Short Notes for FLUID MECHANICS

Pressure (P):

- If F be the normal force acting on a surface of area A in contact with liquid, then pressure exerted by liquid on this surface is: $P = F / A$
- **Units :** N / m^2 or Pascal (S.I.) and Dyne/cm^2 (C.G.S.)
- **Dimension :** $[P] = \frac{[F]}{[A]} = \frac{[MLT^{-2}]}{[L^2]} = [ML^{-1}T^{-2}]$
- **Atmospheric pressure:** Its value on the surface of the earth at sea level is nearly $1.013 \times 10^5 N / m^2$ or Pascal in S.I. other practical units of pressure are atmosphere, bar and torr (*mm* of *Hg*)
- $1 atm = 1.01 \times 10^5 Pa = 1.01$ bar $= 760$ torr
- **Fluid Pressure at a Point:** $\rho = \frac{dF}{dA}$

Density (ρ):

- In a fluid, at a point, density $\boldsymbol{\rho}$ is defined as: $\rho = \lim_{\Delta V \to 0} \frac{\Delta m}{\Delta V} = \frac{dm}{dV}$
- In case of homogenous isotropic substance, it has no directional properties, so is a scalar.
- It has dimensions $[ML^{-3}]$ and S.I. unit kg/m^3 while C.G.S. unit g/cc with $1 g / cc = 10^3 kg / m^3$
- Density of body = Density of substance
- Relative density or specific gravity which is defined as : $RD = \frac{\text{Density of body}}{\text{Density of water}}$
- If m_1 mass of liquid of density ρ_1 and m_2 mass of density ρ_2 are mixed, then as

  $m = m_1 + m_2$ and $V = (m_1 / \rho_1) + (m_2 / \rho_2)$ [As $V = m / \rho$]

  $$\rho = \frac{m}{V} = \frac{m_1 + m_2}{(m_1 / \rho_1) + (m_2 / \rho_2)} = \frac{\Sigma m_i}{\Sigma(m_i / p_i)}$$

  If $m_1 = m_2$, $\rho = \frac{2\rho_1\rho_2}{\rho_1 + \rho_2} =$ Harmonic mean

- If V_1 volume of liquid of density ρ_1 and V_2 volume of liquid of density ρ_2 are mixed, then as: $m = \rho_1 V_1 + \rho_2 V_2$ and $V = V_1 + V_2$ [As $\rho = m / V$]

  If $V_1 = V_2 = V$ $\rho = (\rho_1 + \rho_2)/2 =$ Arithmetic Mean

- With rise in temperature due to thermal expansion of a given body, volume will increase while mass will remain unchanged, so density will decrease, *i.e.*,

$$\frac{\rho}{\rho_0} = \frac{(m/V)}{(m/V_0)} = \frac{V_0}{V} = \frac{V_0}{V_0(1+\gamma\Delta\theta)} \qquad [\text{As } V = V_0(1+\gamma\Delta\theta)]$$

or

$$\rho = \frac{\rho_0}{(1+\gamma\Delta\theta)} \simeq \rho_0(1-\gamma\Delta\theta)$$

- With increase in pressure due to decrease in volume, density will increase, *i.e.*,

$$\frac{\rho}{\rho_0} = \frac{(m/V)}{(m/V_0)} = \frac{V_0}{V} \qquad [\text{As } \rho = \frac{m}{V}]$$

- By definition of **bulk-modulus**: $B = -V_0 \frac{\Delta p}{\Delta V}$ *i.e.*, $V = V_0\left[1 - \frac{\Delta p}{B}\right]$

$$\rho = \rho_0\left(1 - \frac{\Delta p}{B}\right)^{-1} \simeq \rho_0\left(1 + \frac{\Delta p}{B}\right)$$

Specific Weight (w):

- It is defined as the weight per unit volume.
- Specific weight $= \frac{Weight}{Volume} = \frac{m.g}{Volume} = \rho.g$

Specific Gravity or Relative Density (s):

- It is the ratio of specific weight of fluid to the specific weight of a standard fluid. Standard fluid is water in case of liquid and H_2 or air in case of gas.

$$s = \frac{\gamma}{\gamma_w} = \frac{\rho.g}{\rho_w.g} = \frac{\rho}{\rho_w}$$

Where, γ_w = Specific weight of water, and ρ_w = Density of water specific.

Specific Volume (v):

- Specific volume of liquid is defined as volume per unit mass. It is also defined as the reciprocal of specific density.
- Specific volume $= \frac{V}{m} = \frac{1}{\rho}$

Inertial force per unit area = $\frac{dp/dt}{A} = \frac{v(dm/dt)}{A} = \frac{v\,Av\,\rho}{A} = v^2\rho$

Viscous force per unit area: $F/A = \frac{\eta v}{r}$

Reynold's number: $N_R = \frac{\text{Inertial force per unit area}}{\text{Viscous force per unit area}} = \frac{v^2 \rho}{\eta v / r} = \frac{v \rho r}{\eta}$

Pascal's Law: $p_x = p_y = p_z$; where, p_x, p_y and p_z are the pressure at point x,y,z respectively.

Hydrostatic Law:

- $\frac{\partial p}{\partial z} = pg$ or $dp = pg\ dz$
- $\int_o^p dp = pg \int_o^h dz$
- $p = pgh$ and $h = \frac{p}{pg}$; where, h is known as **pressure head.**

Pressure Energy	Potential energy	Kinetic energy
It is the energy possessed by a liquid by virtue of its pressure. It is the measure of work done in pushing the liquid against pressure without imparting any velocity to it.	It is the energy possessed by liquid by virtue of its height or position above the surface of earth or any reference level taken as zero level.	It is the energy possessed by a liquid by virtue of its motion or velocity.
Pressure energy of the liquid PV	Potential energy of the liquid mgh	Kinetic energy of the liquid $mv^2/2$
Pressure energy per unit mass of the liquid P/ρ	Potential energy per unit mass of the liquid gh	Kinetic energy per unit mass of the liquid $v^2/2$
Pressure energy per unit volume of the liquid P	Potential energy per unit volume of the liquid ρgh	Kinetic energy per unit volume of the liquid $\rho\ v^2/2$

Quantities that Satisfy a Balance Equation						
Quantity	mass	x momentum	y momentum	z momentum	Energy	Species
Φ	m	mu	mv	mw	$E + m\mathbf{V}^2/2$	$m^{(K)}$
ϕ	1	u	v	w	$e + \mathbf{V}^2/2$	$W^{(K)}$
In this table, u, v, and w are the x, y and z velocity components, E is the total thermodynamic internal energy, e is the thermodynamic internal energy per unit mass, and $m^{(K)}$ is the mass of a chemical species, K, $W^{(K)}$ is the mass fraction of species K.						

The other energy term, $m\mathbf{V}^2/2$, is the kinetic energy.

- $Storage = \frac{\partial \Phi}{\partial t} = \frac{\partial (m\varphi)}{\partial t} = \frac{\partial (\rho \Delta x \Delta y \Delta z \varphi)}{\partial t} = \frac{\partial (\rho\varphi)}{\partial t}\Delta x \Delta y \Delta z$
- $Inflow = \rho\, u\varphi\big|_x \Delta y\, \Delta z \quad + \rho\, v\varphi\big|_y \Delta x\, \Delta z \quad + \rho\, w\varphi\big|_z \Delta y\, \Delta x$
- $Outflow = \rho\, u\varphi\big|_{x+\Delta x} \Delta y\, \Delta z \quad + \rho\, v\varphi\big|_{y+\Delta y} \Delta x\, \Delta z \quad + \rho\, w\varphi\big|_{z+\Delta z} \Delta y\, \Delta x$
- $Source = S_\varphi\, \Delta x\, \Delta y\, \Delta z$
- $\frac{\partial \rho\varphi}{\partial t} + \frac{\rho\, u\varphi\big|_{x+\Delta x} - \rho\, u\varphi\big|_x}{\Delta x} + \frac{\rho\, v\varphi\big|_{y+\Delta y} - \rho\, v\varphi\big|_y}{\Delta y} \quad + \frac{\rho\, w\varphi\big|_{z+\Delta z} - \rho\, w\varphi\big|_z}{\Delta z} = S_\varphi$
- $\frac{\partial \rho\varphi}{\partial t} + \frac{\partial \rho\, u\varphi}{\partial x} + \frac{\partial \rho\, v\varphi}{\partial y} + \frac{\partial \rho\, w\varphi}{\partial z} = S_\varphi^*$
- $S_\varphi^* = \lim_{\Delta x \Delta y \Delta z \to 0} S_\varphi$

The Mass Balance Equations:

- $\frac{\partial \rho}{\partial t} + \frac{\partial \rho\, u_i}{\partial x_i} = 0$
- $\frac{\partial \rho}{\partial t} + \frac{\partial \rho\, u}{\partial x} + \frac{\partial \rho\, v}{\partial y} + \frac{\partial \rho\, w}{\partial z} = 0$
- $\frac{\partial \rho}{\partial t} + u_i \frac{\partial \rho}{\partial x_i} + \rho \frac{\partial u_i}{\partial x_i} = 0$
- $\frac{\partial \rho}{\partial t} + \rho\left[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right] + u\frac{\partial \rho}{\partial x} + v\frac{\partial \rho}{\partial y} + w\frac{\partial \rho}{\partial z} = 0$
- $\frac{D\Psi}{Dt} = \frac{\partial \Psi}{\partial t} + u\frac{\partial \Psi}{\partial x} + v\frac{\partial \Psi}{\partial y} + w\frac{\partial \Psi}{\partial z} \quad or \quad \frac{D\Psi}{Dt} = \frac{\partial \Psi}{\partial t} + u_i\frac{\partial \Psi}{\partial x_i}$
- $\frac{D\rho}{Dt} + \rho\left[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right] = 0 \qquad \frac{D\rho}{Dt} + \rho\frac{\partial u_i}{\partial x_i} = 0 \qquad \frac{D\rho}{Dt} + \rho\Delta = 0$
- $\Delta \equiv \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad or \quad \Delta \equiv \frac{\partial u_i}{\partial x_i} = 0$

- $$\rho\frac{\partial\varphi}{\partial t}+\varphi\left[\frac{\partial\rho}{\partial t}+\frac{\partial\rho u_i}{\partial x_i}\right]+\rho u_i\frac{\partial\varphi}{\partial x_i}=S_\varphi$$
- $$\rho\frac{\partial\varphi}{\partial t}+\rho u_i\frac{\partial\varphi}{\partial x_i}=S_\varphi$$

Momentum Balance Equation:

- $$Net\ \ j-direction\ \ source\,term=\frac{\partial\sigma_{1j}}{\partial x_1}+\frac{\partial\sigma_{2j}}{\partial x_2}+\frac{\partial\sigma_{3j}}{\partial x_3}+\rho B_j=\frac{\partial\sigma_{ij}}{\partial x_i}+\rho B_j$$
- $$\frac{\partial\rho u_j}{\partial t}+\frac{\partial\rho u_i u_j}{\partial x_i}=\frac{\partial\sigma_{ij}}{\partial x_i}+\rho B_j\qquad j=1,\dots 3$$
- For a Newtonian fluid, the stress, σ_{ij}, is given by the following equation:

$$\sigma_{ij}=-P\delta_{ij}+\mu\left[\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right]+(\kappa-\frac{2}{3}\mu)\Delta\delta_{ij}$$

- $$\frac{\partial\rho u_j}{\partial t}+\frac{\partial\rho u_i u_j}{\partial x_i}=\frac{\partial}{\partial x_i}\left[-P\delta_{ij}+\mu\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)+(\kappa-\frac{2}{3}\mu)\Delta\delta_{ij}\right]+\rho B_j\qquad j=1,\dots 3$$
- $$\frac{\partial\rho u_j}{\partial t}+\frac{\partial\rho u_i u_j}{\partial x_i}=-\frac{\partial P}{\partial x_j}+\frac{\partial}{\partial x_i}\left[\mu\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)\right]+\frac{\partial}{\partial x_j}\left[(\kappa-\frac{2}{3}\mu)\Delta\right]+\rho B_j\qquad j=1,\dots 3$$
- $$\frac{\partial\rho u}{\partial t}+\frac{\partial\rho uu}{\partial x}+\frac{\partial\rho vu}{\partial y}+\frac{\partial wu}{\partial z}=\rho B_x$$
- $$-\frac{\partial P}{\partial x}+2\frac{\partial}{\partial x}\mu\left(\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}\left[\mu\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)\right]+\frac{\partial}{\partial z}\left[\mu\left(\frac{\partial w}{\partial x}+\frac{\partial u}{\partial z}\right)\right]+\frac{\partial}{\partial x}\left[(\kappa-\frac{2}{3}\mu)\Delta\right]$$

Energy Balance Equation:

- This directional heat flux is given the symbol q_i: $q_i=-k\dfrac{\partial T}{\partial x_i}$
- $$\frac{Net\ \ xDirection heat}{Unit\,Volume}=-\frac{q_x|_{x+\Delta x}-q_x|_x}{\Delta x\Delta y\Delta z}\Delta y\Delta z\quad=-\frac{q_x|_{x+\Delta x}-q_x|_x}{\Delta x}$$
- $$\underset{\Delta x\to 0}{Limit}\frac{Net\ \ xDirection\,heat\,source}{Unit\,Volume}=-\frac{\partial q_x}{\partial x}$$

- $Heat\ Rate = -\frac{\partial q_x}{\partial x} - \frac{\partial q_y}{\partial y} - \frac{\partial q_x}{\partial x} = -\frac{\partial q_i}{\partial x_i}$

- *Body-force work rate* $= \rho(uB_x + vB_y + wB_z) = \rho u_i B_i$

- The work term on each face is given by the following equation:

$$y\text{-face surface force work} = (u\sigma_{yx} + v\sigma_{yy} + w\sigma_{yz})\Delta x\, \Delta z = u_i\sigma_{iy}\, \Delta x\, \Delta z$$

- $Net\ yFace\ Surface\ Force\ Work = \frac{\partial(u\sigma_{yx} + v\sigma_{yy} + w\sigma_{yz})}{\partial y} = \frac{\partial u_i \sigma_{yi}}{\partial y}$

- $Net\ Surface\ Force\ Work = \frac{\partial u_i \sigma_{xi}}{\partial x} + \frac{\partial u_i \sigma_{yi}}{\partial y} + \frac{\partial u_i \sigma_{zi}}{\partial z} = \frac{\partial u_i \sigma_{ji}}{\partial x_j}$

- Energy balance equation:

$$\frac{\partial \rho(e + \mathbf{V}^2/2)}{\partial t} + \frac{\partial \rho u_i (e + \mathbf{V}^2/2)}{\partial x_i} = -\frac{\partial q_i}{\partial x_i} + \frac{\partial u_i \sigma_{ji}}{\partial x_j} + \rho u_i B_i$$

Substitutions for Stresses and Heat Flux:

Using only the Fourier Law heat transfer, the source term involving the heat flux in the energy balance equation:

- $-\frac{\partial q_i}{\partial x_i} = -\frac{\partial}{\partial x_i}\left(-k\frac{\partial T}{\partial x_i}\right) = \frac{\partial}{\partial x_i} k \frac{\partial T}{\partial x_i} = \frac{\partial}{\partial x} k \frac{\partial T}{\partial x} + \frac{\partial}{\partial y} k \frac{\partial T}{\partial y} + \frac{\partial}{\partial z} k \frac{\partial T}{\partial z}$

- $\frac{\partial \rho e}{\partial t} + \frac{\partial \rho u_i e}{\partial x_i} = \frac{\partial}{\partial x_i} k \frac{\partial T}{\partial x_i} + \left[-P\delta_{ij} + \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) + (\kappa - \frac{2}{3}\mu)\Delta\delta_{ij}\right]\frac{\partial u_i}{\partial x_j}$

- $\delta_{ij}\frac{\partial u_i}{\partial x_j} = \frac{\partial u_j}{\partial x_j} = \Delta$

- $\frac{\partial \rho e}{\partial t} + \frac{\partial \rho u_i e}{\partial x_i} = \frac{\partial}{\partial x_i} k \frac{\partial T}{\partial x_i} - P\Delta + \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)\frac{\partial u_i}{\partial x_j} + (\kappa - \frac{2}{3}\mu)\Delta^2$

Dissipation to avoid confusion with the general quantity in a balance equation:

- $\boldsymbol{\Phi_D} = \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)\frac{\partial u_i}{\partial x_j} + (\kappa - \frac{2}{3}\mu)\Delta^2$

- $$\frac{\partial \rho e}{\partial t} + \frac{\partial \rho u_i e}{\partial x_i} = \frac{\partial}{\partial x_i} k \frac{\partial T}{\partial x_i} - P\Delta + \boldsymbol{\Phi}_D$$

The temperature gradient in the Fourier law conduction term may also be written as a gradient of enthalpy or internal energy:

- $$\frac{\partial T}{\partial x_i} = \frac{1}{c_v} \frac{\partial e}{\partial x_i} + \frac{1}{c_v} \left[\frac{T\beta_P}{\kappa_T} - P \right] \frac{1}{\rho^2} \frac{\partial \rho}{\partial x_i}$$
- $$\frac{\partial T}{\partial x_i} = \frac{1}{c_p} \frac{\partial h}{\partial x_i} - \frac{1 - T\beta_P}{\rho c_p} \frac{\partial P}{\partial x_i}$$

- $$\frac{\partial \rho e}{\partial t} + \frac{\partial \rho u_i e}{\partial x_i} = \frac{\partial}{\partial x_i} \frac{k}{c_v} \frac{\partial e}{\partial x_i} - P\Delta + \boldsymbol{\Phi}_D + \frac{\partial}{\partial x_i} \frac{1}{c_v} \left[\frac{T\beta_P}{\kappa_T} - P \right] \frac{1}{\rho^2} \frac{\partial \rho}{\partial x_i}$$
- $$\frac{\partial \rho h}{\partial t} + \frac{\partial \rho u_i h}{\partial x_i} = \frac{\partial}{\partial x_i} \frac{k}{c_p} \frac{\partial h}{\partial x_i} + \boldsymbol{\Phi}_D + \frac{\partial}{\partial x_i} \left[\frac{1 - T\beta_P}{\rho c_p} \right] \frac{\partial P}{\partial x_i} + \frac{DP}{Dt}$$
- $$c_p \left[\frac{\partial \rho T}{\partial t} + \frac{\partial \rho u_i T}{\partial x_i} \right] = \frac{\partial}{\partial x_i} k \frac{\partial T}{\partial x_i} + \boldsymbol{\Phi}_D + \beta_P T \frac{DP}{Dt}$$

General Balance Equations			
φ	c	$\Gamma^{(\varphi)}$	$S^{(\varphi)}$
1	1	0	0
$u = u_x = u_1$	1	μ	$-\frac{\partial P}{\partial x}+\frac{\partial}{\partial x}\mu\frac{\partial u}{\partial x}+\frac{\partial}{\partial y}\mu\frac{\partial v}{\partial x}+\frac{\partial}{\partial z}\mu\frac{\partial w}{\partial x}+\frac{\partial}{\partial x}\left[(\kappa-\frac{2}{3}\mu)\Delta\right]+\rho B_x$
$v = u_y = u_2$	1	μ	$-\frac{\partial P}{\partial y}+\frac{\partial}{\partial x}\mu\frac{\partial u}{\partial y}+\frac{\partial}{\partial y}\mu\frac{\partial v}{\partial y}+\frac{\partial}{\partial z}\mu\frac{\partial w}{\partial y}+\frac{\partial}{\partial y}\left[(\kappa-\frac{2}{3}\mu)\Delta\right]+\rho B_y$
$w = u_z = u_3$	1	μ	$-\frac{\partial P}{\partial z}+\frac{\partial}{\partial x}\mu\frac{\partial u}{\partial z}+\frac{\partial}{\partial y}\mu\frac{\partial v}{\partial z}+\frac{\partial}{\partial z}\mu\frac{\partial w}{\partial z}+\frac{\partial}{\partial z}\left[(\kappa-\frac{2}{3}\mu)\Delta\right]+\rho B_z$
e	1	k/c_v	$-P\Delta+\boldsymbol{\Phi_D}+\frac{\partial}{\partial x_i}\frac{1}{c_v}\left[\frac{T\beta_P}{\kappa_T}-P\right]\frac{1}{\rho^2}\frac{\partial\rho}{\partial x_i}$
h	1	k/c_P	$\boldsymbol{\Phi_D}+\frac{\partial}{\partial x_i}\left[\frac{1-T\beta_P}{\rho c_p}\right]\frac{\partial P}{\partial x_i}+\frac{DP}{Dt}$
T	c_P	k	$\boldsymbol{\Phi_D}+\beta_P T\frac{DP}{Dt}$
T	c_v	k	$\boldsymbol{\Phi_D}+\frac{T\beta_P}{\kappa_T}\Delta$
$W^{(K)}$	1	$\rho D_{K,Mix}$	$r^{(K)}$

Momentum equation:

- $$\frac{\partial\rho u_j}{\partial t}+\frac{\partial\rho u_i u_j}{\partial x_i}=\frac{\partial}{\partial x_i}\mu\frac{\partial u_j}{\partial x_i}+S^{(j)}=-\frac{\partial P}{\partial x_i}+\frac{\partial}{\partial x_i}\mu\frac{\partial u_j}{\partial x_i}+S^{*(j)}$$

General Momentum Equations			
φ	c	$\Gamma^{(\varphi)}$	$S^{*(\varphi)}$
1	1	0	0
$u = u_x = u_1$	1	μ	$\frac{\partial}{\partial x}\mu\frac{\partial u}{\partial x}+\frac{\partial}{\partial y}\mu\frac{\partial v}{\partial x}+\frac{\partial}{\partial z}\mu\frac{\partial w}{\partial x}+\frac{\partial}{\partial x}\left[(\kappa-\frac{2}{3}\mu)\Delta\right]+\rho B_x$
$v = u_y = u_2$	1	μ	$+\frac{\partial}{\partial x}\mu\frac{\partial u}{\partial y}+\frac{\partial}{\partial y}\mu\frac{\partial v}{\partial y}+\frac{\partial}{\partial z}\mu\frac{\partial w}{\partial y}+\frac{\partial}{\partial y}\left[(\kappa-\frac{2}{3}\mu)\Delta\right]+\rho B_y$
$w = u_z = u_3$	1	μ	$\frac{\partial}{\partial x}\mu\frac{\partial u}{\partial z}+\frac{\partial}{\partial y}\mu\frac{\partial v}{\partial z}+\frac{\partial}{\partial z}\mu\frac{\partial w}{\partial z}+\frac{\partial}{\partial z}\left[(\kappa-\frac{2}{3}\mu)\Delta\right]+\rho B_z$

Bernoulli's Equation:

This equation has four variables: velocity (v), elevation (z), pressure (p), and density (ρ). It also has a constant (g), which is the acceleration due to gravity. Here is Bernoulli's equation:

- $\frac{v^2}{2} + gz + \frac{p}{\rho} = \text{constant}$
- $P + \rho gh + \frac{1}{2}\rho v^2 = \text{constant}$
- $\frac{P}{\rho g} + h + \frac{v^2}{2g} = \text{constant}$; $\frac{P}{\rho g}$ is called pressure head, h is called gravitational head and $\frac{v^2}{2g}$ is called velocity head.

Factors that influence head loss due to friction are:

- Length of the pipe (l)
- Effective diameter of the pipe (D_h)
- Velocity of the water in the pipe (v)
- Acceleration of gravity (g)
- Friction from the surface roughness of the pipe (λ)
- *The head loss due to the pipe is estimated by the following equation:*

$$h_{f,major} = \lambda \frac{l v^2}{2 D_h g}$$

- To estimate the total head loss in a piping system, one adds the head loss from the fittings and the pipe:

$$h_{f,total} = \sum h_{f,minor} + \sum h_{f,major}$$

- Note that the summation symbol ($\sum$) means to add up the losses from all the different sources. A less compact-way to write this equation is:

$$h_{f,total} = h_{f,minor\,1} + h_{f,minor\,2} + h_{f,minor\,3} + \cdots$$
$$h_{f,major\,1} + h_{f,major\,2} + h_{f,major\,3} + \cdots$$

Combining Bernoulli's Equation With Head Loss:

$$\frac{p_1}{\gamma} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{v_2^2}{2g} + z_2 + h_{f,total}$$

Relation between coefficient of viscosity and temperature:

$$\text{Andrade formula } \eta = \frac{A e^{C\rho/T}}{\rho^{-1/3}}$$

Stoke's Law: $F = 6\pi\eta rv$

Terminal Velocity:

- Weight of the body $(W) = mg = (volume \times density) \times g = \frac{4}{3}\pi r^3 \rho g$
- Upward thrust (T) = weight of the fluid displaced

$$= (volume \times density) \text{ of the fluid} \times g = \frac{4}{3}\pi r^3 \sigma g$$

- Viscous force $(F) = 6\pi\eta rv$
- When the body attains terminal velocity the net force acting on the body is zero.
- $W - T - F = 0$ or $F = W - T$
- $6\pi\eta rv = \frac{4}{3}\pi r^3 \rho g - \frac{4}{3}\pi r^3 \sigma g = \frac{4}{3}\pi r^3 (\rho - \sigma) g$
- Terminal velocity $v = \frac{2}{9}\frac{r^2(\rho - \sigma)g}{\eta}$
- Terminal velocity depend on the radius of the sphere so if radius is made n - fold, terminal velocity will become n^2 times.
- Greater the density of solid greater will be the terminal velocity
- Greater the density and viscosity of the fluid lesser will be the terminal velocity.
- If $\rho > \sigma$ then terminal velocity will be positive and hence the spherical body will attain constant velocity in downward direction.
- If $\rho < \sigma$ then terminal velocity will be negative and hence the spherical body will attain constant velocity in upward direction.

Poiseuille's Formula:

- $V \propto \frac{P r^4}{\eta l}$ or $V = \frac{KP r^4}{\eta l}$
- $V = \frac{\pi P r^4}{8\eta l}$; where $K = \frac{\pi}{8}$ is the constant of proportionality.

Buoyant Force:

- Buoyant force = Weight of fluid displaced by body

- Buoyant force on cylinder =Weight of fluid displaced by cylinder
- $V_{s_{in}}$ = Value of immersed part of solid
- $F_B = p_{water} \times g \times$ Volume of fluid displaced
- $F_B = p_{water} \times g \times$ Volume of cylinder immersed inside the water
- $F_B = mg$
- $F_B = p_w g \frac{\pi}{4} d^2$ $(\because w = mg = pVg)$
- $V_{s_{in}} plg = V_s p_s g$
- $p_w g \frac{\pi}{4} d^2 x = p_{cylinder} g \frac{\pi}{4} d^2 h$
- $p_w x = p_{cylinder} h$

Relation between B, G and M:

- GM $= \frac{I}{V} - BG$; where I = Least moment of inertia of plane of body at water surface, G = Centre of gravity, B = Centre of buoyancy, and M = Metacentre.
- $I = \min(I_{xx}, I_{yy})$, $I_{xx} = \frac{bd^3}{12}$, $I_{yy} = \frac{bd^3}{12}$
- $V = bdx$

Energy Equations:

- $F_{net} = F_g + F_p + F_v + F_c + F_t$; where Gravity force F_g, Pressure force F_p, Viscous force F_v , Compressibility force F_c , and Turbulent force F_t.
- If fluid is incompressible, then $F_c = 0$
  $\therefore F_{net} = F_g + F_p + F_v + F_t$; This is known as **Reynolds equation** of motion.
- If fluid is incompressible and turbulence is negligible, then
  $F_c = 0, F_t = 0 \quad \therefore F_{net} = F_g + F_p + F_v$; This equation is called as **Navier-Stokes equation**.
- If fluid flow is considered ideal then, viscous effect will also be negligible. Then
  $F_{net} = F_g + F_p$; This equation is known as **Euler's equation**.
- Euler's equation can be written as: $\frac{dp}{\rho} + gdz + vdv = 0$

Dimensional analysis:

Quantity	Symbol	Dimensions
Mass	m	M
Length	l	L
Time	t	T
Temperature	T	θ
Velocity	u	LT^{-1}
Acceleration	a	LT^{-2}
Momentum/Impulse	mv	MLT^{-1}
Force	F	MLT^{-2}
Energy - Work	W	$ML^{2}T^{-2}$
Power	P	$ML^{2}T^{-3}$
Moment of Force	M	$ML^{2}T^{-2}$
Angular momentum	-	$ML^{2}T^{-1}$
Angle	η	$M^{0}L^{0}T^{0}$
Angular Velocity	ω	T^{-1}
Angular acceleration	α	T^{-2}
Area	A	L^{2}
Volume	V	L^{3}
First Moment of Area	Ar	L^{3}
Second Moment of Area	I	L^{4}
Density	ρ	ML^{-3}
Specific heat- Constant Pressure	C_p	$L^{2}T^{-2}\theta^{-1}$
Elastic Modulus	E	$ML^{-1}T^{-2}$
Flexural Rigidity	EI	$ML^{3}T^{-2}$
Shear Modulus	G	$ML^{-1}T^{-2}$
Torsional rigidity	GJ	$ML^{3}T^{-2}$
Stiffness	k	MT^{-2}

Angular stiffness	T/η	$ML^{2}T^{-2}$
Flexibiity	1/k	$M^{-1}T^{2}$
Vorticity	-	T^{-1}
Circulation	-	$L^{2}T^{-1}$
Viscosity	μ	$ML^{-1}T^{-1}$
Kinematic Viscosity	τ	$L^{2}T^{-1}$
Diffusivity	-	$L^{2}T^{-1}$
Friction coefficient	f /μ	$M^{0}L^{0}T^{0}$
Restitution coefficient		$M^{0}L^{0}T^{0}$
Specific heat- **Constant volume**	C_v	$L^{2}T^{-2}\theta^{-1}$

Boundary layer:

- **Reynolds number** $= \frac{\rho v \cdot x}{\mu} (\text{Re})_x = \frac{v \cdot x}{v}$
- **Displacement Thickness (δ *):** $\delta^* = \int_0^{\delta} \left(1 - \frac{u}{U}\right) dy$
- **Momentum Thickness (θ):** $\theta = \int_0^{\delta} \frac{u}{U}\left(1 - \frac{u}{U}\right) dy$
- **Energy Thickness (δ **):** $\delta^{**} = \int_0^{\delta} \frac{u}{U}\left(1 - \frac{u^2}{U^2}\right) dy$
- **Boundary Conditions for the Velocity Profile:** Boundary conditions are as
  - (a) $At\ y = 0, u = 0, \frac{du}{dy} \neq 0$; (b) $At\ y = \delta, u = U, \frac{du}{dy} = 0$

Turbulent flow:

- **Shear stress in turbulent flow:** $\tau = \tau_v + \tau_t = \mu \frac{d\bar{u}}{dy} + \eta \frac{d\bar{u}}{dy}$
- **Turbulent shear stress by Reynold:** $\tau = \rho u'v'$
- **Shear stress in turbulent flow due to Prndtle :** $\tau = \rho l^2 \left(\frac{du}{dy}\right)^2$