SCHOOL NAME

GOVT. SR. SEC. SMART SCHOOL KAMAHI DEVI

SUBJECT : MATHEMATICS
TOPIC : DERIVATIVE
SUBMITTED BY : NEHA
CLASS : +2 (Non-Med.)
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UNIT-III : CALCULUS

1. Continuity and Differentiability

A real valued function is finitely derivable at any point of its domain, it is necessarily continuous at that point The converse is not true

1) DEFINITIONS :-

(1) Left Derivative :- A function 'f' is said to possess left derivative at $x = c$ iff f is defined in $[c-\sigma, c]$ and $\lim_{h \to 0^-} \frac{f(c+h) - f(c)}{h}$ exists finitely.

(2) Right Derivative :- A function 'f' is said to possess right derivative at $x = c$ iff f is defined in $[c, c+\sigma]$ and $\lim_{h \to 0^+} \frac{f(c+h) - f(c)}{h}$ exists finitely.

(3) Derivative :- A function 'f' is said to possess derivative at $x = c$ iff f is is defined in $[c-\sigma, c+\sigma]$ and $\lim_{h \to 0} \frac{f(c+h) - f(c)}{h}$ exists finitely.

Ques: Point: If $y=(x)^n$, $n \in \mathbb{R}$ then

$$\frac{dy}{dx} = nx^{n-1}$$

If $y = (f(x))^n$, $n \in \mathbb{R}$ then :

$$\frac{dy}{dx} = n(f(x))^{n-1} \frac{d(f(x))}{dx}$$

(1.) $y = (3x^2+4x+5)^6$ | $x^n \rightarrow nx^{n-1}$

$$\frac{dy}{dx} = 6(3x^2+4x+5)^5 \frac{d}{dx}(3x^2+4x+5)$$

$$\therefore \frac{dy}{dx} = 6(3x^2+4x+5)^5 (6x+4)$$

(2.) $y = ((\log x)^3$ | $x^n \longrightarrow nx^{n-1}$

$$\therefore \frac{dy}{dx} = 3(\log x)^2 \frac{d}{dx}(\log x)$$

$$\therefore \frac{dy}{dx} = 3(\log x)^2 \times \frac{1}{x}$$

Point: (1.) If $y = (x)^n$, $n \in \mathbb{R}$, then $\frac{dy}{dx} = nx^{n-1}$

(2.) If $y = a^x$, then $\frac{dy}{dx} = a^x \log a$

(3.) If $y = e^x$, then $\frac{dy}{dx} = e^x \log e = e^x.1$

(4.) If $y = \log x$, than $\frac{dy}{dx} = 1/x$

1. If $y = (f(x))^n,\ n \in \mathbb{R}$, then $\frac{dy}{dx} = n\,(f(x))^{n-1}\,\frac{d f(x)}{dx}$

2. If $y = a^{f(x)}$, then $\frac{dy}{dx} = a^{f(x)} \log a \cdot \frac{d f(x)}{dx}$

3. If $y = e^{f(x)}$, then $\frac{dy}{dx} = e^{f(x)} \cdot \frac{d(f(x))}{dx}$

4. If $y = \log(f(x))$, then $\frac{dy}{dx} = \frac{1}{f(x)}\,\frac{d(f(x))}{dx}$

Ques: Diff. following w.r.to x

1. $(3x^2-9x+5)^9$

Solve Let $y = \underbrace{(3x^2-9x+5)^9}_{f(x)}$

$x^n \to n x^{n-1}$

$f(x)^n \to n\,(f(x))^{n-1} \cdot \frac{d f(x)}{dx}$

diff w.r.to x

$$\frac{dy}{dx} = 9\,(3x^2-9x+5)^8\,\frac{d}{dx}(3x^2-9x+5)$$

Note: $\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$

$$\therefore \frac{dy}{dx} = 9\,(3x^2-9x+5)^8\,(3(2x)-9(1)+0)$$

$$\therefore \frac{dy}{dx} = 9\,(3x^2-9x+5)^8\,(6x-9)$$

Ques: Diff. following w.r.to x

2. $\sqrt{15x^2-x+1}$

$\sqrt{x} \to \frac{1}{2\sqrt{x}}$ $\quad \frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$

$\sqrt{f(x)} \to \frac{1}{2\sqrt{f(x)}}\,\frac{d}{dx}(f(x))$

Solve Let $y = \sqrt{15x^2-x+1}$

Diff. w.r.to x,

$$\frac{dy}{dx} = \frac{1}{2\sqrt{15x^2-x+1}}\,\frac{d}{dx}(15x^2-x+1)$$

$$\therefore \frac{dy}{dx} = \frac{1}{2\sqrt{15x^2-x+1}} \times (15(2x)-1+0)$$

$$\therefore \frac{dy}{dx} = \frac{30x-1}{2\sqrt{15x^2-x+1}}$$

Ques. Diff. following w.r.to x

(3.) $\sqrt{\frac{x-1}{x+1}}$

$\sqrt{x} \rightarrow \frac{1}{2\sqrt{x}}$ $\sqrt{f(x)} \rightarrow \frac{1}{2\sqrt{f(x)}}$

$\frac{d}{dx}(f(x))$

Solve Let $y = \sqrt{\frac{x-1}{x+1}}$

$\therefore$ diff w.r. to x

$$\frac{dy}{dx} = \frac{1}{2\sqrt{\frac{x-1}{x+1}}} \frac{d}{dx}\left(\frac{x-1}{x+}\right) \quad \left(\frac{u}{v}\right)$$

$$\therefore \frac{dy}{dx} = \frac{1}{2\sqrt{\frac{x-1}{x+1}}} \left(\frac{(x+1)\frac{d}{dx}(x-1)-(x-1)\frac{d}{dx}(x+1)}{(x+1)^2}\right)$$

$$\therefore \frac{dy}{dx} = \frac{1}{2\sqrt{\frac{x-1}{x+1}}} \left(\frac{(x+1)(1-0)-(x-1)(1+0)}{(x+1)^2}\right)$$

$$\therefore \frac{dy}{dx} = \frac{1}{2\sqrt{\frac{x-1}{x+1}}} \times \left(\frac{x+1-x+1}{(x+1)^2}\right)$$

$$\therefore \frac{dy}{dx} = \frac{1}{\cancel{2}\sqrt{\frac{x-1}{x+1}}} \times \left(\frac{\cancel{2}}{(x+1)^2}\right)$$

$$\therefore \frac{dy}{dx} \quad \frac{1}{(x-1)^{1/2}(x+1)^{3/2}}$$

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Ques 4. Diff. following w.r.to x

$$e^{x(1+\log x)}$$

Solve Let $y = e^{x(1+\log x)}$

$$\therefore \frac{dy}{dx} = e^{x(1+\log x)} \frac{d}{dx}\left(x(1+\log x)\right) \quad (u \cdot v)$$

$$\therefore \frac{dy}{dx} = e^{x(1+\log x)} \left[x \cdot \frac{d}{dx}(1+\log x) + (1+\log x) \cdot \frac{d}{dx}(x)\right]$$

$$\therefore \frac{dy}{dx} = e^{x(1+\log x)} \left[x \cdot \left(0 + \frac{1}{x}\right) + (1+\log x)(1)\right]$$

$$\therefore \frac{dy}{dx} = e^{x(1+\log x)} \left[2 + \log x\right]$$

Art ÷) Let $y = f(x)$ be any $f\ x^n$

y → Dependent variable

x → Independent variable

Diff. y w.r.to x

$$\therefore \frac{dy}{dx} = \frac{d}{dx}(y)$$

$$\therefore \frac{dy}{dx} = f'(x)$$

Deri → Formula

(1.) If $y = x^n$, then $\frac{dy}{dx} = nx^{n-1}$, $n \in R$

(2.) If $y = x$, then $\frac{dy}{dx} = 1$

(3.) If $y = c$, c is constant, then $\frac{dy}{dx} = 0$

(4.) If $y = a^x$, then $\frac{dy}{dx} = a^x \log a$, $a > 0$

(5) If $y = e^x$, then $\frac{dy}{dx}$ $e^x \log e$

(6.) If $y = c\,(f(x))$, then $\frac{dy}{dx} = \frac{d}{dx}(c f(x))$ c is constant

$$\therefore \frac{dy}{dx} = \frac{cd}{dx}(f(x))$$

(7.) If $y = u + v$, then $\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$

(8.) If $y = u - v$, then $\frac{dy}{dx} = \frac{du}{dx} - \frac{dv}{dx}$

(9.) If $y = u \cdot v$ (product rule)

then $\frac{dy}{dx} = v \cdot \frac{du}{dx} + u \cdot \frac{dv}{dx}$

or

$$\frac{dy}{dx} = u \cdot \frac{dv}{dx} + v \cdot \frac{du}{dx}$$

10. If $y = u \cdot v \cdot w$, then

$$\frac{dy}{dx} = v \cdot w \frac{du}{dx} + uw \frac{dv}{dx} + uv \frac{dw}{dx}$$

11. (Quotient Rule)

If $y = \frac{u}{v}$

then $$\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

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* $x^n \rightarrow (\text{Variable})^{\text{constant}}$ $\quad a^x \rightarrow (\text{Constant})^{\text{Variable}}$

$x^5 \cdot (\text{Variable})^{(\text{Constant})}$

$\rightarrow \frac{d}{dx}(x^5) = 5x^4$

$5^x (\text{constant})^{\text{variable}} \rightarrow \frac{dy}{dx}(5^x) = 5^x \log 5$

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12. If $y = \log_a x$, then $\frac{dy}{dx} = \frac{1}{x} \log_a e$

13. If $y = \log_a x$, then $\frac{dy}{dx} = \frac{1}{x} \log e = \frac{1}{x}$

Ques Diff. following w.r.to x

(1) $x^4+7x^3+8x^2+3x+2$

Solve Let $y=x^4+7x^3+8x^2+3x+2$ | $x^n \longrightarrow nx^{n-1}$

$\therefore$ diff. w.r.to x,

$$\frac{dy}{dx}=\frac{d}{dx}\left(x^4+7x^3+8x^2+3x+2\right)$$

$$\therefore \frac{dy}{dx}=\frac{d}{dx}(x^4)+7\frac{d}{dx}(x^3)+\frac{8d}{dx}(x^2)+\frac{3d}{dx}(x)+\frac{d}{dx}(2)$$

$$\therefore \frac{dy}{dx}=4x^3+7(3x^2)+8(2x)+3(1)+0$$

$$\therefore \frac{dy}{dx}=4x^3+21x^2+16x+3$$

Ques Diff. following w.r.to x

(2) $(x^2-4x+5)\ (x^3-2)$

(U.V) Product Rule if $y=U.V$ then $\frac{dy}{dx}\ U\frac{dv}{dx}+V\frac{du}{dx}$

Solve Let $y\ (x^2-4x+5)\ (x^3-2)$

Diff. w.r.to x

$$\frac{dy}{dx}=(x^2-4x+5)\frac{d}{dx}(x^3-2)+(x^3+2)\frac{d}{dx}(x^2-4x+5)$$

$$\therefore \frac{dy}{dx}=(x^2-4x+5)\ (3x^2-0)+(x^2-2)\ (2x-4(1)+0)$$

$$\therefore \frac{dy}{dx}=3x^4-12x^3+15x^2+2x^3-4x^2-4x+8$$

$$\therefore \frac{dy}{dx}=3x^4-10x^3+11x^2-4x+8$$

Ques. Diff. following w.r.to x,

(3) $\dfrac{(2x^2+4x+3)}{x^2+1}$

Solve: Let $y = \dfrac{2x^2+4x+3}{x^2+1}$

$\left(\frac{u}{v}\right)$ if $y = \frac{u}{v}$, then (Quotient Rule)

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$\therefore$ Diff. w.r.to x,

$$\frac{dy}{dx} = \frac{(x^2+1)\frac{d}{dx}(2x^2+4x+3) - (2x^2+4x+3)\frac{d}{dx}(x^2+1)}{(x^2+1)^2}$$

$$\therefore \frac{dy}{dx} = \frac{(x^2+1)\,(2(2x)+4(1)+0) - (2x^2+4x+3)(2x+0)}{(x^2+1)^2}$$

$$\therefore \frac{dy}{dx} = \frac{(x^2+1)\ (4x+4) - (2x^2+4x+3)\ (2x)}{(x^2+1)^2}$$

$$\therefore \frac{dy}{dx} = \frac{4x^3+4x^2+4x+4-4x^3-8x^2-6x}{(x^2+1)^2}$$

$$\therefore \frac{dy}{dx} = \frac{-4x^2-2x+4}{(x^2+1)^2}$$

$$\therefore \frac{dy}{dx} = -\frac{(4x^2+2x-4)}{(x^2+1)^2}$$

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Ques: Diff. following w.r.to x,

(4.) $x^2+e^x+3^x$

(1) $\frac{dy}{dx}(x^n) = \frac{d}{dx}(x^n) = nx^{n-1}$

(2) $\frac{d}{dx}(e^x) = e^x$

(3) $\frac{d}{dx}(a^x) = a^x \log a$

Solve Let $y = x^2+e^x+3^x$

$\therefore$ Diff. w.r.to x,

$$\frac{dy}{dx} = \frac{d}{dx}(x^2+e^x+3^x)$$

$$\therefore \frac{dy}{dx} = \underset{\downarrow\, x^n}{\frac{d}{dx}(x^2)} + \underset{\downarrow\, e^x}{\frac{d}{dx}(e^x)} + \underset{\downarrow\, a^x}{\frac{d}{dx}(3^x)}$$

$$\therefore \frac{dy}{dx} = (2x) + e^x + 3^x \log 3$$

Ques: Diff. following w.r.to x,

(5) $4x^3+3e^x+\log x+2^x$

Solve Let $y = 4x^3+3e^x+\log x+2^x$

$\therefore$ Diff w.r.to x,

$$\therefore \frac{dy}{dx} = 4(3x^2)+3(e^x)+\frac{1}{x}+2^x \log 2$$

$$\therefore \frac{dy}{dx} = 12x^2+3e^x+\frac{1}{x}+2^x \log 2$$

NOTE:

$x^a \longrightarrow ax^{a-1}$ (Variable) constant $\quad x^5 \longrightarrow 5x^4$

$a^x \longrightarrow a^x \log a$ (constant) variable $\quad 5^x \longrightarrow 5^x \log 5$

CHAIN RULE

① Let $y = x^n = (x)^n$

then $\frac{dy}{dx} = n(x)^{n-1}$

② If $y = (\text{something})^n$

then $\frac{dy}{dx} = n(\text{something})^{n-1} \frac{d}{dx}(\text{something})$

e.g ÷ If $y = [f(x)]^n$

then $\frac{dy}{dx} = n(f(x))^{n-1} \frac{d}{dx}(f(x))$ [f(x) → something]

e.g ÷ If $y = (\log x)^6$ $[x^n]$

then $\frac{dy}{dx} = 6(\log x)^5 \left(\frac{d}{dx}\right)(\log x)$

$\therefore \frac{dy}{dx} = 6(\log x)^5 \left(\frac{1}{x}\right)$

Que: Diff following w.r.to x

$\sqrt{\log(\log x)}$

Solve Let $y = \sqrt{\log(\log x)}$

diff w.r.to x

$\frac{dy}{dx} = \frac{1}{2\sqrt{\log(\log x)}} \cdot \frac{d}{dx}(\log(\log x))$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{\log(\log x)}}\left[\frac{1}{\log x}\frac{d}{dx}(\log x)\right] \quad \left[\log x = \frac{1}{x}\right]$$

$$\therefore \frac{dy}{dx} = \frac{1}{2\sqrt{\log(\log x)}} \times \frac{1}{\log x} \times \frac{1}{x}$$

Ques 2. $x\sqrt{x^2+1} + \log(x+\sqrt{x^2+1})$

Solve Let $y = x\sqrt{x^2+1} + \log(x+\sqrt{x^2+1})$

$$\frac{dy}{dx} = \frac{d}{dx}\left(\underbrace{x\sqrt{x^2+1}}_{uv}\right) + \frac{d}{dx}\left(\log(x+\sqrt{x^2+1})\right)$$

$$\frac{dy}{dx} = x\frac{d}{dx}\left(\sqrt{x^2+1}\right) + \left(\sqrt{x^2+1}\frac{d}{dx}(x) + \frac{1}{x+\sqrt{x^2+1}}\cdot\frac{d}{dx}\right.$$

$$(x+\sqrt{x^2+1})$$

$$\therefore \frac{dy}{dx} = x\cdot\frac{1}{2\sqrt{x^2+1}}\frac{d}{dx}(x^2+1) + \sqrt{x^2+1}(1) + \frac{1}{x+\sqrt{x^2+1}} =$$

$$\left[1 + \frac{1}{2\sqrt{x^2+1}}\frac{d}{dx}(x^2+1)\right]$$

$$\frac{dy}{dx} = x\frac{1}{2\sqrt{x^2+1}}(2x+0) + \sqrt{x^2+1} + \frac{1}{x+\sqrt{x^2+1}}\left[1 + \frac{1}{2\sqrt{x^2+1}}(2x+1)\right]$$

$$\frac{dy}{dx} = \frac{x^2}{\sqrt{x^2+1}} + \sqrt{x^2+1} + \frac{1}{x+\sqrt{x^2+1}}\left[\frac{\sqrt{x^2+1}+x}{\sqrt{x^2+1}}\right]$$

$$\frac{dy}{dx} = \frac{x^2}{\sqrt{x^2+1}} + \sqrt{x^2+1} + \frac{1}{\sqrt{x^2+1}}$$

$$\frac{dy}{dx} = \frac{x^2+(x^2+1)+1}{\sqrt{x^2+1}}$$

$$\therefore \frac{dy}{dx} = \frac{2x^2+2}{\sqrt{x^2+1}}$$

$$\therefore \frac{dy}{dx} = 2\left(\frac{x^2+1}{\sqrt{x^2+1}}\right)$$

$$\therefore \frac{dy}{dx} = 2\sqrt{x^2+1}$$

Ques: Diff. $\sqrt{\frac{x-1}{x+1}}$, with x

Solve Let $y = \sqrt{\frac{x-1}{x+1}}$

diff. with x

$$\frac{dy}{dx} = \frac{1}{2\sqrt{\frac{x-1}{x+1}}} \cdot \frac{d}{dx}\left(\frac{x-1}{x+1}\right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2\sqrt{\frac{x-1}{x+1}}}\left[\frac{(x+1)\frac{d}{dx}(x-1)-(x-1)\frac{d}{dx}(x+1)}{(x+1)^2}\right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2\sqrt{\frac{x-1}{x+1}}}\left[\frac{(x+1)(1-0)-(x-1)(1+0)}{(x+1)^2}\right]$$

$$\therefore \frac{dy}{dx} = \frac{1}{2\sqrt{\frac{x-1}{x+1}}}\left(\frac{2}{(x+1)^2}\right)$$

$$\therefore \frac{dy}{dx} = \frac{(x+1)^{1/2}}{(x-1)^{1/2}} \times \frac{1}{(x+1)^2}$$

$$\therefore \frac{dy}{dx} = \frac{1}{(x-1)^{1/2}(x+1)^{3/2}}$$

Ques: Diff - $\frac{1}{x+\sqrt{1+x^2}}$ w.r.to x,

Solve Let $y = \frac{1}{x+\sqrt{1+x^2}}$

$$\therefore \quad y = \frac{1}{x+\sqrt{1+x^2}} \times \frac{x-\sqrt{1+x^2}}{x-\sqrt{1+x^2}}$$

$$\therefore \quad y = \frac{x-\sqrt{1+x^2}}{x^2-(1+x^2)}$$

$$\therefore \quad y = \frac{x-\sqrt{1+x^2}}{\cancel{x^2}-1-\cancel{x^2}}$$

$$y = \frac{x-\sqrt{1+x^2}}{-1}$$

$$y = -x+\sqrt{1+x^2}$$

Diff. w.r.to x,

$$\frac{dy}{dx} = (-1) + \frac{1}{2\sqrt{1+x^2}} \frac{d}{dx}(1+x^2)$$

$$\therefore \frac{dy}{dx} = -1 + \frac{1}{\cancel{2}\sqrt{1+x^2}}(0+\cancel{2}x)$$

$$\therefore \frac{dy}{dx} = -1 + \frac{x}{\sqrt{1+x^2}}$$

Ques: If $y = (x+\sqrt{x^2+a^2})^n$, prove that $\frac{dy}{dx} = \frac{ny}{\sqrt{x^2+a^2}}$

Solve Let $y = (x+\sqrt{x^2+a^2})^n$ - - - (1.)

$x^n \rightarrow n(x)^{n-1}$

diff w.r. to x

$$\frac{dy}{dx} = n(x+\sqrt{x^2+a^2})^{n-1} \frac{d}{dx}(x+\sqrt{x^2+a^2})$$

$$\therefore \frac{dy}{dx} = n(x+\sqrt{x^2+a^2})^{n-1}\left[1+\frac{1}{2\sqrt{x^2+a^2}}\frac{d}{dx}(x^2+a^2)\right]$$

$$\therefore \frac{dy}{dx} = n(x+\sqrt{x^2+a^2})^{n-1}\left[1+\frac{1}{\cancel{2}\sqrt{x^2+a^2}}(\cancel{2}x+0)\right]$$

$$\therefore \frac{dy}{dx} = n(x+\sqrt{x^2+a^2})^{n-1}\left[\frac{\sqrt{x^2+a^2}+x}{\sqrt{x^2+a^2}}\right]$$

$$\therefore \frac{dy}{dx} = \frac{n(x+\sqrt{x^2+a^2})^n}{\sqrt{x^2+a^2}} = \frac{ny}{\sqrt{x^2+a^2}}$$

$$\therefore \frac{dy}{dx} = \frac{ny}{\sqrt{x^2+a^2}}$$

Ques: If $y = \log\left(x+4\sqrt{x^2+8x+4}\right)$, then find $\frac{dy}{dx}$

Solve Let $y = \log\left(x+4+\sqrt{x^2+8x+4}\right)$

diff. with x

$$\frac{dy}{dx} = \frac{1}{x+4+\sqrt{x^2+8x+4}}\left[\frac{d}{dx}\left(x+4+\sqrt{x^2+8x+4}\right)\right]$$

$$\frac{dy}{dx} = \frac{1}{x+4+\sqrt{x^2+8x+4}}\left[1+0+\frac{1}{2\sqrt{x^2+8x+4}}(2x+8)\right]$$

$$\frac{dy}{dx} = \frac{1}{(x+4+\sqrt{x^2+8x+4})}\left[1+\frac{1}{\cancel{2}\sqrt{x^2+8x+4}}(\cancel{2}(x+4))\right]$$

$$\frac{dy}{dx} = \frac{1}{x+4+\sqrt{x^2+8x+4}}\left[\frac{\sqrt{x^2+8x+4}+x+4}{\sqrt{x^2+8x+4}}\right]$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{x^2+8x+4}}$$

Ques: Diff. following in to x

$5^{\sqrt{x^2+1}} + (\sqrt{x^2+1})^5$

1. $a^x \to a^x \log a$
2. $x^n \to n x^{n-1}$

Solve Let $y = 5^{\sqrt{x^2+1}} + (\sqrt{x^2+1})^5$

$$y = 5^{\sqrt{x^2+1}} + (x^2+1)^{5/2}$$

diff. with x,

$$\frac{dy}{dx} = 5^{\sqrt{x^2+1}} \log a \frac{d}{dx}\left(\sqrt{x^2+1} + \frac{5}{2}(x^2+1)^{\frac{5}{2}-1} \cdot \frac{d}{dx}(x^2+1)\right)$$

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$$\frac{dy}{dx} = 5^{\sqrt{x^2+1}} \log a \left[\frac{1}{2\sqrt{x^2+1}}(2x+0)\right] + \frac{5}{2}(x^2+1)^{3/2}(2x+0)$$

$$\frac{dy}{dx} = 5^{\sqrt{x^2+1}} \log a \frac{x}{\sqrt{x^2+1}} + 5x(x^2+1)^{\frac{3}{2}}$$

Ques $\sqrt{\log(x^3+e^x)}$

$\sqrt{x} = \frac{1}{2\sqrt{x}}$

Solve Let $y = \sqrt{\log(x^3+e^x)}$

diff. w.r.t. x

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2\sqrt{\log(x^3+e^x)}} \frac{d}{dx}(\log(x^3+e^x))$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2\sqrt{\log(x^3+e^x)}}\left[\frac{1}{(x^3+e^x)}\frac{d}{dx}\cdot[x^3+e^x]\right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2\sqrt{\log(x^3+e^x)}}\left[\frac{1}{(x^3+e^x)}(3x^2+e^x)\right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{(3x^2+e^x)}{2(x^3+e^x)\sqrt{\log(x^3-e^x)}}$$

Ques $\log\left[e^{3x}\left(\frac{5x-3}{4x+2}\right)^{\frac{1}{3}}\right]$

Solve Let $y = \log\left[e^{3x}\left(\frac{5x-3}{4x+2}\right)^{1/3}\right]$

$$y = \log(e^{3x}) + \log\left(\frac{5x-3}{4x+2}\right)^{\frac{1}{3}}$$

1. $\log(mn) = \log m + \log n$
2. $\log(m/n) = \log m - \log n$
3. $\log(m)^n = n\log m$

$$y = 3x \log e + \frac{1}{3}\log\left(\frac{5x-3}{4x+2}\right)$$

$$y = 3x + \frac{1}{3}\log\left(\frac{5x-3}{4x+2}\right) \quad \Big| \ \log_e e = 1$$

$$y = 3x + \frac{1}{3}\left[\log(5x-3)\right] - \frac{1}{3}\left(\log(4x+2)\right. \cdot \quad \frac{d}{dx}\log(5x-3) = \frac{1}{5x-3}(5-0)$$

diff. w.r.t x

$$\frac{dy}{dx} = 3 + \frac{1}{3}\left[\frac{1}{5x-3}(5)\right] - \frac{1}{3}\left[\frac{1}{4x+2}(4)\right]$$

$$\frac{dy}{dx} = 3 + \frac{1}{3}\left[\frac{5}{5x-3} - \frac{4}{4x+2}\right] \quad \Big| \ \frac{d}{dx}(x) = 1$$

Ques: If $y = \frac{e^x - e^{-x}}{e^x + e^{-x}}$, prove that $\frac{dy}{dx} = 1 - y^2$

Solve: Let $y = \frac{e^x - e^{-x}}{e^x + e^{-x}}$ $\left(\frac{u}{v}\right)$

Diff. w.r.to x

$$\frac{dy}{dx} = \frac{(e^x+e^{-x})\frac{d}{dx}(e^x-e^{-x}) - (e^x-e^{-x})\frac{d}{dx}(e^x+e^{-x})}{(e^x+e^{-x})^2}$$

$$\therefore \frac{dy}{dx} = \frac{e^x+e^{-x}\,(e^x + e^{-x}(-1)) - (e^x-e^{-x})\,(e^x + e^{-x}(-1))}{(e^x+e^{-x})^2}$$

1. $\dfrac{d}{dx}(e^x) = e^x \cdot 1$

2. $\dfrac{d}{dx}(e^{-x}) = e^{-x}(-1)$

3. $\dfrac{d}{dx}(e^{2x}) = e^{2x} \cdot 2$

$$\therefore \frac{dy}{dx} = \frac{(e^x+e^{-x})(e^x+e^{-x}) - (e^x-e^{-x})(e^x-e^{-x})}{(e^x+e^{-x})^2}$$

$$\therefore \frac{dy}{dx} = \frac{(e^x+e^{-x})^2 - (e^x-e^{-x})^2}{(e^x+e^{-x})^2}$$

$$\therefore \frac{dy}{dx} = \frac{(e^x+e^{-x})^2}{(e^x+e^{-x})^2} - \frac{(e^x-e^{-x})^2}{(e^x+e^{-x})^2} = 1 - \left(\frac{e^x-e^{-x}}{e^x+e^{-x}}\right)^2$$

$$\therefore \frac{dy}{dx} = 1 - y^2$$

PARAMETRIC EQUATION

Ques If $x = f(t)$, and $y = g(t)$, then find $\frac{dy}{dx} = ?$

Sol^n

$$\frac{dx}{dt} = f'(t), \quad \frac{dy}{dx} = g'(t)$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

$x, y \rightarrow t \longrightarrow \frac{dy}{dx} = ?$

$$\therefore \frac{dy}{dx} = \frac{g'(t)}{f'(t)}$$

Ques Find $\frac{dy}{dx}$, when $x = at^2$, $y = 2at$.

Solve We are given

$$x = at^2, \quad y = 2at$$

$$\frac{dx}{dt} = a(2t) = 2at$$

$$y = 2at \therefore \frac{dy}{dt} = 2a(1) = 2a$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2a}{2at} = \frac{1}{t}$$

$$\frac{dy}{dx} = \frac{1}{t}$$

Ques: If $x = \frac{1-t^2}{1+t^2}$, $y = \frac{2t}{1+t^2}$, prove that $\frac{dy}{dx} + \frac{x}{y} = 0$

Solve: We are given

$$x = \frac{1-t^2}{1+t^2} \left(\frac{u}{v}\right), \quad y = \frac{2t}{1+t^2}$$

$$\therefore \frac{dx}{dt} = \frac{(1+t^2)\frac{d}{dx}(1-t^2) - (1-t^2)\frac{d}{dx}(1+t^2)}{(1+t^2)^2}$$

$$\therefore \frac{dx}{dt} = \frac{-2t - 2t^3 - 2t + 2t^3}{(1+t^2)^2} = \frac{-4t}{(1+t^2)^2}$$

Given $y = \frac{2t}{1+t^2}$ $\left(\frac{u}{v}\right)$ Rule.

$$\therefore \frac{dy}{dt} = \frac{(1+x^2)\frac{d}{dt}(2t) - (2t)\frac{d}{dt}(1+t^2)}{(1+t^2)^2}$$

$$\therefore \frac{dy}{dt} = (1+t^2)(2) - (2t)(0+2k)$$

$$\therefore \frac{dy}{dt} = \frac{2-2t^2}{(1+t^2)^2} = \frac{2(1-t^2)}{(1+t^2)}$$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\frac{2(1-t^2)}{\cancel{(1+t^2)^2}}}{\frac{-4t}{\cancel{(1+t^2)^2}}}$$

$$\therefore \frac{dy}{dx} = \frac{-(1-t^2)}{2t} \quad \text{—— (1)}$$

$$L.H.S = \frac{dy}{dx} + \frac{x}{y} = \frac{-(1-t^2)}{2t} + \frac{(1-t^2)}{(1+t^2)}$$

$$\frac{-(1-t^2)}{2t} + \frac{(1-t^2)}{2t} = 0 \quad R.H.S$$

$$\therefore L.H.S = R.H.S$$

Hence proved *

Ques: If $x = \frac{1+\log t}{t^2}$, $y = \frac{3+2\log t}{t}$, $t>0$ find $\frac{dy}{dx}$ and prove That $y \cdot \frac{dy}{dt} - 2x\left(\frac{dy}{dt}\right)^2 = 1$

Solve We are given

$$x = \frac{1+\log t}{t^2}, \left(\frac{u}{v}\right), \quad y = \frac{3+2\log t}{t} \left(\frac{u}{v}\right) t>0$$

$$\frac{dx}{dt} = \frac{(t^2)\frac{d}{dt}(1+\log t) - (1+\log t)\frac{d}{dt}(t^2)}{(t^2)^2}$$

$$\frac{dx}{dt} = \frac{(t^2)(0+1/t) - (1+\log t)(2t)}{t^4}$$

$$\frac{dx}{dt} = \frac{t - 2t(1+\log t)}{t^4}$$

$$\frac{dx}{dt} = \frac{t[1-2-2\log t]}{t^4}$$

$$\frac{dx}{dt} = \frac{-1-2\log t}{t} \quad \text{------} (1.)$$

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$$\frac{dy}{dx} = \frac{t\frac{d}{dt}(3+2\log t) - (3+2\log t)\frac{d}{dt}(t)}{t^2}$$

$$\frac{dy}{dx} = \frac{(t)(0+2/t) - (3+2\log t)}{t^2}$$

$$\frac{dy}{dx} = \frac{-1-2\log t}{t^2} \quad ------ (2.)$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\left(\frac{-1-2\log t}{t^2}\right)}{\left(\frac{1-2\log t}{t^3}\right)}$$

$$\therefore \frac{dy}{dx} = \frac{t^3}{t^2} = t$$

$$\therefore \frac{dy}{dx} = t \quad ------ (3.)$$

$$L.H.S \div y \frac{dy}{dt} - 2x\left(\frac{dy}{dt}\right)^2$$

$$= \left(\frac{3+2\log t}{t}\right)(t) - 2\left(\frac{1+\log t}{t^2}\right)(t)^2$$

$$= (3+2\cancel{\log t}) - (2+2\cancel{\log t})$$

$$= 3-2 = 1 = R.H.S$$

$$= L.H.S = R.H.S$$

Hence the Proved

Ques: Find $\frac{dy}{dx}$, where $y = a^{(t+\frac{1}{t})}$ and $x = (t+\frac{1}{t})^a$

Solve We are given

$$y = a^{(t+1/t)} \quad (a^x), \quad x = (t+\tfrac{1}{t})^a \quad (x)^a$$

$$\therefore \frac{dy}{dt} = a^{(t+\frac{1}{t})} \log a \frac{d}{dt}(t+1/t)$$

$$\therefore \frac{dy}{dt} = a^{t+1/t} \log a \left(1-\frac{1}{t^2}\right)$$

$$\therefore \frac{dy}{dt} = a^{t+\frac{1}{t}} \log a \left(\frac{t^2-1}{t^2}\right) \text{ ---- } (1)$$

$$\therefore x = \left(t+\frac{1}{t}\right)^a \quad | \; x^a$$

$$\frac{dx}{dt} = a\left(t+\frac{1}{t}\right)^{a-1} \frac{d}{dt}\left(t+\frac{1}{t}\right)$$

$$\frac{dx}{dt} = a\left(t+\frac{1}{t}\right)^{a-1}\left(1-\frac{1}{t^2}\right)$$

$$\therefore \frac{dx}{dt} = a\left(t+\frac{1}{t}\right)^{a-1}\left(\frac{t^2-1}{t^2}\right) \text{ ---- } (2)$$

$$\Rightarrow \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{a^{(t+\frac{1}{t})} \log a \cancel{\left(\frac{t^2-1}{t}\right)}}{a\,(t+\frac{1}{t})^{a-1}\cancel{\left(\frac{t^2-1}{t^2}\right)}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{a^{(t+1/t)} \log a}{a(t+1/t)^{a-1}}$$

DIFFERENTIATION OF A FUNCTION INTO ANOTHER FUNCTION

Ques: 1) Diff $(3x^2+2x)$ into x^3

Solve: Let $y = 3x^2+2x$, $u = x^3$

$$\frac{dy}{dx} = \frac{d}{dx}(3x^2+2x)$$

$$\frac{dy}{dx} = 3(2x)+2(1)$$

$$\frac{dy}{dx} = 6x+2$$

$$u = x^3$$

$$\frac{dy}{du} = \frac{\frac{dy}{dx}}{\frac{du}{dx}}$$

$$\frac{dy}{du} = \frac{6x+2}{3x^2}$$

Ques 2. Diff $\log(xe^x)$ into $x\log x$.

Let $y = \log(xe^x)$

$\therefore y = \log(x) + \log e^x$

$y = \log(x) + x\log e$

$y = \log x + x$

$$\frac{dy}{dx} = \frac{1}{x} + 1$$

$$\frac{dy}{dx} = \frac{1+x}{x}$$

$$\frac{dy}{du} = \frac{\frac{dy}{dx}}{\frac{du}{dx}}$$

$$\frac{dy}{du} = \frac{1+x}{x(1+\log x)}$$

$u = x\log x \; (uv)$

$$\frac{du}{dx} = x\frac{d}{dx}(\log x) + \log x\frac{d}{dx}(x)$$

$$\frac{du}{dx} = x\cdot\frac{1}{x} + \log x \cdot 1$$

$$\frac{du}{dx} = 1 + \log x$$

Ques: Diff. $x^2 e^{3x}$ into $(\log x)^2$

Solve Let $y = x^2 e^{3x}$, $u = (\log x)^2$

$$\frac{dy}{dx} = (x^2)\frac{d}{dx}(e^{3x}) + e^{3x}\frac{d}{dx}(x^2)$$

$$\Rightarrow \frac{dy}{dx} = x^2\frac{d}{dx}(e^{3x}) + e^{3x}\frac{d}{dx}(x^2)$$

$$\Rightarrow \frac{dy}{dx} = x^2\left(e^{3x}\frac{d}{dx}(3x)\right) + (e^{3x})(2x)$$

$$\Rightarrow \frac{dy}{dx} = (x^2)\left[e^{3x}(3)\right] + (e^{3x})(2x)$$

$$\therefore \frac{dy}{dx} = e^{3x}\left[3x^2 + 2x\right] \text{ - - - - - (1)}$$

$u = (\log x)^2$ $\quad [x^n \rightarrow n\cdot x^{n-1}]$

$$\frac{du}{dx} = 2(\log x)^1\frac{d}{dx}(\log x)$$

$\left[\log x = \frac{1}{x}\right]$

$$\frac{dy}{du} = 2(\log x)\left(\frac{1}{x}\right) \quad \text{- - - - (2)}$$

$$\frac{dy}{du} = \frac{\frac{dy}{dx}}{\frac{du}{dx}}$$

$$\frac{dy}{du} = \frac{e^{3x}(3x^2+2x)}{2\log x \cdot \frac{1}{x}}$$

$$\therefore = \frac{e^{3x}(3x^2+2x^2)}{2\log x}$$

Ques: Find $\frac{dy}{dx}$, when $x = e^{\theta}\left(\theta + \frac{1}{\theta}\right)$, $y = e^{-\theta}\left(\theta - \frac{1}{\theta}\right)$

Solve We are given

$$x = \underbrace{e^{\theta}\left(\theta + \frac{1}{\theta}\right)}_{u \cdot v \text{ rule}}, \quad y = e^{-\theta}\left(\theta - \frac{1}{\theta}\right)$$

$$\frac{dx}{d\theta} = \frac{e \cdot d}{d\theta}\left(\theta + \frac{1}{\theta}\right) + \left(\theta + \frac{1}{\theta}\right)\frac{d}{d\theta}(e^{\theta})$$

$$\frac{dx}{d\theta} = e^{\theta}\left(1 - \frac{1}{\theta^2}\right) + \left(\theta + \frac{1}{\theta}\right)(e^{\theta})$$

$$\frac{dx}{d\theta} = e^{\theta}\left[1 - \frac{1}{\theta^2} + \theta + \frac{1}{\theta}\right]$$

$$\frac{dx}{d\theta} = e^{\theta}\left[\frac{\theta^2 - 1 + \theta^3 + \theta}{\theta^2}\right]$$

$\frac{1}{\theta} = -\frac{1}{\theta^2}$

$e^x = e^x$

$$dx = e^{\theta}\left[\frac{\theta^3+\theta^2+\theta-1}{\theta^2}\right] \quad ----(1)$$

$$\therefore \frac{dx}{d\theta} = e^{\theta}\left[\frac{\theta^3+\theta^2+\theta-1}{\theta^2}\right] \quad ----$$

$$\therefore y = \underbrace{e^{-\theta}\left(\theta-\frac{1}{\theta}\right)}_{u \cdot v} \quad (u \cdot v)$$

$$\frac{dy}{d\theta} = e^{-\theta}\frac{d}{d\theta}\left(\theta-\frac{1}{\theta}\right) + \left(\theta-\frac{1}{\theta}\right)\frac{d}{d\theta}\left(e^{-\theta}\right)$$

$$\therefore \frac{dy}{d\theta} = e^{-\theta}\frac{d}{d\theta}\left(\theta-\frac{1}{\theta}\right) + \left(\theta-\frac{1}{\theta}\right)\frac{d}{d\theta}\left(e^{-\theta}\right)$$

$$\therefore \frac{dy}{d\theta} - \left(e^{-\theta}\right)\left(1+\frac{1}{\theta^2}\right) + \left(\theta-\frac{1}{\theta}\right)\left(e^{-\theta}\right)(-1)$$

$$\therefore \frac{dy}{d\theta} = e^{-\theta}\left[1+\frac{1}{\theta^2}-\left(\theta-\frac{1}{\theta}\right)\right]$$

$$\therefore \frac{dy}{d\theta} = e^{-\theta}\left[\frac{\theta^2+1-\theta^3+\theta}{\theta^2}\right]$$

$$\therefore \frac{dy}{d\theta} = e^{-\theta}\left[\frac{-\theta^3+\theta^2+\theta+1}{\theta^2}\right] \quad -----(2)$$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{dy}{dx} = \frac{e^{-\theta}\left(-\theta^3+\theta^2+\theta+1\right)}{e^{\theta}\left(\theta^3+\theta^2+\theta-1\right)}$$

(θ^2 cancel)

$$\therefore \frac{dy}{dx} = \frac{e^{-2\theta}\left(-\theta^3+\theta^2+\theta+1\right)}{\theta^3+\theta^2+\theta-1}$$

Derivative Of Implicit Function

Let the given function is ÷

$x^3+y^3+3x^2y+3xy^2=0$

$y = x^3+x$

$y = 2ax$

$x = at^2$

$y \Rightarrow x^2$

$y \rightarrow t$

$x \rightarrow t$

{खिचड़ी FUNCTION}

1. $\frac{d}{dx}(x^n) = n \cdot x^{n-1} \cdot \frac{d}{dx}(x)$
2. $\frac{d}{dy}(x^n) = n \cdot x^{n-1} \cdot \frac{dx}{dy}$
3. $\frac{d}{dx}(y^n) = n y^{n-1} \cdot \frac{dy}{dx}$
4. $\frac{d}{dy}(y^n) = n \cdot y^{n-1} \cdot \frac{dy}{dx}$

Ques: 1 Find $\frac{dy}{dx}$, if $x^6+y^6+6x^2y^2=16$

Solve We have

$$x^6+y^6+6x^2y^2=16 \quad ----(1)$$

Diff. w.r.to x,

$$\frac{dx}{dx}(x^6)+\frac{d}{dy}(y^6)+6\frac{d}{dx}(x^2y^2)=0$$

$$\therefore 6x^5+6y^5\left(\frac{dy}{dx}\right)+6\left[x^2\left(2y\frac{dy}{dx}\right)+y^2(2x)\right]=0$$

$$\frac{dy}{dx}\left[6y^5+6x^2(2y)\right]=-6x^5-6y^2(2x)$$

$$\frac{dy}{dx}=-\frac{6x(x^4+2y^2)}{6y(y^4+2x^2)}$$

$$\frac{dy}{dx}=\frac{-x(x^4+2y^2)}{y(y^4+2x^2)}$$

Ques 2. Find $\frac{dy}{dx}$, when $xy+xe^{-y}+ye^x=x^2$

Solve We have

$$xy+xe^{-y}+ye^x=x^2 \quad ----(1)$$

Diff w.r.t x

$$\frac{d}{dx}(\underbrace{xy}_{uv})+\frac{d}{dx}(\underbrace{xe^{-y}}_{uv})+\frac{d}{dx}(\underbrace{ye^x}_{uv})=\frac{d}{dx}(x^2)$$

$$\left(x\frac{dy}{dx}+y\cdot 1\right)+\left[x\cdot e^{-y}\left(-\frac{dy}{dx}\right)+e^{-y}(1)\right]+\left[y\cdot e^x+e^x\frac{dy}{dx}\right]=2x$$

$$= \frac{dy}{dx}\left[x + (-1)xe^{-y} + e^{x}\right] = -(y + e^{-y} + ye^{x}) + 2x$$

$$= \frac{dy}{dx} = \frac{2x - y - e^{-y} - ye^{x}}{x - xe^{-y} + e^{x}}$$

Ques 3. If $x\sqrt{1+y} + y\sqrt{1+x} = 0$, show that $\frac{dy}{dx} = -(1+x)^{-2}$

Solve We are given

$$x\sqrt{1+y} + y\sqrt{1+x} = 0$$

$$x\sqrt{1+y} = -y\sqrt{1+x}$$

Squaring both side

$$\Rightarrow x^2(1+y) = y^2(1+x)$$

$$\Rightarrow x^2 + x^2y = y^2 + y^2x$$

$$\Rightarrow (x^2 - y^2) + (x^2y - y^2x) = 0$$

$$\Rightarrow (x-y)(x+y) + xy[x-y] = 0$$

$$\Rightarrow (x-y)[x+y+xy] = 0$$

$$\Rightarrow (x-y)[x+y+xy] = 0$$

$$\Rightarrow x + y + xy = 0 \qquad \because x - y \neq 0$$

$$\Rightarrow y + xy = -x$$

$$\Rightarrow y(1+x) = -x$$

$y = \frac{-x}{1+x}$ $(\because)$

Diff. w.r.to x

$\therefore \frac{dy}{dx} = \frac{(1+x)\frac{d}{dx}(-x) - (-x)\frac{d}{dx}(1+x)}{(1+x)^2}$

$\therefore \frac{dy}{dx} = \frac{(1+x)(-1) - (-x)(0+1)}{(1+x^2)}$

$\therefore \frac{dy}{dx} = \frac{-1 \cancel{-x} \cancel{+x}}{(1+x)^2}$

$\therefore \frac{dy}{dx} = \frac{-1}{(1+x)^2}$

Ques: If $x^2+y^2 = t - \frac{1}{t}$, and $x^4+y^4 = t^2+\frac{1}{t^2}$ Show that $\frac{dy}{dx} = \frac{1}{x^3y}$

Solve We are given

$x^2+y^2 + t - \frac{1}{t}$ ------ (1.)

and $x^4+y^4 = t^2+\frac{1}{t^2}$ --- (2.)

Squaring (1.) both side

$\Rightarrow (x^2+y^2)^2 = \left(t-\frac{1}{t}\right)^2$

$\Rightarrow x^4+y^4+2x^2y^2 = t^2+\frac{1}{t^2} - 2(t)\left(\frac{1}{t}\right)$

$\Rightarrow x^4+y^4+2x^2y^2 = t^2+\frac{1}{t^2} - 2$

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$\Rightarrow x^4+y^4+2x^2y^2 = (x^4+y^4)-2$

$\Rightarrow x^4+y^4+2x^2y^2 - (x^4+y^4) = -2$

$$2x^2y^2 = -2$$

$$x^2y^2 = -1$$

$$y^2 = -\frac{1}{x^2}$$

diff w.r.to x,

$$2y\frac{dy}{dx} = -\left[-\frac{2}{x^3}\right]$$

$$y\left[\frac{dy}{dx} = \frac{1}{x^3}\right]$$

$$\therefore \frac{dy}{dx} = \frac{1}{x^3y}$$

1. $\frac{1}{x} \rightarrow -\frac{1}{x^2}$

2. $\frac{1}{x^2} \rightarrow \frac{-2}{x^3}$

3. $\frac{1}{x^3} = \frac{-3}{x^4}$

Ques) If $y=\sqrt{x+\sqrt{x+\sqrt{x+\ldots\ldots\infty}}}$ show that $(2y-1)\frac{dy}{dx}=1$

Solve We are given

$$y=\sqrt{x+\sqrt{x+\sqrt{x+\ldots\ldots\infty}}}$$

$$\therefore y=\sqrt{x+y}$$

Squaring both side

$$y^2 = x+y$$

diff. w.r.to x

$$2y\frac{dy}{dx} = 2^x \log 2 + \frac{dy}{dx}$$

$$(2y-1)\frac{dy}{dx} = 2^x \log 2$$

$x^a \rightarrow ax^{a-1}$

$a^x \rightarrow a^x \log x$

Ques: If $y\sqrt{x^2+1} = \log(x+\sqrt{x^2+1})$, show that $(x^2+1)\frac{dy}{dx} + xy - 1 = 0$

Solve We are given

$$\underset{u}{y}\,\underset{v}{\sqrt{x^2+1}} = \log(x+\sqrt{x^2+1})$$

$$\therefore y\cdot\frac{d}{dx}\left(\sqrt{x^2+1}\right) + \sqrt{x^2+1}\,\frac{d}{dy}(y) = \frac{1}{x+\sqrt{x^2+1}}\,\frac{d}{dx}\left(x+\sqrt{x^2+1}\right)$$

$$\therefore y\cdot\frac{1}{2\sqrt{x^2+1}}(2x+0) + \sqrt{x^2+1}\,\frac{dy}{dx} = \frac{1}{x+\sqrt{x^2+1}} = \left[1+\frac{1}{2\sqrt{x^2+1}}(2x+0)\right]$$

$$\Rightarrow y\cdot\frac{1}{\sqrt{x^2+1}}(x) + \sqrt{x^2+1}\,\frac{dy}{dx} = \frac{1}{\cancel{x+\sqrt{x^2+1}}} = \left[\frac{\cancel{\sqrt{x^2+1}+x}}{\sqrt{x^2+1}}\right]$$

$$\Rightarrow \frac{xy}{\sqrt{x^2+1}} + \sqrt{x^2+1}\,\frac{dy}{dx} = \frac{1}{\sqrt{x^2+1}}$$

$$xy + (x^2+1)\frac{dy}{dx} = 1$$

$$(x^2+1)\frac{dy}{dx} + xy - 1 = 0$$

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Ques: $ax^2+2hxy+by^2=1$ Verify that $\frac{dy}{dx} \times \frac{dx}{dy}=1$

Solve We are given

$$ax^2+2hxy+by^2=1 \quad ----(1)$$

Diff (1) w.r.to x

$L.H.S = \frac{dy}{dx} \times \frac{dx}{dy}$

$1 = 1$

$$\therefore a(2x)+2h\left[x\cdot\frac{dy}{dx}+y\cdot 1\right]+b\left(2y\frac{dy}{dx}\right)=0$$

$$\therefore 2(hx+2by]\frac{dy}{dx}=-2ax-2hy$$

$$\therefore \frac{dy}{dx}=\frac{-2(ax+hy)}{2(hx+by)}$$

$$\frac{dy}{dx}=\frac{-(ax+hy)}{hx+by} \quad -----(2)$$

diff (1) into y,

$$a\left(2x\frac{dx}{dy}\right)+2h\left[x(1)+y\left(\frac{dx}{dy}\right)\right]+\left[b(2y)\right]=0$$

$$\frac{dx}{dy}\left[2ax+2hy\right]=-2hx-2by$$

$$\frac{dx}{dy}=\frac{-2(hx+by)}{2(ax+hy)}$$

$$\frac{dx}{dy}=\frac{-(hx+by)}{ax+hy} \quad -----(3)$$

$$\frac{dy}{dx}\times\frac{dx}{dy}=\left[-\frac{(ax+hy)}{hx+by}\right]\times\left[-\left(\frac{hx+by}{ax+hy}\right)\right]$$

$$\therefore \frac{dy}{dx}\times\frac{dx}{dy}=1$$

	x^2	xy	y^2	1
W.r.to x ←	$2x$]	$x \cdot \frac{dy}{dx} + y \cdot 1$	$2y \cdot \frac{dy}{dx}$	0
W.r.to y ←	$2x \frac{dx}{dy}$]	$x \cdot 1 + y \cdot \frac{dx}{dy}$	$2y$	0

FORMULAE

1. $\log(m^n) = n \log m$
2. $\log(m \cdot n) = \log m + \log n$
3. $\log\left(\frac{m}{n}\right) = \log m - \log n$
4. $\log x = \frac{1}{x}$, x^x = taking log

Logarithmic Differentiation

$x^a \longrightarrow (\text{variable})^{\text{costant}} \longrightarrow a \cdot x^{a-1}$

$a^x \longrightarrow (\text{constant})^{\text{variable}} \longrightarrow a^x \cdot \log a$

$a^a \longrightarrow (\text{constant})^{\text{constant}} \longrightarrow 0$ zero

$x^x \longrightarrow (\text{variable})^{\text{variable}} \longrightarrow$ (taking log)

4: If $y = x + \cfrac{1}{x + \cfrac{1}{x + \cdots \infty}}$ prove that $\frac{dy}{dx} = \frac{y}{2y - x}$

Solve: We are given

$$y = x + \cfrac{1}{x + \cfrac{1}{x + \cdots \infty}}$$

$$\therefore y = x + \frac{1}{y}$$

$$y = \sqrt{x + \sqrt{x + \sqrt{x + \cdots \infty}}}$$

$$\therefore y^2 = xy + 1$$

$$\therefore y = \sqrt{x + y}$$

$\therefore$ diff w.r.t x,

$$2y \frac{dy}{dx} = x \frac{dy}{dx} + y \cdot 1 + 0$$

$$\therefore (2y - x)\left(\frac{dy}{dx}\right) = y$$

$$\therefore \frac{dy}{dx} = \frac{y}{2y - x}$$

$$\therefore \frac{dy}{dx} = \frac{y}{2y - x}$$

Ques: If $e^x + e^y = e^{x+y}$, prove that $\frac{dy}{dx} + e^{y-x} = 0$

Solve: We are given

$$e^x + e^y = e^{x+y} \quad \text{—— (1)}$$

$$\therefore \frac{e^x}{e^{x+y}} + \frac{e^y}{e^{x+y}} = 1$$

$$\therefore \frac{e^x}{e^x \cdot e^y} + \frac{e^y}{e^x \cdot e^y} = 1$$

$$\therefore e^{-y} + e^{-x} = 1$$

Diff w.r.to x

$$\therefore e^{-y}\left(-\frac{dy}{dx}\right) + e^{-x}(-1) = 0$$

$$\therefore -\frac{1}{e^y} \cdot \frac{dy}{dx} - \frac{1}{e^x} = 0$$

$$\therefore \frac{dy}{dx} + e^{y-x} = 0$$

$$\frac{d}{dx}(e^{-y}) = e^{-y}\left(\frac{d}{dx}(-y)\right)$$

$$\therefore = e^{-y}\left(-\frac{dy}{dx}\right)$$

$$\frac{d}{dx}(e^{-x}) = e^{-x}\left(\frac{d}{dx}(-x)\right)$$

$$\therefore = e^{-x}(-1)$$

Ques: If $e^x + e^y = e^{x+y}$, prove that $\frac{dy}{dx} = -\frac{e^x(e^y-1)}{e^y(e^x-1)}$

Solve: We are given

$$e^x + e^y = e^{x+y}$$

$\therefore$ Diff w.r.to x,

$$\frac{d}{dx}(e^x) + \frac{d}{dx}(e^y) = \frac{d}{dx}(e^{x+y})$$

(1) $\frac{d}{dx}(e^x) = e^x \cdot 1 = e^x$

(2) $\frac{d}{dx}(e^y) = e^y \cdot \frac{dy}{dx}$

(3) $\frac{d}{dx}(e^{x+y}) = e^{x+y}\left(1 + \frac{dy}{dx}\right)$

$$\frac{d}{dx}(e^x) + \frac{d}{dx}(e^y) = \frac{d}{dx}(e^{x+y})$$

$$\therefore e^x + e^y \cdot \frac{dy}{dx} = e^{x+y}\left[1 + \frac{dy}{dx}\right]$$

$$\therefore e^x + e^y \frac{dy}{dx} = e^{x+y} + e^{x+y}\frac{dy}{dx}$$

$$\therefore \frac{dy}{dx}\left[e^y - e^{x+y}\right] = e^{x+y} - e^x$$

$e^{x+y} = e^x \cdot e^y$

$$\therefore \frac{dy}{dx}\left[e^y(1-e^x)\right] = e^x(e^y-1)$$

$$\therefore \frac{dy}{dx} = \frac{e^x(e^y-1)}{e^y(1-e^x)}$$

$$\therefore \frac{dy}{dx} = \frac{e^x(e^y-1)}{e^y(e^x-1)}$$

Ques Diff x^x w.r.to x.

Solve Let $y = x^x$ —— (1)

$\therefore$ Taking log both side

$\log y = \log(x^x)$

$\therefore \log y = x(\log x)$ (U.V)

$\therefore$ diff. w.r.to x,

$$\frac{1}{y}\frac{dy}{dx} = x \cdot \frac{d}{dx}(\log x) + \log x \frac{d}{dx}(x)$$

$$\therefore \frac{1}{y}\frac{dy}{dx} = x \cdot \frac{1}{x} + \log x \cdot 1$$

$$\therefore \frac{1}{y}\frac{dy}{dx} = 1 + \log x$$

$$\therefore \frac{dy}{dx} = y\,(1+\log x)$$

$$\therefore \frac{dy}{dx} = x^x(1+\log x)$$

1.) $x^a \longrightarrow a x^{a-1}$

2.) $a^x \longrightarrow a^x \log a$

3.) $a^a \longrightarrow 0$

4.) $x^x \longrightarrow$ Taking log

FORMULAE

Ques.) Diff following w.r.to x

(1.) x^{x^x}

Solve Let $y = x^{x^x}$ ---- (1.)

$\therefore$ Taking log both side

$$\log y = \log(x^{x^x})$$

$$\therefore \log y = x^x \log x \text{ --- } (2)$$

$\log m^n = n \log m$

Taking log both x

$\therefore \log(\log y) = \log(x^x \log x)$ | $\log(m \cdot n) = \log m + n$

$\therefore \log(\log y) = \log(x^x) + \log(\log x)$

$\therefore \log(\log y) = x \log x + \log(\log x)$

diff. w.r.to x,

$\therefore \frac{1}{\log y} \cdot \frac{1}{y} \frac{dy}{dx} = x \cdot \frac{1}{x} + \log x \cdot 1 + \frac{1}{\log x} \cdot \frac{1}{x} \cdot 1$

$\therefore \frac{1}{\log y \cdot y} \frac{dy}{dx} = 1 + \log x + \frac{1}{x + \log x}$

$\therefore \frac{dy}{dx} = y \cdot \log y \left[1 + \log x + \frac{1}{x \log x}\right]$

$\therefore \frac{dy}{dx} = x^{x^x} \cdot x^x \log x \left[1 + \log x + \frac{1}{x \log x}\right]$

$\frac{d}{dx}(\log(\log y)) \longrightarrow \frac{1}{\log y} \cdot \frac{1}{y} \cdot \frac{dy}{dx}$

$\frac{d}{dx}(\log(\log x)) \longrightarrow \frac{1}{\log x} \cdot \frac{1}{x} \cdot 1 = \frac{1}{\log x}$

Q: Diff following w.r.to x,

(2.) $(x)^{x^x}$

Solve Let $y = (x^x)^x = x^{x^2}$ ------- (1.)

Taking log both side,

$\therefore \log y = x^2 \log x$ ------- (2.)

diff. both side w.r.to x,

$$\frac{1}{y} \cdot \frac{dy}{dx} = x^2 \frac{d}{dx}(\log x) + (\log x)\frac{d}{dx}(x^2)$$

$$\therefore \frac{1}{y} \cdot \frac{dy}{dx} = x^{\cancel{2}} \cdot \frac{1}{\cancel{x}} + (\log x)(2x)$$

$$\therefore \frac{1}{y} \frac{dy}{dx} = x + (2x)\log x$$

$$\therefore \frac{dy}{dx} = y\left[x + 2x \log x\right]$$

$$\therefore \frac{dy}{dx} = (x^x)^x \left(x + 2^x \log\right)$$

Ques. Diff. $(\log x)^{\log x}$, $x > 1$ w.r.t x.

Solve. Let $y = (\log x)^{\log x}$ ---- (1.)

Taking log both side

$\log y = \log (\log x)^{\log x}$ (Variable)$^{\text{Variable}}$

$\therefore \log y = \log x \cdot \log(\log x)$ (U.V)

$\therefore$ diff w.r.to x,

$$\frac{1}{y}\cdot\frac{dy}{dx} = (\log x)\cdot\frac{d}{dx}\left[\log(\log x)\right] + \log(\log x)\frac{d}{dx}(\log x)$$

$$\therefore \frac{1}{y}\frac{dy}{dx} = \cancel{(\log x)}\left(\frac{1}{\cancel{\log x}}\cdot\frac{1}{x}\cdot 1\right) + \log(\log x)\cdot\frac{1}{x}\cdot 1$$

$$\therefore \frac{1}{y}\frac{dy}{dx} = \frac{1}{x} + \log(\log x)\cdot\frac{1}{x}$$

$$\therefore \frac{dy}{dx} = y\left[\frac{1}{x} + \log(\log x)\frac{1}{x}\right]$$

$$\therefore \frac{dy}{dx} = (\log x)^{\log x}\left[\frac{1}{x} + \log(\log x\cdot\frac{1}{x})\right]$$

Ques: If $x^y = e^{x-y}$, prove that $\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$ or $\frac{\log y}{(\log[xe)]^2}$

Solve We are given

$$x^y = e^{x-y}$$

$\therefore$ Taking log both side we get

$$\log(x^y) = \log(e^{x-y})$$

$$\therefore y\log x = (x-y)\log e \quad \Big| \quad \therefore \log e = 1$$

$$y\log x = x-y$$

$$y\log x + y = x$$

$y \log x + y = x$

$\therefore y(\log x + 1) = x$

$\therefore y = \dfrac{x}{(1+\log x)}$ $\left[\dfrac{u}{v}\right]$

$\therefore$ diff w.r.t. x,

$\therefore \dfrac{dy}{dx} \dfrac{(1+\log x)\frac{d}{dx}(x) - x\frac{d}{dx}(1+\log x)}{(1+\log x)^2}$

$\therefore \dfrac{dy}{dx} \dfrac{(1+\log x)(1) - (x)(0+\frac{1}{x})}{(1+\log x)^2}$

$\therefore \dfrac{dy}{dx} = \dfrac{(1+\log x) - 1}{(1+\log x)^2}$

$\therefore \dfrac{dy}{dx} = \dfrac{\log x}{(1+\log x)^2}$

$\therefore \dfrac{dy}{dx} = \dfrac{\log x}{[\log e + \log x]^2}$

$\therefore \dfrac{dy}{dx} = \dfrac{\log x}{(\log(xe))^2}$

$\log m \cdot n = \log m + \log n$

Ques: If $y^x = e^{y-x}$ prove that $\frac{dy}{dx} = \frac{(1+\log y)^2}{\log y}$

Solve) We are given

$$y^x = e^{y-x} \text{ --- } (1.)$$

Taking log both side

$$\log(y^x) = \log(e^{y-x})$$

$$\therefore x \log y = (y-x)$$

$$\therefore x \log y = y-x$$

$$\therefore x \log y + x = y$$

$$\therefore x[1+\log y] = y$$

$$\therefore x = \frac{y}{(1+\log y)} \quad \left(\frac{u}{v}\right)$$

$$\therefore \frac{dx}{dy} = \frac{(1+\log y)\frac{d}{dy}(y) - y\{\frac{dy}{dx}(1+\log x)}{(1+\log y)^2}$$

$$\therefore \frac{dx}{dy} = \frac{(1+\log y)\cdot 1 - y\left(0+\frac{1}{y}\right)}{(1+\log y)^2}$$

$$\frac{dx}{dy} = ?$$

$$\frac{dy}{dx} = \frac{1}{dx/dy}$$

$$\therefore \frac{dx}{dy} = \frac{(1+\log y)\cdot 1 - 1}{(1+\log y)^2}$$

$$\frac{d}{dx}(1+\log y) = 0 + \frac{1}{y}\frac{dy}{dx}$$

$$\therefore \frac{dy}{dx} = \left(\frac{1}{\frac{dx}{dy}}\right)$$

$$\frac{d}{dy}(1+\log y) = 0 + \frac{1}{y}\cdot 1$$

$$\therefore \frac{dy}{dx} = \frac{(1+\log y)^2}{\log y}$$

Que If $y = x^{x^{x^{\cdots\infty}}}$, prove that $x\frac{dy}{dx} = \frac{y^2}{1-y\log x}$

Solve We are given

$$y = x^{x^{x^{\cdots\infty}}}$$

$$\therefore y = x^y$$

$\therefore$ Taking log both side

$$\log y = \log(x^y)$$

$$\therefore \log y = y\log(x) \quad (U\cdot V)$$

$\therefore$ diff w.r.to x

$$\therefore \frac{1}{y}\frac{dy}{dx} = y\cdot\frac{d}{dx}(\log x) + (\log x)\cdot\frac{dy}{dx}$$

$$\therefore \frac{1}{y}\frac{dy}{dx} = y\cdot\frac{1}{x} + (\log x)\cdot\frac{dy}{dx}$$

$\therefore \left(\frac{1}{y} - \log x\right) \frac{dy}{dx} = \frac{y}{x}$

$\therefore \left(\frac{1 - y\log x}{y}\right) \frac{dy}{dx} = \frac{y}{x}$

$\therefore x \frac{dy}{dx} = \frac{y^2}{(1 - y\log x)}$

Hence the Result

Ques If $y = a^{x^{a^{x^{\cdots \infty}}}}$, Prove that $\frac{dy}{dx} = \frac{y^2 \log y}{x(1 - y\log x - \log y)}$

Solve We are given

$y = a^{x^{a^{x^{\cdots \infty}}}}$

$\therefore y = a^{x^y}$ ------ (1)

$\therefore$ Taking log both side

$\log y = \log (a^{x^y})$

$\therefore \log y = x^y \log a$

$\therefore$ Again Taking Log both side

$\log (\log y) = (x^y . \log a)$

$\therefore \log(\log y) = \log(x^y) + \log(\log a)$

$\therefore \log(\log y) = y\log x + \log(\log a)$

$\therefore$ diff. w.r.to x,

$$\therefore \frac{1}{\log y}\cdot\frac{1}{y}\cdot\frac{dy}{dx} = \left(y\cdot\frac{d}{dx}(\log x) + \log x\,\frac{d}{dx}(y)\right) + 0$$

$$\therefore \frac{1}{\log y}\cdot\frac{1}{y}\cdot\frac{dy}{dx} = \left[y\cdot\frac{1}{x} + \log x\cdot\frac{dy}{dx}\right]$$

$$\therefore \frac{dy}{dx}\left[\frac{1}{y(\log y)} - \log x\right] = \frac{y}{x}$$

$$\therefore \frac{dy}{dx}\left[\frac{1 - y\log x\log y}{y\log y}\right] = \frac{y}{x}$$

$$\therefore \frac{dy}{dx} = \frac{y^2\log y}{x(1 - y\log x - \log y)}$$

Ques: If $x^y = y^x$ prove that $\frac{dy}{dx} = \frac{\frac{y}{x} - \log y}{\frac{x}{y} - \log x}$

Solve We are given

$x^y = y^x$ ------ (1.)

$\therefore$ Taking log both side

$\log(x^y) = \log(y^x)$

$\therefore y \log x = x \log y$

$\therefore$ diff w.r.to x

① $\log x$ (U·V)

$y \cdot \frac{1}{x} + \log x \cdot \frac{dy}{dx}$

② $x \log y$ (U·V)

$x \cdot \frac{1}{y} \cdot \frac{dy}{dx} + \log y \cdot 1$

$$\therefore y \frac{d}{dx}(\log x) + \log x \frac{dy}{dx} = x \frac{d}{dx}(\log y) + \log y \frac{d}{dx}(x)$$

$$y \cdot \frac{1}{x} + \log x \frac{dy}{dx} = x \frac{1}{y} \frac{dy}{dx} + \log y \cdot 1$$

$$\frac{dy}{dx}\left[\log x - \frac{x}{y}\right] = \log y - \frac{y}{x}$$

$$\frac{dy}{dx} = \frac{\log y - y/x}{\log - x/y} = \frac{y/x - \log y}{x/y - \log x}$$

$$\therefore \frac{dy}{dx} = \frac{y/x - \log y}{x/y - \log x}$$

Hence the Result.

Ques: If $y = a^{xy}$, prove that $\frac{dy}{dx} = \frac{y^2 \log a}{1 - xy \log a}$

Solve: We are given

$$y = a^{xy} \quad \text{----- (1)}$$

Taking log both side

$$\log y = \log(a^{xy})$$

① $\log(m \cdot n) = \log m + \log n$

② $\log(m/n) = \log m - \log n$

③ $\log(m^n) = n \log m$

$\therefore \log y = xy \log a$

$\therefore$ diff w.r.to x,

$$\therefore \frac{1}{y}\frac{dy}{dx} = (\log a)\left[x\cdot\frac{dy}{dx} + y\cdot 1\right]$$

$$\therefore \frac{dy}{dx}\left[\frac{1}{y} - x\log a\right] = y\log a$$

$$\therefore \frac{dy}{dx}\left[\frac{1 - xy\log a}{y}\right] = y\log a$$

$$\therefore \frac{dy}{dx} = \frac{y^2\log a}{1 - xy\log a}$$

NOTE ⇒ $xy\log a = (\log a)[xy]$

$$= (\log a)\left[x\cdot\frac{dy}{dx} + y\cdot 1\right]$$

Ques: Diff. $\sqrt{\frac{(x-1)\cdot(x-2)}{(x-3)\cdot(x-4)(x-5)}}$ w.r.to x.

Solve Let $y = \sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)(6x-5)}}$ ---- (1)

Taking log both side

$$\log y = \log\left[\sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}}\right]$$

$$\log y = \frac{1}{2}\left[\log\left[\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}\right]\right]$$

$$\log y = \frac{1}{2}\left[\log[(x-1)(x-2)] - \log[(x-3)(x-4)(x-5)]\right]$$

$$\therefore \log y = \frac{1}{2}\left[\log(x-1) + \log(x+2) - \log(x-3) - \log(x-4) - \log(x-5)\right]$$

Diff w.r.to x,

$$\therefore \frac{1}{y}\frac{dy}{dx} = \frac{1}{2}\left[\frac{1}{x-1} + \frac{1}{x-2} - \frac{1}{x-3} - \frac{1}{x-4} - \frac{1}{x-5}\right]$$

$$\therefore \frac{dy}{dx} = \frac{1}{2}y\left[\frac{1}{x-1} + \frac{1}{x-2} - \frac{1}{x-3} - \frac{1}{x-4} - \frac{1}{x-5}\right]$$

$$\therefore \frac{dy}{dx} = \frac{1}{2}\sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}}\left[\frac{1}{x-1} + \frac{1}{x-2} + \frac{1}{x-3} - \frac{1}{x-4} - \frac{1}{x-5}\right]$$

Ques: If $x^p \cdot y^q = (x+y)^{p+q}$, Show that $\frac{dy}{dx} = \frac{y}{x}$

Solve We are given

$$x^p \cdot y^q = (x+y)^{p+q} \quad \text{----} \quad (1)$$

$\therefore$ Taking log both side

$$\log[x^p \cdot y^q] = \log[(x+y)^{p+q}]$$

$$\therefore \log(x^p) + \log(y^q) = (p+q)\log(x+y)$$

$$\therefore p\log x + q\log y = (p+q)\log(x+y)$$

$\therefore$ diff. w.r.to x

$$\therefore p\frac{1}{x}+q\cdot\frac{1}{y}\frac{dy}{dx}=(p+q)\left[\frac{1}{x+y}\left(1+\frac{dy}{dx}\right)\right]$$

$$\therefore \frac{p}{x}+\frac{q}{y}\frac{dy}{dx}=\frac{p+q}{x+y}+\frac{p+q}{x+y}\frac{dy}{dx}$$

$$\therefore \frac{dy}{dx}\left[\frac{q}{y}-\frac{p+q}{x+y}\right]=\frac{p+q}{x+y}-\frac{p}{x}$$

$$\therefore \frac{dy}{dx}\left[\frac{q(x+y)-y(p+q)}{y(x+y}\right]=\frac{(p+q)x-p(x+y)}{x(x+y)}$$

$$\therefore \frac{dy}{dx}\left[\frac{q(x+y)-y(p+q)}{y(x+y)}\right]=(p+q)x-p(x+y)$$

$$\therefore \frac{dy}{dx}=\left(\frac{qx-py}{x}\right)\times\left(\frac{y}{qx-yp}\right)$$

$$\therefore \frac{dy}{dx}=\frac{y}{x}$$

Ques: Diff. the following function w.r.to x,

(1) $x^x+x^{\log x}$

Solve Let $y=x^x+x^{\log x}$ —— (1)

$\therefore y=u+v$

where $u=x^x$, $v=x^{\log x}$

$$\therefore \frac{dy}{dx}=\frac{du}{dx}+\frac{dv}{dx} \quad \text{----- (2)}$$

Ques: $\therefore u = x^x$

$\therefore \log u = \log(x^x)$

$\therefore \log u = x \log x$ [U·V]

$\therefore$ diff. w.r.to x

$\frac{1}{u}\frac{du}{dx} = x \cdot \frac{1}{x} + \log x \cdot 1$

$\therefore \frac{du}{dx} = x^x [1 + \log x]$

$V = x^{\log x}$

$\therefore \log V = \log(x^{\log x})$

$\therefore \log V = \log x \cdot \log x$

$\therefore \log = (\log x^2)$

$\therefore$ diff. w.r.to. x

$\frac{1}{v}\frac{dv}{dx} = 2(\log x)$

$\therefore \frac{dv}{dx} = 2x^{\log x}\left(\frac{\log x}{x}\right)$

$\therefore \frac{dy}{dx} = x^x(1 + \log u) + 2x^{\log u - 1}(\log x)$

Ques: Diff. the following into x,

(1.) $(\log x)^x + (1+x)^{2x}$

Solve Let $y = (\log x)^x + (1+x)^{2x}$ — (1.)

$\therefore$ Let $y = u + V$

$\therefore dy = \frac{dv}{dx} + \frac{dv}{dx}$ --- (2.)

$\therefore u = (\log x)^x$

$\therefore \log u = \log(\log x)^x$

$\therefore \log u = x \log(\log x)$

$V = (1+x)^{2x}$

$\therefore \log V = \log(1+x)^{2x}$

$\therefore \log V = (2x)\log(1+x)$

∴ diff. w.r.t x

$$\frac{1}{u}\frac{du}{dx} = x\left(\frac{1}{\log x}\cdot\frac{1}{x}\cdot 1\right) + \log(\log x) - 1$$

$$\frac{du}{dx} = (\log x)^x\left[\frac{1}{\log} + \log(\log x)\right]$$

∴ diff. w.r.t x

$$\frac{1}{v}\frac{dv}{dx} = (2x)\frac{1}{1+x} + \log(1+x)$$

$$\therefore \frac{dv}{dx} = (1+2x)^{2x}\left[\frac{2x}{1+x} + 2\log(1+x)\right]$$

$$\therefore ② \Rightarrow \frac{dy}{dx} = (\log x)^x\left[\frac{1}{\log x} + \log(\log x)\right] + (1+2x)^{2x}\left[\frac{2x}{1+x} + 2\log(1+x)\right]$$

NOTE :→

$$\frac{d}{dx}(\log(\log x)) = \frac{1}{\log x}\cdot\frac{1}{x}\cdot 1$$

⇒ Let $y = x^x + x^a + a^x + a^{a^x}$, $a > 0$

∴ Let $y = u + v + w + t$

$$\therefore \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} + \frac{dw}{dx} + \frac{d(t)}{dx} \quad \text{—— ①}$$

∴ $u = x^x$, $v = x^a$

$w = a^x$, $t = a^a$

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$\therefore u = x^x$

$\therefore \log u = \log(x^x)$

$\therefore \log u = x \log x$

$\therefore$ diff. w.r.to x,

$v = x^a$

$\therefore \frac{dv}{dx} = ax^{a-1}$

$w = a^x$

$\frac{dw}{dx} = a^x \log a$

$t = a^a$

$\frac{dt}{dx} = 0$

$\therefore \frac{1}{u}\frac{du}{dx} = \left(x \cdot \frac{1}{x} + \log x \cdot 1\right)$

$\therefore \frac{du}{dx} = x^n (1 + \log x)$

$\therefore (1) \Rightarrow \frac{dy}{dx} = x^x (1 + \log x) + ax^{a-1} + \log a + 0$

$\therefore \frac{dy}{dx} = x^x (1 + \log x) + ax^{a-1} + a^x \log a$

Ques Find $\frac{dy}{dx}$, when $x^y + x^x = a^b$

$x^y + y^x = 1$

Solve We are given

$x^y + y^x = a^b$ ------ (1)

$\therefore u + v = a^b$ ------ (2)

$\therefore \frac{du}{dx} + \frac{dv}{dx} = 0$ --- (3)

$\frac{d}{dx}(y(\log x))$

$= y \cdot \frac{1}{x} + \log x \cdot \frac{dy}{dx}$

$\frac{d}{dx}(x \log y)$

$= x \cdot \frac{1}{y}\frac{dy}{dx} + \log y$

Let $u = x^y$

$\therefore \log u = \log (x)^y$

$\therefore \log u = y \log x$

$\therefore$ diff w.r.to x,

$$\frac{1}{u}\frac{du}{dx} = y \cdot \frac{1}{x} + \log x \cdot \frac{dy}{dx}$$

$$\therefore \frac{du}{dx} = x^y\left[\frac{y}{x} + \log x \cdot \frac{dy}{dx}\right]$$

$V = y^x$

$\therefore \log V = \log(y^x)$

$\therefore \log V = x \log y$

$\therefore$ diff w.r.to x

$$\frac{1}{V}\frac{dv}{dx} = x \cdot \frac{1}{y}\frac{dy}{dx} + \log y \cdot 1$$

$$\therefore \frac{dv}{dx} = y^x\left[\frac{x}{y}\frac{dy}{dx} + \log y\right]$$

$$\therefore (3) \Rightarrow x^y\left[\frac{y}{x} + \log x \cdot \frac{dy}{dx}\right] + y^x\left[\frac{x}{y}\frac{dy}{dx} + \log y\right] = 0$$

$$\therefore \frac{dy}{dx}\left[x^y \log x + x y^{x-1}\right] = -x^y\left(\frac{y}{x}\right) - y^x(\log y)$$

$$\therefore \frac{dy}{dx} = -\left(\frac{x^{y-1} \cdot y + y^x \log y}{x^y \log x + x y^{x-1}}\right)$$

Ques: If $(x-y)e^{x/x-y} = a$, prove that $y\frac{dy}{dx} + x = 2y$

Solve We are given

$$(x-y)e^{\frac{x}{x-y}} = a$$

$$\therefore \log\left[(x-y)e^{\frac{x}{x-y}}\right] = \log a$$

$\therefore \log(x-y) + \log\left(e^{\frac{x}{x-y}}\right) = \log a$

$\therefore \log(x-y) + \frac{x}{x-y} \log e = \log a$

$\therefore$ diff w.r.t x

$$\frac{1}{(x-y)}\left[1-\frac{dy}{dx}\right] + \frac{(x-y)\cdot 1 - x\left(1-\frac{dy}{dx}\right)}{(x-y)^2} = 0$$

$$\therefore \frac{1}{x-1}\left[1-\frac{dy}{dx}\right] + \left[\frac{x-y}{(x-y^2)^2} - \frac{x}{(x-y)^2}\left(1-\frac{dy}{dx}\right)\right]$$

$$\therefore \left[\frac{1}{(x-y)} - \frac{x}{(x-y)^2}\right]\left(1-\frac{dy}{dx}\right) = -\frac{\cancel{(x-y)}}{(x-y)^{\cancel{2}}}$$

$$\therefore \left[\frac{\cancel{x}-y) - \cancel{x}}{(x-y)^2}\right]\left(1-\frac{dy}{dx}\right) = \frac{-1}{(x-y)}$$

$$\therefore \left(+\frac{y}{(x-y)^{\cancel{2}}}\right)\left(1-\frac{dy}{dx}\right) = +\frac{1}{(x-\cancel{y})}$$

$$\therefore \left(\frac{y}{x-y}\right)\cdot\left(1-\frac{dy}{dx}\right) = 1$$

$$\therefore 1-\frac{dy}{dx} = \frac{x-y}{y}$$

$$\therefore \cancel{1} - \frac{dy}{dx} = \frac{x}{y} - \frac{\cancel{y}}{\cancel{y}}$$

$\therefore 1-\frac{dy}{dx}=\frac{x}{y}-1$

$\therefore \frac{dy}{dx}+\frac{x}{y}=2$

$\therefore y\frac{dy}{dx}+x=2y$

Ques If $y=\frac{ax^2}{(x-a)(x-b)(x-c)}+\frac{bx}{(x-b)(x-c)}+\frac{c}{(x-c)}+1$,

Prove that $\frac{y'}{y}=\frac{1}{x}\left(\frac{a}{a-x}+\frac{b}{b-c}+\frac{c}{c-x}\right)$

$y'=\frac{dy}{dx}$

Solve We are given

$$y=\frac{ax^2}{(x-a)(x-b)(x-c)}+\frac{bx}{(x-b)(x-c)}+\left[\frac{c}{x-c}+1\right]$$

$$\therefore y=\frac{ax^2}{(x-b)(x-b)(x-c)}+\frac{bx}{(x-b)(x-1)}+\frac{\cancel{c}+x\cancel{c}}{x-c}$$

$$\therefore y=\frac{ax^2}{(x-a)(x-b)(x-x)}+\left[\frac{bx}{(x-b)(x-c)}+\frac{x}{(x-c)}\right]$$

$$\therefore y=\frac{ax^2}{(x-a)(x-b)(x-c)}+\left[\frac{bx+x(x-b)}{(x-b)(x-c)}\right]$$

$$\therefore y=\frac{ax^2}{(x-a)(x-b)(x-c)}+\frac{\cancel{bx}+x-\cancel{xb}}{(x-b)(x-c)}$$

$$\therefore y = \frac{ax^2}{(x-a)(x-b)(x-c)} + \frac{x^2}{(x-b)(x-c)}$$

$$\therefore y = \frac{ax^2 + x^2(x-a)}{(x-a)(x-b)(x-c)}$$

$$\therefore y = \frac{\cancel{ax^2} + x^3 - \cancel{x^2 a}}{(x-a)(x-b)(x-c)}$$

$$\therefore y = \frac{x^3}{(x-a)(x-b)(x-c)}$$

Taking log both side x,

$$\therefore \log y = \log\left[\frac{x^2}{(x-a)(x-b)(x-c)}\right]$$

$$\therefore \log y = \log(x^3) - \log[(x-a)(x-b)(x-c)]$$

$$\therefore \log y = 3\log x - \log(x-a) - \log(x-b) - \log(x-c)$$

$\therefore$ diff w.r.to x,

$$\frac{1}{y}\frac{dy}{dx} = 3 \times \frac{1}{x} - \frac{1}{x-a} - \frac{1}{x-b} - \frac{1}{x-c}$$

$$\therefore \frac{y'}{y} = \left[\left(\frac{1}{x} - \frac{1}{x-a}\right) + \left(\frac{1}{x} - \frac{1}{x-b}\right) + \left(\frac{1}{x} - \frac{1}{x-c}\right)\right]$$

$$\therefore \frac{y'}{y} = \left[\left(\frac{x-a-x}{x(x-a)}\right) + \left(\frac{x-b-x}{x(x-b)}\right) + \left(\frac{x-c-x}{x(x-c)}\right)\right]$$

$$\therefore \frac{y'}{y} = \left[\frac{-a}{x(x-a)} + \frac{-b}{x(x-b)} + \frac{-c}{x(x-c)}\right]$$

$$\therefore \frac{y'}{y} = \frac{1}{x}\left[\frac{a}{a-x} + \frac{b}{b-x} + \frac{c}{c-x}\right]$$

Ques: Find $\frac{dy}{dx}$, if $y^x + x^y + x^x = a^b$

Solve We are given

$$y^x + x^y + x^x = a^b \quad \text{----} (1)$$

$$\therefore u + v + w = a^b$$

$$\therefore \frac{du}{dx} + \frac{dv}{dx} + \frac{dw}{dx} = 0 \quad \text{----} (2)$$

When $u = y^x$, $v = x^y$, $w = x^x$

$$\Rightarrow \frac{1}{u} \cdot \frac{du}{dx} = 1 \cdot \log y + x \cdot \frac{1}{y}\frac{dy}{dx}$$

$$\Rightarrow \frac{du}{dx} = y^x \left(\log y + \frac{x}{y} \cdot \frac{dy}{dx}\right)$$

$$v = x^y \Rightarrow \log v = y \cdot \log x$$

$$\Rightarrow \frac{1}{v} \cdot \frac{dv}{dx} = \frac{dy}{dx} \cdot \log x + \frac{y}{x}$$

$$\Rightarrow \frac{dv}{dx} = x^y\left(\frac{dy}{dx}\cdot \log x + \frac{y}{x}\right)$$

$$w = x^x \Rightarrow \log w = x\cdot \log x$$

$$\Rightarrow \frac{1}{w}\cdot\frac{dw}{dx} = \log x + 1$$

$$\Rightarrow \frac{dw}{dx} = x^x\,(1+\log x)$$

$$\therefore\ (*) \Rightarrow y^x\left(\log y + \frac{x}{y}\cdot\frac{dy}{dx}\right) + x^y\left[\frac{dy}{dx}\cdot \log x + \frac{y}{x}\right] + x^x(1+\log x) = 0$$

$$\Rightarrow \frac{dy}{dx}\left(y^x\cdot\frac{x}{y} + x^y\cdot\frac{y}{x}\right) = -x^x(1+\log x) - y^x\log y - x^y\cdot y/x$$

$$\Rightarrow \frac{dy}{dx}\left(x\cdot y^{x-1} + x^y\log x\right) = -x^x(1+\log x) - y^x\log y - y\cdot x^{y-1}$$

$$\Rightarrow \frac{dy}{dx} = \left[\frac{-x^x(1+\log x) + y^x\log y + y\cdot x^{y-1}}{x\cdot y^{x-1} + x^y\log x}\right]$$

Derivatives Of Inverse Trigonometric

Pageno→62

1.) If $y = \sin x$, then $\frac{dy}{dx} = \cos x$

2.) If $y = \cos x$, then $\frac{dy}{dx} = -\sin x$

3.) If $y = \tan x$, then $\frac{dy}{dx} = \sec^2 x$

4.) If $y = \cot x$, then $\frac{dy}{dx} = -\operatorname{cosec}^2 x$

5.) If $y = \sec$, then $\frac{dy}{dx} = \sec x \tan x$

6.) If $y = \operatorname{cosec} x$, then $\frac{dy}{dx} = -\operatorname{cosec} \cot x$

Ques: If $(x^2+7x+2)(e^x - \sin x)$ w.r.to x

Solve: Let $y = (x^2+7x+2)(e^x-\sin x)$

∴ diff w.r.t x,

$$\frac{dy}{dx} = (x^2+7x+2)\frac{d}{dx}(e^x-\sin x) + (e^x-\sin x)\frac{d}{dx}(x^2+7x+2)$$

$$\frac{dy}{dx} = (x^2+7x+2)(e^x-\cos x) + (e^x-\sin x)(2x+7)$$

Ques: Diff. following w.r.t. x. Page no - 62

(1) $\sin(3x)$

Solve: Let $y = \sin 3x$

$$\therefore \frac{dy}{dx} = \cos(3x) \cdot 3$$

$$\frac{d}{dx}(\sin x) = \cos x$$

Ques: Diff. following w.r.t. x

(2) $\sin^4 x$ (3) $\sin^6(4x)$

Solve: Let $y = \sin^4 x = (\sin x)^4$

$$\therefore \frac{dy}{dx} = 4\,(\sin x)^3 \frac{d}{dx}(\sin x)$$

$$\therefore \frac{dy}{dx} = 4 \sin^3 x \times \cos x$$

Let $y = \sin^6(4x) = (\sin(4x))^6$

$$\therefore \frac{dy}{dx} = 6\,(\sin(4x))^5 \frac{d}{dx}(\sin(4x))$$

$$\therefore \frac{dy}{dx} = 6\,(\sin^5(4x))\,(\cos(4x))\,\frac{d}{dx}(4x)$$

$$\therefore \frac{dy}{dx} = 6 \sin^5(4x) \cos(4x) \cdot 4$$

P · T · A

Power

Trigonometric

Angle

Ques:- If $y = \sqrt{\cos x + \sqrt{\cos x + \sqrt{\cos x + ---- \alpha}}}$ Prove that

$(2y-1)\frac{dy}{dx} = -\sin x$

Page no → 64

Solve We are given

$$y = \sqrt{\cos x + \sqrt{\cos x + \sqrt{\cos x + ---- \alpha}}}$$

$$\therefore y = \sqrt{\cos x + y} \quad \text{— (1)}$$

Squaring both side.

$$y^2 = \cos x + y$$

$\therefore$ diff. w.r.to x

$$2y\frac{dy}{dx} = -\sin x + \frac{dy}{dx}$$

$$\therefore (2y-1)\frac{dy}{dx} = -\sin x$$

Ques. Diff. $\cos(x^3)\cdot\sin^2(x^5)$ w.r.to x

Solve Let $y = \cos(x^3) - \sin^2(x^5)$ (u.v)

$\therefore$ diff. w.r.to x,

$$\therefore \frac{dy}{dx} = \cos(x^3)\frac{d}{dx}(\sin^2(x^5)) + \sin^2(x^5)\frac{d}{dx}(\cos x^5)$$

$$\therefore \frac{dy}{dx} = \cos(x^3)\left[2(\sin(x^5)\frac{d}{dx}(\sin(x^5)\right] + \sin^2(x^5)$$

$$\left[-\sin(x^3)\frac{d}{dx}(x^3)\right]$$

$$\therefore \frac{dy}{dx} = \cos(x^3)\left[2\sin(x^5)\cos(x^5)(5x^4)\right] + \cos^2(x^5)\left[-\sin(x^3)(3x^2)\right]$$

Very Important Formulae

1. $\sin(A+B) = \sin A \cdot \cos B + \cos A \cdot \sin B$
2. $\sin(A-B) = \sin A \cdot \cos B - \cos A \cdot \sin B$
3. $\cos(A+B) = \cos A \cdot \cos B - \sin A \cdot \sin B$
4. $\cos(A-B) = \cos A \cdot \cos B + \sin A \cdot \sin B$
5. $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$
6. $\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A \cdot \tan B}$
7. $\tan(45^\circ + \theta) = \dfrac{1 + \tan\theta}{1 - \tan\theta}$
8. $\tan(45^\circ - \theta) = \dfrac{1 - \tan\theta}{1 + \tan\theta}$

A, B FORMULAE

1.) $2\sin A \cdot \cos B = \sin(A+B) + \sin(A-B)$

2.) $2\cos A \cdot \sin B = \sin(A+B) - \sin(A-B)$

3.) $2\cos A \cdot \cos B = \cos(A+B) + \cos(A-B)$

4.) $2\sin A \cdot \sin B = \cos(A-B) - \cos(A+B)$

C, D FORMULAE

1.) $\sin C + \sin D = 2\sin \frac{C+D}{2} \cdot \cos \frac{C-D}{2}$

2.) $\sin C - \sin D = 2\cos \frac{C+D}{2} \cdot \sin \frac{C-D}{2}$

3.) $\cos C + \cos D = 2\cos \frac{C+D}{2} \cdot \cos \frac{C-D}{2}$

4.) $\cos C - \cos D = 2\sin \frac{C+D}{2} \cdot \sin \frac{D-C}{2}$

IMPORTANT FORMULAE

(1) $\sin 2\theta = 2\sin\theta \cdot \cos\theta$

(2) $\sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta}$

(1) $\cos 2\theta = 1 - 2\sin^2\theta$

(2) $\cos 2\theta = 2\cos^2\theta - 1$

(3) $\cos 2\theta = \cos^2\theta - \sin^2\theta$

(4) $\cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$

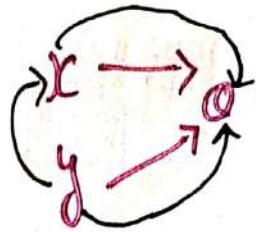
(1) $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

(2) $1+\cos 2\theta = 2\cos^2\theta$

(3) $1-\cos 2\theta = 2\sin^2\theta$

Ques Find $\frac{dy}{dx}$, when

(1.) $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$



Solve We are given

$$x = a(\theta + \sin\theta),\ y = (1 - \cos\theta)$$

$$\therefore \frac{dx}{d\theta} = a(1 + \cos\theta),\ \frac{dy}{d\theta} = a(0 + \sin\theta)$$

$$\therefore \frac{dy}{d\theta} = a\sin\theta$$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{\cancel{a}\sin\theta}{\cancel{a}(1+\cos\theta)}$$

$$\therefore \frac{dy}{dx} = \frac{\sin\theta}{1+\cos\theta}$$

$\sin 2A = 2\sin A\cos$

$$\therefore \frac{dy}{dx} = \frac{\cancel{2}\sin\theta/2\,\cos\theta/2}{\cancel{2}\cos^2(\theta/2)}$$

$$\therefore \frac{dy}{dx} = \tan(\theta/2)$$

Ques If $y = (\cos x)^{(\cos x)^{\cos x \cdots \infty}}$ Prove that

$$\frac{dy}{dx} = \frac{-y^2 \tan x}{1 - y \log \cos x}$$

Solve We are given

$$y = (\cos x)^{\cos x^{\cos x \cdots \infty}}$$

$\therefore y = (\cos x)^y$

$\therefore$ Taking log both side

$\therefore \log y = \log (\cos x)^y$

$\therefore \log y = y \log(\cos x)$ (U.V)

$\therefore$ diff w.r.to x,

$$\therefore \frac{1}{y}\frac{dy}{dx} = y \cdot \frac{d}{dx}(\log(\cos x)) + \log(\cos x)\frac{dy}{dx}$$

$$\therefore \frac{1}{y}\frac{dy}{dx} = y \cdot \frac{1}{\cos x}(-\sin x) + \log(\cos x)\frac{dy}{dx}$$

$$\therefore \left[\frac{1}{y} - \log(\cos x)\right]\frac{dy}{dx} = -y \tan x$$

$$\therefore \left[\frac{1 - y\log(\cos x)}{y}\right]\frac{dy}{dx} = -y \tan x$$

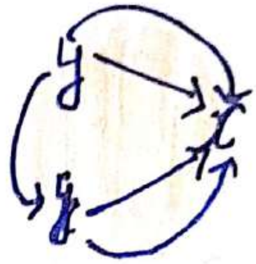
$$\therefore \frac{dy}{dx} = \frac{-y^2 \tan x}{1 - y \log(\cos x)}$$

$$\therefore \frac{dy}{dx} = \frac{-y^2 \tan x}{1 - y \log(\cos x)}$$

Ques Diff. $x^{\sin x}$ into $(\sin x)^x$

Solve Let $y = x^{\sin x}$ ---- (1)

$u = (\sin x)^x$ ---- (2)



$\therefore y = x^{\sin x}$

$\therefore \log y = \log(x^{\sin x})$

$\therefore y = x^{\sin x}$

$\therefore \log y = \log(x^{\sin x})$

$\therefore \log y = (\sin x) \log x$

$\therefore$ diff w.r. to x,

$$\frac{1}{y} \frac{dy}{dx} = (\sin x) \cdot \frac{1}{x} + (\log x) \cdot \cos x$$

$$\therefore \frac{dy}{dx} = y \left[\frac{\sin x}{x} + (\log x) \cdot \cos x \right]$$

$$\therefore \frac{dy}{dx} = x^{\sin x} \left[\frac{\sin x}{x} + (\log x) \cdot \cos x \right] \text{ ---- (3)}$$

Let $u = (\sin x)^x$

$\therefore \log u = \log (\sin x)^x$ P.T.O.

$\therefore \log u = x \log(\sin x)$ (U.V)

$\therefore$ diff. w.r.to x,

$$\frac{1}{u}\frac{du}{dx} = x \frac{1}{\sin x} \times \cos x + \log(\sin x)\cdot 1$$

$$\therefore \frac{1}{u}\frac{du}{dx} = x\left[x \cot x + \log(\sin x)\right]$$

$$\therefore \frac{du}{dx} = (\sin x)^x \left[x \cot x + \log(\sin x)\right] \text{ ---- (4.)}$$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{dx}}{\frac{du}{dx}}$$

$$\therefore \frac{dy}{dx} = \frac{x^{\sin x}\left[\frac{\sin x}{x} + (\log x)\cdot\cos x\right]}{(\sin x)^x \left[x \cot x + \log(\sin x)\right]}$$

Ques Diff following w.r.to x

(1.) $x^{\sin x} + (\sin x)^x$

Solve Let $y = x^{\sin x} + (\sin x)^{\cos x}$ ----- (1.)

Let $y = u + v$ ----- (1.)

$$\therefore \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \text{ — (2.)}$$

Let $y = u + v$

$$\therefore \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

$u = x^{\sin x}$

$\therefore \log u = \log(x^{\sin x})$

$\therefore \log u = \sin x \cdot \log$

$\therefore$ diff. w.r.t x,

$\frac{1}{u}\frac{du}{dx} = \sin x \cdot \frac{1}{x} + \log x \cdot (\cos x)$

$\therefore \frac{du}{dx} = x^{\sin x}\left[\frac{\cos x}{x} + \log x \cdot \cos x\right]$

$v = (\sin x)^{\cos x}$

$\therefore \log v = \log(\sin x)^{\cos x}$

$\therefore \log v = (\cos x) \cdot \log(\sin x)$

$\therefore$ diff w.r.t. to x

$\frac{1}{v}\frac{dv}{dx} = (\cos x) \cdot \frac{1}{\sin x} \times \cos x + \log$

$(\sin x)(-\sin)$

$\therefore \frac{dv}{dx} = (\sin x)^{\cos x}\left[\frac{\cos^2 x}{\sin x} - (\sin x)\log(\sin x)\right]$

$\therefore \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$

$\therefore \frac{dy}{dx} = (x)^{\sin x}\left[\frac{\sin x}{x} + \log x \cdot \cos x\right] + (\sin x)^{\cos x}$

$\left[\frac{\cos^2 x}{\sin x} - (\sin x)\log(\sin x)\right]$

Ques! Diff following w.r.to x

(1.) $\sqrt{\dfrac{1-\cos x}{1+\cos x}}$

Solve Let $y = \sqrt{\dfrac{1-\cos x}{1+\cos x}}$

$1-\cos 2x = 2\sin^2 x$

$1+\cos 2x = 2\cos^2 x$

$\therefore\ y = \sqrt{\dfrac{2\sin^2 x}{2\cos^2 x}}$

$\therefore\ y = \sqrt{\tan^2 x} = \tan x$

$\therefore\ y = \tan x$

$\therefore$ diff w.r.to x

$\therefore\ \dfrac{dy}{dx} = \sec^2 x$

Ques! Diff following w.r.to x,

(2.) $\dfrac{\sin x}{1+\cos x}$

Solve Let $y = \dfrac{\sin x}{1+\cos x}$

$= \dfrac{2\sin(x/2)\cos(x/2)}{2\cos^2(x/2)}$

$\therefore\ y = \tan(x/2)$

P.T.A ⇒ $\tan(x/2)$

∴ diff. w.r.to x

$$\frac{dy}{dx} = \sec^2\left(\frac{x}{2}\right)\frac{d}{dx}\left(\frac{x}{2}\right)$$

$$\therefore \frac{dy}{dx} = \sec^2\left(\frac{x}{2}\right)\cdot\frac{1}{2}$$

$$\tan\left(\frac{x}{2}\right) \downarrow \sec^2\left(\frac{x}{2}\right)\cdot\frac{1}{2}$$

Qus: Diff. following w.r.to x (3.) $\sqrt{\frac{1-\tan x}{1+\tan x}}$

Solve Let $y = \sqrt{\frac{1-\tan x}{1+\tan x}}$

$$\frac{1-\tan x}{1+\tan x} = \tan\left(\frac{\pi}{4}-x\right)$$

P.T.A

$$\sqrt{x} \rightarrow \frac{1}{2\sqrt{x}}$$

$$\therefore y = \sqrt{\tan\left(\pi/4 - x\right)}$$

∴ diff w.r.to x

$$\frac{dy}{dx} = \frac{1}{2\sqrt{\tan\left(\frac{\pi}{4}-x\right)}}\frac{d}{dx}\left(\tan\left(\pi/4 - x\right)\right)$$

$$\therefore \frac{dy}{dx} = \frac{1}{2\sqrt{\tan\left(\pi/4 - x\right)}} \times \sec^2\left(\pi/4 - x\right)\frac{d}{dx}\left(\pi/4 - x\right)$$

$$\therefore \frac{dy}{dx} = \frac{1}{2\sqrt{\tan\left(\pi/4 - x\right)}} \times \sec^2\left(\pi/4 - x\right) \times (0-1)$$

$$\therefore \frac{dy}{dx} = \frac{1}{2\sqrt{\tan\left(\pi/4 - x\right)}} \times \sec^2\left(\pi/4 - x\right) \times (-1)$$

Ques. Diff. following w.r.to x,

(4) $\dfrac{(\cot^2 x-1)}{(\cot^2 x+1)}$

Solve Let $= \dfrac{(\cot^2 x-1)}{(\cot^2 x+1)} = \dfrac{\frac{1}{\tan x}-1}{\frac{1}{\tan}+1}$

$\therefore y = \dfrac{1-\tan^2 x}{1+\tan^2 x} = \cos(2x)$

$\therefore y = \cos(2x)$ D.R.A

$\therefore$ diff w.r.to x

$\therefore \dfrac{dy}{dx} = -\sin(2x)\dfrac{d}{dx}(2x)$

$\therefore \dfrac{dy}{dx} = -2\sin(2x)$

Ques. Diff. following w.r.to x,

(5) $\sqrt{\dfrac{1-\sin x}{1+\sin x}}$

Solve Let $y = \sqrt{\dfrac{1-\sin x}{1+\sin x}} = \sqrt{\dfrac{1-\sin x}{1+\sin x} \times \dfrac{1-\sin x}{1+\sin x}}$

$\therefore y = \sqrt{\dfrac{(1-\sin x)^2}{1-\sin^2 x}} = \sqrt{\dfrac{(1-\sin x)^2}{\cos^2 x}}$

$\therefore y = \frac{1-\sin x}{\cos x} = \frac{1}{\cos x} - \frac{\sin x}{\cos x}$

$\therefore y = \sec x - \tan x$

$\therefore$ diff. w.r.to x

$\frac{dy}{dx} = \sec x \tan x - \sec^2 x$

Ques If $y = \sqrt{\frac{1-\sin 2x}{1+\sin 2x}}$, show that $\frac{dy}{dx} + \sec^2\left(\frac{\pi}{4} - x\right) = 0$

Solve Let $y = \sqrt{\frac{1-\sin 2x}{1+\sin 2x}}$

$\therefore y = \sqrt{\frac{\cos^2 x + \sin^2 x - 2\sin x \cos x}{\cos^2 x + \sin^2 x + 2\sin x \cos x}}$

$\therefore y = \sqrt{\frac{(\cos x - \sin x)^2}{(\cos x + \sin x)^2}}$

$y = \frac{\cos x - \sin x}{\cos x + \sin x}$

$\therefore y = \frac{1-\tan x}{1+\tan x}$

$\therefore y = \tan\left(\pi/4 - x\right)$ | P. T. A

$\therefore$ diff w.r. to x

$$\frac{dy}{dx} = sec^2\left(\frac{\pi}{4}-x\right)\frac{d}{dx}\left(\frac{\pi}{4}-x\right)$$

$$\therefore \frac{dy}{dx} = sec^2\left(\frac{\pi}{4}-4\right)(0-1)$$

$$\therefore \frac{dy}{dx} = -sec^2\left(\frac{\pi}{4}-x\right)$$

$$\therefore \frac{dy}{dx} + sec^2\left(\frac{\pi}{4}-x\right) = 0$$

Ques: If $\sin y = x \sin(a+y)$ then prove that $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$

Solve We are given (y) $\therefore$

$$\sin y = x\sin(a+y)$$

$$\therefore x = \frac{\sin y}{\sin(a+y)}$$

$$\therefore \text{diff w.r.to } , \frac{dx}{dy}$$

$$\therefore \frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$$

$$\therefore \frac{dy}{dx} = \frac{\sin^2(a+b)}{\sin a}$$

$$\therefore \frac{dx}{dy} = \frac{\sin(a+y)\frac{d}{dy}(\sin y) - \sin y\frac{d}{dy}\sin(a+y)}{\sin^2(a+y)}$$

$$\therefore \frac{dx}{dy} = \frac{\sin(a+y)\cdot a+y - \sin y\cos(a+y)(0+1)}{\sin^2(a+y)}$$

$$\frac{dx}{dx} = \frac{\sin(a+y-y)}{\sin^2(a+y)} = \frac{\sin a}{\sin^2(a+y)}$$

Ques: Find $\frac{dy}{dx}$, if $y = (\cos x)^x + (\sin x)^{1/x}$

Solve We are given

$$y = (\cos x)^x + (\sin x)^{1/x} \quad \text{---- (1.)}$$

$$\therefore y = u + v \quad \text{---- (2.)}$$

$$\therefore \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \quad \text{---- (3)}$$

where $u = \cos(x)^x$

$\therefore \log u = \log (\cos x)^x$

$\therefore \log u = \log (\cos x)$

$\therefore$ diff w.r.to x

$$\therefore \frac{1}{u}\frac{du}{dx} = x\left[\frac{1}{\cos x}\cdot(-\sin x)\right] + \log((\cos x)\cdot 1$$

$$\therefore \frac{du}{dx} = (\cos x)^x\left[-x\tan x + \log(\cos x)\right]$$

$V = (\sin x)^{1/x}$

$\therefore \log u = \log (\sin x)^{1/x}$

$\therefore \log v = \frac{1}{x}\log(\sin x)$

$\therefore$ diff w.r.to x

$$\frac{1}{V}\frac{dv}{dx} = \frac{1}{x} \times \frac{1}{\sin x} \times \cos x + \log(\sin x)\left(\frac{-1}{x^2}\right)$$

$$\therefore \frac{dv}{dx} = (\sin x)^{1/x}\left[\frac{\cot x}{x} - \frac{\log(\sin x)}{x^2}\right]$$

$$\therefore (1) \Rightarrow \frac{dy}{dx} = (\cos x)^x\left[-x\tan x + \log(\cos x)\right] + (\sin x)^{1/x}\left[\frac{\cot x}{x} - \frac{\log(\sin x)}{x^2}\right]$$

Ques: Find $\frac{dy}{dx}$, when $(\cos x)^y = (\cot y)^x$

Solve We are given

$$(\cos x)^y = (\cot y)^x$$

$\therefore$ taking log both side,

$$\log(\cos x)^y = \log(\cot y)^x$$

$$\therefore y(\log(\cos x)) = x \log \cot y$$

$\therefore$ diff w.r.to x,

$$\therefore y \times \frac{1}{\cos x} \times (-\sin x) + \log(\cos x)\cdot\frac{d}{dx} = x\frac{1}{\cos y}(-\sin y)$$

$$\frac{dy}{dx} + \log(\cos y) - 1$$

$$y\log(\cos x) \longrightarrow y\cdot\frac{1}{\cos x}\cdot(-\sin x) + \log \cos x \frac{dy}{dx}$$

$$x\log(\cos y) \longrightarrow x\cdot\frac{1}{\cos y}\cdot(-\sin y)\frac{dy}{dx} + \log(\cos y)$$

$$\therefore \frac{dy}{dx}\left[\log(\cos x) + x\tan y)\right] = \log(\cot y) + y\tan x$$

$$\therefore \frac{dy}{dx} = \frac{\log(\cot y) + y\tan x}{\log(\cot x) + x\tan y}$$

Ques) Diff $x^{\tan} + \sqrt{\frac{x^2+1}{x}}$ w.r.to x,

Solve Let $y = x^{\tan x} + \sqrt{\frac{x^2+1}{x}}$ ------ (1.)

$\therefore y = u+v$ ------ (2.)

$\therefore \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$ ----- (3.)

$\therefore u = \cancel{x_0}^{\tan x}$

$\therefore \log u = \log(x^{\tan x})$

$\therefore \log u = \tan \cdot \log x$

$\therefore$ diff w.r.to x

$\frac{1}{u} \frac{du}{dx} = \tan x \cdot \frac{1}{x} + \log \cdot \sec^2 x$

$\therefore \frac{du}{dx} = x^{\tan x}\left[\frac{\tan x}{x} + \log x - \sec^2 x\right]$

$V = \sqrt{\frac{x^2+1}{x}}$

$\log V = \log\left[\sqrt{\frac{x^2+1}{x}}\right]$

$\therefore \log V = \frac{1}{2}\log\left(\frac{x^2+1}{x}\right)$

$\therefore \log V = \frac{1}{2}\left[\log(x^2+1) - \log(x)\right]$

$\therefore$ diff w.r.to x,

$\frac{1}{V} \cdot \frac{dv}{dx} = \frac{1}{2}\left[\frac{1}{x^2+1}(2x) \cdot \frac{1}{x}(1)\right]$

$\therefore \frac{dv}{dx} = \sqrt{\frac{x^2+1}{x}} \times \frac{1}{2}\left[\frac{2x}{x^2+1} \cdot \frac{1}{x}\right]$

(3.) $\Rightarrow \frac{dy}{dx} = x^{\tan x}\left[\frac{\tan x}{x} + \log x \cdot \sec^2 x\right] + \sqrt{\frac{x^2+1}{x}} \times \frac{1}{2}$

$\left[\frac{2x}{x^2+1} - \frac{1}{x}\right]$

Que) Diff. following w.r.t. to x.

(1.) $x^x - 2^{\sin x}$

Solve: Let $y = x^x - 2^{\sin x}$ ......(1)

$\therefore y = u - v$ ......(2)

$\therefore \frac{dy}{dx} = \frac{du}{dx} - \frac{dv}{dx}$ ......(3)

$\therefore u = x^x, \; v = 2^{\sin x}$

$\therefore \log u = \log(x^x)$

$\therefore \log u = x \log x$

$\therefore$ diff w.r.t. to x,

$\frac{1}{u} \cdot \frac{du}{dx} = x \cdot \frac{1}{x} + \log x \cdot 1$

$\therefore \frac{du}{dx} = x^x \left[1 + \log x\right]$

$\therefore v = 2^{\sin x}$

$a^x \to a^x \log a$

diff w.r.t. to x

$\therefore \frac{dv}{dx} = 2^{\sin x} \cdot \log 2 \, \frac{d}{dx}(\sin x)$

$\therefore \frac{dv}{dx} = 2^{\sin x} \cdot \log 2 \, (\cos x)$

$\therefore$ (3) $\Rightarrow \frac{dy}{dx} = x^x (1 + \log x) - 2^{\sin x} (\log 2) \cos x.$

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Ques: Diff following w.r.to x

(2) $(\sin x)^x + \sin^{-1}(\sqrt{x}$

Solve Let $y = (\sin x)^x + \sin^{-1}[\sqrt{x}]$ --- (1)

$\therefore y = u + v$ --- (2)

$\therefore \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$ --- (3)

$\therefore u = (\sin x)^x$

$\therefore \log u = \log(\sin x)^x$

$\therefore \log u = x \log(\sin x)$

$\therefore$ diff w.r.to x,

$\frac{1}{u}\frac{du}{dx} = x \cdot \frac{1}{\sin x} \cdot \cos x + \log(\sin x) \cdot 1$

$\therefore \frac{du}{dx} = (\sin(x))^x \left(x \cot x + \log(\sin x)\right)$

(3) $\Rightarrow \frac{d}{dx}(\log(\sin x)) \rightarrow \frac{1}{\sin x} \cdot \cos x = \cot x$

$V = \sin^{-1}(\sqrt{x})$

$\therefore$ diff w.r.to x

$\frac{dv}{dx} = \frac{1}{\sqrt{1-(\sqrt{x})^2}} \times \frac{d}{dx}(\sqrt{x})$

$\therefore \frac{dv}{dx} = \frac{1}{\sqrt{1-x}} \times \frac{1}{2\sqrt{x}}$

$\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$

$\frac{d}{d}(\cos^{-1}x) = \frac{-1}{\sqrt{1-x^2}}$

$\frac{d}{dx}(\log \cos x) = \frac{1}{\cos x} \times (-\sin x) = -\tan x$

$\frac{dy}{dx} = (\sin x)^x \left[x \cot x + \log(\sin x)\right] + \frac{1}{\sqrt{1-x}} \times \frac{1}{2\sqrt{x}}$

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Derivative of Inverse

1. $\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$, whose $-1<x<1$

2. $\frac{d}{dx}(\cos^{-1}x) = \frac{-1}{\sqrt{1-x^2}}$, whose $-1<x<1$

3. $\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$, whose $-\alpha<x<\alpha$

4. $\frac{d}{dx}(\cot^{-1}x) = \frac{-1}{1+x^2}$, whose $-\alpha<x<\alpha$

5. $\frac{d}{dx}(\sec^{-1}x) = \frac{1}{|x|\sqrt{x^2-1}}$, whose $x>1$ and $x<-1$

6. $\frac{d}{dx}(\operatorname{cosec}^{-1}x) = -\frac{1}{|x|\sqrt{x^2-1}}$, whose $x>1$ and $x<-1$

Ques: Diff. $\sin^{-1}(e^x)$ w.r.to x

Solve Let $y = \sin^{-1}(e^x)$

$\therefore$ diff w.r.to x

$\therefore \frac{dy}{dx} = \frac{1}{\sqrt{1-(e^x)^2}} \frac{d}{dx}(e^x)$

$\therefore \frac{dy}{dx} = \frac{1}{\sqrt{1-e^{2x}}}(e^x)$

$$\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\cos^{-1}x) = \frac{-1}{\sqrt{1-x^2}}$$

Ques Find that $\cot^{-1}(x) + \cot^{-1}\left(\frac{1}{x}\right)$

Solve Let $y = \cot^{-1}(x) + \cot^{-1}\left(\frac{1}{x}\right)$

$\therefore$ diff w.r.to x

$\frac{dy}{dx} = 0$

$\cot^{-1}(x) \to \frac{-1}{1+x^2}$

$$\frac{dy}{dx} = -\frac{1}{1+x^2} + \left(\frac{-1}{1+\left(\frac{1}{x}\right)^2}\frac{d}{dx}\left(\frac{1}{x}\right)\right)$$

$$\therefore \frac{dy}{dx} = \frac{-1}{1+x^2} - \frac{1}{1+\frac{1}{x^2}}\left(-\frac{1}{x^2}\right)$$

$$\therefore \frac{dy}{dx} = \frac{-1}{1+x^2} + \frac{x^2}{x^2+1} \times \frac{1}{x^2} = 0$$

$$\therefore \frac{dy}{dx}(y) = 0$$

$\therefore$ y is constant

$\therefore \cot^{-1}(x) + \cot^{-1}\left(\frac{1}{x}\right)$ constant

Ques. If $y = \sec^{-1}\left(\frac{x+1}{x-1}\right) + \sin^{-1}\left(\frac{x-1}{x+1}\right)$, find $\frac{dy}{dx}$ Page → 85

Solve 1. $\sin^{-1}(x) + \cos^{-1}(x) = \pi/2$

2. $\operatorname{cosec}^{-1}(x) + \sec^{-1}(x) = \pi/2$

3. $\tan^{-1}(x) + \cot^{-1}(x) = \pi/2$

1. $\sin^{-1}(x) = \operatorname{cosec}^{-1}(1/x)$

2. $\tan^{-1}(x) = \cot^{-1}(1/x)$, $x > 0$

3. $\cos^{-1}(x) = \sec^{-1}(1/x)$

:- Let $y = \sec^{-1}\left(\frac{x+1}{x-1}\right) + \sin^{-1}\left(\frac{x-1}{x+1}\right)$

$\therefore y = \cos^{-1}\left(\frac{x-1}{x+1}\right) + \sin^{-1}\left(\frac{x-1}{x+1}\right)$ $\left[\because \sec^{-1}(x) = \cos^{-1}(1/x)\right]$

$\therefore y = \pi/2$ $\left[\because \sin^{-1}x + \cos^{-1}x = \pi/2\right]$

$\therefore$ diff. w.r.to x

$\therefore \frac{dy}{dx} = 0$

Ques: Diff the following w.r.to x,

So ① $\tan^{-1}\left[\left(\frac{1-\cos x}{1+\cos x}\right)^{1/2}\right]$

Solve Let $y = \tan^{-1}\left[\left(\frac{1-\cos x}{1+\cos x}\right)^{1/2}\right]$

$\therefore y = \tan^{-1}\left[\left(\frac{2\sin^2(x/2)}{2\cos^2(x/2)}\right)^{1/2}\right]$

$\therefore y = \tan^{-1}\left[\tan(x/2)\right] = x/2$

$\therefore$ diff w.r.to x

$\therefore \frac{dy}{dx} = 1/2$

Ques: Diff the following $\tan^{-1}\left[\sqrt{\frac{1+\sin x}{1-\sin x}}\right]$ w.r.to x

Solve Let $y = \tan^{-1}\left[\sqrt{\frac{1+\sin x}{1-\sin x}}\right]$

$\therefore y = \tan^{-1}\left[\sqrt{\frac{\cos^2(x/2) + \sin^2(x/2) + 2\sin(x/2)\cos(x/2)}{\cos^2(x/2) + \sin^2(x/2) - 2\sin(x/2)\cos}}\right]$

$$\therefore y = \tan^{-1}\left[\sqrt{\frac{(\cos(x/2) + \sin(x/2))^2}{(\cos(x/2) - \sin(x/2))^2}}\right]$$

$$\therefore y = \tan^{-1}\left[\frac{\cos x/2 + \sin x/2}{\cos x/2 - \sin x/2}\right]$$

$$\therefore y = \tan^{-1}\left[\frac{1 + \tan(x/2)}{1 - \tan(x/2)}\right]$$

$$\therefore y = \tan^{-1}\left[\tan\left(\pi/4 + x/2\right)\right] = \pi/4 + x/2$$

$\therefore$ diff w.r.to x,

$$\therefore \frac{dy}{dx} = 0 + 1/2$$

$$\therefore \frac{dy}{dx} = 1/2$$

Ques: Diff following w.r.to x

$$\sin^{-1}\left(\frac{2x}{1+x^2}\right)$$

Solve Let $y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

∴ Put $x = \tan\theta$

∴ $\theta = \tan^{-1}x$

∴ $y = \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$

∴ $y = \sin^{-1}(\sin 2\theta)$

∴ $y = 2\theta$

∴ $y = 2\tan^{-1}x$

∴ diff w.r.to x

$$\frac{dy}{dx} = 2 \times \frac{1}{1+x^2}$$

∴ $\theta = \tan^{-1}x$

$x = \tan \cancel{x}\theta$

$x = \tan\theta$

Que. 3 Diff. following w.r.to x

$\sin^{-1}\left(\frac{2^{x+1}}{1+4^x}\right)$

Solve: Let $y = \sin^{-1}\left(\frac{2^{x+1}}{1+4^x}\right)$

$\therefore y = \sin^{-1}\left(\frac{2 \cdot 2^x}{1+(2^x)^2}\right)$

$4^x = (2^2)^x$
$= (2^x)^2$

$\therefore$ Put $2^x = \tan\theta$

$\therefore y = \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$

$\sin 2A = \frac{2\tan A}{1+\tan^2 A}$

$\therefore y = \sin^{-1}(\sin 2\theta) = 2\theta$

$\therefore y = 2\tan^{-1}(2^x)$

$\therefore$ diff. w.r.to x,

$\frac{d}{dx}\tan^{-1}x = \frac{1}{1+x^2}$

$\therefore \frac{dy}{dx} = 2 \times \frac{1}{1+(2^x)^2} \frac{d}{dx}(2^x)$

$\therefore \frac{dy}{dx} = 2 \times \frac{1}{1+(2^x)^2} \times 2^x \log 2$

$\therefore \frac{dy}{dx} = 2 \times \frac{1}{1+4^x} \times 2^x \log 2$

$\therefore \frac{dy}{dx} = 2^{x+1} \times \frac{1}{1+4^x} \times \log 2$

$(2^x)^2 = (2^2)^x$
$= 4^x$

Ques: Diff. following w.r.to x

(2.) $\sin^{-1}\left(\frac{2^{x+1}\,3^x}{1+(36)^x}\right)$

Solve: Let $y = \sin^{-1}\left(\frac{2^{x+1}\cdot 3^x}{1+(36)^x}\right)$

$(36)^x = (6^2)^x$
$= (6^x)^2$

$\therefore y = \sin^{-1}\left(\frac{2\cdot 2^x\cdot 3^x}{1+(6^x)^2}\right)$

$\therefore y = \sin^{-1}\left(\frac{2\cdot (6)^x}{1+(6^x)^2}\right)$

Put $6^x = \tan\theta$

$\therefore \theta = \tan^{-1}(6^x)$

$\therefore y = \sin^{-1}(\sin 2\theta)$

$\therefore y = 2\theta$

$\therefore y = 2\tan^{-1}(6^x)$

$\therefore \frac{dy}{dx} = 2 \times \frac{1}{1+(6^x)^2} \times \frac{d}{dx}(6^x)$

$\therefore \frac{dy}{dx} = 2 \times \frac{1}{1+(36)^x} \times (6^x \log 6)$

Ques. $\cos^{-1}\left(\frac{2^{x+1}}{1+4^x}\right)$

Solve. Let $y = \cos^{-1}\left(\frac{2^{x+1}}{1+4^x}\right) = \cos^{-1}\left(\frac{2\cdot 2^x}{1+(2^x)^2}\right)$

$\therefore y = \cos^{-1}\left(\frac{2\cdot 2^x}{1+(2^x)^2}\right)$

$\therefore$ put $2^x = \tan\theta$

$\theta = \tan^{-1}(2^x)$

$\therefore y = \cos^{-1}(\sin 2\theta)$ $2\theta \rightarrow \pi/2 - 2\theta$

$\therefore y = \cos^{-1}\left[\cos\left(\pi/2 - 2\theta\right)\right]$

$\therefore y = \pi/2 - 2\theta$

$\therefore y = \pi/2 - 2\times\tan^{-1}(2^x)$

$\therefore$ diff. w.r.to x,

$\therefore \frac{dy}{dx} = 0 - 2\times\frac{1}{1+(2^x)^2}\times\frac{d}{dx}(2^x)$

$\therefore \frac{dy}{dx} = -2\times\frac{1}{1+(2^x)^2}\times 2^x\log 2$

$\therefore \frac{dy}{dx} = -2^{x+1}\times\frac{1}{1+4^x}\times\log 2.$

Some Important Substution

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1.) In case of :-

2.) $a^2 + x^2$, put $x = a\tan\theta$

3.) $\sqrt{a^2 - x^2}$, put $x = a\sin\theta$

4.) $\sqrt{x^2 - a^2}$, put $x = a\sec\theta$

5.) Both $1+x^2$, and $1-x^2$, put $x^2 = \cos\theta$

Ques: Diff $\tan^{-1}\left[\frac{\sqrt{1+x^2}-1}{x}\right]$, $0<x<1$, w.r.to x

Solve Let $y = \tan^{-1}\left[\frac{\sqrt{1+x^2}-1}{x}\right]$

$$\text{put } x = 1\tan\theta$$

$$\theta = \tan^{-1}(x)$$

$$\therefore y = \tan^{-1}\left[\frac{\sqrt{1+\tan^2\theta}-1}{\tan\theta}\right]$$

$$\therefore y = \tan^{-1}\left[\frac{\sec\theta - 1}{\tan\theta}\right]$$

$$\therefore y = \tan^{-1}\left[\frac{\frac{1}{\cot\theta} - 1}{\frac{\sin\theta}{\cos\theta}}\right]$$

$$\therefore y = \tan^{-1}\left[\frac{1-\cos\theta}{\sin\theta}\right]$$

$$\therefore y = \tan^{-1}\left[\frac{1-\cos\theta}{\sin\theta}\right]$$

$$\therefore y = \tan^{-1}\left[\frac{\cancel{2}\sin^{\cancel{2}}(\theta/2)}{\cancel{2}\sin(\theta/2)\cos(\theta/2)}\right]$$

$$\therefore y = \tan^{-1}\left[\tan(\theta/2)\right] = \theta/2$$

$$\therefore y = 1/2\tan^{-1}(x)$$

$\therefore$ diff w.r.t x

$$\frac{dy}{dx} = 1/2 \times \frac{1}{1+x^2}$$

$\cos\left(\frac{1-x^2}{1+x^2}\right)$
$\downarrow$
Put $x = \tan\theta$

Ques: If $y = \tan^{-1}\left[\dfrac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right]$ find $\dfrac{dy}{dx}$.

Solve: Let $y = \tan^{-1}\left[\dfrac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right]$

Put $x^2 = \cos\theta$

$\theta = \cos^{-1}(x^2)$

$$\therefore y = \tan^{-1}\left[\frac{\sqrt{1+\cos\theta}-\sqrt{1-\cos\theta}}{\sqrt{1+\cos\theta}+\sqrt{1-\cos\theta}}\right]$$

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$$\therefore y = \tan^{-1}\left[\frac{\sqrt{2\cos^2(\theta/2)} - \sqrt{2\sin^2(\theta/2)}}{\sqrt{2\cos^2(\theta/2)} + \sqrt{2\sin^2(\theta/2)}}\right]$$

$$\therefore y = \tan^{-1}\left[\frac{\cos(\theta/2) - \sin(\theta/2)}{\cos(\theta/2) + \sin(\theta/2)}\right]$$

$$\therefore y = \tan^{-1}\left[\frac{1-\tan(\theta/2)}{1+\tan(\theta/2)}\right]$$

$$\therefore y = \tan^{-1}\left[\tan\left[\frac{\pi}{4} - \frac{\theta}{2}\right]\right]$$

$$\therefore y = \pi/4 - \theta/2 = \pi/4 - \frac{1}{2}\cos^{-1}(x^2)$$

$\therefore$ diff w.r.to x,

$$\therefore \frac{dy}{dx} = 0 - \frac{1}{2} \times \left(\frac{-1}{\sqrt{1-(x^2)^2}} \times \frac{d}{dx}(x^2)\right)$$

$$\therefore \frac{dy}{dx} = +\frac{1}{2} \times \frac{1}{\sqrt{1-x^4}} \times (2x) \qquad \frac{d}{dx}(\cos^{-1}x) = \frac{-1}{\sqrt{1-x^2}}$$

$$\therefore \frac{dy}{dx} = \frac{x}{\sqrt{1-x^4}}$$

Qus: If $y = \tan^{-1}\left(\frac{5ax}{a^2-6x^2}\right)$, prove that

$$\frac{dy}{dx} = \frac{3a}{a^2+9x^2} + \frac{2a}{a^2+4x^2}$$

Solve Let $y = \tan^{-1}\left(\frac{5ax}{a^2-6x^2}\right) = \tan^{-1}\left(\frac{5\,x/a}{1-\frac{6x^2}{a^2}}\right)$

$\frac{5x}{a} \to \frac{3x}{a} + \frac{2x}{a}$

$\left(3\frac{x}{a}\right)\left(2\frac{x}{a}\right)$

$= 6x/a$

$$\boxed{\tan^{-1}(x) + \tan^{-1}(y) = \tan^{-1}\left(\frac{x+y}{1-xy}\right)}$$

$x+y$

xy

$$\therefore y = \tan^{-1}\left(\frac{5x/a}{1-6x^2/a^2}\right)$$

$$\therefore y = \tan^{-1}\left[\frac{3x/a \longrightarrow 2x/a}{1-\left(3x/a\right)\left(2x/a\right)}\right]$$

$$\therefore y = \tan^{-1}\left(\frac{3x}{a}\right) + \tan^{-1}\left[\frac{2x}{a}\right]$$

$\therefore$ diff w.r.t. to x,

$\tan^{-1}\left(\frac{x+y}{1-xy}\right)$
$= \tan^{-1}(x) + \tan^{-1}(y)$

$$\frac{dy}{dx} = \frac{1}{1+\left(3x/a\right)^2}\frac{d}{dx}\left(3x/a\right) + \frac{1}{1+\left(2x/a\right)^2}\frac{d}{dx}\left(2x/a\right)$$

$$\therefore \frac{dy}{dx} = \frac{1}{1+\frac{9x^2}{a^2}}\left(3/a\right) + \frac{1}{1+\frac{4x^2}{a^2}} \times 2/a$$

$\therefore \frac{dy}{dx} = \frac{a^{\cancel{2}}}{a^2+9x^2} \times \left(\frac{3}{\cancel{a}}\right) + \frac{a^{\cancel{2}}}{a^2+4x^2} \times \frac{2}{\cancel{a}}$

$\therefore \frac{dy}{dx} = \frac{3a}{a^2+9x^2} + \frac{2a}{a^2+4x^2}$

Ques: Diff following w.r.to x, $\sin^{-1}(2x\sqrt{1^2-x^2})$, $-\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$

Solve let $y = \sin^{-1}(2x\sqrt{1-x^2})$

Put $x = \sin\theta$

$\theta = \sin^{-1}x$

$\therefore y = \sin^{-1}(2\sin\theta\sqrt{1-\sin^2\theta})$

$\therefore y = \sin^{-1}(2\sin\theta \cdot \cos\theta)$

$\therefore y = \sin^{-1}(\sin 2\theta) = 2\theta = 2\sin^{-1}x$

$\therefore y = 2\sin^{-1}x$

$\therefore$ diff. w.r.to x

$\frac{dy}{dx} = 2x \frac{1}{\sqrt{1-x}}$

$\frac{d}{dx}(\sin^{-1}x) \cdot \frac{1}{\sqrt{1-x^2}}$

$\frac{d}{dx}(\cos^{-1}x) = \frac{-1}{\sqrt{1-x^2}}$

Ques Diff following w.r.to x

(2) → $\sin^{-1}(6x\sqrt{1-9x})$, $-\frac{1}{3\sqrt{}} < x < \frac{1}{3\sqrt{}}$

Solve: let $y = \sin^{-1}(6x\sqrt{1-9x^2})$

$\therefore y = \sin^{-1}(2(3x)\sqrt{1-(3x)^2})$

$\therefore$ put $3x = \sin\theta$

$\therefore \theta = \sin^{-1}(3x)$

$\therefore y = \sin^{-1}(2\sin\theta\sqrt{1-\sin^2\theta}$

$\therefore y = \sin^{-1}(2\sin\theta\cos\theta)$

$\therefore y = \sin^{-1}(\sin 2\theta)$

$\therefore y = 2\theta = 2\sin^{-1}(3x)$

$\therefore$ diff w.r.to x

$$\frac{dy}{dx} = 2\times\frac{1}{\sqrt{1-(3x)^2}}\times\frac{d}{dx}(3x)$$

$$\therefore \frac{dy}{dx} = 2\times\frac{1}{\sqrt{1-9x^2}}\times(3)$$

Ques: Diff · following w.r.t.x
$\cos^{-1}(2x^2-1)$

Solve Let $y = \cos^{-1}(2x^2-1)$

① $\cos 2A = 2\cos^2 A - 1 = 2x^2-1$

② $\cos 2A = 1 - 2\sin^2 A$
$= 1-2x^2$

$\therefore$ put $x = \cos\theta$
$\theta = \cos^{-1}x$

$\therefore y = \cos^{-1}(2\cos^2\theta - 1)$

$\therefore y = \cos^{-1}(\cos 2\theta) = 2\theta$

$\therefore y = 2\theta = 2\cos^{-1}x$

$\therefore$ diff w.r.t.o x

$$\frac{dy}{dx} = 2x\left(-\frac{1}{\sqrt{1-x^2}}\right)$$

$$\therefore \frac{dy}{dx} = \frac{-2}{\sqrt{1-x^2}}$$

Ques: Diff following w.r.t.x
$\cos^{-1}(1-2x^2)$ put $x = \sin\theta$

Solve Let $\cos^{-1}(1-2x^2)$ put $x = \sin\theta$

$y = \cos^{-1}(1-2x^2)$

put $x = \sin\theta$

$\therefore y = \cos^{-1}(1-2\sin^2\theta)$

$= \cos^{-1}(\cos 2\theta)$

$= 2\theta$

$= 2\sin^{-1}x$

$\because 1-\cos 2\theta = 2\sin^2\theta$

$\Rightarrow \cos 2\theta = 1-2\sin^2\theta$

$\therefore \dfrac{dy}{dx} = \dfrac{2}{\sqrt{1-x^2}}$

Ques: $\sin^{-1}(3x-4x^3)$

Solve: Let $y = \sin^{-1}(3x-4x^3)$

$\sin 3A = 3\sin A - 4\sin^3 A$

$\cos 3A = 4\cos^3 A - 3\cos A$

$\therefore$ put $x = \sin\theta$

$\theta = \sin^{-1}x$

$\therefore y = \sin^{-1}(3\sin\theta - 4\sin^3\theta)$

$\therefore y = \sin^{-1}(\sin 3\theta) = 3\theta = 3\sin^{-1}x$

$\therefore y = 3\sin^{-1}x$

$\therefore$ diff w.r.to x

$\dfrac{dy}{dx} = 3x \dfrac{1}{\sqrt{1-x^2}}$

Ques: Diff following w.r.t x : $\cos^{-1}(4x^3-3x)$

Solve let $y = \cos^{-1}(4x^3-3x)$

$x = \cos\theta$

$y = \cos^{-1}(4\cos^3\theta - 3\cos\theta)$

$y = \cos^{-1}(\cos 3\theta)$

$y = 3\theta = 3\cos^{-1}x$

$\therefore$ diff w.r.t. to x

$\frac{dy}{dx} = \frac{-3}{\sqrt{1-x^2}}$

Ques: Diff. following w.r.t x $\operatorname{cosec}^{-1}\left(\frac{1+x^2}{2x}\right)$

Solve let $y = \operatorname{cosec}^{-1}\left(\frac{1+x^2}{2x}\right)$

$\therefore y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

$\therefore$ put $x = \tan\theta$

$\theta = \tan^{-1}x$

$\therefore y = \sin^{-1}(\sin 2\theta) = 2\theta = 2\tan^{-1}x$

$\therefore$ diff w.r.t. to x

$\therefore \frac{dy}{dx} = 2 \times \frac{1}{1+x}$

Ques: Diff. following w.r.t x $\sec^{-1}\left(\frac{1}{1-2x^2}\right)$

Solve Let $y = \sec^{-1}\left(\frac{1}{1-2x^2}\right)$

$\therefore y = \cos^{-1}(1-2x^2)$

$\therefore$ put $x = \sin\theta$

$\theta = \sin^{-1} x$

$\therefore y = \cos^{-1}(1-2\sin^2\theta)$

$\therefore y = \cos^{-1}(\cos 2\theta) = 2\theta$

$\therefore y = 2\sin^{-1} x$

$\therefore$ diff w.r.to x

$\therefore \frac{dy}{dx} = 2 \times \frac{1}{\sqrt{1+x}}$

$\cos 2A = \cos^2 A - \sin^2 A$

$= 2\cos^2 A - 1$

$= 1 - 2\sin^2 A$

$= \frac{1-\tan^2 A}{1+\tan^2 A}$

Ques: Diff. following w.r.to x $\sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

Solve Let $y = \sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

$\therefore$ Put $x = \tan\theta$

$\cos 2A = \frac{1-\tan^2 A}{1+\tan^2 A}$

$\theta = \tan^{-1} x$

$\therefore y = \sin^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$

$\therefore y = \sin^{-1}(\cos 2\theta) = \sin^{-1}(\sin(\pi/2 - 2\theta))$

$\therefore y = \pi/2 - 2\theta$

$\therefore y = \pi/2 - 2\tan^{-1}(x)$

$\therefore$ diff w.r.to x,

$\therefore \frac{dy}{dx} = 0 - 2 \times \frac{1}{1+x^2}$

$\therefore \frac{dy}{dx} = \frac{-2}{1+x^2}$

Ques: Diff following w.r.to x

$\cos^{-1}\left(\frac{2x}{1+x^2}\right)$

Solve Let $y = \cos^{-1}\left(\frac{2x}{1+x^2}\right)$

$\sin A = \frac{2\tan A}{1+\tan^2 A}$

$\therefore$ put $x = \tan\theta$

$\therefore \theta = \tan^{-1}(x)$

$\therefore y = \cos^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$

$\therefore y = \cos^{-1}(\sin 2\theta)$

$\therefore y = \cos^{-1}(\cos \pi/2 - 2\theta)$

$\therefore y = \pi/2 - 2\theta = \pi/2 - 2\tan^{-1}(x)$

$\therefore$ diff w.r.to x,

$$\therefore \frac{dy}{dx} = 0 - 2 \times \frac{1}{1+x^2}$$

$$\therefore \frac{dy}{dx} = -\frac{2}{1+x^2}$$

Ques: Diff. following w.r.to x

$\cos^{-1}(2x\sqrt{1-x^2})$

Solve Let $y = \cos^{-1}(2x\sqrt{1-x^2})$

$\therefore$ Put $x = \sin\theta$

$\therefore \theta = \sin^{-1} x$

$\therefore y = \cos^{-1}(2\sin\theta\sqrt{1-\sin^2\theta})$

$y = \cos^{-1}(2\sin\theta\cos\theta)$

$\therefore y = \cos^{-1}(\sin 2\theta)$

$\therefore y = \cos^{-1}\left(\cos\left(\frac{\pi}{2} - 2\theta\right)\right)$

$\therefore y = \frac{\pi}{2} - 2\theta$

$\therefore y = \frac{\pi}{2} - 2 \sin^{-1} x$

$\therefore$ diff w.r.t. x

$$\frac{dy}{dx} = 0 - 2 \times \frac{1}{\sqrt{1-x}}$$

$$\therefore \frac{dy}{dx} = \frac{-2}{\sqrt{1-x}}$$

Ques: If $y = \sin^{-1}\left(x^2\sqrt{1-x^2} + x\sqrt{1-x}\right)$ Show that $\frac{dy}{dx} - \frac{2x}{\sqrt{1-x}} = \frac{1}{\sqrt{1-x^2}}$

Solve Let $y = \sin^{-1}\left(x^2\sqrt{1-x^2} + x\sqrt{1-x}\right)$

Put $x = \sin\theta$, $x^2 = \sin\theta$

$\theta = \sin^{-1}(x)$, $\theta = \sin^{-1}(x^2)$

$\therefore y = \sin^{-1}\left(\sin\theta\sqrt{1-\sin^2\theta} + \sin\theta\sqrt{1-\sin^2\theta}\right)$

$\therefore y = \sin^{-1}\left(\sin\theta\cos\theta + \sin\theta\cos\theta\right)$

$\therefore y = \sin^{-1}\left(\sin\theta\cos\theta + \sin\theta\cos\theta\right)$

$\therefore y = \sin^{-1}(x) + \sin^{-1}(x^2)$

$\therefore$ diff w.r.to x

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}} + \frac{1}{\sqrt{1-(x^2)^2}} \frac{d}{dx}(x^2)$$

$$\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$$

$$\therefore \frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}} + \frac{2x}{\sqrt{1-x^4}}$$

$$\therefore \frac{dy}{dx} - \frac{2x}{\sqrt{1-x^4}} = \frac{1}{1-x^2}$$

Ques: Diff. $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ w.r.to $\tan^{-1}x$

Solve Let $y = \sin^{-1}\left(\frac{2x}{1+x}\right)$, $u = \tan^{-1}(x)$

Put $x = \tan\theta$

$\theta = \tan^{-1}(x)$

$$\sin 2A = \frac{2\tan A}{1+\tan^2 A}$$

$$\therefore y = \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$$

$$\therefore y = \sin^{-1}(\sin 2\theta) = 2\theta = 2\tan$$

$$\therefore y = 2\tan^{-1}x$$

$$\therefore \frac{dy}{dx} = 2x\frac{1}{1+x^2}(2\tan^{-1}(x))$$

1. $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$
2. $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$
3. $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$

$\therefore u = \tan^{-1}(u)$

$$\therefore \frac{du}{dx} = \frac{1}{1+x^2}$$

$$\therefore \frac{dy}{du} = \frac{\frac{dy}{dx}}{\frac{du}{dx}}$$

$$\therefore \frac{dy}{dx} = \frac{2 \times \frac{1}{1+x^2}}{\frac{1}{1+x^2}} = 2$$

$$\therefore \frac{dy}{dx} = 2$$

Ques: Diff. $\tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$ w.r.to $\sin^{-1}x$

Solve Let $y = \tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$ ------ (1)

$u = \sin^{-1}x$ ----- (2)

Put $x = \sin\theta$ in -- (1)

$$y = \tan^{-1}\left(\frac{\sin\theta}{\sqrt{1-\sin^2\theta}}\right)$$

$$= \tan^{-1}\left(\frac{\sin\theta}{\cos\theta}\right)$$

$= \tan^{-1}(\tan\theta)$

$= \theta = \sin^{-1}x$

$\left[\because \text{ If } x = \sin\theta \text{ then } \theta = \sin^{-1}x\right]$

$\therefore \dfrac{dy}{dx} = \dfrac{1}{\sqrt{1-x^2}}$

diff. (2) w.r.t x,

$\dfrac{du}{dx} = \dfrac{1}{\sqrt{1-x^2}}$

Now, $\dfrac{dy}{dx} = \dfrac{\frac{dy}{dx}}{\frac{du}{dx}}$

$\Rightarrow \dfrac{1/\sqrt{1-x^2}}{1/\sqrt{1-x^2}} = 1$

Ques: (3) Diff $\tan^{-1}\left(\dfrac{3x-x^3}{1-3x^2}\right)$ w.r.to $\sin^{-1}\left(\dfrac{2x}{1+x^2}\right)$

Solve Let $\tan^{-1}\left(\dfrac{3x-x^3}{1-3x^2}\right)$, $u = \sin^{-1}\left(\dfrac{2x}{1+x^2}\right)$

$\therefore$ Put $x = \tan\theta$

$\theta = \tan^{-1}(x)$

$\therefore y = \tan^{-1}\left(\dfrac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}\right)$

$\therefore$ Put $x = \tan\theta$

$\theta = \tan(x)$

$\therefore u = \sin^{-1}\left(\dfrac{2\tan\theta}{1+\tan^2\theta}\right)$

$\therefore y = \tan^{-1}(\tan 3\theta)$

$\therefore y = 3\theta = 3\tan^{-1}(u)$

$\frac{dy}{dx} = 3 \times \frac{1}{1+x^2}$

$\therefore \frac{dy}{dx} = \frac{\frac{dy}{dx}}{\frac{du}{dx}} = 2 \times \frac{1}{1+x^2}$

$\therefore \frac{dy}{du} = 3/2$

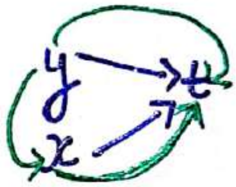
$\therefore u = \sin^{-1}(\sin^2\theta)$

$\therefore u = 2\theta$

$\therefore u = 2\tan^2 x$

$\therefore \frac{du}{dx} = \frac{2}{1+x^2}$

$3 \times \frac{1}{1+x^2}$



Ques: Diff $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ w.r.to $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$

Solve Let $y = \cos^{-1}\left(\frac{1-x}{1+x^2}\right)$ ------ (1)

$u = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$ ------ (2)

Put $x = \tan\theta$ in (1)

$y = \cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$

$= \cos^{-1}(\cos 2\theta)$ $\left[\because \frac{1-\tan^2\theta}{1+\tan^2\theta} = \cos^2\theta\right]$

$= 2\theta$

$= 2\tan^{-1}x$

$\therefore \frac{dy}{dx} = 2 \cdot \frac{1}{1+x^2}$

Now put $x = \tan\theta$ in (2),

$$u = \tan^{-1}\left(\frac{2\tan\theta}{1-\tan^2\theta}\right)$$

$$= \tan^{-1}(\tan 2\theta) \qquad \left[\because \frac{2\tan\theta}{1+\tan^2\theta} = \tan 2\theta\right]$$

$$= 2\theta$$

$$= 2\tan^{-1}x$$

$$\therefore \frac{du}{dx} = 2\cdot\frac{1}{1+x^2}$$

$$\Rightarrow \frac{dy}{du} = \frac{\frac{dy}{dx}}{\frac{du}{dx}} = \frac{2\cdot\frac{1}{1+x^2}}{2\cdot\frac{1}{1+x^2}} = 1$$

$$\therefore \frac{dy}{dx} = 1$$ Hence the Result

Ques: If $\sqrt{1-x^4} + \sqrt{1-y^4} = a(x^2-y^2)$ prove that $\frac{dy}{dx} = \frac{x}{y}$

$$\frac{\sqrt{1-y^4}}{\sqrt{1-x^4}}$$

Solve: The given that

$$\sqrt{1-x^4} + \sqrt{1y^4} = a(x^2-y^2)$$

Let $x^2 = \cos\alpha$, $y^2 = \cos\beta$

$$\sqrt{1-\cos^2\alpha} + \sqrt{1-\cos^2\beta} = a(\cos\alpha - \cos\beta)$$

$$\Rightarrow \sin\alpha + \sin\beta = a(\cos\alpha - \cos\beta)$$

$$\cancel{2}\sin\left(\cancel{\frac{\alpha+\beta}{2}}\right)\cos\left(\frac{\alpha-\beta}{2}\right) = a\,\cancel{2}\sin\left(\cancel{\frac{\alpha+\beta}{2}}\right)\sin\left(\frac{\beta-\alpha}{2}\right)$$

$$\cos\left(\frac{\alpha-\beta}{2}\right) = -a\sin\left(\frac{\alpha-\beta}{2}\right)$$

$$\Rightarrow \frac{\cos\left(\frac{\alpha-\beta}{2}\right)}{\sin\left(\frac{\alpha-\beta}{2}\right)} = (-a)$$

$$\Rightarrow \cot\left(\frac{\alpha-\beta}{2}\right) = (-a)$$

$$\Rightarrow \left(\frac{\alpha-\beta}{2}\right) = \cot^{-1}(-a)$$

$$= \alpha - \beta = 2\cot^{-1}(-a)$$

$x^2 = \cos\alpha$	$y^2 = \cos\beta$
$\alpha = \cos^{-1}x^2$	$\beta = \cos^{-1}y^2$

$$\cos^{-1}x^2 - \cos^{-1}y^2 = 2\cot^{-1}(-a)$$

$$\Rightarrow \frac{-1\ (2x)}{\sqrt{1-(x^2)^2}} - \left(-\frac{1}{\sqrt{1-(y^2)^2}}\right)\cdot(2y)\cdot\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{-2x}{\sqrt{1-x^4}} + \frac{2y}{\sqrt{1-y^4}}\ \frac{dy}{dx} = 0$$

$$\Rightarrow \frac{\cancel{y}}{\sqrt{1-y^4}} \frac{dy}{dx} = \frac{\cancel{x}}{\sqrt{1-x^4}}$$

$$\Rightarrow \frac{dy}{dx} = x/y \sqrt{\frac{1-y^4}{1-x^4}}$$

Hence the Result :)

Ques: Diff $\tan^{-1}\left[\frac{\sqrt{1+x^2}-\sqrt{1-x}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right]$, w.r.to $\cos^{-1}(x^2)$

Solve Let $y = \tan^{-1}\left[\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right]$

$\therefore$ put $x^2 = \cos\theta$ $\qquad u = \cos^{-1}(x^2)$

$\theta = \cos^{-1}(x^2)$

$$\therefore y = \tan^{-1}\left[\frac{\sqrt{1+\cos\theta}-\sqrt{1-\cos\theta}}{\sqrt{1+\cos\theta}+\sqrt{1-\cos\theta}}\right]$$

$$\therefore y = \tan^{-1}\left[\frac{\sqrt{2\cos^2(\theta/2)}-\sqrt{2\sin^2(\theta/2)}}{\sqrt{2\cos^2(\theta/2)}+\sqrt{2\sin^2(\theta/2)}}\right]$$

$$y = \tan^{-1}\left[\frac{\cos\theta/2-\sin\theta/2}{\cos\theta/2+\sin\theta/2}\right]$$

$$\therefore y = \tan^{-1}\left[\frac{1-\tan(\theta/2)}{1+\tan(\theta/2)}\right]$$

$$\therefore y = \tan^{-1}\left[\tan\left(\pi/4 - \theta/2\right)\right]$$

$$\therefore y = \pi/4 - \theta/2 = \pi/4 - 1/2 \cos^{-1}(x^2)$$

$\therefore$ diff w.r.to x, $\quad \frac{d}{dx}(\cos^{-1}(x)) = -\frac{1}{\sqrt{1-x^2}}$

$$\frac{dy}{dx} = 0 - 1/2\left[-\frac{1}{\sqrt{1-(x^2)^2}} \times \frac{d}{dx}(x^2)\right]$$

$$\therefore \frac{dy}{dx} = \cancel{1/2} \times \frac{1}{\sqrt{1-x^4}} \times (\cancel{2}x)$$

$$\therefore \frac{dy}{dx} = \frac{x}{\sqrt{1-x^4}}$$

$$\therefore u = \cos^{-1}(x^2)$$

$$\therefore \frac{du}{dx} = \frac{-1}{\sqrt{1-(x^2)^2}} \times \frac{d}{dx}(x^2)$$

$$\therefore \frac{du}{dx} = \frac{-1}{\sqrt{1-x^4}} \times (2x)$$

$$\therefore \frac{du}{dx} = \frac{-2x}{\sqrt{1-x^4}}$$

$$\therefore \frac{dy}{du} = \frac{\frac{dy}{dx}}{\frac{du}{dx}}$$

$$\therefore \frac{dy}{du} = \frac{\frac{x}{\sqrt{1-x^4}}}{\frac{-2x}{\sqrt{1-x^4}}} = -\frac{1}{2}$$

$$\therefore \frac{dy}{dx} = -\frac{1}{2}$$

Ques: Diff the following function w.r.to x

(1) $\tan^{-1}(\sqrt{1+x^2}-x)$

Solve Let $y = \tan^{-1}(\sqrt{1+x^2}-x)$ put $x = \tan\theta$

$\theta = \tan^{-1}x$

$$\therefore y = \tan^{-1}(\sqrt{1+\tan^2\theta} - \tan\theta)$$

$$\therefore y = \tan^{-1}(\sec\theta - \tan\theta)$$

$$\therefore y = \tan^{-1}\left(\frac{1}{\cot\theta} - \frac{\sin\theta}{\cos\theta}\right)$$

$$\therefore y = \tan^{-1}\left(\frac{1-\sin\theta}{\cos\theta}\right)$$

$$\therefore y = \tan^{-1}\left(\frac{1-\sin\theta}{\cos\theta}\right) = \tan^{-1}\left[\frac{\cos^2\theta/2 + \sin\theta/2 - 2\sin\theta/2 \cdot \cos\theta/2}{\cos^2(\theta/2) - \sin^2(\theta/2)}\right]$$

$$y = \tan^{-1}\left[\frac{(\cos\theta/2 - \sin\theta/2)^2}{(\cos(\theta/2) - \sin(\theta/2))(\cos\theta/2) + \sin\theta/2)}\right].$$

$$\therefore y = \tan^{-1}\left[\frac{\cos(\theta/2) - \sin(\theta/2)}{\cos(\theta/2) + \sin\theta/2}\right]$$

$$\therefore y = \tan^{-1}\left[\frac{1 - \tan\theta/2}{1 + \tan\theta/2}\right]$$

$$\therefore y = \tan^{-1}\left[\tan\left[\pi/4 - \theta/2\right]\right]$$

$$\therefore y = \pi/4 - \theta/2 = \pi/4 - 1/2\ \tan^{-1}(x)$$

$$\therefore y = \pi/4 - 1/2\ \tan^{-1}(x)$$

$\therefore$ diff w.r.to x

$$\frac{dy}{dx} = 0 - 1/2 \times \frac{1}{1+x^2}$$

$$\therefore \frac{dy}{dx} = -\frac{1}{2(1+x^2)}$$

Ques: Diff. the following function w.r.t. to x, Page no:=>115

(9) $\tan^{-1}\left(\sqrt{1+x^2}+x\right)$

Solve Let $y = \tan^{-1}\left(\sqrt{1+x^2}+x\right)$

$$\text{Put } x = \tan\theta$$

$$\theta = \tan^{-1}(x)$$

$$\therefore y = \tan^{-1}\left(\sqrt{1+\tan^2\theta}+\tan\theta\right)$$

$$\therefore y = \tan^{-1}\left(\sec\theta + \tan\theta\right)$$

$$\therefore y = \tan^{-1}\left(\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}\right)$$

$$\therefore y = \tan^{-1}\left(\frac{1+\sin\theta}{\cos\theta}\right)$$

$$\therefore y = \tan^{-1}\left(\frac{\cos^2(\theta/2) + \sin^2\theta/2 + 2\sin\theta/2\cos\theta/2}{\cos^2(\theta/2) - \sin^2(\theta/2)}\right)$$

$$\therefore y = \tan^{-1}\left(\frac{(\cos\theta/2 + \sin\theta/2)^2}{(\cos\theta/2 - \sin\theta/2)(\cos\theta/2 + \cos\theta/2)}\right)$$

$$\therefore y = \tan^{-1}\left(\frac{\cos\theta/2 + \sin\theta/2}{\cos\theta/2 - \sin\theta/2}\right) = \tan^{-1}\left(\frac{1+\tan\theta/2}{1-\tan\theta/2}\right)$$

$\therefore y = \cot^{-1}\left[\frac{1-\sin\theta}{\cos\theta}\right] = \cot^{-1}\left[\frac{\cos^2\theta/2 + \sin^2\theta/2 - 2\sin\theta/2\cos\theta/2}{\cos^2\theta/2 - \sin^2\theta/2}\right]$

$\therefore y = \cot^{-1}\left[\frac{(\cos\theta/2 - \sin\theta/2)^{\cancel{2}}}{\cancel{(\cos\theta/2 - \sin\theta/2)}(\cos\theta/2 + \sin\theta/2)}\right]$

$\therefore y = \cot^{-1}\left[\frac{\cos\theta/2 - \sin\theta/2}{\cos\theta/2 + \sin\theta/2}\right]$

$\frac{1-\tan\theta}{1+\tan\theta} = \tan(\pi/4 - \theta) = \cot(\pi/4 + \theta)$

$\therefore y = \cot^{-1}\left[\frac{1-\tan\theta/2}{1+\tan\theta/2}\right]$

$\frac{1+\tan\theta}{1-\tan\theta} = \tan(\pi/4 + \theta) = \cot(\pi/4 - \theta)$

$\therefore y = \cot^{-1}\left[\cot\left(\pi/4 + \theta/2\right)\right]$

$\therefore y = \pi/4 + \theta/2 = \pi/4 + \frac{1}{2}\tan^{-1}(x)$

diff. w.r.to x

$\frac{dy}{dx} = 0 + \frac{1}{2} \times \frac{1}{1+x^2} = \frac{1}{2(1+x^2)}$

Ques $\cot^{-1}\left[\sqrt{1+x^2} + x\right]$ Diff. the following function w.r.to x

Solu Let $y = \cot^{-1}\left[\sqrt{1+x^2} + x\right]$

$y = \cot^{-1}\left[\sqrt{1+\cot^2} + \cot\theta\right]$

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$$y = \cot^{-1}\left[\sqrt{\operatorname{cosec}^2\theta} + \cot\theta\right]$$

$x = \cot\theta$

$\theta = \cot^{-1}x$

$\frac{d\theta}{dx} = \frac{-1}{1+x^2}$

$$y = \cot^{-1}\left[\frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}\right]$$

$$y = \cot^{-1}\left[\frac{1+\cos\theta}{\sin\theta}\right]$$

$$y = \cot^{-1}\left[\frac{\cancel{1} + \cancel{2}\cos^{\cancel{2}}\theta/2 - \cancel{1}}{\cancel{2}\sin\theta/2 \cdot \cos\theta/2}\right]$$

$$y = \cot^{-1}\left(\cot\theta/2\right)$$

diff. w.r.to x,

$$y = \theta/2$$

$$\therefore dy = 1/2\left(\frac{d\theta}{dx}\right)$$

$$\therefore \frac{dy}{dx} = -1/2 \cdot \frac{1}{1+x^2}$$

Derivate Of Higher Order

Ques: 1 Find $\frac{d^3y}{dx^3}$, when $y = \frac{\log x}{x}$

Solve Let $y = \log\left(\frac{u}{v}\right)$

$$\therefore \frac{dy}{dx} = \frac{x\frac{d}{dx}(\log x) - \log x \frac{d}{dx}(x)}{x^2}$$

$$\therefore \frac{dy}{dx} = \frac{x \cdot \frac{1}{x} - \log x \cdot 1}{x^2}$$

$$\therefore \frac{dy}{dx} = \frac{1 - \log x}{x^2} \quad \left(\frac{u}{v}\right)$$

again diff. w.r.to x.

$$\frac{d^2y}{dx^2} = \frac{x^2 \frac{d}{dx}(1-\log x) - (1-\log x)\frac{d}{dx}(x^2)}{(x^2)^2}$$

$$\therefore \frac{d^2y}{dx^2} = \frac{x^2(0-\frac{1}{x}) - (1-\log x)(2x)}{x^4} = \frac{d^2y}{dx^2} = \frac{-x - 2x + 2x\log x}{x^4}$$

$$\frac{d^2y}{dx^2} = \frac{-3x + 2x\log x}{x^4} \quad \left(\frac{u}{v}\right)$$

$$\therefore \frac{d^2y}{dx^2} = \frac{-3+2\log x}{x^3} \quad \left(\frac{u}{v}\right)$$

$$\therefore \frac{d^3y}{dx^3} = \frac{(x^3)\frac{d}{dx}(-3+2\log x) - (-3+2\log x)\frac{d}{dx}(x^3)}{(x^3)^2}$$

$$\therefore \frac{d^3y}{dx^3} = \frac{(x^3)^2(0+2\times\frac{1}{x}) - (-3+2\log x)(3x^2)}{x^6}$$

$$\therefore \frac{d^3y}{dx^3} = \frac{2x^2 - 3x^2(-3+2\log x)}{x^6}$$

$$\therefore \frac{d^3y}{dx^3} = \frac{2-3(-3+2\log x)}{x^4}$$

$$\therefore \frac{d^3y}{dx^3} = \frac{2-3(-3+2\log x)}{x^4}$$

$$\therefore \frac{d^3y}{dx^3} = \frac{2+9-6\log x}{x^4}$$

$$\therefore \frac{d^3y}{dx^3} = \frac{11-6\log x}{x^4}$$

Ques: If $y = a\sin(\log x) + b\cos(\log x)$ then prove that, $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

Solve We are given $y = a\sin(\log x) + b\cos(\log x)$ --- (1)

diff. w.r.to x,

$$\frac{dy}{dx} = a\left(\cos(\log x)\cdot\frac{1}{x}\right) + b\left(-\sin(\log x)\cdot\frac{1}{x}\right)$$

$$\therefore x\frac{dy}{dx} = a\cos(\log x) - b\sin(\log x)$$

$\therefore$ Again diff. w.r.to x,

$$\therefore x\frac{d^2y}{dx^2} + \frac{dy}{dx}\cdot 1 = a\left[-\sin(\log x)\frac{1}{x}\right] b\left[\cos(\log x)\cdot\frac{1}{x}\right]$$

$$\therefore x\frac{d^2y}{dx^2} + \frac{dy}{dx} = \frac{-1}{x}\left[a\sin(\log x) + b\cos(\log x)\right]$$

$$\therefore x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} = -y \quad (\because \text{ and } (1))$$

$$\therefore x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$$

Hence the Result

Part: Derivative of Higher Order :-

Ques: If y is a function of x, then $y \rightarrow \frac{dy(x)}{dx} = \frac{dy}{dx}$

Solve:- $\frac{d}{dx}\left(\frac{d^2y}{dx^2}\right) \leftarrow \frac{d}{dx}\left[\frac{dy}{dx}\right] = \frac{d^2y}{dx^2}$

$\frac{d^3y}{dx^3}$ and so on

Let $y = x^4$

$\frac{dy}{dx} = 4x^3$

$\frac{d^2y}{dx^2} = 12x^2$

$\frac{d^3y}{dx^3} = 24x$

$\frac{d^4y}{dx^4} = 24$

$\frac{d^5y}{dx^5} = 0$

Ques. If $y = Ae^{mx} + Be^{nx}$, show that $\frac{d^2y}{dx^2} - (m+n)\frac{dy}{dx} + mny = 0$

Solve Let $y = Ae^{mx} + Be^{nx}$ ---- (1)

$\therefore$ Diff w.r.t x,

$$\frac{dy}{dx} = Ae^{mx}(m) + Be^{nx}(n)$$

$$\therefore \frac{dy}{dx} = Ame^{mx} + Bne^{nx}$$

Again diff w.r.t x,

$$\frac{d^2y}{dx^2} = Am(e^{mx}(m)) + Bn(e^{nx}(n))$$

$$\frac{d^2y}{dx^2} = Am^2e^{mx} + Bn^2e^{nx} \text{ ---- (2)}$$

$$\therefore L.H.S = \frac{d^2y}{dx^2} - (m+n)\frac{dy}{dx} + mny$$

$$= (Am^2e^{mx} + Bn^2e^{nx}) - (m+n)(Ame^{mx} + Bne^{nx}) + mn(Ae^{mx} + Be^{nx})$$

$$= \cancel{Am^2e^{mx}} + \cancel{Bn^2e^{nx}} - \cancel{Am^2e^{mx}} - \cancel{Bnme^{nx}} - \cancel{Amne^{mx}} - \cancel{Bn^2e^{nx}} + \cancel{Amne^{mx}} + \cancel{Bmne^{nx}}$$

$$\Rightarrow 0 = R.H.S$$

$\therefore L.H.S = R.H.S$ Hence Proved

Ques: If $y=\left[x+\sqrt{x^2+1}\right]^p$, Prove $(x^2+1)y_2+xy_1-p^2y=0$

Solve: We are given $y=\left[x+\sqrt{x^2+1}\right]^p$ ---- (1)

$\therefore$ diff w.r.to x,

$$\frac{dy}{dx}=p\left[x+\sqrt{x^2+1}\right]^{p-1}\frac{d}{dx}\left(x+\sqrt{x^2+1}\right)$$

$$\therefore \frac{dy}{dx}=p\left[x+\sqrt{x^2+1}\right]^{p-1}\left[1+\frac{1}{\cancel{2}\sqrt{x^2+1}}(\cancel{2}x+0)\right]$$

$$\therefore \frac{dy}{dx}=p\left[x+\sqrt{x^2+1}\right]^{p-1}\left[1+\frac{x}{\sqrt{x^2+1}}\right]$$

$$\therefore \frac{dy}{dx}=p\left[x+\sqrt{x^2+1}\right]^{p-1}\left[\frac{\sqrt{x^2+1}+x}{\sqrt{x^2+1}}\right]$$

$$\therefore \frac{dy}{dx}=\frac{p\left(x+\sqrt{x^2+1}\right)^p}{\sqrt{x^2+1}}$$

$$\therefore \frac{dy}{dx}=p\frac{y}{\sqrt{x^2+1}} \quad [\because \text{ of } (1)]$$

$$\therefore \sqrt{x^2+1}\,\frac{dy}{dx}=py \text{ ---- } (2)$$

$\therefore$ Squaring Both side, x

$$\therefore (x^2+1)\left(\frac{dy}{dx}\right)^2=p^2y^2$$

$\therefore$ diff w.r.to x

$$(x^2+1)\,2\left(\frac{dy}{dx}\right)\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2(2x)=p^2(2y)\,\frac{dy}{dx}$$

$$\therefore (x^2+1)\frac{d^2y}{dx^2}+x\frac{dy}{dx}=p^2y$$

$$\therefore (x^2+1)\frac{d^2y}{dx^2}+x\frac{dy}{dx}-p^2y=0$$

Hence the Result

Ques: If $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, show that $\frac{d^2y}{dx^2}=-\frac{b^4}{a^2y^3}$

Solve Let

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \quad \text{------} (1.)$$

∴ diff. w.r.to x,

$$\frac{2x}{a^2}+\frac{1}{b^2}\left(2y\frac{dy}{dx}\right)=0$$

$$\therefore \frac{2y}{b^2}\frac{dy}{dx}=-\frac{2x}{a^2}$$

$$\therefore \frac{dy}{dx}=\frac{-b^2}{a^2}\times\frac{2x}{2y}$$

$$\therefore \frac{dy}{dx}=-\frac{b^2}{a^2}\left(\frac{x}{y}\right) \quad \text{----} (2.)$$

$$\therefore \frac{d^2y}{dx^2}=\frac{-b^2}{a^2}\left[\frac{y.1-x.\frac{dy}{dx}}{y^2}\right]$$

$$\therefore \frac{d^2y}{dx^2}=\frac{-b^2}{a^2}\left[\frac{y-x\left(-\frac{\cancel{y}}{a^2}\left(\frac{x}{y}\right)\right)}{y^2}\right]$$

$$\therefore \frac{d^2y}{dx^2} = \frac{-b^2}{a^2}\left[\frac{y^2 + \frac{b^2}{a^2}x^2}{y^3}\right] \quad (\because \text{from } (1))$$

$$\therefore \frac{d^2y}{dx^2} = \frac{-b^4}{a^2}\left[\frac{\frac{y^2}{b^2} + \frac{x^2}{a^2}}{y^3}\right] = \frac{-b^2}{a^2y^3}$$

$$\therefore \frac{d^2y}{dx^2} = \frac{-b^2}{a^2y^3}$$

Ques: If $y = x\log\left(\frac{x}{ax+bx}\right)$, prove that $x^3\frac{d^2y}{dx^2} = \left(x\frac{dy}{dx} - y\right)^2$

Solve Let we are given

$$y = x\log\left(\frac{x}{ax+bx}\right) = x\left(\log x - \log(ax+bx)\right) \quad \text{---} (1)$$

$$\therefore \frac{y}{x} = \left[\log x - \log(a+bx)\right]$$

$\therefore$ diff w.r.to x

$$\frac{x\cdot\frac{dy}{dx} - y\cdot 1}{x^2} = \left[\frac{1}{x} - \frac{1}{a+bx}(b)\right]$$

$$\therefore x\frac{dy}{dx}-y = x^2\left[\frac{a+bx-bx}{(x)(a+bx)}\right]$$

$$\therefore x\frac{dy}{dx}-y = \frac{ax}{a+bx} \quad \text{---} (3) \quad (\tfrac{u}{v})$$

$\therefore$ again diff. w.r.to x,

$$\left(x\cdot\frac{d^2y}{dx}+\frac{dy}{dx}\cdot 1\right)-\frac{dy}{dx} = \frac{(a+bx)\cdot a - (ax)\cdot(0+b)}{(a+bx)^2}$$

$$\therefore x\frac{d^2y}{dx}+\cancel{\frac{dy}{dx}}-\cancel{\frac{dy}{dx}} = \frac{a^2+\cancel{abx}-\cancel{abx}}{(a+bx)^2}$$

$$\therefore x\frac{d^2y}{dx^2} = \frac{a^2}{(a+bx)^2}$$

$$\therefore x^3\cdot\frac{d^2y}{dx^2} = \frac{a^2x^2}{(a+bx)^2}$$

$$\therefore x^3\frac{d^2y}{dx^2} = \left(\frac{ax}{a+bx}\right)^2$$

$$\therefore x^3\frac{d^2y}{dx^2} = \left(x\frac{dy}{dx}-y\right)^2$$

Ques: If $y = x^x$ prove that $\frac{d^2y}{dx^2} - \frac{1}{y}\left(\frac{dy}{dx}\right)^2 - \frac{y}{x} = 0$

Solve We are given

$$y = x^x \text{ ------ } (1)$$

$\therefore$ Taking log both side we get,

$$\log y = \log(x^x)$$

$$\therefore \log y = \log(x^x)$$

$$\therefore \log y = x \log x$$

$\therefore$ diff w.r.to x,

$$\therefore \frac{1}{y}\frac{dy}{dx} = x \cdot \frac{1}{x} + \log x \cdot 1$$

$$\therefore \frac{dy}{dx} = y\left[1 + \log x\right] \text{ ------ } (2)$$

again diff. w.r.to x,

$$\frac{d^2y}{dx^2} = y\frac{d}{dx}(1+\log x) + (1+\log x)\cdot\frac{dy}{dx}$$

$$\therefore \frac{d^2y}{dx^2} = y\left[0 + \frac{1}{x}\right] + (1+\log x)\,(y(1+\log x))\;(\because \text{of } (2))$$

$$\therefore \frac{d^2y}{dx^2} = \frac{y}{x} + y\,(1+\log x)^2$$

$$\therefore \frac{d^2y}{dx^2} = y/x + y\left[\frac{1}{y}\frac{dy}{dx}\right]^2 \quad (\because \text{ of } 2)$$

$$\therefore \frac{d^2y}{dx^2} - \frac{y}{x} - y \cdot \frac{1}{y^2}\left(\frac{dy}{dx}^2\right) = 0$$

$$\therefore \frac{d^2y}{dx^2} - \frac{1}{y}\left(\frac{dy}{dx}\right)^2 - \frac{y}{x} = 0$$

Ques: If $2y = x\left(1+\frac{dy}{dx}\right)$, show that $\frac{d^2y}{dx^2}$ is constant

Solve We are given

$$2y = x\left(1+\frac{dy}{dx}\right)$$

$\therefore$ diff. w.r.to x,

$\frac{d^2y}{dx^2}$ is constant

$$\frac{d}{dx}\left(\frac{d^2y}{dx^2}\right) = 0$$

$$\therefore \frac{d^3y}{dx^3} = 0$$

$$2\frac{dy}{dx} = x\frac{d}{dx}\left(1+\frac{dy}{dx}\right) + \left(1+\frac{dy}{dx}\right)\frac{d}{dx}(x)$$

$$\therefore 2\frac{dy}{dx} = x\left[0+\frac{d^2y}{dx^2}\right] + \left(1+\frac{dy}{dx}\right)(1)$$

$$\therefore \frac{dy}{dx} = x\frac{d^2y}{dx^2}$$

$\therefore$ again diff. w.r.to x,

$$\therefore \frac{d^2y}{dx^2} = x\frac{d}{dx}\left(\frac{d^2y}{dx^2}\right) + \frac{d^2y}{dx^2}\left(\frac{d}{dx}(x)\right)$$

$$\therefore \frac{d^2y}{dx^2} = x\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2}\cdot 1$$

$$\therefore x\frac{d^3y}{dx^3} = 0$$

$$\therefore \frac{d^3y}{dx^3} = 0$$

$$\therefore \frac{d}{dx}\left(\frac{d^2y}{dx^2}\right) = 0$$

$\therefore \frac{d^2y}{dx^2}$ is constant

Ques: If $y = \sin^{-1}x$, then show that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0$

Solve: We are given

$$y = \sin^{-1}x \text{ ---- } (1.)$$

∴ diff w.r.to x

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$$

$$\therefore \sqrt{1-x^2}\,\frac{dy}{dx} = 1 \text{ ---- } (2.)$$

∴ Squaring both sides,

$$(1-x^2)\left(\frac{dy}{dx}\right)^2 = 1$$

∴ again diff. w.r.to x,

$$(1-x^2)\left[2\left(\frac{dy}{dx}\right)\frac{d^2y}{dx^2}\right] + \left(\frac{dy}{dx}\right)^2\left[0-2x\right] = 0$$

$$\therefore (1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0$$

Que: If $y = (\tan^{-1}(x))^2$, show that $(x^2+1)^2 y_2 + 2x(x^2+1)y_1 = 2$

Solve: We are given $y = (\tan^{-1}x)^2$ ------ (1)

$y_1 \to \frac{dy}{dx}$

$y_2 \to \frac{d^2y}{dx^2}$

$\therefore$ diff w.r.to x,

$y_1 = 2(\tan^{-1}x) \frac{d}{dx}(\tan^{-1}(x))$

$\therefore y_1 = 2(\tan^{-1}x) \times \frac{1}{1+x^2}$

$\therefore (1+x^2) y_1 = 2(\tan^{-1}(x))$ ---- (2)

$\therefore$ squaring both sides,

$\therefore (1+x^2)^2 y_1^2 = 4(\tan^{-1}x)^2$

$\therefore (1+x^2)^2 y_1^2 = 4y$ [$\because$ of (1)]

$\therefore (1+x^2)^2 y_1^2 = 4y$

$\left(\frac{dy}{dx}\right)^2 \to 2\left(\frac{d}{dx}\right)\left(\frac{d^2y}{dx^2}\right)$

$y_1^2 \to 2y_1 y_2$

$\therefore$ diff w.r.to x,

$(1+x^2)^2[2y_1y_2] + y_1^2(2(1+x^2)'(2x)) = 4y_1$

$\therefore (1+x^2)^2 y_2 + 2x(1+x^2)y_1 = 2.$

Ques: If $y = (\sin^{-1}x)^2$, Prove that $(1-x^2)y_2 - xy_1 - 2 = 0$

Solve We are given $y = (\sin^{-1}x)^2$ . . . (1)

∴ diff w.r.to x,

$$\therefore y_1 = 2(\sin^{-1}x) \cdot \frac{1}{\sqrt{1-x^2}}$$

$$\therefore \sqrt{1-x^2}\, y_1 = 2(\sin^{-1}x)$$

∴ squaring both side x,

$$(1-x^2)y_1^2 = 4(\sin^{-1})^2$$

$$\therefore (1-x^2)y_1^2 = 4y \quad [\because \text{of } (1)]$$

∴ again diff w.r.to x,

$$(1-x^2)(2y_1 y_2) + y_1^2(0-2x) = 4y_1$$

$$\therefore (1-x^2)y_2 - xy_1 = 2$$

$$\therefore (1-x^2)y_2 - xy_1 - 2 = 0$$

Ques) If $y = \sin(2\sin^{-1}x)$, prove that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + 4y$

Solve We are given

$$y = \sin(2\sin^{-1}x) \quad \text{----} (1)$$

$\therefore$ diff w.r.to x (P.T.A)

$$\frac{dy}{dx} = \cos(2\sin^{-1}x)\ \frac{d}{dx}(2\sin^{-1}x)$$

$$\therefore \frac{dy}{dx} = \cos(2\sin^{-1}x) \times 2 \times \frac{1}{\sqrt{1-x^2}}$$

$$\therefore \sqrt{1-x^2}\,\frac{dy}{dx} = 2\cos(2\sin^{-1}x)$$

$$\therefore (1-x^2)\left(\frac{dy}{dx}\right)^2 = 4\cos^2(2\sin^{-1}x)$$

$$\therefore (1-x^2\left(\frac{dy}{dx}\right)^2 = 4\left[1-\sin^2(2\sin^{-1}(x))\right]$$

$$\therefore (1-x^2)\left(\frac{dy}{dx}\right)^2 = 4(1-y^2)$$

again diff w.r.to x

$$\therefore (1-x^2)\left[2\left(\frac{dy}{dx}\right)\frac{d^2y}{dx^2}\right] + \left(\frac{dy}{dx}\right)^2(0-2x) = 4\left(0-2y\frac{dy}{dx}\right)$$

$$\therefore (1-x^2)\frac{d^2y}{dx} - x\ \frac{dy}{dx} = -4y$$

$$\therefore (1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + 4y = 0$$

Hence the Result ∴

Ques. If $x = a(\theta - \sin\theta)$, $y = a(1+\cot\theta)$, find $\frac{d^2y}{dx^2}$

Solve We are given $x = a(\theta - \sin\theta)$

$$y = a(1+\cos\theta)$$

$x \rightarrow \theta$, $y \rightarrow \theta$

$$\therefore \frac{dx}{d\theta} = a(1-\cot\theta)$$

$$\therefore \frac{dy}{d\theta} = a(0-\sin\theta) = -a\sin\theta$$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$$

$$\therefore \frac{dy}{dx} = \frac{-\cancel{a}\sin\theta}{\cancel{a}(1-\cos\theta)}$$

$$\therefore \frac{dy}{dx} = \frac{-(\cancel{2}\sin\theta/\cancel{2}\cos\theta/2)}{\cancel{2}\sin^2(\theta/2)}$$

$$\therefore \frac{dy}{dx} = -\cot(\theta/2)$$

$\therefore$ diff w.r.to x,

$$\frac{d^2y}{dx^2} = -\left[-\operatorname{cosec}^2(\theta/2)\ \tfrac{1}{2}\ \frac{d\theta}{dx}\right]$$

$$\therefore \frac{d^2y}{dx^2} = \tfrac{1}{2}\operatorname{cosec}^2(\theta/2) \times \frac{1}{a(1-\cos\theta)}$$

$$\therefore \frac{d^2y}{dx^2} = \frac{1}{2a}\operatorname{cosec}^2(\theta/2) \times 1/2\sin^2(\theta/2)$$

$$\therefore \frac{d^2y}{dx^2} = 1/4a\ \operatorname{cosec}^4(\theta/2)$$

Ques: If $y = [\log(x+\sqrt{x^2+1})^2]$, prove that $(1+x^2)y_2 + xy_1 = 2$

Solve We are given

$$y = [\log(x+\sqrt{x^2+1})^2] \text{ ---- (1)}$$

$\hookrightarrow x^n$

$\therefore$ diff w.r.to x

$$y_1 = 2[\log(x+\sqrt{x^2+1})]\ \frac{d}{dx}(\log(x+\sqrt{x^2+1}))\ \log$$

$$\therefore y_1 = 2[\log(x+\sqrt{x^2+1})] \times \frac{1}{x+\sqrt{x^2+1}}\ \frac{d}{dx}(x+\sqrt{x^2+1})$$

$$\therefore y_1 = 2[\log(x+\sqrt{x^2+1})] \times \frac{1}{x+\sqrt{x^2+1}}\left[1+\frac{1}{\cancel{2}\sqrt{x^2+1}} \times \cancel{2}(x)\right]$$

$$\therefore y_1 = 2[\log(x+\sqrt{x^2+1}] \times \frac{1}{\cancel{x+\sqrt{x^2+1}}}\left[\frac{\cancel{\sqrt{x^2+1}+x}}{\sqrt{x^2+1}}\right]$$

$\therefore y_1 = 2x\left[\log(x+\sqrt{x^2+1})\right] \times \frac{1}{\sqrt{x^2+1}}$

$\therefore \sqrt{x^2+1}\, y_1 = 2\left[\log(x+\sqrt{x^2+1})\right]$

$\therefore$ Squaring both sides,

$(x^2+1)\, y_1^2 = 4\left[\log(x+\sqrt{x^2+1})\right]^2$

$\therefore (x^2+1)y_1^2 = 4y \quad [\because \text{of } 1]$

$\therefore$ diff. w.r.to x,

$(x^2+1)\,(2y_1 y_2) + y_1^2\,(2x) = (4y_1)$

$\therefore (x^2+1)\,y_2 + xy_1 = 2$

$\therefore (x^2+1)\,y_2 + xy_1 = 2$

Ques If $y = e^{a\cos^{-1}x}$, show that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2y = 0$

Solve We are given

$y = e^{a\cos^{-1}x}$ --- (1)

$\therefore$ diff w.r.to x,

$\frac{dy}{dx} = e^{a\cos^{-1}x}\frac{d}{dx}(a\cos^{-1}x)$

$\therefore \frac{dy}{dx} = e^{a\cos^{-1}x}\left[ax\left(\frac{-1}{\sqrt{1+x^2}}\right)\right]$

$e^x \longrightarrow e^x$

$e^{f(x)} \longrightarrow e^{f(x)}\frac{d}{dx}(f(x))$

$\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$

$\frac{d}{dx}(\cos^{-1}x) = -\frac{1}{\sqrt{1-x^2}}$

$\therefore \frac{dy}{dx} = e^{a\cos^{-1}x}\left[\frac{-a}{\sqrt{1-x^2}}\right]$

$\therefore \sqrt{1-x^2}\,\frac{dy}{dx} = -a\,e^{a\cos^{-1}x}$

$\therefore \sqrt{1-x^2}\,\frac{dy}{dx} = -ay \quad [\because \text{of } ①]$

$\therefore$ Squaring both sides,

$\therefore (1-x^2)\left(\frac{dy}{dx}\right)^2 = a^2y^2$

$\therefore$ diff w.r. to x

$(1-x^2)\left(2\left(\frac{dy}{dx}\right)\left(\frac{d^2y}{dx^2}\right)\right) + \left(\frac{dy}{dx}\right)^2(0-2x) = a^2\left(2y\frac{dy}{dx}\right)$

$\therefore (1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = a^2y$

$\therefore (1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2y = 0$