$$

$$\text{We know } \frac{d}{dx}(\sin x) = \cos x$$

$$\therefore \frac{df}{dx} = \cos x$$

$$\text{When } x = \frac{\pi}{2}, \text{ we have } \frac{df}{dx} = \cos\left(\frac{\pi}{2}\right).$$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=\frac{\pi}{2}} = 0$$

Recall that if $y = f(x)$ and Δx is a small increment in x , then the corresponding increment in y , $\Delta y = f(x + \Delta x) - f(x)$, is approximately given as

$$\Delta y = \left(\frac{dy}{dx}\right) \Delta x$$

$$\text{Here, } \frac{df}{dx} = 0 \text{ and } \Delta x = \frac{22}{14} - \frac{\pi}{2}$$

$$\Rightarrow \Delta f = (0) \left(\frac{22}{14} - \frac{\pi}{2}\right)$$

$$\therefore \Delta f = 0$$

$$\text{Now, we have } f\left(\frac{22}{14}\right) = f\left(\frac{\pi}{2}\right) + \Delta f$$

$$\Rightarrow f\left(\frac{22}{14}\right) = \sin\left(\frac{\pi}{2}\right) + 0$$

$$\Rightarrow f\left(\frac{22}{14}\right) = \sin\left(\frac{\pi}{2}\right)$$

$$\therefore f\left(\frac{22}{14}\right) = 1$$

$$\text{Thus, } \sin\left(\frac{22}{14}\right) \approx 1$$

9 N. Question

Using differentials, find the approximate values of the following:

$$\cos\left(\frac{11\pi}{36}\right)$$

Answer

Let us assume that $f(x) = \cos x$

$$\text{Let } x = \frac{12\pi}{36} = \frac{\pi}{3} \text{ so that } x + \Delta x = \frac{11\pi}{36}$$

$$\Rightarrow \frac{\pi}{3} + \Delta x = \frac{11\pi}{36}$$

$$\Rightarrow \Delta x = -\frac{\pi}{36}$$

$$\Rightarrow \Delta x = -\frac{22}{36}$$

$$\therefore \Delta x = -0.0873$$

On differentiating $f(x)$ with respect to x , we get

$$\frac{df}{dx} = \frac{d}{dx}(\cos x)$$

$$\text{We know } \frac{d}{dx}(\cos x) = -\sin x$$

$$\therefore \frac{df}{dx} = -\sin x$$

$$\text{When } x = \frac{\pi}{3}, \text{ we have } \frac{df}{dx} = -\sin\left(\frac{\pi}{3}\right).$$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=\frac{\pi}{3}} = -0.86603$$

Recall that if $y = f(x)$ and Δx is a small increment in x , then the corresponding increment in y , $\Delta y = f(x + \Delta x) - f(x)$, is approximately given as

$$\Delta y = \left(\frac{dy}{dx}\right) \Delta x$$

$$\text{Here, } \frac{df}{dx} = -0.86603 \text{ and } \Delta x = -0.0873$$

$$\Rightarrow \Delta f = (-0.86603)(-0.0873)$$

$$\therefore \Delta f = 0.07560442$$

$$\text{Now, we have } f\left(\frac{11\pi}{36}\right) = f\left(\frac{\pi}{3}\right) + \Delta f$$

$$\Rightarrow f\left(\frac{11\pi}{36}\right) = \cos\left(\frac{\pi}{3}\right) + 0.07560442$$

$$\Rightarrow f\left(\frac{11\pi}{36}\right) = 0.5 + 0.07560442$$

$$\therefore f\left(\frac{11\pi}{36}\right) = 0.57560442$$

$$\text{Thus, } \cos\left(\frac{11\pi}{36}\right) \approx 0.57560442$$

9 O. Question

Using differentials, find the approximate values of the following:

$$(80)^{1/4}$$

Answer

Let us assume that $f(x) = x^{\frac{1}{4}}$

Also, let $x = 81$ so that $x + \Delta x = 80$

$$\Rightarrow 81 + \Delta x = 80$$

$$\therefore \Delta x = -1$$

On differentiating $f(x)$ with respect to x , we get

$$\frac{df}{dx} = \frac{d}{dx}\left(x^{\frac{1}{4}}\right)$$

$$\text{We know } \frac{d}{dx}(x^n) = nx^{n-1}$$

$$\Rightarrow \frac{df}{dx} = \frac{1}{4}x^{\frac{1}{4}-1}$$

$$\Rightarrow \frac{df}{dx} = \frac{1}{4}x^{-\frac{3}{4}}$$

$$\therefore \frac{df}{dx} = \frac{1}{4x^{\frac{3}{4}}}$$

$$\text{When } x = 81, \text{ we have } \frac{df}{dx} = \frac{1}{4(81)^{\frac{3}{4}}}$$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=81} = \frac{1}{4(3^4)^{\frac{3}{4}}}$$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=81} = \frac{1}{4(3^3)}$$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=81} = \frac{1}{4(27)}$$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=81} = \frac{1}{108}$$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=81} = 0.00926$$

Recall that if $y = f(x)$ and Δx is a small increment in x , then the corresponding increment in y , $\Delta y = f(x + \Delta x) - f(x)$, is approximately given as

$$\Delta y = \left(\frac{dy}{dx}\right) \Delta x$$

Here, $\frac{df}{dx} = 0.00926$ and $\Delta x = -1$

$$\Rightarrow \Delta f = (0.00926)(-1)$$

$$\therefore \Delta f = -0.00926$$

Now, we have $f(80) = f(81) + \Delta f$

$$\Rightarrow f(80) = (81)^{\frac{1}{4}} - 0.00926$$

$$\Rightarrow f(80) = (3^4)^{\frac{1}{4}} - 0.00926$$

$$\Rightarrow f(80) = 3 - 0.00926$$

$$\therefore f(80) = 2.99074$$

Thus, $(80)^{1/4} \approx 2.99074$

9 P. Question

Using differentials, find the approximate values of the following:

$$(29)^{1/3}$$

Answer

Let us assume that $f(x) = x^{\frac{1}{3}}$

Also, let $x = 27$ so that $x + \Delta x = 29$

$$\Rightarrow 27 + \Delta x = 29$$

$$\therefore \Delta x = 2$$

On differentiating $f(x)$ with respect to x , we get

$$\frac{df}{dx} = \frac{d}{dx} \left(x^{\frac{1}{3}} \right)$$

We know $\frac{d}{dx} (x^n) = nx^{n-1}$

$$\Rightarrow \frac{df}{dx} = \frac{1}{3} x^{\frac{1}{3}-1}$$

$$\Rightarrow \frac{df}{dx} = \frac{1}{3} x^{-\frac{2}{3}}$$

$$\therefore \frac{df}{dx} = \frac{1}{3x^{\frac{2}{3}}}$$

When $x = 27$, we have $\frac{df}{dx} = \frac{1}{3(27)^{\frac{2}{3}}}$

$$\Rightarrow \left(\frac{df}{dx} \right)_{x=27} = \frac{1}{3(3^3)^{\frac{2}{3}}}$$

$$\Rightarrow \left(\frac{df}{dx} \right)_{x=27} = \frac{1}{3 \times 3^2}$$

$$\Rightarrow \left(\frac{df}{dx} \right)_{x=27} = \frac{1}{3(9)}$$

$$\Rightarrow \left(\frac{df}{dx} \right)_{x=27} = \frac{1}{27}$$

$$\Rightarrow \left(\frac{df}{dx} \right)_{x=27} = 0.03704$$

Recall that if $y = f(x)$ and Δx is a small increment in x , then the corresponding increment in y , $\Delta y = f(x + \Delta x) - f(x)$, is approximately given as

$$\Delta y = \left(\frac{dy}{dx} \right) \Delta x$$

Here, $\frac{df}{dx} = 0.03704$ and $\Delta x = 2$

$$\Rightarrow \Delta f = (0.03704)(2)$$

$$\therefore \Delta f = 0.07408$$

Now, we have $f(29) = f(27) + \Delta f$

$$\Rightarrow f(29) = (27)^{\frac{1}{3}} + 0.07408$$

$$\Rightarrow f(29) = (3^3)^{\frac{1}{3}} + 0.07408$$

$$\Rightarrow f(29) = 3 + 0.07408$$

$$\therefore f(29) = 3.07408$$

Thus, $(29)^{1/3} \approx 3.07408$

9 Q. Question

Using differentials, find the approximate values of the following:

$$(66)^{1/3}$$

Answer

Let us assume that $f(x) = x^{\frac{1}{3}}$

Also, let $x = 64$ so that $x + \Delta x = 66$

$$\Rightarrow 64 + \Delta x = 66$$

$$\therefore \Delta x = 2$$

On differentiating $f(x)$ with respect to x , we get

$$\frac{df}{dx} = \frac{d}{dx} \left(x^{\frac{1}{3}} \right)$$

We know $\frac{d}{dx} (x^n) = nx^{n-1}$

$$\Rightarrow \frac{df}{dx} = \frac{1}{3} x^{\frac{1}{3}-1}$$

$$\Rightarrow \frac{df}{dx} = \frac{1}{3} x^{-\frac{2}{3}}$$

$$\therefore \frac{df}{dx} = \frac{1}{3x^{\frac{2}{3}}}$$

When $x = 64$, we have $\frac{df}{dx} = \frac{1}{3(64)^{\frac{2}{3}}}$

$$\Rightarrow \left(\frac{df}{dx} \right)_{x=64} = \frac{1}{3(4^3)^{\frac{2}{3}}}$$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=64} = \frac{1}{3 \times 4^2}$$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=64} = \frac{1}{3(16)}$$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=64} = \frac{1}{48}$$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=64} = 0.02083$$

Recall that if $y = f(x)$ and Δx is a small increment in x , then the corresponding increment in y , $\Delta y = f(x + \Delta x) - f(x)$, is approximately given as

$$\Delta y = \left(\frac{dy}{dx}\right) \Delta x$$

Here, $\frac{df}{dx} = 0.02083$ and $\Delta x = 2$

$$\Rightarrow \Delta f = (0.02083)(2)$$

$$\therefore \Delta f = 0.04166$$

Now, we have $f(66) = f(64) + \Delta f$

$$\Rightarrow f(66) = (64)^{\frac{1}{3}} + 0.04166$$

$$\Rightarrow f(66) = (4^3)^{\frac{1}{3}} + 0.04166$$

$$\Rightarrow f(66) = 4 + 0.04166$$

$$\therefore f(66) = 4.04166$$

Thus, $(66)^{1/3} \approx 4.04166$

9 R. Question

Using differentials, find the approximate values of the following:

$$\sqrt{26}$$

Answer

Let us assume that $f(x) = \sqrt{x}$

Also, let $x = 25$ so that $x + \Delta x = 26$

$$\Rightarrow 25 + \Delta x = 26$$

$$\therefore \Delta x = 1$$

On differentiating $f(x)$ with respect to x , we get

$$\frac{df}{dx} = \frac{d}{dx}(\sqrt{x})$$

$$\Rightarrow \frac{df}{dx} = \frac{d}{dx}\left(x^{\frac{1}{2}}\right)$$

We know $\frac{d}{dx}(x^n) = nx^{n-1}$

$$\Rightarrow \frac{df}{dx} = \frac{1}{2}x^{\frac{1}{2}-1}$$

$$\Rightarrow \frac{df}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$$

$$\therefore \frac{df}{dx} = \frac{1}{2\sqrt{x}}$$

When $x = 25$, we have $\frac{df}{dx} = \frac{1}{2\sqrt{25}}$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=25} = \frac{1}{2 \times 5}$$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=25} = \frac{1}{10}$$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=25} = 0.1$$

Recall that if $y = f(x)$ and Δx is a small increment in x , then the corresponding increment in y , $\Delta y = f(x + \Delta x) - f(x)$, is approximately given as

$$\Delta y = \left(\frac{dy}{dx}\right) \Delta x$$

Here, $\frac{df}{dx} = 0.1$ and $\Delta x = 1$

$$\Rightarrow \Delta f = (0.1)(1)$$

$$\therefore \Delta f = 0.1$$

Now, we have $f(26) = f(25) + \Delta f$

$$\Rightarrow f(26) = \sqrt{25} + 0.1$$

$$\Rightarrow f(26) = 5 + 0.1$$

$$\therefore f(26) = 5.1$$

Thus, $\sqrt{26} \approx 5.1$

9 S. Question

Using differentials, find the approximate values of the following:

$$\sqrt{37}$$

Answer

Let us assume that $f(x) = \sqrt{x}$

Also, let $x = 36$ so that $x + \Delta x = 37$

$$\Rightarrow 36 + \Delta x = 37$$

$$\therefore \Delta x = 1$$

On differentiating $f(x)$ with respect to x , we get

$$\frac{df}{dx} = \frac{d}{dx}(\sqrt{x})$$

$$\Rightarrow \frac{df}{dx} = \frac{d}{dx}\left(x^{\frac{1}{2}}\right)$$

We know $\frac{d}{dx}(x^n) = nx^{n-1}$

$$\Rightarrow \frac{df}{dx} = \frac{1}{2} x^{\frac{1}{2}-1}$$

$$\Rightarrow \frac{df}{dx} = \frac{1}{2} x^{-\frac{1}{2}}$$

$$\therefore \frac{df}{dx} = \frac{1}{2\sqrt{x}}$$

When $x = 36$, we have $\frac{df}{dx} = \frac{1}{2\sqrt{36}}$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=36} = \frac{1}{2 \times 6}$$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=36} = \frac{1}{12}$$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=36} = 0.08333$$

Recall that if $y = f(x)$ and Δx is a small increment in x , then the corresponding increment in y , $\Delta y = f(x + \Delta x) - f(x)$, is approximately given as

$$\Delta y = \left(\frac{dy}{dx}\right) \Delta x$$

Here, $\frac{df}{dx} = 0.08333$ and $\Delta x = 1$

$$\Rightarrow \Delta f = (0.08333)(1)$$

$$\therefore \Delta f = 0.08333$$

Now, we have $f(37) = f(36) + \Delta f$

$$\Rightarrow f(37) = \sqrt{36} + 0.08333$$

$$\Rightarrow f(37) = 6 + 0.08333$$

$$\therefore f(37) = 6.08333$$

Thus, $\sqrt{37} \approx 6.08333$

9 T. Question

Using differentials, find the approximate values of the following:

$$\sqrt{0.48}$$

Answer

Let us assume that $f(x) = \sqrt{x}$

Also, let $x = 0.49$ so that $x + \Delta x = 0.48$

$$\Rightarrow 0.49 + \Delta x = 0.48$$

$$\therefore \Delta x = -0.01$$

On differentiating $f(x)$ with respect to x , we get

$$\frac{df}{dx} = \frac{d}{dx}(\sqrt{x})$$

$$\Rightarrow \frac{df}{dx} = \frac{d}{dx}\left(x^{\frac{1}{2}}\right)$$

We know $\frac{d}{dx}(x^n) = nx^{n-1}$

$$\Rightarrow \frac{df}{dx} = \frac{1}{2}x^{\frac{1}{2}-1}$$

$$\Rightarrow \frac{df}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$$

$$\therefore \frac{df}{dx} = \frac{1}{2\sqrt{x}}$$

When $x = 0.49$, we have $\frac{df}{dx} = \frac{1}{2\sqrt{0.49}}$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=0.49} = \frac{1}{2 \times 0.7}$$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=0.49} = \frac{1}{1.4}$$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=0.49} = 0.7143$$

Recall that if $y = f(x)$ and Δx is a small increment in x , then the corresponding increment in y , $\Delta y = f(x + \Delta x) - f(x)$, is approximately given as

$$\Delta y = \left(\frac{dy}{dx}\right) \Delta x$$

Here, $\frac{df}{dx} = 0.7143$ and $\Delta x = -0.01$

$$\Rightarrow \Delta f = (0.7143)(-0.01)$$

$$\therefore \Delta f = -0.007143$$

Now, we have $f(0.48) = f(0.49) + \Delta f$

$$\Rightarrow f(0.48) = \sqrt{0.49} + 0.08333$$

$$\Rightarrow f(0.48) = 0.7 - 0.007143$$

$$\therefore f(0.48) = 0.692857$$

Thus, $\sqrt{0.48} \approx 0.692857$

9 U. Question

Using differentials, find the approximate values of the following:

$$(82)^{1/4}$$

Answer

Let us assume that $f(x) = x^{\frac{1}{4}}$

Also, let $x = 81$ so that $x + \Delta x = 82$

$$\Rightarrow 81 + \Delta x = 82$$

$$\therefore \Delta x = 1$$

On differentiating $f(x)$ with respect to x , we get

$$\frac{df}{dx} = \frac{d}{dx}\left(x^{\frac{1}{4}}\right)$$

We know $\frac{d}{dx}(x^n) = nx^{n-1}$

$$\Rightarrow \frac{df}{dx} = \frac{1}{4} x^{\frac{1}{4}-1}$$

$$\Rightarrow \frac{df}{dx} = \frac{1}{4} x^{-\frac{3}{4}}$$

$$\therefore \frac{df}{dx} = \frac{1}{4x^{\frac{3}{4}}}$$

When $x = 81$, we have $\frac{df}{dx} = \frac{1}{4(81)^{\frac{3}{4}}}$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=81} = \frac{1}{4(3^4)^{\frac{3}{4}}}$$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=81} = \frac{1}{4(3^3)}$$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=81} = \frac{1}{4(27)}$$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=81} = \frac{1}{108}$$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=81} = 0.00926$$

Recall that if $y = f(x)$ and Δx is a small increment in x , then the corresponding increment in y , $\Delta y = f(x + \Delta x) - f(x)$, is approximately given as

$$\Delta y = \left(\frac{dy}{dx}\right) \Delta x$$

Here, $\frac{df}{dx} = 0.00926$ and $\Delta x = 1$

$$\Rightarrow \Delta f = (0.00926)(1)$$

$$\therefore \Delta f = 0.00926$$

Now, we have $f(82) = f(81) + \Delta f$

$$\Rightarrow f(82) = (81)^{\frac{1}{4}} + 0.00926$$

$$\Rightarrow f(82) = (3^4)^{\frac{1}{4}} + 0.00926$$

$$\Rightarrow f(82) = 3 + 0.00926$$

$$\therefore f(82) = 3.00926$$

Thus, $(82)^{1/4} \approx 3.00926$

9 V. Question

Using differentials, find the approximate values of the following:

$$\left(\frac{17}{81}\right)^{1/4}$$

Answer

Let us assume that $f(x) = x^{\frac{1}{4}}$

Also, let $x = \frac{16}{81}$ so that $x + \Delta x = \frac{17}{81}$

$$\Rightarrow \frac{16}{81} + \Delta x = \frac{17}{81}$$

$$\therefore \Delta x = \frac{1}{81}$$

On differentiating $f(x)$ with respect to x , we get

$$\frac{df}{dx} = \frac{d}{dx} \left(x^{\frac{1}{4}} \right)$$

We know $\frac{d}{dx} (x^n) = nx^{n-1}$

$$\Rightarrow \frac{df}{dx} = \frac{1}{4} x^{\frac{1}{4}-1}$$

$$\Rightarrow \frac{df}{dx} = \frac{1}{4} x^{-\frac{3}{4}}$$

$$\therefore \frac{df}{dx} = \frac{1}{4x^{\frac{3}{4}}}$$

When $x = \frac{16}{81}$, we have $\frac{df}{dx} = \frac{1}{4 \left(\frac{16}{81} \right)^{\frac{3}{4}}}$

$$\Rightarrow \left(\frac{df}{dx} \right)_{x=\frac{16}{81}} = \frac{1}{4 \left(\left(\frac{2}{3} \right)^4 \right)^{\frac{3}{4}}}$$

$$\Rightarrow \left(\frac{df}{dx} \right)_{x=\frac{16}{81}} = \frac{1}{4 \left(\frac{2}{3} \right)^3}$$

$$\Rightarrow \left(\frac{df}{dx} \right)_{x=\frac{16}{81}} = \frac{1}{4 \left(\frac{8}{27} \right)}$$

$$\Rightarrow \left(\frac{df}{dx} \right)_{x=\frac{16}{81}} = \frac{27}{32}$$

$$\Rightarrow \left(\frac{df}{dx} \right)_{x=\frac{16}{81}} = 0.84375$$

Recall that if $y = f(x)$ and Δx is a small increment in x , then the corresponding increment in y , $\Delta y = f(x + \Delta x) - f(x)$, is approximately given as

$$\Delta y = \left(\frac{dy}{dx} \right) \Delta x$$

Here, $\frac{df}{dx} = 0.84375$ and $\Delta x = \frac{1}{81}$

$$\Rightarrow \Delta f = (0.84375) \left(\frac{1}{81} \right)$$

$$\therefore \Delta f = 0.0104166$$

Now, we have $f\left(\frac{17}{81}\right) = f\left(\frac{16}{81}\right) + \Delta f$

$$\Rightarrow f\left(\frac{17}{81}\right) = \left(\frac{16}{81}\right)^{\frac{1}{4}} + 0.0104166$$

$$\Rightarrow f\left(\frac{16}{81}\right) = \left(\left(\frac{2}{3}\right)^4\right)^{\frac{1}{4}} + 0.0104166$$

$$\Rightarrow f\left(\frac{16}{81}\right) = \frac{2}{3} + 0.0104166$$

$$\Rightarrow f\left(\frac{16}{81}\right) = 0.666666 + 0.0104166$$

$$\therefore f\left(\frac{16}{81}\right) = 0.6778026$$

$$\text{Thus, } \left(\frac{17}{81}\right)^{1/4} \approx 0.6778026$$

9 W. Question

Using differentials, find the approximate values of the following:

$$(33)^{1/5}$$

Answer

Let us assume that $f(x) = x^{\frac{1}{5}}$

Also, let $x = 32$ so that $x + \Delta x = 33$

$$\Rightarrow 32 + \Delta x = 33$$

$$\therefore \Delta x = 1$$

On differentiating $f(x)$ with respect to x , we get

$$\frac{df}{dx} = \frac{d}{dx} \left(x^{\frac{1}{5}} \right)$$

$$\text{We know } \frac{d}{dx} (x^n) = nx^{n-1}$$

$$\Rightarrow \frac{df}{dx} = \frac{1}{5} x^{\frac{1}{5}-1}$$

$$\Rightarrow \frac{df}{dx} = \frac{1}{5} x^{-\frac{4}{5}}$$

$$\therefore \frac{df}{dx} = \frac{1}{5x^{\frac{4}{5}}}$$

$$\text{When } x = 32, \text{ we have } \frac{df}{dx} = \frac{1}{5(32)^{\frac{4}{5}}}$$

$$\Rightarrow \left(\frac{df}{dx} \right)_{x=32} = \frac{1}{5(2^5)^{\frac{4}{5}}}$$

$$\Rightarrow \left(\frac{df}{dx} \right)_{x=32} = \frac{1}{5(2^4)}$$

$$\Rightarrow \left(\frac{df}{dx} \right)_{x=32} = \frac{1}{5(16)}$$

$$\Rightarrow \left(\frac{df}{dx} \right)_{x=32} = \frac{1}{80}$$

$$\Rightarrow \left(\frac{df}{dx} \right)_{x=32} = 0.0125$$

Recall that if $y = f(x)$ and Δx is a small increment in x , then the corresponding increment in y , $\Delta y = f(x + \Delta x) - f(x)$, is approximately given as

$$\Delta y = \left(\frac{dy}{dx}\right) \Delta x$$

Here, $\frac{df}{dx} = 0.0125$ and $\Delta x = 1$

$$\Rightarrow \Delta f = (0.0125)(1)$$

$$\therefore \Delta f = 0.0125$$

Now, we have $f(33) = f(32) + \Delta f$

$$\Rightarrow f(33) = (32)^{\frac{1}{5}} + 0.0125$$

$$\Rightarrow f(33) = (2^5)^{\frac{1}{5}} + 0.0125$$

$$\Rightarrow f(33) = 2 + 0.0125$$

$$\therefore f(33) = 2.0125$$

Thus, $(33)^{1/5} \approx 2.0125$

9 X. Question

Using differentials, find the approximate values of the following:

$$\sqrt{36.6}$$

Answer

Let us assume that $f(x) = \sqrt{x}$

Also, let $x = 36$ so that $x + \Delta x = 36.6$

$$\Rightarrow 36 + \Delta x = 36.6$$

$$\therefore \Delta x = 0.6$$

On differentiating $f(x)$ with respect to x , we get

$$\frac{df}{dx} = \frac{d}{dx}(\sqrt{x})$$

$$\Rightarrow \frac{df}{dx} = \frac{d}{dx}\left(x^{\frac{1}{2}}\right)$$

We know $\frac{d}{dx}(x^n) = nx^{n-1}$

$$\Rightarrow \frac{df}{dx} = \frac{1}{2}x^{\frac{1}{2}-1}$$

$$\Rightarrow \frac{df}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$$

$$\therefore \frac{df}{dx} = \frac{1}{2\sqrt{x}}$$

When $x = 36$, we have $\frac{df}{dx} = \frac{1}{2\sqrt{36}}$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=36} = \frac{1}{2 \times 6}$$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=36} = \frac{1}{12}$$

$$\Rightarrow \left(\frac{df}{dx} \right)_{x=36} = 0.0833333$$

Recall that if $y = f(x)$ and Δx is a small increment in x , then the corresponding increment in y , $\Delta y = f(x + \Delta x) - f(x)$, is approximately given as

$$\Delta y = \left(\frac{dy}{dx} \right) \Delta x$$

Here, $\frac{df}{dx} = 0.0833333$ and $\Delta x = 0.6$

$$\Rightarrow \Delta f = (0.0833333)(0.6)$$

$$\therefore \Delta f = 0.05$$

Now, we have $f(36.6) = f(36) + \Delta f$

$$\Rightarrow f(36.6) = \sqrt{36} + 0.05$$

$$\Rightarrow f(36.6) = 6 + 0.05$$

$$\therefore f(36.6) = 6.05$$

Thus, $\sqrt{36.6} \approx 6.05$

9 Y. Question

Using differentials, find the approximate values of the following:

$$25^{1/3}$$

Answer

Let us assume that $f(x) = x^{\frac{1}{3}}$

Also, let $x = 27$ so that $x + \Delta x = 25$

$$\Rightarrow 27 + \Delta x = 25$$

$$\therefore \Delta x = -2$$

On differentiating $f(x)$ with respect to x , we get

$$\frac{df}{dx} = \frac{d}{dx} \left(x^{\frac{1}{3}} \right)$$

We know $\frac{d}{dx} (x^n) = nx^{n-1}$

$$\Rightarrow \frac{df}{dx} = \frac{1}{3} x^{\frac{1}{3}-1}$$

$$\Rightarrow \frac{df}{dx} = \frac{1}{3} x^{-\frac{2}{3}}$$

$$\therefore \frac{df}{dx} = \frac{1}{3x^{\frac{2}{3}}}$$

When $x = 27$, we have $\frac{df}{dx} = \frac{1}{3(27)^{\frac{2}{3}}}$

$$\Rightarrow \left(\frac{df}{dx} \right)_{x=27} = \frac{1}{3(3^3)^{\frac{2}{3}}}$$

$$\Rightarrow \left(\frac{df}{dx} \right)_{x=27} = \frac{1}{3 \times 3^2}$$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=27} = \frac{1}{3(9)}$$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=27} = \frac{1}{27}$$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=27} = 0.03704$$

Recall that if $y = f(x)$ and Δx is a small increment in x , then the corresponding increment in y , $\Delta y = f(x + \Delta x) - f(x)$, is approximately given as

$$\Delta y = \left(\frac{dy}{dx}\right) \Delta x$$

Here, $\frac{df}{dx} = 0.03704$ and $\Delta x = 2$

$$\Rightarrow \Delta f = (0.03704)(-2)$$

$$\therefore \Delta f = -0.07408$$

Now, we have $f(25) = f(27) + \Delta f$

$$\Rightarrow f(25) = (27)^{\frac{1}{3}} - 0.07408$$

$$\Rightarrow f(25) = (3^3)^{\frac{1}{3}} - 0.07408$$

$$\Rightarrow f(25) = 3 - 0.07408$$

$$\therefore f(25) = 2.92592$$

Thus, $(25)^{1/3} \approx 2.92592$

9 Z. Question

Using differentials, find the approximate values of the following:

$$\sqrt{49.5}$$

Answer

Let us assume that $f(x) = \sqrt{x}$

Also, let $x = 49$ so that $x + \Delta x = 49.5$

$$\Rightarrow 49 + \Delta x = 49.5$$

$$\therefore \Delta x = 0.5$$

On differentiating $f(x)$ with respect to x , we get

$$\frac{df}{dx} = \frac{d}{dx}(\sqrt{x})$$

$$\Rightarrow \frac{df}{dx} = \frac{d}{dx}\left(x^{\frac{1}{2}}\right)$$

We know $\frac{d}{dx}(x^n) = nx^{n-1}$

$$\Rightarrow \frac{df}{dx} = \frac{1}{2}x^{\frac{1}{2}-1}$$

$$\Rightarrow \frac{df}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$$

$$\therefore \frac{df}{dx} = \frac{1}{2\sqrt{x}}$$

When $x = 49$, we have $\frac{df}{dx} = \frac{1}{2\sqrt{49}}$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=49} = \frac{1}{2 \times 7}$$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=49} = \frac{1}{14}$$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=49} = 0.0714286$$

Recall that if $y = f(x)$ and Δx is a small increment in x , then the corresponding increment in y , $\Delta y = f(x + \Delta x) - f(x)$, is approximately given as

$$\Delta y = \left(\frac{dy}{dx}\right) \Delta x$$

Here, $\frac{df}{dx} = 0.0714286$ and $\Delta x = 0.5$

$$\Rightarrow \Delta f = (0.0714286)(0.5)$$

$$\therefore \Delta f = 0.0357143$$

Now, we have $f(49.5) = f(49) + \Delta f$

$$\Rightarrow f(49.5) = \sqrt{49} + 0.0357143$$

$$\Rightarrow f(49.5) = 7 + 0.0357143$$

$$\therefore f(49.5) = 7.0357143$$

Thus, $\sqrt{49.5} \approx 7.0357143$

9 A1. Question

Using differentials, find the approximate values of the following:

$$(3.968)^{3/2}$$

Answer

Let us assume that $f(x) = x^{\frac{3}{2}}$

Also, let $x = 4$ so that $x + \Delta x = 3.968$

$$\Rightarrow 4 + \Delta x = 3.968$$

$$\therefore \Delta x = -0.032$$

On differentiating $f(x)$ with respect to x , we get

$$\frac{df}{dx} = \frac{d}{dx} \left(x^{\frac{3}{2}} \right)$$

We know $\frac{d}{dx} (x^n) = nx^{n-1}$

$$\Rightarrow \frac{df}{dx} = \frac{3}{2} x^{\frac{3}{2}-1}$$

$$\Rightarrow \frac{df}{dx} = \frac{3}{2} x^{\frac{1}{2}}$$

$$\therefore \frac{df}{dx} = \frac{3}{2} \sqrt{x}$$

When $x = 4$, we have $\frac{df}{dx} = \frac{3}{2} \sqrt{4}$

$$\Rightarrow \left(\frac{df}{dx} \right)_{x=4} = \frac{3}{2} \times 2$$

$$\Rightarrow \left(\frac{df}{dx} \right)_{x=4} = 3$$

Recall that if $y = f(x)$ and Δx is a small increment in x , then the corresponding increment in y , $\Delta y = f(x + \Delta x) - f(x)$, is approximately given as

$$\Delta y = \left(\frac{dy}{dx} \right) \Delta x$$

Here, $\frac{df}{dx} = 3$ and $\Delta x = -0.032$

$$\Rightarrow \Delta f = (3)(-0.032)$$

$$\therefore \Delta f = -0.096$$

Now, we have $f(3.968) = f(4) + \Delta f$

$$\Rightarrow f(3.968) = (4)^{\frac{3}{2}} - 0.096$$

$$\Rightarrow f(3.968) = (2^2)^{\frac{3}{2}} - 0.096$$

$$\Rightarrow f(3.968) = 2^3 - 0.096$$

$$\Rightarrow f(3.968) = 8 - 0.096$$

$$\therefore f(3.968) = 7.904$$

Thus, $(3.968)^{3/2} \approx 7.904$

9 B1. Question

Using differentials, find the approximate values of the following:

$$(1.999)^5$$

Answer

Let us assume that $f(x) = x^5$

Also, let $x = 2$ so that $x + \Delta x = 1.999$

$$\Rightarrow 2 + \Delta x = 1.999$$

$$\therefore \Delta x = -0.001$$

On differentiating $f(x)$ with respect to x , we get

$$\frac{df}{dx} = \frac{d}{dx}(x^5)$$

We know $\frac{d}{dx}(x^n) = nx^{n-1}$

$$\Rightarrow \frac{df}{dx} = 5x^{5-1}$$

$$\therefore \frac{df}{dx} = 5x^4$$

When $x = 2$, we have $\frac{df}{dx} = 5(2)^4$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=2} = 5 \times 16$$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=2} = 80$$

Recall that if $y = f(x)$ and Δx is a small increment in x , then the corresponding increment in y , $\Delta y = f(x + \Delta x) - f(x)$, is approximately given as

$$\Delta y = \left(\frac{dy}{dx}\right) \Delta x$$

Here, $\frac{df}{dx} = 80$ and $\Delta x = -0.001$

$$\Rightarrow \Delta f = (80)(-0.001)$$

$$\therefore \Delta f = -0.08$$

Now, we have $f(1.999) = f(2) + \Delta f$

$$\Rightarrow f(1.999) = 2^5 - 0.08$$

$$\Rightarrow f(1.999) = 32 - 0.08$$

$$\therefore f(1.999) = 31.92$$

Thus, $(1.999)^5 \approx 31.92$

9 C1. Question

Using differentials, find the approximate values of the following:

$$\sqrt{0.082}$$

Answer

Let us assume that $f(x) = \sqrt{x}$

Also, let $x = 0.09$ so that $x + \Delta x = 0.082$

$$\Rightarrow 0.09 + \Delta x = 0.082$$

$$\therefore \Delta x = -0.008$$

On differentiating $f(x)$ with respect to x , we get

$$\frac{df}{dx} = \frac{d}{dx}(\sqrt{x})$$

$$\Rightarrow \frac{df}{dx} = \frac{d}{dx}\left(x^{\frac{1}{2}}\right)$$

We know $\frac{d}{dx}(x^n) = nx^{n-1}$

$$\Rightarrow \frac{df}{dx} = \frac{1}{2}x^{\frac{1}{2}-1}$$

$$\Rightarrow \frac{df}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$$

$$\therefore \frac{df}{dx} = \frac{1}{2\sqrt{x}}$$

When $x = 0.09$, we have $\frac{df}{dx} = \frac{1}{2\sqrt{0.09}}$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=0.09} = \frac{1}{2 \times 0.3}$$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=0.09} = \frac{1}{0.6}$$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=0.09} = 1.6667$$

Recall that if $y = f(x)$ and Δx is a small increment in x , then the corresponding increment in y , $\Delta y = f(x + \Delta x) - f(x)$, is approximately given as

$$\Delta y = \left(\frac{dy}{dx}\right) \Delta x$$

Here, $\frac{df}{dx} = 1.6667$ and $\Delta x = -0.008$

$$\Rightarrow \Delta f = (1.6667)(-0.008)$$

$$\therefore \Delta f = -0.013334$$

Now, we have $f(0.082) = f(0.09) + \Delta f$

$$\Rightarrow f(0.082) = \sqrt{0.09} - 0.013334$$

$$\Rightarrow f(0.082) = 0.3 - 0.013334$$

$$\therefore f(0.082) = 0.286666$$

Thus, $\sqrt{0.082} \approx 0.286666$

10. Question

Find the approximate value of $f(2.01)$, where $f(x) = 4x^2 + 5x + 2$.

Answer

Given $f(x) = 4x^2 + 5x + 2$

Let $x = 2$ so that $x + \Delta x = 2.01$

$$\Rightarrow 2 + \Delta x = 2.01$$

$$\therefore \Delta x = 0.01$$

On differentiating $f(x)$ with respect to x , we get

$$\frac{df}{dx} = \frac{d}{dx}(4x^2 + 5x + 2)$$

$$\Rightarrow \frac{df}{dx} = \frac{d}{dx}(4x^2) + \frac{d}{dx}(5x) + \frac{d}{dx}(2)$$

$$\Rightarrow \frac{df}{dx} = 4 \frac{d}{dx}(x^2) + 5 \frac{d}{dx}(x) + \frac{d}{dx}(2)$$

We know $\frac{d}{dx}(x^n) = nx^{n-1}$ and derivative of a constant is 0.

$$\Rightarrow \frac{df}{dx} = 4(2x) + 5(1) + 0$$

$$\therefore \frac{df}{dx} = 8x + 5$$

When $x = 2$, we have $\frac{df}{dx} = 8(2) + 5$

$$\Rightarrow \left(\frac{df}{dx} \right)_{x=2} = 21$$

Recall that if $y = f(x)$ and Δx is a small increment in x , then the corresponding increment in y , $\Delta y = f(x + \Delta x) - f(x)$, is approximately given as

$$\Delta y = \left(\frac{dy}{dx} \right) \Delta x$$

Here, $\frac{df}{dx} = 21$ and $\Delta x = 0.01$

$$\Rightarrow \Delta f = (21)(0.01)$$

$$\therefore \Delta f = 0.21$$

Now, we have $f(2.01) = f(2) + \Delta f$

$$\Rightarrow f(2.01) = 4(2)^2 + 5(2) + 2 + 0.21$$

$$\Rightarrow f(2.01) = 16 + 10 + 2 + 0.21$$

$$\therefore f(2.01) = 28.21$$

Thus, $f(2.01) = 28.21$

11. Question

Find the approximate value of $f(5.001)$, where $f(x) = x^3 - 7x^2 + 15$.

Answer

Given $f(x) = x^3 - 7x^2 + 15$

Let $x = 5$ so that $x + \Delta x = 5.001$

$$\Rightarrow 5 + \Delta x = 5.001$$

$$\therefore \Delta x = 0.001$$

On differentiating $f(x)$ with respect to x , we get

$$\frac{df}{dx} = \frac{d}{dx}(x^3 - 7x^2 + 15)$$

$$\Rightarrow \frac{df}{dx} = \frac{d}{dx}(x^3) + \frac{d}{dx}(-7x^2) + \frac{d}{dx}(15)$$

$$\Rightarrow \frac{df}{dx} = \frac{d}{dx}(x^3) - 7 \frac{d}{dx}(x^2) + \frac{d}{dx}(15)$$

We know $\frac{d}{dx}(x^n) = nx^{n-1}$ and derivative of a constant is 0.

$$\Rightarrow \frac{df}{dx} = 3x^2 - 7(2x) + 0$$

$$\therefore \frac{df}{dx} = 3x^2 - 14x$$

When $x = 5$, we have $\frac{df}{dx} = 3(5)^2 - 14(5)$

$$\Rightarrow \left(\frac{df}{dx} \right)_{x=5} = 75 - 70$$

$$\Rightarrow \left(\frac{df}{dx} \right)_{x=5} = 5$$

Recall that if $y = f(x)$ and Δx is a small increment in x , then the corresponding increment in y , $\Delta y = f(x + \Delta x)$

- $f(x)$, is approximately given as

$$\Delta y = \left(\frac{dy}{dx} \right) \Delta x$$

Here, $\frac{df}{dx} = 5$ and $\Delta x = 0.001$

$$\Rightarrow \Delta f = (5)(0.001)$$

$$\therefore \Delta f = 0.005$$

Now, we have $f(5.001) = f(5) + \Delta f$

$$\Rightarrow f(5.001) = 5^3 - 7(5)^2 + 15 + 0.005$$

$$\Rightarrow f(5.001) = 125 - 175 + 15 + 0.005$$

$$\Rightarrow f(5.001) = -35 + 0.005$$

$$\therefore f(5.001) = -34.995$$

Thus, $f(5.001) = -34.995$

12. Question

Find the approximate value of $\log_{10} 1005$, given that $\log_{10} e = 0.4343$.

Answer

Let us assume that $f(x) = \log_{10} x$

Also, let $x = 1000$ so that $x + \Delta x = 1005$

$$\Rightarrow 1000 + \Delta x = 1005$$

$$\therefore \Delta x = 5$$

On differentiating $f(x)$ with respect to x , we get

$$\frac{df}{dx} = \frac{d}{dx} (\log_{10} x)$$

$$\Rightarrow \frac{df}{dx} = \frac{d}{dx} \left(\frac{\log_e x}{\log_e 10} \right) \left[\because \log_b a = \frac{\log_c a}{\log_c b} \right]$$

$$\Rightarrow \frac{df}{dx} = \frac{d}{dx} (\log_e x \times \log_{10} e) \left[\because \frac{1}{\log_a b} = \log_b a \right]$$

$$\Rightarrow \frac{df}{dx} = \log_{10} e \times \frac{d}{dx} (\log_e x)$$

$$\Rightarrow \frac{df}{dx} = 0.4343 \frac{d}{dx} (\log_e x)$$

We know $\frac{d}{dx} (\log_e x) = \frac{1}{x}$

$$\Rightarrow \frac{df}{dx} = 0.4343 \times \frac{1}{x}$$

$$\therefore \frac{df}{dx} = \frac{0.4343}{x}$$

When $x = 1000$, we have $\frac{df}{dx} = \frac{0.4343}{1000}$

$$\Rightarrow \left(\frac{df}{dx} \right)_{x=1000} = 0.0004343$$

Recall that if $y = f(x)$ and Δx is a small increment in x , then the corresponding increment in y , $\Delta y = f(x + \Delta x) - f(x)$, is approximately given as

$$\Delta y = \left(\frac{dy}{dx} \right) \Delta x$$

Here, $\frac{df}{dx} = 0.0004343$ and $\Delta x = 5$

$$\Rightarrow \Delta f = (0.0004343)(5)$$

$$\therefore \Delta f = 0.0021715$$

Now, we have $f(1005) = f(1000) + \Delta f$

$$\Rightarrow f(1005) = \log_{10} 1000 + 0.0021715$$

$$\Rightarrow f(1005) = \log_{10} 10^3 + 0.0021715$$

$$\Rightarrow f(1005) = 3 \times \log_{10} 10 + 0.0021715$$

$$\Rightarrow f(1005) = 3 + 0.0021715 [\because \log_a a = 1]$$

$$\therefore f(1005) = 3.0021715$$

Thus, $\log_{10} 1005 = 3.0021715$

13. Question

If the radius of a sphere is measured as 9 cm with an error of 0.03 m, find the approximate error in calculating its surface area.

Answer

Given the radius of a sphere is measured as 9 cm with an error of 0.03 m = 3 cm.

Let x be the radius of the sphere and Δx be the error in measuring the value of x .

Hence, we have $x = 9$ and $\Delta x = 3$

The surface area of a sphere of radius x is given by

$$S = 4\pi x^2$$

On differentiating S with respect to x , we get

$$\frac{dS}{dx} = \frac{d}{dx} (4\pi x^2)$$

$$\Rightarrow \frac{dS}{dx} = 4\pi \frac{d}{dx} (x^2)$$

We know $\frac{d}{dx} (x^n) = nx^{n-1}$

$$\Rightarrow \frac{dS}{dx} = 4\pi (2x)$$

$$\therefore \frac{dS}{dx} = 8\pi x$$

When $x = 9$, we have $\frac{dS}{dx} = 8\pi (9)$.

$$\Rightarrow \left(\frac{dS}{dx} \right)_{x=9} = 72\pi$$

Recall that if $y = f(x)$ and Δx is a small increment in x , then the corresponding increment in y , $\Delta y = f(x + \Delta x) - f(x)$, is approximately given as

$$\Delta y = \left(\frac{dy}{dx} \right) \Delta x$$

Here, $\frac{dS}{dx} = 72\pi$ and $\Delta x = 3$

$$\Rightarrow \Delta S = (72\pi)(3)$$

$$\therefore \Delta S = 216\pi$$

Thus, the approximate error in calculating the surface area of the sphere is $216\pi \text{ cm}^2$.

14. Question

Find the approximate change in the surface area of cube of side x meters caused by decreasing the side by 1%.

Answer

Given a cube whose side x is decreased by 1%.

Let Δx be the change in the value of x .

Hence, we have $\Delta x = -\frac{1}{100} \times x$

$$\therefore \Delta x = -0.01x$$

The surface area of a cube of radius x is given by

$$S = 6x^2$$

On differentiating A with respect to x , we get

$$\frac{dS}{dx} = \frac{d}{dx}(6x^2)$$

$$\Rightarrow \frac{dS}{dx} = 6 \frac{d}{dx}(x^2)$$

We know $\frac{d}{dx}(x^n) = nx^{n-1}$

$$\Rightarrow \frac{dS}{dx} = 6(2x)$$

$$\therefore \frac{dS}{dx} = 12x$$

Recall that if $y = f(x)$ and Δx is a small increment in x , then the corresponding increment in y , $\Delta y = f(x + \Delta x) - f(x)$, is approximately given as

$$\Delta y = \left(\frac{dy}{dx} \right) \Delta x$$

Here, $\frac{dS}{dx} = 12x$ and $\Delta x = -0.01x$

$$\Rightarrow \Delta S = (12x)(-0.01x)$$

$$\therefore \Delta S = -0.12x^2$$

Thus, the approximate change in the surface area of the cube is $0.12x^2 \text{ m}^2$.

15. Question

If the radius of a sphere is measured as 7 m with an error of 0.02m, find the approximate error in calculating its volume.

Answer

Given the radius of a sphere is measured as 7 m with an error of 0.02 m.

Let x be the radius of the sphere and Δx be the error in measuring the value of x .

Hence, we have $x = 7$ and $\Delta x = 0.02$

The volume of a sphere of radius x is given by

$$V = \frac{4}{3}\pi x^3$$

On differentiating V with respect to x , we get

$$\begin{aligned}\frac{dV}{dx} &= \frac{d}{dx}\left(\frac{4}{3}\pi x^3\right) \\ \Rightarrow \frac{dV}{dx} &= \frac{4\pi}{3} \frac{d}{dx}(x^3)\end{aligned}$$

We know $\frac{d}{dx}(x^n) = nx^{n-1}$

$$\begin{aligned}\Rightarrow \frac{dV}{dx} &= \frac{4\pi}{3}(3x^2) \\ \therefore \frac{dV}{dx} &= 4\pi x^2\end{aligned}$$

When $x = 7$, we have $\frac{dV}{dx} = 4\pi(7)^2$.

$$\Rightarrow \left(\frac{dV}{dx}\right)_{x=7} = 4\pi \times 49$$

$$\Rightarrow \left(\frac{dV}{dx}\right)_{x=7} = 196\pi$$

Recall that if $y = f(x)$ and Δx is a small increment in x , then the corresponding increment in y , $\Delta y = f(x + \Delta x) - f(x)$, is approximately given as

$$\Delta y = \left(\frac{dy}{dx}\right) \Delta x$$

Here, $\frac{dV}{dx} = 196\pi$ and $\Delta x = 0.02$

$$\Rightarrow \Delta V = (196\pi)(0.02)$$

$$\therefore \Delta V = 3.92\pi$$

Thus, the approximate error in calculating the volume of the sphere is $3.92\pi \text{ m}^3$.

16. Question

Find the approximate change in the volume of a cube of side x meters caused by increasing the side by 1%.

Answer

Given a cube whose side x is increased by 1%.

Let Δx be the change in the value of x .

Hence, we have $\Delta x = \frac{1}{100} \times x$

$$\therefore \Delta x = 0.01x$$

The volume of a cube of radius x is given by

$$V = x^3$$

On differentiating A with respect to x , we get

$$\frac{dV}{dx} = \frac{d}{dx}(x^3)$$

We know $\frac{d}{dx}(x^n) = nx^{n-1}$

$$\Rightarrow \frac{dV}{dx} = 3x^{3-1}$$

$$\therefore \frac{dV}{dx} = 3x^2$$

Recall that if $y = f(x)$ and Δx is a small increment in x , then the corresponding increment in y , $\Delta y = f(x + \Delta x) - f(x)$, is approximately given as

$$\Delta y = \left(\frac{dy}{dx}\right) \Delta x$$

Here, $\frac{dV}{dx} = 3x^2$ and $\Delta x = 0.01x$

$$\Rightarrow \Delta V = (3x^2)(0.01x)$$

$$\therefore \Delta V = 0.03x^3$$

Thus, the approximate change in the volume of the cube is $0.03x^3 \text{ m}^3$.

MCQ

1. Question

Mark the correct alternative in the following:

If there is an error of 2% in measuring the length of a simple pendulum, then percentage error in its period is:

- A. 1%
- B. 2%
- C. 3%
- D. 4%

Answer

given $(\Delta L/L) \times 100 = 2$ (if we let the length of pendulum is L)

we all know the formula of period of a pendulum is $T = 2\pi \times \sqrt{L/g}$

By the formula of approximation in derivation, we get-

$$\left(\frac{\Delta T}{T}\right) \times 100 = \frac{1}{2} \times \left(\frac{\Delta L}{L}\right) \times 100$$

$$= \left(\frac{1}{2}\right) \times (2)$$

$$= 1\%$$

2. Question

Mark the correct alternative in the following:

If there is an error of $a\%$ in measuring the edge of a cube, then percentage error in its surface is:

- A. $2a\%$
- B. $\frac{a}{2}\%$

C. 3a%

D. none of these

Answer

given that

% Error in measuring the edge of a cube $[(\Delta L/L) \times 100]$ is = a (if L is edge of the cube)

We have to find out $(\Delta A/A) \times 100 = ?$ (If let the surface of the cube is A)

By the formula of approximation of derivation we get,

$$\left(\frac{\Delta A}{A}\right) \times 100 = 2 \times \left(\frac{\Delta L}{L}\right) \times 100$$

$$= 2 \times a$$

$$= 2a$$

3. Question

Mark the correct alternative in the following:

If an error of k% is made in measuring the radius of a sphere, then percentage error in its volume is

A. k%

B. 3k%

C. 2k%

D. $\frac{k}{3}\%$

Answer

given % error in measuring the radius of a sphere $\Delta r/r \times 100 = k$ (if let r is radius)

Find out : $(\Delta v/v) \times 100 = ?$

We know by the formula of the volume of the sphere

$$V = \frac{4}{3} \pi r^3$$

$$\text{So, } dV = \frac{4}{3} \pi \times 3r^2 dr$$

$$\text{So, } \frac{\Delta V}{V} = \frac{\frac{4}{3} \pi \times 3r^2 dr}{\frac{4}{3} \pi r^3}$$

$$\text{So, } \left(\frac{\Delta v}{v}\right) \times 100 = 3 \times \left(\frac{\Delta r}{r}\right) \times 100$$

$$= 3 \times k$$

$$= 3k\%$$

4. Question

Mark the correct alternative in the following:

The height of a cylinder is equal to the radius. If an error of $\alpha\%$ is made in the height, then percentage error in its volume is:

A. $\alpha\%$

B. $2\alpha\%$

C. $3\alpha\%$

D. none of these

Answer

let height of a cylinder = h = radius of that cylinder = r

% error in height $\Delta h/h \times 100 = a$ (given)

Volume of cylinder = $V = (1/3) \times \pi r^2 h$

We have given that $h = r$

Then

$$V = \frac{1}{3} \pi r^2 h = \frac{1}{3} \pi h^3$$

$$\text{So, } \Delta V = \frac{1}{3} \pi h^2 dh$$

Finally

$$\left(\frac{\Delta V}{V} \right) \times 100 = 3 \times \left(\frac{\Delta h}{h} \right) \times 100$$

$$= 3 \times a$$

$$= 3a\%$$

5. Question

Mark the correct alternative in the following:

While measuring the side of an equilateral triangle an error of $k\%$ is made, the percentage error in its area is

A. $k\%$

B. $2k\%$

C. $\frac{k}{2}\%$

D. $3k\%$

Answer

we know that the area of an equilateral triangle is $A = (\sqrt{3}/4) \times a^2$

Where a = side of equilateral triangle

So by the formula of approximation of derivation, we get,

$$\left(\frac{\Delta A}{A} \right) \times 100 = 2 \times \left(\frac{\Delta a}{a} \right) \times 100$$

$$= 2 \times k$$

$$= 2k\% \text{ ans}$$

6. Question

Mark the correct alternative in the following:

If $\log_e 4 = 1.3868$, then $\log_e 4.01 =$

A. 1.3968

B. 1.3898

C. 1.3893

D. none of these

Answer

let $y=f(x)=\log x$

Let $x=4$,

$X+\Delta x=4.01$,

$\Delta x=0.01$,

For $x=4$,

$Y=\log 4=1.3868$,

$y=\log x$

$$\frac{dy}{dx} = \frac{1}{x} = \frac{1}{4}$$

$\Delta y=dy$

$$= \left(\frac{dy}{dx} \right) \cdot dx$$

$$= \left(\frac{1}{4} \right) \times 0.01$$

$\Delta y=0.0025$

So, $\log(4.01)=y+\Delta y$

$=1.3893$

7. Question

Mark the correct alternative in the following:

A sphere of radius 100 mm shrinks to radius 98 mm, then the approximate decrease in its volume is

A. $12000\pi \text{ mm}^3$

B. $800\pi \text{ mm}^3$

C. $80000\pi \text{ mm}^3$

D. $120\pi \text{ mm}^3$

Answer

we know that volume of sphere = $v = (4/3) \times \pi r^3$ (r is radius of sphere)

$r = 100\text{mm}$

$$\Delta v = \left(\frac{4}{3} \right) \times \pi \times 3r^2 \Delta r$$

$$= 4\pi r^2 \Delta r$$

$$\Delta r = (98-100)$$

$$= -2$$

$$\Delta v = 4\pi(100)^2 \times (-2)$$

$$\Delta v = -80,000\pi \text{ mm}^3 \text{ans}$$

8. Question

Mark the correct alternative in the following:

If the ratio of base radius and height of a cone is 1 : 2 and percentage error in radius is $\lambda\%$, then the error in its volume is:

- A. $\lambda\%$
- B. $2\lambda\%$
- C. $3\lambda\%$
- D. none of these

Answer

given that the radius is half then the height of the cone so

Let $h = 2r$ (where r is radius and h is height of the cone)

Volume of the cone = v

$$= \left(\frac{1}{3}\right) \times \pi r^2 \times h$$

$$= \left(\frac{2}{3}\right) \times \pi r^3 \text{ (because } h = 2r \text{)}$$

$$\Delta v = \left(\frac{2}{3}\right) \pi \times 3r^2 \Delta r$$

$$\Delta v = 2\pi r^2 \Delta r$$

So finally ,

$$\left(\frac{\Delta v}{v}\right) \times 100 = 3 \cdot \left(\frac{\Delta r}{r}\right) \times 100$$

$$= 3 \times \lambda$$

$$= 3\lambda\%$$

9. Question

Mark the correct alternative in the following:

The pressure P and volume V of a gas are connected by the relation $PV^{1/4} = \text{constant}$. The percentage increase in the pressure corresponding to a deminition of $1/2\%$ in the volume is

A. $\frac{1}{2}\%$

B. $\frac{1}{4}\%$

C. $\frac{1}{8}\%$

- D. none of these

Answer

let $pv^{1/4}=k$ (constant)

$$Pv^{1/4}=k$$

$$P=k.v^{-1/4}$$

$$\log(p) = \log(k.v^{1/4})$$

$$\log(p) = \log(k) - (1/4)\log(v)$$

$$\frac{dP}{P} = 0 - \frac{1}{4} \times \frac{dV}{V}$$

$$\frac{dP}{P} = -\frac{1}{4} \times -\frac{1}{2} \%$$

$$= \frac{1}{8} \%$$

10. Question

Mark the correct alternative in the following:

If $y = x^n$, then the ratio of relative errors in y and x is

A. 1 : 1

B. 2 : 1

C. 1 : n

D. n : 1

Answer

given $y = x^n$

$$\Delta y = n \cdot x^{n-1} \cdot \Delta x = x$$

$$\frac{\frac{\Delta y}{y}}{\frac{\Delta x}{x}} = \frac{x}{y} \cdot \frac{\Delta y}{\Delta x}$$

$$= \frac{x}{y} \times \frac{n \cdot x^{n-1} \cdot \Delta x}{\Delta x}$$

$$= \frac{n \cdot x^n}{x^n}$$

$$= \frac{n}{1}$$

So finally ratio is = n:1

11. Question

Mark the correct alternative in the following:

The approximate value of $(33)^{1/5}$ is

A. 2.0125

B. 2.1

C. 2.01

D. none of these

Answer

$$f(x) = x^{1/5}$$

$$F'(x) = (1/5) \cdot x^{-4/5}$$

$$F(a+h) = f(a) + h \times f'(a)$$

$$(a+h)^{\frac{1}{5}} = a^{\frac{1}{5}} + h \times \left(\frac{1}{5}\right) \times (a)^{-\frac{4}{5}}$$

Now

Let $a = 32$ & $h=1$

$$(32 + 1)^{\frac{1}{5}} = (32)^{\frac{1}{5}} + 1 \times \left(\frac{1}{5}\right) \times (32)^{-\frac{4}{5}}$$

$$= 2 + 1 \times \left(\frac{1}{5}\right) \times (2)^{-4}$$

$$= 2 + \left(\frac{1}{80}\right)$$

$$= \frac{161}{80}$$

$$= 2.0125$$

12. Question

Mark the correct alternative in the following:

The circumference of a circle is measured as 28 cm with an error of 0.01 cm. The percentage error in the area is

A. $\frac{1}{14}$

B. 0.01

C. $\frac{1}{7}$

D. none of these

Answer

given that circumference is $= C = 2\pi r = 28$ cm

That's mean $r=14/\pi$

$$\Delta C = 2\pi \Delta r = 0.01$$

$$\Delta r = (0.01/2\pi)$$

We all know that area of a circle is $= A = \pi r^2$

$$\Delta A = 2\pi r \times dr$$

So finally,

$$\left(\frac{\Delta A}{A}\right) \times 100 = 2 \times \frac{\frac{0.01}{2\pi}}{\frac{14}{\pi}} \times 100$$

$$= 1/14$$

Very short answer

1. Question

For the function $y = x^2$, if $x = 10$ and $\Delta x = 0.1$. Find Δy .

Answer

by the formula of differentiation we all know that-

$$\frac{dy}{dx} = \frac{\Delta y}{\Delta x} \dots \dots \dots \text{eq(1)}$$

If $y=x^2$ then

$\frac{dy}{dx} = 2x$, so put the value of $\frac{dy}{dx}$ in eq(1), we get-

$$2x = \frac{\Delta y}{0.1}$$

$$\Delta y = 2 \times 10 \times (0.1)$$

$$\Delta y = 2$$

2. Question

If $y = \log_e x$, then find Δy when $x = 3$ and $\Delta x = 0.03$.

Answer

given that

$$Y = \log x \text{ then } y' = 1/x$$

$$\Delta y = ?$$

$$X = 3$$

$$\Delta x = 0.03$$

By putting the values of above in the formula $\frac{dy}{dx} = \frac{\Delta y}{\Delta x}$ we get

$$\frac{1}{x} = \frac{\Delta y}{0.03}$$

$$\frac{1}{3} = \frac{\Delta y}{0.03}$$

$$\Delta y = 0.01$$

3. Question

If the relative error in measuring the radius of a circular plane is α , find the relative error measuring its area.

Answer

given that

$$\frac{\Delta r}{r} = \alpha \text{ (if let } r \text{ is radius)}$$

$$\frac{\Delta A}{A} = ? \text{ (if let } A \text{ is area of circle)}$$

We know that the area of a circle (A) = πr^2 then

$$dA = 2\pi r \times dr$$

now

$$\frac{dA}{A} = \frac{2\pi r \times dr}{A}$$

$$\frac{dA}{A} = \frac{2\pi r \times dr}{\pi r^2}$$

$$\frac{dA}{A} = 2 \times \frac{dr}{r}$$

we know that if there is a little approximation in variables then,

$$\frac{dA}{A} = \frac{\Delta A}{A}$$

$$= 2 \times \frac{\Delta r}{r}$$

$$=2 \times a$$

$$=2a$$

4. Question

If the percentage error in the radius of a sphere is α , find the percentage error in its volume.

Answer

given that

$$\left(\frac{\Delta r}{r}\right) \times 100 = a \text{ (if let } r \text{ is a radius of a sphere)}$$

$$\left(\frac{\Delta v}{v}\right) \times 100 = ?$$

$$\text{We know that } v = \left(\frac{4}{3}\right) \pi r^3$$

$$\text{Then, } dv = (4\pi r^2) \times dr$$

$$\text{Finally } \left(\frac{\Delta v}{v}\right) \times 100 = 3 \times \left(\frac{\Delta r}{r}\right) \times 100$$

$$=3 \times a$$

$$=3a\%$$

5. Question

A piece of ice is in the form of a cube melts so that the percentage error in the edge of cube is a , then find the percentage error in its volume.

Answer

$$\text{given that cube edge error } \%[(\Delta x/x) \times 100] = a$$

$$\text{Volume } \% = ?$$

Let the edge of cube is x ,

$$\text{Volume; } v = x^3$$

$$\text{Then, } dv = 3x^2 \cdot dx$$

$$\text{So finally } \frac{\Delta v}{v} \times 100 = \frac{(3x^2)\Delta x}{x^3} \times 100$$

$$= 3 \cdot \left(\frac{\Delta x}{x}\right) \times 100$$

$$=3 \times a$$

$$=3a$$