Understanding Quadrilaterals

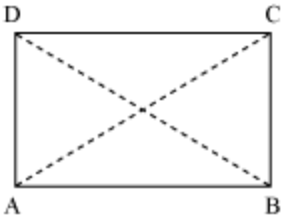
- **Polygons**

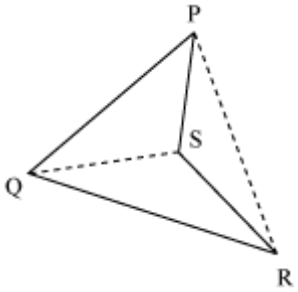
  - A simple closed curve made up of line segments only is called a **polygon.**

  - **Polygons can be classified according to their number of sides (or vertices).**

Number of side/vertices	Classification
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon
.	.
.	.
n	*n* – gon

- The line segment connecting two non-consecutive vertices of a polygon are called **diagonals.**





For polygon ABCD, AC and BD are diagonals and for polygon PQRS, QS and PR are diagonals.

- The polygon, none of whose diagonals lie in its exterior, is called a **convex polygon.** In the given figure, ABCD is a convex polygon.

  The polygon whose atleast one of the diagonals lie in its exterior is called a **concave polygon.** PQRS is a concave polygon.

- A polygon, which is both equiangular and equilateral, is called a **regular polygon.** Otherwise, it is an **irregular polygon.**

Example: Square is a regular polygon but rectangle is an irregular polygon.

- The sum of all the interior angles of an *n*-sided polygon is given by, (*n* – 2) × 180°.

  Example: What is the number of sides of a polygon whose sum of all interior angles is 720°?

  Solution: It is known that,

  $(n-2)180° = 720°$

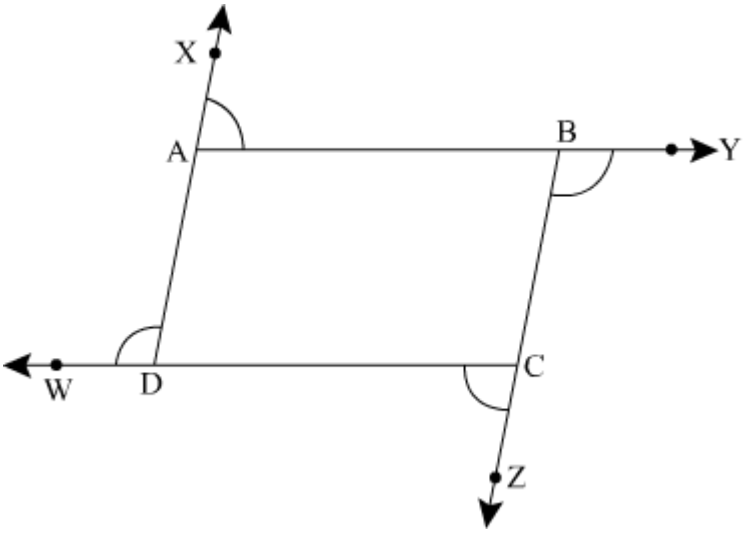
  $\Rightarrow (n-2) = \frac{720°}{180°} = 4$

  $\Rightarrow n = 6$

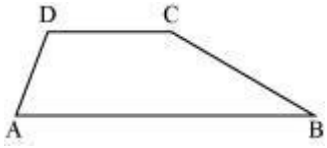
  Thus, the required polygon is six-sided.

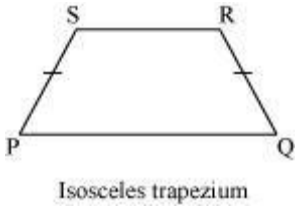
- The sum of measures of all exterior angles of a polygon is 360°.

  For example, in the quadrilateral given below,
  ∠XAB + ∠YBC + ∠ZCD + ∠WDA = 360°



  - **Trapezium**
    - A quadrilateral with a pair of parallel sides is called a trapezium.
    - A trapezium whose non-parallel sides are equal is called an isosceles trapezium.

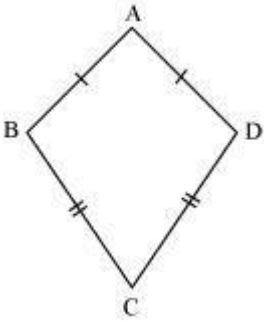




Isosceles trapezium

- **Kite**

A kite is a quadrilateral with exactly two distinct consecutive pairs of sides of equal lengths.



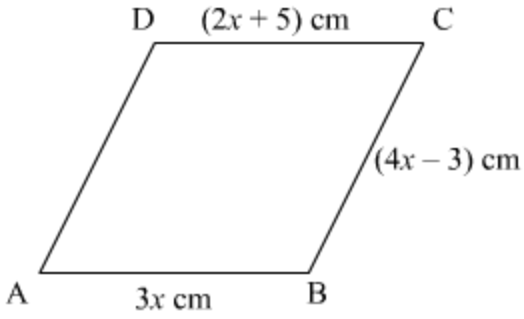
ABCD is a kite with AB = AD and BC = CD.

- **Parallelogram**

1. A parallelogram is a quadrilateral whose opposite sides are parallel and equal.
2. Its opposite angles are of equal measure.
3. The adjacent angles in a parallelogram are supplementary.
4. The diagonals of a parallelogram are not equal. However, they bisect each other.

- Opposite sides in a parallelogram are equal. Conversely, in a quadrilateral, if each pair of opposite sides are equal then the quadrilateral is a parallelogram.

Example:
In the following figure, ABCD is a parallelogram. Find the length of each sides.



Solution: We know, the opposite sides of a parallelogram are equal in length.

Therefore, AB = CD

$3x = 2x + 5$

$\Rightarrow 3x - 2x = 5$

$\therefore x = 5$

Thus, $AB = 3x = 3 \times 5 = 15$ cm

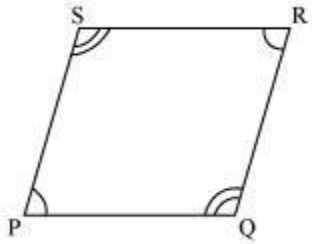
$BC = 4x - 3 = 4 \times 5 - 3 = 17$ cm

$CD = 2x + 5 = 2 \times 5 + 5 = 15$ cm

Also, BC = AD [opposite sides of parallelogram]

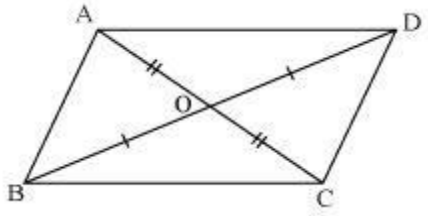
$\therefore$ AD = 17 cm

- In a parallelogram, opposite angles are equal. Conversely in a quadrilateral, if pair of opposite angles is equal, then the quadrilateral is a parallelogram.



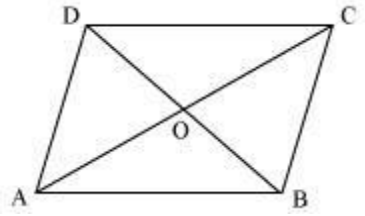
If in the quadrilateral PQRS, $\angle P = \angle R$ and $\angle Q = \angle S$ as shown in the above figure, then the quadrilateral is a parallelogram.

- The diagonals of a parallelogram bisect each other. Conversely, if the diagonals of a quadrilateral bisect each other, then it is a parallelogram.
Suppose ABCD is a quadrilateral. The diagonals of the quadrilateral intersect at O such that AO = OC and DO = OB



Therefore, ABCD is a parallelogram.

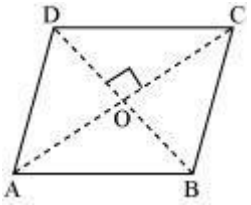
Example: In the given figure, ABCD is a parallelogram. If OD = (3x – 2) cm and OB = (2x + 3) cm, then find x and length of diagonal BD.



Solution: We know that the diagonals of a parallelogram bisect each other.
$\therefore$ OD = OB
$\Rightarrow 3x - 2 = 2x + 3$
$\Rightarrow 3x - 2x = 3 + 2$
$\Rightarrow x = 5$
Thus, the value of x is 5.
Length of BD = OD + OB
$= (3x - 2) + (2x + 3)$
$= (3\times5 - 2) + (2\times5 + 3)$
$= 13 + 13$
$= 26$ cm

- **Rhombus:** A quadrilateral whose opposite sides are parallel and all sides are of equal lengths.

  - Its opposite angles are of equal measure.
  - Its diagonals are perpendicular bisectors of one another.

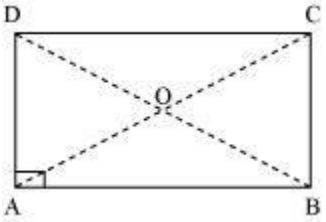


In rhombus ABCD, OA = OC and OB = OD. Also, AC $\perp$ BD.

- A quadrilateral is a rhombus if its diagonals bisect each other at right angles.

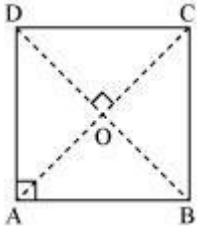
- **Rectangle:** A parallelogram whose each interior angle is a right angle.

- Its diagonals are equal and bisect each other.



In rectangle ABCD, AC = BD. Also, OA = OC and OB = OD

- A parallelogram is a rectangle if its diagonals are equal.
- **Square:** A square is a rectangle with equal sides.
- Its diagonals are equal and are perpendicular bisectors of each other.



In square ABCD, AC = BD and AC ⊥ BD. Also, OA = OC and OB = OD.

- A quadrilateral is a square, if its diagonals are equal and bisect each other at right angles.