Random Variables and Noise

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Theory of Random Signals and Process

1. If p(x) is the probability density function

$$\int_{-\infty}^{\infty} p(x)dx = 1$$

2. Cumulative distribution function (CDF)

$$P(X) = \begin{cases} \int_{-\infty}^{x} p(x)dx & , X \le x \\ 1 & , X < \infty \end{cases}$$

3. Avg. or dc value of signal/mean

$$\bar{X} = E(X) = m = \int_{-\infty}^{\infty} x\,p(x)dx$$

4. DC power contained in the given signal

$$P_{dc} = m^2$$

5. Total power contained in the given signal/mean square value

$$\bar{X}^2 = E(X^2) = \int_{-\infty}^{\infty} x^2 p(x)dx$$

6. AC power contained in the signal/variance

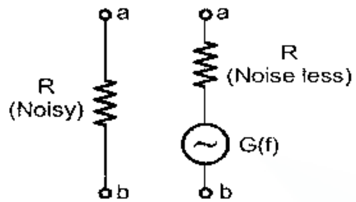
$$P_{AC} = \sigma_X^2 = \left[E(X^2) - m^2\right]$$

where, σ_x = Standard deivation

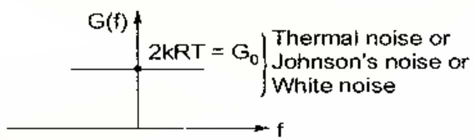
7. RMS voltage

$$V_{rms} = \sqrt{\bar{x}^2} = \sqrt{E(x^2)}$$

Noise



- Power spectral density of thermal noise



$$\boxed{G(f) = 2kRT}$$

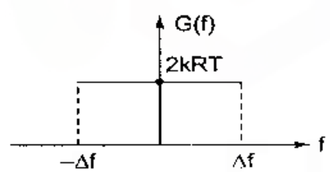
where k = Boltzmann constant

$= 1.38 \times 10^{-23}$ J/°K

- Noise power

$$\boxed{N_0 = \int_{-\infty}^{\infty} G(f)df = \int_{-\infty}^{\infty} 2kRTdf = \infty}$$

- The thermal noise requires infinite amount of power for its generation and infinite amount of BW for its transmission.
- Band limited thermal noise.



$\boxed{G(f) = 2kRT}$ W/Hz ...... $-\Delta f < f < \Delta f$

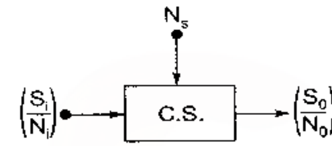
$$\boxed{N_0 = \int_{-\Delta f}^{+\Delta f} G(f)df = 4kRT\Delta f} \text{ W}$$

$$\boxed{V_{rms} = \sqrt{N_0} = \sqrt{4kRT\Delta f}} \text{ V}$$

Noise Figure of a Communication System

- The noise figure of any communication system is a measure of total noise power contributed by that communication system.
- For an ideal communication system or for a noise-less system the noise figure has a minimum value of unit.

$$\boxed{F = \frac{\text{Input SNR}}{\text{Output SNR}}}$$



$$\boxed{F = 1 + \frac{N_s}{AN_i} = 1 + \frac{T_{eq}}{T_0} = 1 + \frac{R_{eq}}{R_s}}$$

where, A = Power gain

N_s = System noise power

N_i = Input noise power

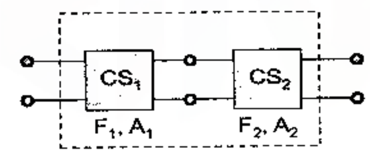
T_{eq} = Equivalent noise temperature

T_0 = Room temperature

R_{eq} = Equivalent input noise resistance

R_s = Source resistance

- Noise figure of cascaded Communication Systems



$$\boxed{F = F_1 + \frac{F_2 - 1}{A_1} + \frac{F_3 - 1}{A_1A_2} + \ldots}$$

Note: ....

- Higher the value of noise figure, higher is the contribution of the noise by that system.
- The noise figure of any system does not depend upon signal power at the input. Therefore the noise figure remain same irrespective of the variation in the input signal power.